Selection of Optimum Nonlinear Confusion Component of Information Confidentiality Mechanism Using Grey Theory Based Decision-Making Technique

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Abstract: In this age of internet communication, the security of digital information is one of the main issues. The privacy of data depends upon the encryption using some secure algorithm. The selection of robust cryptosystems to ensure confidentiality is a major concern to decrease the risk of cryptographic attacks. In this article, we have implemented a grey theory-based decision-making technique for the election of a robust cryptosystem that complies with all the cryptographic parameters. Six different already proposed encryption algorithms are selected as the alternatives of the decision-making problem and the parameters concerned for the decision are entropy, correlation coefficient, the number of pixels changing rate (NPCR), unified average changing intensity (UACI). The algorithm ranked as first by using grey-based decision-making method can be utilized for secure data encryption.

Keywords: Grey theory, Grey set, MCDM, Decision making problem, Cryptosystem, Encryption algorithm.

1. Introduction

In the current phase of digital media, the fast spread of confidential data by using the internet is usual. In the arena of information security, with the widespread communication ways, the security of multimedia devices becomes a major issue. Cryptographic means are the best possibility to secure the internet data by some encryption process. The information transferred by some insecure channels of communication can be protected by implementing some secure encryption algorithm. There exist many encryption algorithms in the literature that claim to provide confidentiality.

Cryptographic algorithms are divided into two major branches concerning the private key of the algorithm; the first one is the symmetric algorithms in which the encryption and decryption are done by using the same key; the second one is the asymmetric algorithms in which private key for encryption and decryption is different. Many conventional symmetric encryption systems are applied as a standard to secure communication such as Data Encryption Standard (DES) [1], Triple Data Encryption Standard (TDES) [2], and Advanced Encryption Standard (AES) [3]. Some of these structures are easy to attack by some cryptanalyst. The security of encryption structures depends on the private key of the algorithm. There are also many techniques proposed by researchers which support the requirements of secure encryption structures [4-15]. Unfortunately, all the offered encryption approaches cannot impart robustness. Recently, numerous encryption structures were attacked by cryptanalysts due to the weak implementation of private keys [16-20].

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The major issue in the implementation of an encryption algorithm is the choice of a secure algorithm. Due to the presence of many encryption algorithms the choice of the best algorithm is difficult to reduce the risk of information leakage.

Multiple criteria decision making (MCDM) is a means for handling complex problems of engineering and science with incomplete and uncertain information. Many decision-making approaches are utilized in real-world problems [21-32,37,38]. There are numerous MCDM techniques, and each approach has its limitation according to the known information of the problem. Generally, decision-making approaches are executed on the problems with uncertain information in which the selection of the best alternative is hard by using some numerical features. Grey theory is one of the decision-making procedures utilized for the selection of best selection of alternatives by the means of uncertain information [33]. The grey theory also links with the conditions of fuzziness under some specific conditions [34-36].

In this paper, we have discussed the grey decision-making method for the election of a robust cryptosystem. We have selected some already existing encryption techniques which can be utilized for secure communication. As the selection of a secure encryption structure is a tough choice due to some uncertainty in the cryptographic properties of the algorithm. Some encryption algorithm successfully achieves the idealness of one property but may fail to achieve the standard in other features. Therefore, manual selection of cryptosystems to provide the best security is a very hard criterion. The grey theory is executed on some statistical properties of cryptosystems which are entropy, correlation coefficient, NPCR, and UACI. These parameters are considered as the attributes and six cryptosystems are taken as alternatives to the problem. The algorithm ranked as first by using grey theory-based decision-making method is declared as a secure and robust encryption system.

The persisting manuscript is arranged as follows: Section 2 offers some elementary definitions; In section 3 we have presented the statistical properties of some understudy cryptosystems; section 4 offers the basic idea of grey theory-based decision-making problem; in the next section, we have implemented grey theory for the selection of robust cryptosystem; finally, the conclusion is drawn.

2. Some Basic Preliminaries

Grey theory was initially proposed in 1982 by Deng [33]. This theory leads towards the solution of uncertainty problems with incomplete information and discrete data by using grey sets. In this segment, we have described some fundamental definitions related to the grey theory.

