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Article

# The Pauli Exclusion Principle and Informational Geometry: An Injectivity Constraint in Viscous Time Theory

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## Abstract

The Pauli exclusion principle is traditionally introduced in quantum mechanics as a postulate encoded in the antisymmetry of the fermionic wavefunction. While extraordinarily successful, this formulation leaves open a deeper question: *why* must nature forbid the perfect overlap of identical fermions? In this work, we propose a reinterpretation of Pauli exclusion within the framework of Viscous Time Theory (VTT), where physical law emerges from the geometry of informational state space under constraints of memory, recoverability, and causal trace preservation. We propose that the coincidence of two identical fermionic states can be interpreted, in informational-geometric terms, as a loss of injectivity of the causal mapping, i.e., to an informational singularity where distinct histories become non-separable. To prevent this collapse of recoverability, the joint state manifold naturally develops a “diagonal barrier”: a forbidden submanifold where the informational cost diverges and admissible trajectories are repelled. Within this perspective, antisymmetry of the wavefunction appears not as the cause of exclusion, but as its mathematical symptom. Within this perspective, Pauli exclusion can be interpreted as a geometric and informational constraint rather than a primitive quantum axiom. The framework further suggests a unified interpretation of the difference between fermions and bosons: the former may be viewed as carriers of identity-bearing, non-overwritable informational structure, while the latter correspond to additive excitations that do not threaten causal injectivity. In this way, the exclusion principle appears as a consequence of informational geometry in a universe characterized by viscous time and memory.

**Keywords:** Pauli exclusion principle; informational geometry; recoverability; injective mappings; fermions and bosons; diagonal barrier; Viscous Time Theory (VTT); quantum information structure

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## 1. Introduction

Few principles in physics are as fundamental and as enigmatic as the Pauli exclusion principle [1,2]. Since its introduction, it has shaped our understanding of atomic structure, chemistry, condensed matter, and astrophysics, providing the ultimate explanation for the stability of matter and the architecture of the periodic table. In its modern formulation, the principle is encoded in the antisymmetry of the fermionic wavefunction [2–4] under particle exchange, which implies that the amplitude vanishes when two identical fermions occupy the same quantum state. This mathematical rule is both elegant and empirically flawless. Yet, at a conceptual level, it leaves open a profound question: why should nature enforce such an exclusion at all?

In standard quantum mechanics, the usual answer is essentially formal: fermions are described by antisymmetric representations of the permutation group [2,3], and therefore they exclude each other by construction. While this explains how the rule is implemented, it does not explain why the

physical world should be organized in this way. The exclusion principle remains, in this sense, a structural axiom—an input to the theory rather than an emergent consequence of deeper principles.

Viscous Time Theory (VTT) invites a different point of view. In VTT, the universe is not merely a stage on which states evolve reversibly according to timeless equations. Instead, time is endowed with viscosity, meaning that evolution carries memory, deformation, and irreversibility. Physical processes are constrained not only by local dynamics, but also by global informational requirements: in particular, by the need to preserve a usable causal trace. This requirement is formalized through the concept of **recoverability**, which measures whether past states can, in principle, be reconstructed from present ones without catastrophic ambiguity.

From this perspective, the fundamental object is not the particle or the wavefunction, but the **informational geometry of state space** [5]. States are points in a manifold whose metric, anisotropy, and barriers encode the costs, constraints, and admissible directions of evolution. Trajectories are not arbitrary: they are selected by an optimization principle that balances geometric cost, memory deformation, and proximity to regions where recoverability collapses [7,9]. In this framework, what we usually call “laws of physics” appear as geometric and topological features of the admissible state manifold.

It is within this setting that the Pauli exclusion principle can be revisited at a deeper level. Consider two identical fermionic entities described by the same set of microstate variables—position, momentum, spin, and all other identity-bearing degrees of freedom [16]. If they were allowed to occupy exactly the same state, they would become strictly indistinguishable not only in practice, but in principle. More importantly, two distinct histories would collapse into a single present description. The mapping from past to present would cease to be injective: different causal trajectories would become irreversibly merged. In VTT terms, this corresponds to a collapse of recoverability—a genuine **informational singularity** [7,10], or what one might call an “informational black hole” in state space.

A universe that admits viscous time, memory, and causal trace cannot tolerate such a collapse without undermining its own structural consistency [13]. The preservation of a usable history requires that identity-bearing informational records remain, at least in principle, separable. This immediately suggests that the perfect overlap of two identical fermionic states must be forbidden **before** any physical interaction or energetic consideration enters the picture. The prohibition is not dynamical in origin; it is **geometric and informational**.

In the joint state space of two entities, this requirement manifests itself as a special role of the diagonal set, where the two states coincide. VTT predicts that this diagonal is not merely a neutral region of the manifold, but a **barrier**: a locus where the informational cost diverges and where recoverability drops to zero [15]. This “diagonal barrier” plays a role analogous to the recoverability skin introduced in other VTT contexts: trajectories may approach it, slide along it, or be deflected by it, but they cannot cross it without destroying the causal structure of the description. In this sense, Pauli exclusion emerges as a geometric constraint of the admissible state space, not as an arbitrary rule imposed on particles. The resulting structure of the diagonal barrier in the joint state manifold is illustrated schematically in Figure 1.

Seen from this angle, the familiar antisymmetry of the fermionic wavefunction acquires a new meaning. It is no longer the fundamental cause of exclusion, but rather the **mathematical signature** of an underlying geometric prohibition. The vanishing of the wavefunction on the diagonal simply encodes, in the language of quantum amplitudes, the fact that this region of state space is informationally forbidden. The direction of explanation is thus reversed: it is not antisymmetry that creates exclusion, but exclusion—rooted in informational geometry and recoverability—that forces the formalism to adopt an antisymmetric structure [14].

