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Observer-Dependent Navigability in Swarm Intelligence: A Path-Theoretic Decomposition of Performance into Perception and Distortion

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Abstract

Why do different swarm algorithms achieve different performance on the same fitness landscape? This paper proposes that navigability—the structural capacity to find improving paths—is *observer-dependent*: different algorithms perceive different navigability on identical landscapes, and this difference is irreducible to landscape properties alone. We formalise this through the decomposition F = P/D, where Perception (P) measures an algorithm's differentiation capacity and Distortion (D) measures structural resistance. The ratio form is derived uniquely from three axioms (monotonicity, scale-covariance, separability). Three claims are advanced and tested across five experiments on the Deucalion supercomputer, totalling over 200,000 simulated trials. **Claim 1 (Distortion is multiplicative):** D compounds geometrically, not additively (R^2 = 0.993 vs. 0.856; n = 250 cross-algorithm trials). **Claim 2 (Perception is observer-dependent):** Six navigation strategies on the same 9,913 graphs yield six different P values; a hidden variable model reconstructing P from graph features and strategy identity achieves only R^2 = 0.058 (n = 9,470 strategy–graph pairs). In the CEC optimisation domain, the same hidden variable test yields R^2 = 0.403 (n = 50 algorithm–function pairs), indicating a domain-dependent boundary. **Claim 3 (Alignment dominates):** Step-wise alignment—the fraction of moves that reduce distance to the optimum—predicts navigation efficiency at R^2 = 0.82 across 57,518 trials, outperforming all tested graph-theoretic and landscape metrics (maximum alternative R^2 = 0.03). Cross-domain validation spans graph navigation (10,000 graphs, 6 strategies), CEC-2017 benchmarks (10 functions, 5 algorithms), 2D continuous landscapes (79,956 trials, mediation analysis), PSO parameter sweeps (5,000 runs), and ACO pheromone dynamics (2,987 runs). Six counterfactual tests and a mediation analysis support the framework. All results are simulation-based. What fails is reported with the same rigour as what succeeds: P alone outperforms P/D at the graph level (ϱ = 0.343 vs. 0.108), the FLRP multiplicative decomposition is dead (R^2 = 0.0002), and the scalar F-field fails in continuous space (R^2 = 0.004). Twelve falsification criteria are specified. The framework is a hypothesis under test, not a proven law.

Keywords: observer-dependent navigability; swarm intelligence; path-theoretic decomposition; F = P/D; alignment; fitness landscape analysis; perception; distortion; falsification

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1. Introduction

Metaheuristic optimisation has produced dozens of algorithms—Particle Swarm Optimisation (PSO), Differential Evolution (DE), Grey Wolf Optimiser (GWO), Ant Colony Optimisation (ACO), and many others—each evaluated primarily by benchmark performance. The CEC competition suites provide standardised functions, statistical tests identify significant differences, and Friedman rankings order algorithms. Yet a structural question remains: *why* does one algorithm outperform another on a given landscape? The No Free Lunch theorems (Wolpert & Macready, 1997) establish that no algorithm dominates across all problems, but they do not explain the mechanism by which a specific algorithm succeeds or fails on a specific problem instance.

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Fitness landscape analysis (FLA) offers a partial answer by computing statistical features—ruggedness, deceptiveness, information content—from sampled fitness values (Mersmann et al., 2011; Kerschke & Trautmann, 2019). These features predict algorithm performance with moderate accuracy and underpin algorithm selection systems (Rice, 1976; Kerschke et al., 2019). However, FLA treats the landscape as observer-independent: one landscape, one set of features, regardless of which algorithm will navigate it. This paper challenges that assumption.

We propose that the answer lies in *navigability*—the structural capacity to find improving paths—and that navigability is *observer-dependent*. The same fitness landscape, viewed by different algorithms, presents different navigability. A greedy algorithm perceives high navigability on smooth, unimodal landscapes but low navigability on deceptive, multimodal ones. A stochastic algorithm may perceive moderate navigability in both cases. The landscape does not change; the observer does.

We formalise this through the ratio F = P/D, derived uniquely from three axioms (Section 3.1). Here, P (Perception) measures the algorithm's differentiation capacity—its ability to distinguish improving from non-improving moves—and D (Distortion) measures the structural resistance of the landscape to traversal. The decomposition is designed to be non-tautological: P is measured from decision quality (alignment), while D is measured from the cost structure (geometric mean of fitness values). Different instruments measure different quantities.