Definition: 2.1

Consider U be the universal set consisting of all possible alternatives. The grey set C over U is defined by two mappings $\bar{\gamma}_C(u)$ and $\underline{\gamma}_C(u)$. Mathematically, mapping can be defined as:

$$\begin{cases} \bar{\gamma}_C(u): u \rightarrow [0,1], \\ \underline{\gamma}_C(u): u \rightarrow [0,1]. \end{cases} \quad (1)$$

With the condition $\bar{\gamma}_C(u) \geq \underline{\gamma}_C(u)$ for all $u \in U$, $\bar{\gamma}_C(u)$ and $\underline{\gamma}_C(u)$ are the upper and lower membership functions in C respectively.

Definition: 2.2

The grey set C over the universal set U turn out to be a fuzzy set if the upper and lower membership functions are the same. Mathematically it can be written as $\bar{\gamma}_C(u) = \underline{\gamma}_C(u)$. It indicates that grey theory can also link with the condition of fuzziness.

Definition: 2.3

A number with uncertain information can be indicated as the grey number. For example, if the attributes ratings are defined in some linguistic variable; there with an interval stating it numerically. This interval will encompass vague data. Usually, the grey number is written as $\otimes C = C \Big|_{\underline{\gamma}}^{\bar{\gamma}}$.

Definition: 2.4

The grey set C can be described as the grey number with a lower limit and upper limit by $\otimes C = [\underline{C}, \infty)$ and $\otimes C = (\infty, \bar{C})$ respectively.

Definition: 2.5

The grey set with the upper and lower limits can be defined as an interval grey number by $\otimes C = [\underline{C}, \bar{C}]$.

Definition: 2.6 [37]

R. E, Moore's developed the interval operations [38]. Grey number operation is specified on the set of intervals, instead of real numbers. The fundamental operations for two grey numbers $\otimes C_1 = [\underline{C}_1, \bar{C}_1]$ and $\otimes C_2 = [\underline{C}_2, \bar{C}_2]$ can be defined mathematically as:

$$\otimes C_1 + \otimes C_2 = [\underline{C}_1 + \underline{C}_2, \bar{C}_1 + \bar{C}_2], \quad (2)$$

$$\otimes C_1 - \otimes C_2 = [\underline{C}_1 - \underline{C}_2, \bar{C}_1 - \bar{C}_2], \quad (3)$$

$$\otimes C_1 \times \otimes C_2 = [\min(\underline{C}_1 \underline{C}_2, \underline{C}_1 \bar{C}_2, \bar{C}_1 \underline{C}_2, \bar{C}_1 \bar{C}_2), \max(\underline{C}_1 \underline{C}_2, \underline{C}_1 \bar{C}_2, \bar{C}_1 \underline{C}_2, \bar{C}_1 \bar{C}_2)], \quad (4)$$

$$\otimes C_1 \div \otimes C_2 = [\underline{C}_1, \bar{C}_1] \times \left[\frac{1}{\underline{C}_2}, \frac{1}{\bar{C}_2} \right]. \quad (5)$$

Definition: 2.7

The length of the upper and lower limit of a grey number $\otimes C = [\underline{C}, \bar{C}]$ can be calculated by

$$L(\otimes C) = [\bar{C} - \underline{C}]. \quad (6)$$

Definition: 2.8 [37]

The possibility degree $\otimes C_1 \leq \otimes C_2$ for two grey numbers $\otimes C_1 = [\underline{C}_1, \bar{C}_1]$ and $\otimes C_2 = [\underline{C}_2, \bar{C}_2]$ can be determined by the following mathematical expression:

$$P\{\otimes C_1 \leq \otimes C_2\} = \frac{\max(0, L^* - \max(0, \bar{C}_1 - \underline{C}_2))}{L^*}, \quad (7)$$

Where $L^* = L(\otimes C_1) + L(\otimes C_2)$.

The possibility degree lies on the real axis in the following four conditions:

1. If $\bar{C}_1 = \bar{C}_2$ and $\underline{C}_1 = \underline{C}_2$, or in other words, we can say that $\otimes C_1$ and $\otimes C_2$ are equal. Then possibility degree $P\{\otimes C_1 \leq \otimes C_2\} = 0.5$.
2. If $\bar{C}_2 < \underline{C}_1$, we can say that $\otimes C_2$ is smaller than $\otimes C_1$. Then $P\{\otimes C_1 \leq \otimes C_2\} = 0$.
3. If $\underline{C}_2 < \bar{C}_1$, we say that $\otimes C_2$ is greater than $\otimes C_1$. Then $P\{\otimes C_1 \leq \otimes C_2\} = 1$.
4. If $P\{\otimes C_1 \leq \otimes C_2\} > 0.5$ we can say that $\otimes C_2 > \otimes C_1$ and if $P\{\otimes C_1 \leq \otimes C_2\} < 0.5$ we can say that $\otimes C_2 < \otimes C_1$.