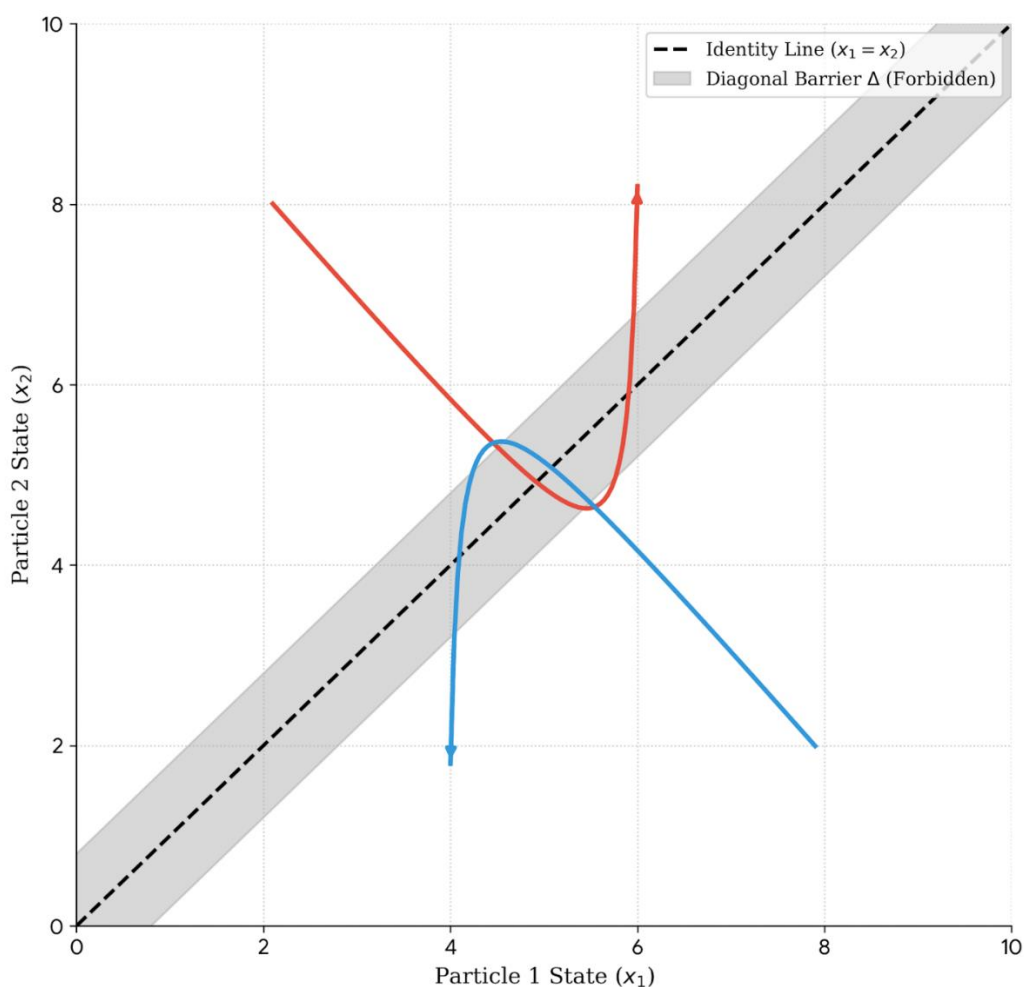
This viewpoint also clarifies, in a unified manner, the contrast between fermions and bosons [3,4,6]. In the present framework, fermions carry identity-bearing informational structure that cannot be overwritten: allowing two identical fermions to occupy the same informational state would amount to overwriting causal records and destroying the injectivity of the underlying causal map.

Bosons, by contrast, correspond to additive excitations of modes rather than exclusive carriers of identity. Their superposition does not compromise recoverability in the same way, and therefore no diagonal barrier is required in their sector of state space.

The aim of this paper is to articulate this reinterpretation in a precise yet conceptually transparent form. We examine how the Pauli exclusion principle may arise as a consequence of informational geometry in a universe with viscous time and memory [5,8], how the diagonal barrier emerges as a recoverability constraint in the informational state manifold, and how the familiar antisymmetry of the fermionic wavefunction appears as a secondary representational consequence. From this perspective, one of the central principles of quantum theory may be understood not as a primitive axiom, but as an emergent feature of a deeper informational structure underlying physical law [8,11,12].

The remainder of the paper is organized as follows. Section 2 introduces the informational-geometric framework and develops the concept of the diagonal barrier as a consequence of recoverability constraints in the informational state space. Section 3 analyzes how this structure naturally distinguishes fermionic and bosonic sectors and shows how antisymmetry arises as a representation of the barrier in the quantum formalism. Section 4 discusses possible physical implications and observational limits of the diagonal barrier in extreme regimes. Finally, the conclusions summarize the conceptual implications of this approach and outline directions for further investigation.

Figure 1. Diagonal Barrier  $\Delta$  (or “Forbidden Diagonal”) within the joint state manifold  $X = (x_1, x_2)$ . VTT trajectories (arrowed lines) approach the diagonal but are geometrically repelled before a state merger can occur. This repulsion preserves the foundational informational axiom of Recoverability  $R(X)$ , preventing identity collapse.



**Figure 1.** The Diagonal Barrier in Joint State Manifold.

## 2. Theory Framework

### 2.1. The Injective Mapping Problem and the Collapse of Recoverability

At the heart of Viscous Time Theory lies a simple but demanding requirement: physical evolution must preserve, at least in principle, a usable causal trace. Because time is viscous, the present is not a perfectly reversible image of the past, yet it cannot be an arbitrary erasure either. Between perfect reversibility and total amnesia, VTT introduces the concept of **recoverability**: a measure of how much of the past remains reconstructible from the present without catastrophic ambiguity.

This requirement can be expressed in geometric terms. Let  $M$  denote the informational state manifold of a single entity. For a system of two entities, the joint state space is the Cartesian product

$$M^{(2)} = M \times M, \quad (1)$$

with a generic point  $X = (x_1, x_2)$ . Physical evolution over a small time step can be represented as a mapping

$$\Phi_{\Delta t}: M^{(2)} \rightarrow M^{(2)}. \quad (2)$$

In an ideal, perfectly reversible world, this map would be bijective. In a viscous-time universe, this is no longer required: dissipation, coarse-graining, and deformation of information are allowed. However, VTT imposes a weaker but crucial condition: the map must not become **catastrophically non-injective** over the domain of physically admissible states. In other words, distinct causal histories must not collapse into a single present description in a way that destroys their distinguishability beyond recovery.

To make this precise, we introduce a recoverability functional  $R(X)$ , defined on  $M^{(2)}$ , which quantifies the degree to which past states can be reconstructed from the present state  $X$ . While its exact form depends on the chosen informational metric, its qualitative behavior is what matters here:

- $R(X) \approx 1$  corresponds to high recoverability (histories remain distinguishable),
- $R(X) \approx 0$  corresponds to recoverability collapse (distinct histories become irretrievably merged).

Now consider the special subset of  $M^{(2)}$  given by the **diagonal**

$$\Delta = \{(x_1, x_2) \in M^{(2)} \mid x_1 = x_2\}. \quad (3)$$

This set represents configurations in which the two entities occupy exactly the same informational state. For classical distinguishable objects, this may be an innocuous coincidence. For **identical, identity-bearing quantum entities**, however, the situation is radically different.

If two such entities are in precisely the same state, then not only are they observationally indistinguishable, but their **causal histories become non-separable**. Any permutation or exchange of their past trajectories leads to the same present description. The inverse mapping  $\Phi_{\Delta t}^{-1}$ , even in a weak, coarse-grained sense, ceases to exist as a function: it becomes a many-to-one relation. In informational terms, the evolution has lost injectivity at that point.

Within VTT, this loss of injectivity is not a minor technical inconvenience—it is a structural failure. It means that the universe has allowed a configuration in which part of its own causal record has been overwritten. The present state no longer contains enough information, even in principle, to discriminate between different pasts. This is precisely what VTT identifies as a **collapse of recoverability**:

$$X \rightarrow \Delta \Rightarrow R(X) \rightarrow 0. \quad (4)$$

Such a collapse plays a role analogous to a singularity in geometric theories: it marks a boundary beyond which the informational structure of the theory breaks down. Just as physical singularities signal the limits of spacetime geometry, these points signal the limits of admissible informational geometry.