Three specific claims are advanced:

C1 (Distortion is multiplicative). D compounds geometrically, not additively. Independent resistance factors multiply rather than sum. Evidence: $R^2 = 0.993$ (geometric) vs. $R^2 = 0.856$ (additive) across 250 cross-algorithm CEC trials (Section 5.3).

C2 (Perception is observer-dependent). Different algorithms on the same landscape perceive different P, and this difference cannot be fully reconstructed from landscape features and algorithm identity alone. In graph navigation: hidden variable $R^2 = 0.058$ (n = 9,470). In CEC optimisation: $R^2 = 0.403$ (n = 50). The observer effect varies by domain (Section 5.4).

C3 (Alignment dominates). Step-wise alignment—the path-integral of local improving moves—is the dominant predictor of navigation efficiency at $R^2 = 0.82$ across 57,518 graph navigation trials, outperforming all tested graph-theoretic metrics (Section 5.2). Cross-domain: mediation analysis in continuous space confirms alignment as the causal pathway (74.7% reduction in partial correlation). PSO early alignment: $R^2 = 0.078$. ACO late alignment: $R^2 = 0.153$ with stigmergic learning confirmed (Section 5.5).

The experimental evidence is drawn from five experiments on the Deucalion supercomputer (FCT 2025.00020.AIVLAB.DEUCALION), comprising: (A) graph navigation alignment across 9,913 graphs and 57,518 trials with 6 strategies and 6 counterfactual conditions; (B) FLRP multi-layer decomposition across 5,000 graphs and 13,732 trials; (C) 2D continuous landscapes across 1,000 landscapes and 79,956 trials with mediation analysis; (D) PSO alignment across 5,000 runs on three 10D functions; (E) ACO pheromone alignment across 2,987 runs with stigmergic learning analysis. Additionally, a CEC-2017 benchmark (10 functions × 5 algorithms × 5 runs) is reported. Total computation: approximately 3.4 h for the main alignment experiments, plus 3.8 min for the CEC benchmark. All results are simulation-based and reported with full honesty: what survives, what fails, and what remains open.

The paper is organised as follows. Section 2 reviews related work. Section 3 presents the theoretical framework. Section 4 details the experimental design. Section 5 reports results with source attribution. Section 6 discusses implications, limitations, and falsification criteria. Section 7 concludes.

2. Related Work

2.1. Fitness Landscape Analysis

The concept of fitness landscapes, introduced by Wright (1932), has been formalised through exploratory landscape analysis (ELA), which computes statistical features from sampled fitness

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values (Mersmann et al., 2011). Recent advances include deep-ELA using neural network embeddings (Seiler et al., 2024) and cell-based feature computation (Kerschke & Trautmann, 2019). These features predict algorithm performance with moderate accuracy but treat the landscape as observer-independent: one landscape, one feature set, regardless of which algorithm navigates it. The present work challenges this assumption empirically.

2.2. Algorithm Selection and Performance Prediction

The algorithm selection problem (Rice, 1976) seeks mappings from problem features to algorithm choices. Modern approaches use per-instance algorithm selection with landscape features as inputs to meta-learners (Kerschke et al., 2019; Bischl et al., 2016). The AutoML paradigm extends this to hyperparameter configuration (Feurer et al., 2022). These approaches implicitly assume that landscape features are sufficient statistics for performance prediction. Our hidden variable analysis (Section 5.4) tests this assumption directly: if P_agent can be predicted from landscape features and algorithm identity, then observer-dependence is classical (reducible); if not, it is irreducible.

2.3. Search Trajectory Analysis

Search trajectory networks (Ochoa et al., 2014) model optimisation runs as directed graphs where nodes represent search positions and edges represent transitions. Fitness-distance correlation (Jones & Forrest, 1995) measures the relationship between fitness and distance to optima. Algorithm footprints (Smith-Miles & Tan, 2012) characterise performance across instance space. These approaches analyse trajectories *after* execution. The alignment metric proposed here (Section 3.2) is structurally related but can be computed *before* execution from the graph structure alone (structural alignment), offering a predictive rather than retrospective perspective.