Table 1: The linguistic to numerical scale for attribute weight and attribute rating.

Attribute weights		Attribute rating	
Linguistic variable	$\otimes \omega$	Linguistic variable	$\otimes C$
Very less (VL)	[0.0, 0.1]	Very bad (VB)	[0, 1]
Less (L)	[0.1, 0.3]	Bad (B)	[1, 3]
Average less (AL)	[0.3, 0.4]	Average bad (AB)	[3, 4]
Average (A)	[0.4, 0.5]	Fair (F)	[4, 5]
Average high (AH)	[0.5, 0.6]	Average good (AG)	[5, 6]
High (H)	[0.6, 0.9]	Good (G)	[6, 9]
Very high (VH)	[0.9, 1.0]	Very good (VG)	[9, 10]

3. Understudy Cryptosystems and their Statistical Properties

Many cryptographic algorithms claim to be secure for data communication. But all of them do not provide confidentiality. In this segment, we have selected some cryptosystems for the selection of robust algorithms through the grey-based decision-making approach. In Ref. [10] construct encryption procedure built on a chaotic map, elliptic curves, and Substitution-box. Authors in Ref. [11] proposed a cryptosystem based on generalized chaotic maps and fractional-order edge detection. J. B. Lima et al. [12] offered a medical image encryption algorithm by using cosine number transform. In [13] authors presented an improved Elgamal based encryption technique. X. In [14] authors developed an elliptic curves-based chaotic encryption scheme. Zhang and X. Wang [15] proposed a novel cryptosystem based on elliptic curves.

Table 2: Statistical properties of understudy cryptosystems.

Scheme	Entropy	Correlation	NPCR	UACI
S. Ibrahim et. al. [10]	7.9994	0.0088	99.6086	33.4409
S. M. Ismail et al. [11]	7.9972	-0.0001	76.1681	24.2534
J. B. Lima et al. [12]	7.9992	-0.0007	99.6082	33.4682
J. B. Laip. Et al. [13]	7.9994	0.0024	99.6071	33.4423
A. M. Abbas et al. [14]	7.9998	-0.0006	99.6095	33.4621
X. Zhang et al. [15]	7.9986	0.0034	99.9500	33.1100

The evaluation of the security of cryptographic algorithms is carried out by some statistical analysis. The statistical analyses considered for the evaluation of the cryptosystems are, entropy, correlation coefficient, NPCR, and UACI [9]. We have listed the values of some statistical analysis of understudy cryptosystems in Table 2. The bar chart illustration of statistical properties of understudy cryptosystems is shown in Fig. 1.

Fig. 1: Bar chart representation of statistical properties of understudy cryptosystems.

4. Grey Theory Based Decision-Making Method [33]

The grey theory-based decision-making method is for ordering the preferences of any alternatives. This approach is very useful for solving problems with uncertain information. Consider that $C = \{C_1, C_2, \dots, C_n\}$ be the set of all possible options and $A = \{A_1, A_2, \dots, A_m\}$ be the set of all possible parameters of the problem. The weight vector for attributes is taken as $\otimes \omega = \{\otimes \omega_1, \otimes \omega_2, \dots, \otimes \omega_m\}$. In this approach, the weight and ratings of the attributes are taken as linguistic variables. The numerical scale for attribute weight and attribute rating is defined in Table 1. The grey decision-making approach can be reviewed by the subsequent steps:

Step 1: Firstly, the weight of each attribute is assigned by some decision-makers in the linguistic variable. Suppose that there are Q decision-makers who make the decision in linguistic variable and the resultant weight is calculated by converting them into the numerical scale by using the results in Table 1. The weight for attributes is selected as:

$$\otimes \omega_q = \frac{1}{Q} [\otimes \omega_q^1 + \otimes \omega_q^2 + \dots + \otimes \omega_q^Q], \quad (8)$$

Where $\otimes \omega_q^Q, (q=1,2,\dots,m)$ is the attribute weight of the Q th decision-maker and can be stated in the structure of grey numbers as $\otimes \omega_q^Q = [\underline{\omega}_q^Q, \bar{\omega}_q^Q]$.