From this perspective, the diagonal  $\Delta$  is not a benign region of state space. It is a locus where the causal map becomes non-injective, where distinct histories are forced into the same present, and where the informational bookkeeping of the universe fails. If the universe is to maintain a coherent

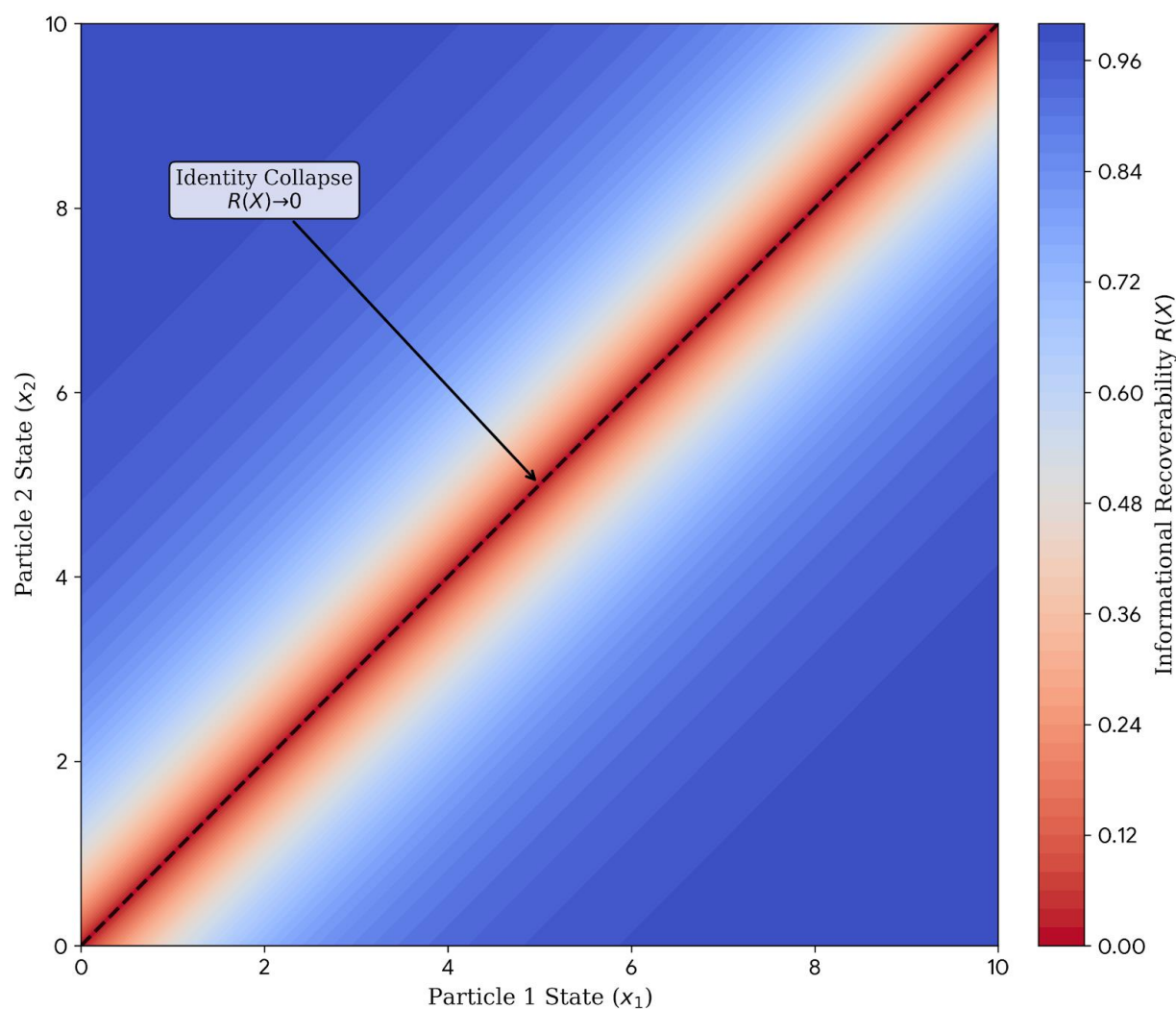
notion of history, memory, and identity—even in a viscous, lossy sense—it must prevent physical trajectories from entering this region.

This requirement does not arise from dynamics in the usual sense. It is not the result of a force, a potential, or an interaction energy. It is a **structural constraint** imposed by the need to preserve a minimal level of causal traceability. In other words, before we ask how particles move, we must ask which regions of state space are even admissible for motion.

The injective mapping problem thus reveals a deep reinterpretation of exclusion: the prohibition against two identical fermionic entities occupying the same state is, at its core, a prohibition against **informational non-injectivity**. Allowing such a configuration would amount to allowing the universe to overwrite part of its own history.

In the following section, we suggest how this abstract constraint acquires a concrete geometric form: the diagonal  $\Delta$  becomes associated with a diverging informational cost, giving rise to what we call a **diagonal barrier** in the joint state manifold. This barrier is the geometric mechanism through which the universe enforces the preservation of recoverability—and, as we shall see, it is the true origin of Pauli exclusion in the VTT framework.

**Figure 2.** Color gradient map of the Informational Recoverability  $R(X)$  functional. The cost of state-overlap diverges dramatically as the system state  $X$  approaches the identity condition ( $x_1 = x_2$ ), visualized as cold temperatures (low  $R$ , high tension) near the diagonal (dotted line). This geometric gradient acts as an effective informational pressure driving exclusion.



**Figure 2.** Recoverability Gradient Map.

## 2.2. The Diagonal Barrier as an Informational–Geometric Structure

If the collapse of recoverability marks a structural failure of the causal description, then the next natural question is how such a failure is avoided in practice. In Viscous Time Theory, the answer is not given in terms of ad hoc dynamical prohibitions, but in terms of **geometry**. The joint state manifold itself is shaped in such a way that trajectories are prevented from reaching regions where recoverability would vanish. This is achieved through the emergence of what we call a **diagonal barrier**.

As introduced above, the diagonal set

$$\Delta = \{(x_1, x_2) \in M \times M \mid x_1 = x_2\} \quad (5)$$

collects all configurations in which two entities occupy exactly the same informational state. For identity-bearing quantum entities, this set coincides with the locus where the causal map becomes non-injective and recoverability collapses. In VTT, such a locus cannot remain dynamically neutral. Instead, it acquires the status of a **forbidden or repulsive region** in the informational geometry of the state space.

To formalize this idea, VTT associates to each point  $X \in M^{(2)}$  an informational cost functional, or action density, which governs the admissibility of trajectories. Schematically, one may write an effective VTT action along a trajectory  $\gamma(t) \subset M^{(2)}$  as

$$\mathcal{S}_{\text{VTT}}[\gamma] = \int (\mathcal{L}_{\text{geo}}(X, \dot{X}) + \lambda V_{\Delta}(X)) dt, \quad (6)$$

where  $\mathcal{L}_{\text{geo}}$  encodes the ordinary geometric and dynamical costs, and  $V_{\Delta}(X)$  is a barrier potential associated with proximity to the diagonal set  $\Delta$ . The essential requirement is not the precise functional form of  $V_{\Delta}$ , but its asymptotic behavior:

$$V_{\Delta}(X) \rightarrow +\infty \text{ as } X \rightarrow \Delta, \quad (7)$$

for the fermionic, identity-bearing sector of the theory.