2.4. Swarm Intelligence Theory

PSO (Kennedy & Eberhart, 1995) and ACO (Dorigo et al., 2006) are among the most studied swarm algorithms. Theoretical analyses have characterised convergence properties (Clerc & Kennedy, 2002), stagnation (van den Bergh & Engelbrecht, 2006), and the exploration–exploitation trade-off (Črepinšek et al., 2013). Recent work examines information flow in swarms (Pang et al., 2024), network-based interaction topologies (Liu et al., 2024), and large-scale multi-objective optimisation (Tian et al., 2024). The present work contributes a structural decomposition that applies across swarm paradigms: PSO particles, ACO ants, and abstract graph navigators all instantiate F = P/D at different observer levels.

2.5. Performance Decomposition

The No Free Lunch theorems (Wolpert & Macready, 1997) motivate understanding why specific algorithms succeed on specific problems. Existing decompositions include landscape difficulty measures (Malan & Engelbrecht, 2013), algorithm complexity profiles (Muñoz et al., 2015), and the separation of exploration and exploitation contributions (Črepinšek et al., 2013). The F = P/D decomposition offers a complementary perspective: it separates the contribution of the navigator (P, observer-dependent) from the contribution of the landscape (D, observer-independent), providing a causal attribution framework rather than a statistical correlation.

3. Theoretical Framework

3.1. The Law of Freedom: Derivation of F = P/D

We define Freedom (F) as the structural availability of improving paths in a navigable system. The central hypothesis is that F decomposes into the ratio of two quantities: Perception (P), measuring the navigator's differentiation capacity, and Distortion (D), measuring the structural resistance of the landscape to traversal.

The ratio form is derived from three axioms:

Axiom C1 (Monotonicity): $\partial F/\partial P > 0$ and $\partial F/\partial D < 0$ for all $P, D > 0$. More Perception yields more Freedom; more Distortion yields less.

Axiom C2 (Scale-covariance): $F(\lambda P, \lambda D) = F(P, D)$ for all $\lambda > 0$. Freedom depends on the ratio of Perception to Distortion, not on absolute magnitudes.

Axiom C3 (Separability): $F(P, D) = g(P) \cdot h(D)$. Perception and Distortion contribute independently. This is a parsimony assumption; interaction terms have not been tested.

Derivation. From C3, $F = g(P) \cdot h(D)$. C2 requires $g(\lambda P) \cdot h(\lambda D) = g(P) \cdot h(D)$, which for $P = D = 1$ gives $g(\lambda) \cdot h(\lambda) = g(1) \cdot h(1) =$ constant. This forces power-law form: $g(P) = P^a$, $h(D) = D^b$ with $a + b = 0$. C1 constrains $a > 0$, $b < 0$. Setting $K = 1$, $a = 1$ yields the minimal hypothesis $F = P/D$. Empirically, fitted exponents are $a \approx 1.29$, $b \approx -1.09$ with 95% confidence intervals excluding 1.0 (Melo de Magalhães, 2026a). The power form $(P/D)^a$ fits better than the strict ratio; both are tested in Section 5.

What the derivation does not prove. The derivation establishes that *if* C1–C3 hold, then $F \propto (P/D)^a$. It does not prove that the axioms are correct, that $\alpha = 1$, or that P and D are the right quantities. The paper tests the conditional: given the axioms, does the predicted form match computation?

3.2. Perception (P): Definition and Observer-Dependence

The key theoretical contribution is that P is not a property of the landscape alone. P is a property of the (landscape, observer) pair. Different observers—different algorithms—on the same landscape measure different P.

At the agent level, P is defined as *alignment*: the fraction of steps where the chosen move reduces distance to the optimum. For a graph $G = (V, E)$ with shortest-path distance d, source s, and target t, a navigation strategy σ that produces a path $x_0, x_1, \ldots, x_k$ has alignment:

$$P\_align(\sigma, G) = (1/k) \sum_{i=1}^{k} \mathbb{1}[d(x_i, t) < d(x_{i-1}, t)]$$

Structural alignment (computable without running any agent) averages over all intermediate nodes: $P\_structural(G) = (1/|V|) \sum_v (|\{u \in N(v) : d(u,t) < d(v,t)\}|/|N(v)|)$, where $N(v)$ is the neighbourhood of v. This is a pure graph property, computable in $O(N^3)$ via all-pairs shortest paths.

A topological proxy avoids the circularity of requiring shortest paths: $P \approx 1/\bar{a}$, where $\bar{a}$ is the mean node degree (Section 5.4; $\varrho = 0.735$ with P_structural, $R^2 = 0.719$ for predicting P_align from 1/avg_degree on 2,456 graphs).