Step 2: The second step is the rating of the attribute by Q decision-makers in linguistic variables which are converted into numerical data by using Table 1. The rating is for each attribute is calculated by:

$$\otimes C_{pq} = \frac{1}{Q} [\otimes C_{pq}^1 + C_{pq}^2 + \dots + C_{pq}^Q], \quad (9)$$

Where $\otimes C_{pq}^Q, (p=1,2,\dots,n; q=1,2,\dots,m)$ denotes the rating attribute of Q th decision-maker and can be stated in the form of grey numbers as $\otimes C_{pq}^Q = [\underline{C}_{pq}^Q, \bar{C}_{pq}^Q]$.

Step 3: Next, the grey decision matrix (GDM) is constructed based on the grey attribute rating obtained in the previous step as follows:

$$GDM = \begin{bmatrix} \otimes C_{11} & \otimes C_{12} & \dots & \otimes C_{1m} \\ \otimes C_{21} & \otimes C_{22} & \dots & \otimes C_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes C_{n1} & \otimes C_{n2} & \dots & \otimes C_{nm} \end{bmatrix}, \quad (10)$$

where $\otimes C_{pq}$ denotes the grey rating attributes.

Step 4: In this step, the grey decision matrix (GDM) obtained from the previous step is normalized and defined mathematically as:

$$Normalized_GDM = \begin{bmatrix} \otimes C_{11}^* & \otimes C_{12}^* & \dots & \otimes C_{1m}^* \\ \otimes C_{21}^* & \otimes C_{22}^* & \dots & \otimes C_{2m}^* \\ \vdots & \vdots & \ddots & \vdots \\ \otimes C_{n1}^* & \otimes C_{n2}^* & \dots & \otimes C_{nm}^* \end{bmatrix}, \quad (11)$$

where for beneficial attribute, $\otimes C_{pq}^*$ is defined as:

$$\otimes C_{pq}^* = \left[\frac{\underline{C}_{pq}}{C_q^{\max}}, \frac{\bar{C}_{pq}}{C_q^{\max}} \right], \quad (12)$$

For $C_q^{\max} = \max_{1 \leq p \leq n} (\{\bar{C}_{pq}\})$. And for non-beneficial attributes $\otimes C_{pq}$ is defined as:

$$\otimes C_{pq}^* = \left[\frac{C_q^{\min}}{C_{pq}}, \frac{C_q^{\min}}{C_{pq}} \right], \quad (13)$$

For $C_q^{\min} = \min_{1 \leq p \leq n} \left(\{C_{pq}\} \right)$.

Step 5: Further, the weight vector is multiplied with normalized GDM to obtain the weighted normalized GDM by:

$$\text{Weighted_normalized_GDM}^* = \begin{bmatrix} \otimes R_{11} & \otimes R_{12} & \cdots & \otimes R_{1m} \\ \otimes R_{21} & \otimes R_{22} & \cdots & \otimes R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes R_{n1} & \otimes R_{n2} & \cdots & \otimes R_{nm} \end{bmatrix}, \quad (14)$$

where $\otimes R_{pq} = \otimes C_{pq}^* \times \omega_q$.

Step 6: The ideal referential alternatives are selected from weighted normalized GDM acquired in step 5. The mathematical expression for ideal referential alternatives is defined as:

$$I^{\max} = \left\{ \left[\max_{1 \leq p \leq n} R_{p1}, \max_{1 \leq p \leq n} \bar{R}_{p1} \right], \left[\max_{1 \leq p \leq n} R_{p2}, \max_{1 \leq p \leq n} \bar{R}_{p2} \right], \dots, \left[\max_{1 \leq p \leq n} R_{pm}, \max_{1 \leq p \leq n} \bar{R}_{pm} \right] \right\}. \quad (15)$$

Step 7: This step involves the calculation of the grey possibility degree between the alternatives $S = \{C_1, C_2, \dots, C_m\}$ and the ideal referential alternatives $I^{\max}$ by the following mathematical expression.

$$P = \{S_i \leq I^{\max}\} = \frac{1}{n} \sum_{q=1}^m P\{\otimes R_{pq} \leq I_q^{\max}\}. \quad (16)$$

Step 8: The last step is the ranking of preference orders. If the grey possible degree is lesser then the ranking of choice is higher. Otherwise, in the case of greater grey possibility degree, the ranking order is worse.