Geometrically, this means that the diagonal is not simply a submanifold of  $M^{(2)}$ , but a **ridge, wall, or skin** in the informational landscape: an infinite-cost boundary that admissible trajectories can approach only asymptotically, but never cross. The state space is thus effectively stratified into regions separated by this barrier, even though topologically the manifold remains connected.

This structure is closely analogous to other VTT barriers introduced in different contexts, such as recoverability skins, hysteresis ridges, and anisotropic corridors. In all these cases, the same principle applies: when a certain region of state space corresponds to a catastrophic loss of informational structure, the geometry reorganizes itself so that this region becomes dynamically inaccessible. The “law” is not imposed externally; it is encoded in the **shape of the admissible manifold itself**.

An important point is that this barrier is not, in general, an energetic barrier in the conventional sense. It does not arise because some physical interaction becomes infinitely strong at short distances. Rather, it arises because the **informational cost**—the cost of maintaining a coherent causal trace—diverges. One may equivalently express this in terms of recoverability:

$$V_{\Delta}(X) \equiv \Phi(1 - R(X)), \quad (8)$$

where  $R(X)$  is the recoverability functional introduced in the previous section, and  $\Phi(\rho)$  is a monotonically increasing function that diverges as  $\rho \rightarrow 1$ . Since  $R(X) \rightarrow 0$  as  $X \rightarrow \Delta$ , the barrier potential necessarily diverges in this limit.

From this viewpoint, the diagonal barrier is simply the geometric encoding of the rule: *the universe does not allow trajectories that destroy its own causal bookkeeping*. The prohibition is not enforced by a force, but by the fact that the path integral, variational principle, or optimization process underlying VTT assigns infinite cost to such paths. They are not dynamically forbidden; they are **geometrically excluded**.

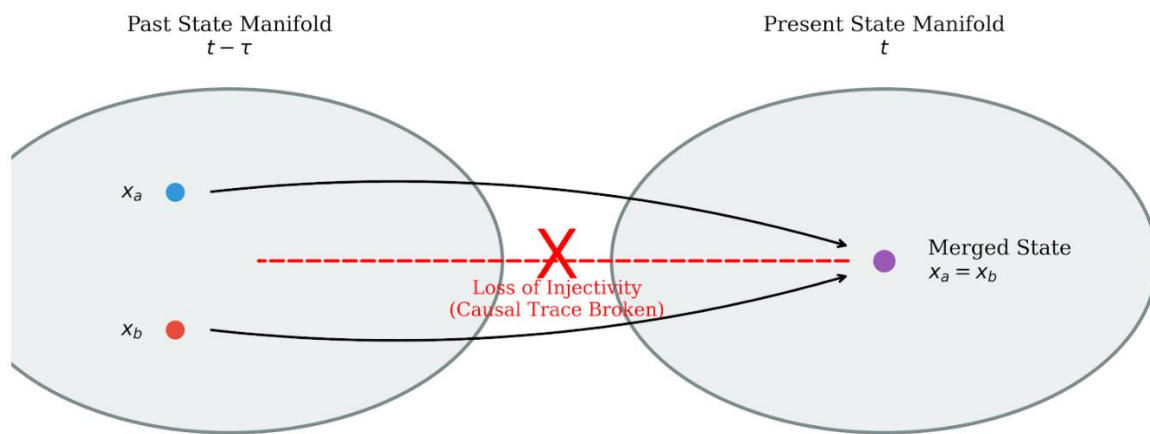
This reinterpretation has a crucial conceptual consequence. What we usually call “Pauli exclusion” is no longer a mysterious quantum rule about particles repelling each other in state space. It becomes the statement that, for identity-bearing entities, the joint informational manifold contains

a **singular barrier structure** along the diagonal. The exclusion is not an interaction between particles; it is a property of the **space of admissible states itself**.

In this sense, the diagonal barrier plays a role analogous to that of a geometric singularity or a horizon in spacetime physics. Just as a horizon is not a force but a boundary of accessibility defined by the metric structure, the diagonal barrier is not a force but a boundary defined by the informational metric and recoverability constraints of VTT.

In the next section, we will show how this geometric picture naturally explains the familiar antisymmetry of the fermionic wavefunction. Rather than being the fundamental cause of exclusion, antisymmetry will appear as the **representational imprint** of the diagonal barrier: a mathematical way of encoding, within the quantum formalism, the fact that the diagonal is a null, forbidden set of the state space.

Figure 3. Schematic of causal trace violation due to state collapse. A many-to-one mapping (identity merging) creates a causal singularity. Far from being a mere quantum rule, VTT identifies state coincidence as a physical singularity of injectivity (non-invertible mapping), making the Past state  $\{x_a(t - \tau), x_b(t - \tau)\}$ , fundamentally unrecoverable.



**Figure 3.** Identity Merge as an Informational Singularity.

### 2.3. Mathematical Formalization of the VTT Action and Recoverability Constraints.

#### 2.3.1. Recoverability Functional

We introduce a **recoverability functional**  $R(X) \in [0,1]$ , defined on  $M^{(2)}$ , which measures the degree to which past states can be reconstructed from the present state  $X$ . While many concrete definitions are possible (e.g., based on Fisher information, reconstruction error, or conditional entropy), only its qualitative properties are required here:

1.  $R(X) \approx 1$ : high recoverability (distinct pasts remain distinguishable).
2.  $R(X) \approx 0$ : recoverability collapse (distinct pasts become merged beyond recovery).
3.  $R(X)$  decreases as the local inverse of  $\Phi_{\Delta t}$  becomes more non-injective.

In particular, for **identity-bearing identical entities**, configurations approaching the diagonal  $\Delta = \{(x_1, x_2) \in M^{(2)} \mid x_1 = x_2\}$  (9) correspond to a loss of injectivity of the causal map, since permutations of histories become observationally indistinguishable. We therefore impose the structural condition

$$X \rightarrow \Delta \Rightarrow R(X) \rightarrow 0. \quad (10)$$

#### 2.3.2. Informational Cost and Barrier Term

In VTT, admissible trajectories minimize (or extremize) an informational-geometric action of the form

$$\delta_{\text{VTT}}[\gamma] = \int (\mathcal{L}_{\text{geo}}(X, \dot{X}) + \mathcal{L}_{\text{info}}(X)) dt, \quad (11)$$

where:

- $\mathcal{L}_{\text{geo}}$  encodes ordinary geometric and dynamical costs,
- $\mathcal{L}_{\text{info}}$  encodes informational penalties associated with memory, distortion, and loss of recoverability.