5. Application of Grey Theory-Based Decision-Making Method in the Selection of Robust Cryptosystem

We have six possible cryptosystems as alternatives from which we need to select the robust one. The strength of cryptographic algorithms can be extinguished by the statistical properties entropy, correlation coefficient, NPCR, and UACI which are named as A_1 , A_2 , A_3 and A_4 respectively for grey-based decision-making method. All the possible cryptosystems are nominated as C_1 , C_2 , C_3 , C_4 , C_5 , and C_6 . The selection of a strong cryptosystem is done by using the following steps:

Step 1: The first step is the calculation of the weight of the parameters A_1 , A_2 , A_3 and A_4 with the decision of four decision-makers DM_1 , DM_2 , DM_3 , and DM_4 . The weights of each parametric attribute

are selected by employing the linguistic to the numeric scale defined in Table 1. By using Table 1 and Eq. (8) the calculated weight is defined in Table 3.

Table 3: Weight of each parameter according to each decision-maker.					
A_i	DM_1	DM_2	DM_3	DM_4	$\otimes \omega_q$
A_1	AH	VH	H	VH	[0.725, 0.875]
A_2	A	A H	A H	H	[0.5, 0.65]
A_3	VH	H	VH	A H	[0.725, 0.875]
A_4	VH	H	H	A H	[0.725, 0.875]

Step 2: The next step is the calculation of attribute rating according to each decision-maker concerning Eq. (9) and the resultant ratings are depicted in Table 4.

Table 4: Rating of each alternative according to four decision-makers.						
A_i	C_i	DM_1	DM_2	DM_3	DM_4	$\otimes \omega_q$
A_1	C_1	G	VG	G	AG	[6.5, 8.5]
	C_2	F	AG	AB	AG	[4.25, 5.25]
	C_3	G	G	AG	F	[5.25, 7.25]
	C_4	G	VG	G	AG	[6.5, 8.5]
	C_5	VG	VG	VG	G	[8.25, 9.75]
	C_6	AG	AG	F	F	[4.5, 5.5]
A_2	C_1	F	AG	AB	F	[4,5]
	C_2	VG	VG	G	G	[7.5, 9.5]
	C_3	VG	VG	VG	VG	[9, 10]
	C_4	AG	G	F	F	[4.75, 6.25]
	C_5	VG	VG	VG	VG	[9, 10]
	C_6	AG	G	F	F	[4.75, 6.25]
A_3	C_1	G	AG	AG	G	[5.5, 7.5]
	C_2	VB	B	B	B	[0.75, 2.5]
	C_3	G	AG	AG	G	[5.5,7.5]

A_3	C_4	G	AG	F	AG	[5, 6.5]
	C_5	G	G	G	G	[6, 9]
	C_6	VG	VG	VG	VG	[9, 10]
A_4	C_1	G	AG	AG	AG	[5.25, 6.75]
	C_2	B	VB	B	VB	[0.5, 2]
	C_3	VG	G	VG	VG	[8.25, 9.75]
	C_4	G	AG	G	AG	[5.5, 7.5]
	C_5	VG	G	VG	VG	[8.25, 9.75]
	C_6	AG	F	AB	F	[4, 5]

Step 3: Next, the grey decision matrix (GDM) is created by applying the attribute ratings calculated in Table 4. The constructed GDM according to Eq. (10) is displayed in Table 5.

Table 5: Grey decision matrix for all possible alternatives.				
C_i	A_1	A_2	A_3	A_4
C_1	[6.5, 8.5]	[4,5]	[5.5, 7.5]	[5.25, 6.75]
C_2	[4.25, 5.25]	[7.5, 9.5]	[0.75, 2.5]	[0.5, 2]
C_3	[5.25, 7.25]	[9, 10]	[5.5,7.5]	[8.25, 9.75]
C_4	[6.5, 8.5]	[4.75, 6.25]	[5, 6.5]	[5.5, 7.5]
C_5	[8.25, 9.75]	[9, 10]	[6, 9]	[8.25, 9.75]
C_6	[4.5, 5.5]	[4.75, 6.25]	[9, 10]	[4, 5]

Step 4: In this step, the normalization of GDM is carried out by using Eq. (11)-(13). For the process of normalization, the entropy, NPCR, and UACI are considered beneficial attributes, and correlation is taken as a non-beneficial attribute. The normalized GDM is demonstrated in Table 6.