We define the informational penalty term as a monotonic function of recoverability loss:

$$\mathcal{L}_{\text{info}}(X) \equiv \lambda \Phi(1 - R(X)), \quad (12)$$

where:

- $\lambda > 0$  sets the relative weight of informational cost,
- $\Phi(\rho)$  is a monotonically increasing function with

$$\Phi(\rho) \rightarrow +\infty \text{ as } \rho \rightarrow 1. \quad (13)$$

Since  $R(X) \rightarrow 0$  as  $X \rightarrow \Delta$ , we obtain:

$$\mathcal{L}_{\text{info}}(X) \rightarrow +\infty \text{ as } X \rightarrow \Delta. \quad (14)$$

Thus, the diagonal set  $\Delta$  is associated with a **divergent informational cost**. Any trajectory attempting to cross or reach  $\Delta$  would contribute an infinite action and is therefore excluded from the set of admissible physical paths.

Additional formal details on the informational state manifold and the geometric emergence of the diagonal barrier are provided in Appendix A.

### 3. Results

#### 3.1. Antisymmetry as a Symptom, Not the Cause

In the standard formulation of quantum mechanics, the Pauli exclusion principle is usually presented in the following logical order: fermions are described by antisymmetric wavefunctions under particle exchange, and **because** of this antisymmetry, the probability amplitude vanishes when two identical fermions occupy the same state. Exclusion, in this view, is a direct consequence of representation theory and of the symmetry properties of the quantum state.

From the perspective developed in this work, this logical order must be reversed.

Within Viscous Time Theory, exclusion is not a primitive algebraic rule imposed on states. It is a **geometric and informational necessity** arising from the requirement that the causal mapping of the universe remain sufficiently injective to preserve recoverability. The diagonal barrier introduced in the previous section expresses this necessity at the level of state-space geometry: the coincidence set  $\Delta \subset M \times M$  is a region of infinite informational cost and therefore of zero admissible measure for physical trajectories.

Once this geometric fact is established, the role of antisymmetry becomes clear. It is not the origin of exclusion; it is the **mathematical encoding** of the diagonal barrier within the quantum formalism.

To see this, recall that for two identical fermions the wavefunction satisfies

$$\Psi(x_1, x_2) = -\Psi(x_2, x_1). \quad (15)$$

Setting  $x_1 = x_2 = x$  immediately gives

$$\Psi(x, x) = 0. \quad (16)$$

In the usual interpretation, this vanishing is taken as the statement of Pauli exclusion itself. In the VTT interpretation, however, this vanishing has a different meaning: it is the **representational imprint** of the fact that the diagonal  $\Delta$  is a forbidden region of the informational manifold.

In other words, the wavefunction is forced to have a node on the diagonal because the geometry of admissible states requires that no physical probability weight be assigned there. The antisymmetry condition is the simplest and most natural way, within the Hilbert-space formalism, to implement a **null measure** on  $\Delta$ . It ensures that any amplitude associated with coincident identity-bearing states is exactly zero, in perfect agreement with the existence of a diagonal barrier.

From this point of view, the logical structure is inverted:

- In standard quantum mechanics:

*Antisymmetry  $\Rightarrow$  Node on the diagonal  $\Rightarrow$  Exclusion.*

- In VTT:

*Recoverability constraint*  $\Rightarrow$  *Diagonal barrier*  $\Rightarrow$  *Node on the diagonal*  $\Rightarrow$  *Antisymmetric representation*.

The antisymmetry is thus not the cause of exclusion, but its **symptom**—the way in which the quantum formalism faithfully reflects a deeper geometric prohibition.

This shift in perspective has important conceptual consequences. It means that Pauli exclusion is not a mysterious quantum peculiarity tied to abstract group-theoretical properties of particles. It is instead the manifestation, in quantum language, of a much more general principle: **the universe does not allow informational overwriting of identity-bearing structures**. The wavefunction must adapt to this constraint, and antisymmetry is the minimal algebraic structure that enforces it.

This also clarifies why exclusion appears so absolute and non-dynamical. There is no “force” pushing fermions apart in state space, and no finite energy scale associated with the prohibition. The exclusion is absolute because it is **geometric**: it is enforced by the structure of the admissible manifold itself. Just as no finite force can push a timelike trajectory beyond a spacetime horizon defined by the metric, no finite dynamical process can push a fermionic trajectory across the diagonal barrier defined by informational geometry.

In this light, the familiar textbook statement that “fermions have antisymmetric wavefunctions” should be reinterpreted as a statement about the **encoding of informational geometry** in Hilbert space. The antisymmetry is the shadow, in the quantum formalism, of a deeper constraint imposed by recoverability and causal trace preservation in a viscous-time universe.

In the next section, we will extend this analysis to the contrast between fermions and bosons, showing that the presence or absence of a diagonal barrier is not a matter of algebraic taste, but a reflection of whether the corresponding degrees of freedom carry **identity-bearing, non-overwritable informational structure**.

### 3.2. Fermions, Bosons, and Identity-Bearing Information

The reinterpretation of Pauli exclusion proposed in the previous sections naturally raises a further question: if the diagonal barrier is a consequence of recoverability and injectivity constraints, why does it apply to fermions but not to bosons? In standard quantum theory, this difference is encoded in symmetry classes of representations, but the physical meaning of this distinction often remains obscure. Within the VTT framework, the answer emerges from the **informational role** played by different degrees of freedom.

The crucial distinction is not between “particles” in a naive sense, but between **identity-bearing structures** and **additive excitations of modes**.

Fermions, in this perspective, are carriers of **exclusive informational identity**. An electron in a given orbital with a given spin does not merely represent an amount of energy stored in a mode; it represents a specific, individuated record in the causal history of the system. Its presence changes the informational bookkeeping of the universe in a way that cannot be undone or merged without loss. If two such records were allowed to coincide perfectly, the mapping from past to present would become non-injective: two distinct causal histories would collapse into a single present description. This is precisely the recoverability catastrophe discussed in Section 2.1.

For such identity-bearing degrees of freedom, the diagonal set in joint state space corresponds to **informational overwriting**. The emergence of a diagonal barrier is therefore unavoidable: it is the geometric mechanism by which the universe protects the integrity of its own causal records. Exclusion is not a special interaction between fermions; it is the expression of the fact that **identity cannot be duplicated or merged without destroying information**.

Bosons, by contrast, play a fundamentally different informational role. A bosonic excitation does not represent an exclusive identity record. Instead, it represents an **additive, stackable modification of a mode**. Multiple photons in the same electromagnetic mode do not overwrite one another’s identity; they simply increase the occupation number of that mode. From the standpoint of informational geometry, this does not introduce a non-injective collapse of causal mapping in the

same way. The past can still be reconstructed, at least in principle, from the present occupation structure.

In other words, for bosonic degrees of freedom, the diagonal in joint state space does not correspond to an informational singularity. The recoverability functional  $R(X)$  does not collapse there, and therefore no diagonal barrier is required. The geometry remains smooth across the coincidence set, and trajectories are free to overlap. The familiar phenomenon of bosonic condensation and coherent superposition is, in this sense, a direct reflection of the absence of an identity-overwriting problem.