Table 6: Normalized grey decision matrix.				
C_i	A_1	A_2	A_3	A_4
C_1	[0.667, 0.872]	[0.400, 0.500]	[0.550, 0.750]	[0.074, 0.095]
C_2	[0.436, 0.538]	[0.750, 0.950]	[0.075, 0.250]	[0.250, 1.000]
C_3	[0.538, 0.744]	[0.900, 1.000]	[0.550, 0.750]	[0.051, 0.061]
C_4	[0.667, 0.872]	[0.475, 0.625]	[0.500, 0.650]	[0.067, 0.091]
C_5	[0.841, 1.000]	[0.900, 1.000]	[0.600, 0.900]	[0.051, 0.061]
C_6	[0.462, 0.564]	[0.475, 0.625]	[0.900, 1.000]	[0.100, 0.125]

Step 5: After the calculation of normalized GDM, the weight vectors are multiplied with it to get weighted normalized GDM. The weight vectors calculated in Table 3 are multiplied with the normalized vectors in Table 5 by using the mathematical expression defined in Eq. (14). The calculated weighted normalized GDM is presented in Table 7.

Table 7: Weighted normalized GDM.				
C_i	A_1	A_2	A_3	A_4
C_1	[0.483, 0.763]	[0.200, 0.325]	[0.399, 0.656]	[0.054, 0.083]
C_2	[0.316, 0.471]	[0.375, 0.618]	[0.054, 0.219]	[0.181, 0.875]
C_3	[0.390, 0.651]	[0.450, 0.650]	[0.399, 0.656]	[0.037, 0.053]
C_4	[0.483, 0.763]	[0.238, 0.406]	[0.363, 0.569]	[0.048, 0.080]
C_5	[0.610, 0.875]	[0.450, 0.650]	[0.435, 0.788]	[0.037, 0.053]
C_6	[0.335, 0.494]	[0.238, 0.406]	[0.653, 0.875]	[0.073, 0.109]

Step 6: The ideal referential alternatives $I^{\max}$ are selected from weighted normalized GDM acquired in step 5. The selection of ideal referential alternatives is done by using the mathematical expression defined in Eq. (15) as:

$$I^{\max} = \{[0.610, 0.875], [0.450, 0.650], [0.653, 0.875], [0.181, 0.875]\}.$$

Step 7: In this step, the gray possibility degree is calculated between all the six alternatives C_i and the ideal referential alternative $I^{\max}$. Corresponding to Eq. (15), the possibility degree for each alternative is calculated as

$$P(C_1 \leq I^{\max}) = 0.928,$$

$$P(C_2 \leq I^{\max}) = 0.780,$$

$$P(C_3 \leq I^{\max}) = 0.854,$$

$$P(C_4 \leq I^{\max}) = 0.930,$$

$$P(C_5 \leq I^{\max}) = 0.691,$$

$$P(C_6 \leq I^{\max}) = 0.875.$$

Step 7: Ranking of liking order is done according to the possibility degree. If the rate of possibility degree is lowest the rank of preference order is highest. According to the possibility degree calculated in the previous step the ranking of all possible alternatives can be done as:

$$C_5 > C_2 > C_3 > C_6 > C_1 > C_4 .$$

From the ranking of preference order, we can say that the cryptosystem C_5 [14] is the best and robust among all the possible cryptosystem inserted in the grey decision-making approach. The bar chart representation of the ranking of each alternative is depicted in Fig. 2 which reflects that the cryptosystem C_5 is best among all the alternatives.

Fig. 2: Bar chart representation of ranking of each alternative.

6. Conclusion

In this work, we have implemented a grey theory-based decision-making method for the election of robust cryptosystem. The grey theory is executed on six cryptosystems considered as all possible alternatives of the problem. The parameter utilized for the selection procedure is entropy, correlation coefficient, NPCR, and UACI. The grey theory-based approach uses some decision-makers for the best selection. In this work, we have involved four decision-makers who decide the priority of each parameter. The first ranked algorithm by using the grey approach is the cryptosystem proposed by A. M. Abbas [14] which depends on elliptic curves. Therefore, we can conclude that the cryptosystem offered in [14] can be used for secure communication and the grey theory-based decision-making method can be successfully implemented for the selection of cryptographic problems.

Acknowledgment

This research was funded by Princess Nourah bint Abdulrahman University Researchers Supporting Project Number (PNURSP2022R87), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Funding

This research was funded by Princess Nourah bint Abdulrahman University Researchers Supporting Project Number (PNURSP2022R87), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Conflicts of interest: The authors declare that they have no conflict of interest.

Ethical Approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability Statement: The authors declare that data supporting the findings of this study are available within the article.

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