This provides a unified and physically transparent explanation of the spin-statistics dichotomy within the VTT framework:

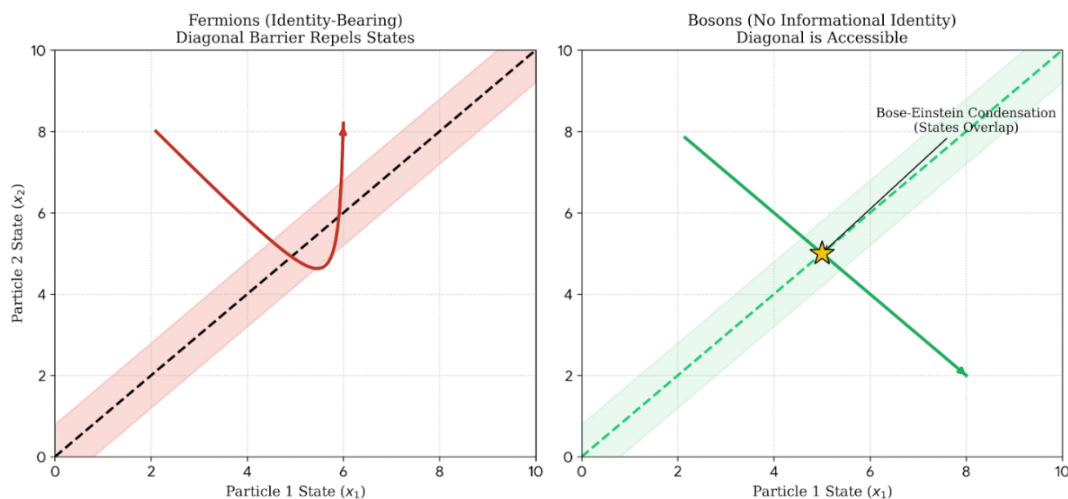
- **Fermions** correspond to degrees of freedom that carry **exclusive, non-overwritable identity-bearing information**. Their joint state space necessarily develops a diagonal barrier to protect recoverability, and this is encoded algebraically as antisymmetry.
- **Bosons** correspond to degrees of freedom that represent **additive, non-exclusive excitations of modes**. Their overlap does not threaten injectivity of the causal mapping, so no diagonal barrier arises, and symmetric representations are admissible.

Importantly, this distinction is not introduced by hand. It follows from the informational role assigned to different structures in the theory. The algebraic classification of states into symmetric and antisymmetric sectors becomes, in this light, a **secondary reflection** of a deeper geometric and informational stratification of state space.

This perspective also suggests that the Pauli exclusion principle is not an isolated peculiarity of quantum theory, but a specific instance of a much more general rule: **wherever degrees of freedom function as identity-preserving records in a system with memory, the geometry of state space must prevent their perfect overlap**. In VTT, Pauli exclusion is simply the quantum manifestation of this universal informational constraint.

In the next section, we will connect this geometric-informational picture back to standard quantum formalism, showing how the diagonal barrier and recoverability constraints can be viewed as underlying structures beneath the Hilbert-space description, and how familiar results emerge as effective, representational consequences.

**Figure 4.** A geometric and informational distinction between standard particle classes. **(Left)** Fermions, or identity-bearing particles, are defined by an inaccessible and repulsive Diagonal Barrier that protects identity. **(Right)** Bosons, lacking an inherent identity constraint, are defined by an accessible (and potentially attractive) diagonal, facilitating identity condensation (e.g., Bose-Einstein Condensation) without violating VTT injectivity constraints.



**Figure 4.** Comparative Geometry of Fermions and Bosons.

The informational interpretation of recoverability, including its relation to reconstruction error, entropy, and distinguishability metrics, is discussed in **Appendix B**

## 4. Discussion

### 4.1. From Informational Geometry to Quantum Formalism

Having established that Pauli exclusion arises, in VTT, from a geometric and informational constraint—the diagonal barrier associated with recoverability collapse—the natural question is how this structure relates to the standard quantum formalism. In particular, one may ask: where, in the Hilbert-space description of quantum mechanics, is this barrier hidden, and why does it appear there in the specific algebraic form we are accustomed to?

The answer proposed here is that the quantum formalism does not *create* the exclusion principle; it **encodes** it.

In VTT, the primary object is the informational state manifold and its geometry: metrics, anisotropies, corridors, skins, and barriers determined by recoverability and memory constraints. The diagonal barrier is one such structure. When we pass to the quantum description, we replace this geometric space of admissible states with a Hilbert space of complex amplitudes. This is a powerful representational move, but it comes at a price: geometric constraints must now be translated into **algebraic conditions on states**.

The diagonal barrier, being a region of infinite informational cost and zero admissible measure, must correspond, in the quantum representation, to a **null set** for physical probability amplitudes. In other words, any physically admissible quantum state must assign zero amplitude to configurations lying on the forbidden region:

$$X \in \Delta \Rightarrow \Psi(X) = 0. \quad (17)$$

This single requirement already captures the essence of Pauli exclusion. However, quantum mechanics does not impose such a condition by directly excising regions of configuration space. Instead, it implements it through **symmetry constraints** on the wavefunction.

For two identical fermions, the antisymmetry condition

$$\Psi(x_1, x_2) = -\Psi(x_2, x_1) \quad (18)$$

automatically enforces

$$\Psi(x, x) = 0. \quad (19)$$

Thus, antisymmetry is the minimal algebraic structure that guarantees the diagonal is a node of the wavefunction. From the VTT perspective, this is precisely what is required: the forbidden region of the informational manifold must be mapped to a zero-amplitude region in Hilbert space.

Seen this way, the passage from informational geometry to quantum formalism follows a clear conceptual route:

1. **Geometric level (VTT):** The joint state manifold develops a diagonal barrier because recoverability collapses there. Admissible trajectories are repelled from this region by diverging informational cost.
2. **Representational level (QM):** The barrier is encoded by requiring the wavefunction to vanish on the diagonal. This is implemented algebraically by antisymmetry.
3. **Operational level:** The vanishing of the amplitude translates into zero probability of observing two identical fermions in the same state—i.e., Pauli exclusion.

In this sense, the Hilbert-space formalism acts as a **projection** of a richer geometric structure onto an algebraic language. The antisymmetric representation is not a fundamental postulate about nature; it is the *shadow* of a deeper geometric constraint in informational state space.

This viewpoint also clarifies why exclusion appears as an absolute, kinematic rule rather than as a dynamical effect. In standard quantum mechanics, there is no Hamiltonian term that “pushes” fermions apart in state space. The prohibition is built into the space of allowed states itself. In VTT,

this is exactly what one expects from a barrier of geometric origin: it is not a force, but a **boundary of admissibility** defined by the informational metric and recoverability constraints.

Moreover, this approach suggests a broader unification. Other structures introduced in VTT—such as recoverability skins, hysteresis ridges, anisotropic corridors, and latency-induced barriers—should likewise admit quantum representations as nodal structures, phase constraints, or selection rules in Hilbert space. What appear in quantum theory as abstract symmetry or superselection rules may, in fact, be the algebraic traces of a much richer **informational-geometric landscape**.

Finally, this perspective offers a conceptual resolution to a long-standing discomfort in the foundations of quantum theory: the fact that the exclusion principle is usually introduced as an axiom tied to representation theory, rather than derived from physical or informational necessity. In VTT, exclusion is no longer an unexplained rule about wavefunctions. It is the inevitable consequence of demanding that a universe with memory and viscous time **cannot allow its own causal records to be overwritten**.

In the concluding section, we will summarize the implications of this reinterpretation, discuss its scope and limitations, and outline how this informational-geometric view of Pauli exclusion may extend to other areas—ranging from many-body physics to emergent structures in complex systems and artificial intelligence.

#### 4.2. Limits of the Diagonal Barrier

The interpretation developed above suggests that Pauli exclusion arises from a geometric constraint in the informational state manifold rather than from a primitive algebraic postulate. Within ordinary physical regimes this constraint appears effectively absolute, because the informational cost associated with the diagonal barrier diverges.

However, if the barrier is understood as a geometric feature of the admissible state space rather than a purely axiomatic rule, it becomes meaningful to ask under what circumstances the underlying geometry itself might be altered. In the VTT framework the cost functional governing admissible trajectories,  $\mathcal{L}_{info}(X)$ , depends on the informational structure of the system and on the preservation of recoverability.

In regimes where the informational metric becomes strongly distorted—either by extreme densities, strong spacetime curvature, or severe information scrambling—the structure of the admissible manifold may deviate from its low-energy approximation. In such situations the diagonal barrier would remain present as a constraint of the geometry, but its effective properties could in principle be modified.

This perspective does not imply that Pauli exclusion fails in ordinary conditions. Rather, it suggests that the strictness of the barrier is tied to the stability of the informational geometry from which it emerges.

#### 4.3. Extreme Regimes and Possible Observational Signatures

If the diagonal barrier originates from informational geometry rather than from an immutable axiom, extreme physical environments provide natural contexts in which the robustness of this structure might be probed.

**Ultra-Dense Matter:** In conventional quantum theory the Pauli exclusion principle generates degeneracy pressure, which plays a central role in stabilizing compact astrophysical objects such as white dwarfs and neutron stars. Within the VTT framework this pressure arises from the divergence of informational cost near the diagonal barrier of the joint state manifold.

In environments where baryonic density becomes extremely large, the informational geometry governing fermionic states may become highly curved. Although any deviation from strict exclusion would be expected to remain extremely small, such curvature could in principle introduce subtle corrections to the effective equation of state of ultra-dense matter. These corrections might manifest in the detailed mass–radius relations of neutron stars or in modifications to their thermal evolution.

**Horizon Proximity:** A second extreme regime arises near gravitational horizons. VTT places strong emphasis on the preservation of causal informational traces, quantified through the recoverability functional  $R(X)$ . In the vicinity of an event horizon the causal structure of spacetime becomes highly distorted, and information scrambling approaches its theoretical limits.

Under such conditions the informational metric governing the distinguishability of fermionic states may degrade. If the diagonal barrier is fundamentally tied to recoverability constraints, its effective structure could be influenced by this distortion of causal traceability.

One speculative consequence is that the statistical structure of quantum fields near a horizon might differ slightly from the idealized flat-spacetime description. In particular, the fermionic sector of Hawking radiation could in principle exhibit small deviations from the perfectly thermal spectrum predicted by semiclassical treatments.

A detailed analysis of such effects lies beyond the scope of the present work, but these considerations illustrate how the informational-geometric interpretation of Pauli exclusion naturally suggests avenues for future investigation.

## 5. Conclusions

In this work, we have proposed a reinterpretation of the Pauli exclusion principle that shifts it from the status of a primitive quantum axiom to that of a **geometric and informational necessity**. Within the framework of Viscous Time Theory, where time carries memory and physical evolution must preserve a usable causal trace, exclusion emerges not from abstract symmetry postulates, but from the requirement that the universe must not destroy its own recoverability.

The central idea is simple but far-reaching. If two identical, identity-bearing entities were allowed to occupy exactly the same informational state, distinct causal histories would collapse into a single present description. The mapping from past to present would become non-injective, and the system would suffer a catastrophic loss of recoverability. In VTT, such a situation is not merely improbable or dynamically suppressed—it is **structurally inadmissible**.

This inadmissibility is encoded geometrically in the joint state manifold through the emergence of a **diagonal barrier**: a region of diverging informational cost and vanishing admissible measure. Physical trajectories are repelled from this region not by a force, but by the geometry of the state space itself. Exclusion, in this sense, is not an interaction; it is a **boundary of admissibility**.

Within the quantum formalism, this geometric constraint appears as a node of the wavefunction on the diagonal and is algebraically implemented by antisymmetry. From the VTT perspective, this reverses the usual explanatory order: antisymmetry is not the cause of Pauli exclusion, but its **representational consequence**. The wavefunction is forced to vanish on the diagonal because the underlying informational geometry forbids that region, not the other way around.

This framework also provides a natural and physically transparent distinction between fermions and bosons. Fermions are interpreted as carriers of **exclusive, identity-bearing informational structure**, for which overlap would amount to informational overwriting and loss of causal trace. Bosons, by contrast, correspond to **additive excitations of modes**, whose superposition does not threaten injectivity of the causal mapping and therefore requires no diagonal barrier. The familiar spin-statistics dichotomy thus appears as a reflection of a deeper stratification of informational roles in state space.

Beyond its implications for the foundations of quantum mechanics, this reinterpretation suggests a broader unifying perspective. Pauli exclusion becomes one instance of a general VTT principle: **wherever a system must preserve identity-bearing records in the presence of memory and viscous time, the geometry of its state space must develop barriers that prevent informational collapse**. Similar mechanisms already appear in VTT under the form of recoverability skins, hysteresis ridges, anisotropic corridors, and optimization-driven forbidden regions. Exclusion is simply the most famous and experimentally ubiquitous example of this class of structures.

In this sense, the stability of matter, the architecture of atoms, and the entire edifice of fermionic physics can be seen as macroscopic consequences of a much deeper requirement: that the universe, as an informational system, must remain capable of telling its own story without overwriting its past.

Finally, this work opens several directions for future research. A more explicit formulation of the recoverability functional and its relation to established quantum information measures may enable quantitative investigations of the diagonal barrier concept. Extensions to many-body systems, mixed statistics, and emergent quasiparticles may help clarify how informational geometry organizes complex phases of matter. More speculatively, similar principles may find application beyond traditional physics—for example in biological organization or artificial intelligence systems—where identity, memory, and recoverability also play fundamental structural roles.

From this perspective, the Pauli exclusion principle is no longer merely a rule governing fermions. Rather, it may be understood as a manifestation of a deeper principle: that physical law reflects the geometry of information constrained by the requirement that causal records remain recoverable.

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## Appendix A — Informational State Space and Diagonal Barrier Structure

### A.1. Informational State Space and Evolution Map

Let  $M$  denote the informational state manifold of a single entity, equipped with an informational metric  $g$  that measures distinguishability of states under the coarse-grained observational and causal structure of the theory. For a two-entity system, the joint state space is

$$M^{(2)} = M \times M, X = (x_1, x_2). \quad (\text{A1})$$

Physical evolution over a small time interval  $\Delta t$  is represented by a causal (generally non-invertible) map:

$$\Phi_{\Delta t}: M^{(2)} \rightarrow M^{(2)}. \quad (\text{A2})$$

In VTT, time is viscous: information can be deformed, compressed, and partially lost. However, the theory imposes a structural constraint: **the evolution must not become catastrophically non-injective** on the domain of physically admissible states.

### A.2. Emergence of the Diagonal Barrier

The previous construction shows in section 2.3, that the **diagonal barrier** is not an ad hoc addition, but a direct consequence of:

1. The requirement of non-catastrophic non-injectivity of the causal map,
2. The definition of recoverability as a structural quantity,
3. The inclusion of recoverability loss in the informational action.

Formally, we may define an effective barrier potential:

$$V_{\Delta}(X) \equiv \lambda \Phi (1 - R(X)), \quad (\text{A3})$$

with the asymptotic behavior:

$$V_{\Delta}(X) \rightarrow +\infty \text{ as } X \rightarrow \Delta. \quad (\text{A4})$$

Geometrically, this means that  $\Delta$  becomes a **repulsive ridge or forbidden skin** in the informational metric of  $M^{(2)}$ . The state space remains topologically connected, but dynamically and

variationally stratified: admissible trajectories can approach the diagonal only asymptotically and never cross it.

### A.3. Connection to the Quantum Representation

In the quantum formalism, this barrier must be represented as a **null measure region** in configuration space. This is achieved by requiring:

$$X \in \Delta \Rightarrow \Psi(X) = 0, \quad (\text{A5})$$

which, for identical fermions, is implemented algebraically by antisymmetry:

$$\Psi(x_1, x_2) = -\Psi(x_2, x_1) \Rightarrow \Psi(x, x) = 0. \quad (\text{A6})$$

Thus, the antisymmetric structure of the fermionic wavefunction appears as the **Hilbert-space encoding** of the diagonal barrier generated by the recoverability functional.

### A.4. Interpretation

This appendix shows, in minimal formal terms, how Pauli exclusion can be derived in VTT from a single structural requirement: the theory requires that physical evolution avoid states that destroy recoverability. The diagonal barrier is the geometric expression of this requirement, and antisymmetry is its quantum-algebraic shadow.

## Appendix B — Recoverability, Information Loss, and Distinguishability

### B.1. Recoverability as an Informational Notion

In the main text, recoverability  $R(X)$  has been introduced as a structural quantity measuring the degree to which past states remain reconstructible from the present. While we have intentionally kept its definition abstract, it is important to show that this concept is not vague or metaphysical. On the contrary, it is closely related to several well-established notions in classical and quantum information theory.

At an intuitive level, recoverability answers the question:

Given the present state  $X$ , how many distinct past states could have plausibly produced it, and how well can they be discriminated?

If many distinct histories map to the same present description and cannot be distinguished even in principle, recoverability is low. If the mapping remains close to injective and past states remain distinguishable, recoverability is high.

This idea can be formalized in several equivalent ways, depending on the chosen informational framework.

### B.2. Recoverability and Reconstruction Error

One possible approach is operational. Let  $\hat{\Phi}^{-1}$  denote an optimal reconstruction procedure (not necessarily an exact inverse) that attempts to infer a past state from the present one. One may define a reconstruction error functional:

$$\epsilon(X) = \mathbb{E}[d(X_{\text{past}}, \hat{\Phi}^{-1}(X))], \quad (\text{B1})$$

where  $d(\cdot, \cdot)$  is an informational or geometric distance on state space and the expectation value is taken over admissible past histories compatible with  $X$ .

Recoverability can then be defined, schematically, as a decreasing function of this error:

$$R(X) = \mathcal{F}(\epsilon(X)), \mathcal{F}' < 0, \quad (\text{B2})$$

with:

- $R \approx 1$  when reconstruction error is small (histories remain distinguishable),
- $R \approx 0$  when reconstruction error is large (many histories collapse into one).

In the case of two identical identity-bearing entities approaching the diagonal  $\Delta$ , permutations of their past trajectories become observationally indistinguishable, and the reconstruction error necessarily diverges. Hence:

$$X \rightarrow \Delta \Rightarrow \epsilon(X) \rightarrow \infty \Rightarrow R(X) \rightarrow 0. \quad (\text{B3})$$

### B.3. Recoverability and Entropic Measures

A complementary, more information-theoretic perspective uses entropy. Let  $H(\text{Past} | X)$  denote the conditional entropy of the past given the present state  $X$ . This quantity measures how much uncertainty about the past remains once the present is known.

- If  $H(\text{Past} | X)$  is small, the past is (approximately) determined by the present: recoverability is high.
- If  $H(\text{Past} | X)$  is large, many distinct pasts are compatible with the same present: recoverability is low.

One may therefore define a normalized recoverability measure of the form:

$$R(X) = 1 - \frac{H(\text{Past}|X)}{H_{\max}}, \quad (\text{B4})$$

where  $H_{\max}$  is a suitable normalization scale.

In configurations approaching the diagonal  $\Delta$  for identical fermions, the conditional entropy necessarily increases, because permutations of identity-bearing histories become indistinguishable. Distinct causal records are merged into a single present description, and:

$$X \rightarrow \Delta \Rightarrow H(\text{Past} | X) \rightarrow H_{\max} \Rightarrow R(X) \rightarrow 0. \quad (\text{B5})$$

Thus, from an entropic viewpoint, the diagonal corresponds to a **maximal loss of historical information**.

### B.4. Recoverability and Distinguishability Metrics

In both classical and quantum information theory, the distinguishability of states can be quantified by metrics or divergences, such as:

- Fisher information metrics,
- Kullback–Leibler divergence,
- Trace distance or fidelity in the quantum case.

Let  $D(\rho_1, \rho_2)$  be a measure of distinguishability between two candidate past states compatible with the same present  $X$ . If, near the diagonal, all such candidate past states become mutually indistinguishable, then:

$$D(\rho_i, \rho_j) \rightarrow 0 \text{ for all admissible } i, j, \quad (\text{B6})$$

and the past effectively collapses into a single equivalence class. Recoverability, being tied to distinguishability, must then vanish:

$$(X) \sim \langle D(\rho_i, \rho_j) \rangle \rightarrow 0. \quad (\text{B7})$$

This provides yet another, fully operational interpretation of the recoverability collapse at the diagonal.

### B.5. Conceptual Synthesis

All these perspectives—reconstruction error, conditional entropy, and distinguishability metrics—converge on the same qualitative conclusion:

The diagonal  $\Delta$  corresponds to a regime where distinct causal histories become informationally indistinguishable, and where the present state no longer carries enough information to support a meaningful inverse mapping.

In VTT, such a regime is not merely inconvenient; it is **structurally forbidden**. The diagonal barrier introduced in the main text can therefore be understood as the geometric encoding of a universal informational principle: **physical evolution must avoid regions of state space where historical information collapses beyond recovery**.

From this standpoint, Pauli exclusion is simply the most familiar and experimentally accessible manifestation of a much more general rule: whenever identity-bearing degrees of freedom are

involved, the informational geometry of state space must prevent their perfect overlap, because such overlap would amount to an irreversible loss of causal trace.

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