3.3. Distortion (D): Observer-Independent Resistance

Distortion measures the structural resistance of the landscape to traversal. For graph navigation, D is the geometric mean of edge costs: $D = \exp(\text{mean}(\log(\text{costs})))$. For continuous landscapes, D is the geometric mean of fitness values encountered.

The geometric (multiplicative) model is chosen because independent barriers compound multiplicatively: a locked door *times* a long corridor, not plus. Empirically, R^2(geometric) = 0.993 vs. R^2(additive) = 0.856 across 250 cross-algorithm CEC trials (Section 5.3).

The critical property is that D is observer-independent: the same landscape imposes the same geometric cost structure on all navigators. This asymmetry—P is observer-dependent, D is observer-independent—is the central structural feature of the decomposition.

3.4. Three Observer Levels

The framework operates at three verified observer levels, each with its own definition of P and its own instrument of measurement:

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Table 1. Three observer levels. The law F = P/D is identical; the measurement of P is not. Sources: [a] Melo de Magalhães (2026a), 80,000 simulations across 7 physics laws. [b] ISC_V2 run, 2,400 graphs, 6 topology families. [c] Present work, Exp. A, 57,518 trials; R^2 varies by strategy: greedy 0.84, random 0.54.

Level	Observer	P Definition	D Definition	R^2
Physics	None (passive)	P = 1 (constant)	Physical resistance	> 0.99 [a]
System	Topology scanner	$P = 1/\bar{A}$ (inv. mean path)	Geo. mean edge costs	0.935 [b]
Agent	Decision-maker	P = alignment fraction	Geo. mean edge costs	0.54–0.84 [c]

At the physics level, the material IS the observer: P = 1, F = 1/D, recovering Ohm, Fick, Fourier, Darcy, and Langevin as special cases ($R^2 > 0.99$; these are mathematical identities, not empirical discoveries). At the system level, $P = 1/\bar{A}$ (inverse mean shortest path on the unweighted graph), D = geometric mean of edge costs, and the two come from different instruments (binary adjacency vs. cost values)—ensuring non-tautology. At the agent level, P = alignment, which varies with the agent: greedy agents align at $P \approx 1.00$, random agents at $P \approx 0.003$ on the same graph.

4. Experimental Design

All experiments were conducted on the Deucalion supercomputer (FCT Project 2025.00020.AIVLAB.DEUCALION, MACC, Guimarães, Portugal), using NumPy 1.26 and SciPy 1.14 with base seed 2026. Results were reproduced with different seeds; Wilcoxon $p > 0.05$ between runs. All results are simulation-based. No empirical sensor data. All graphs are randomly generated (never tested on real networks). These qualifications apply to every number in this paper.

4.1. Experiment A: Graph Navigation Alignment (Main Experiment)

10,000 random graphs were generated with 15–150 nodes, edge density 0.04–0.30, and edge costs drawn uniformly from [1, 30]. After removing disconnected graphs and those with infinite s–t paths, 9,913 valid graphs remained. Six navigation strategies were applied to each graph:

(i) **Greedy:** always choose the cheapest-cost improving neighbour. (ii) **Boltzmann β = 2.0:** softmax selection with high inverse temperature (near-greedy). (iii) **Boltzmann β = 0.5:** moderate stochasticity. (iv) **Noisy greedy (noise = 0.3):** greedy with 30% random perturbation. (v) **Noisy greedy (noise = 0.6):** greedy with 60% random perturbation. (vi) **Random:** uniform random neighbour selection.

For each (graph, strategy) pair, efficiency was measured as the ratio of optimal path cost (Dijkstra) to actual path cost. Alignment was computed as the mean fraction of improving steps. Trials where the agent failed to reach the target were excluded. Total: 57,518 valid trials across 9,913 graphs. Runtime: 66 s.

Six counterfactual conditions were tested on separate graph samples (3,000 graphs each): CF-A1 (flat costs, all edges = 5.0), CF-A2 (extreme bimodal costs, 1 vs. 100), CF-A3 (paired same-graph comparison, greedy vs. random, n = 2,397 pairs), CF-A4 (graph size sweep, n = 20, 50, 100, 150). Runtime: 36 s.

4.2. Experiment B: FLRP Multi-Layer Decomposition

5,000 graphs were generated (same parameters as Exp. A) to test whether a multi-layer decomposition $F = F_F \times F_L$ (topology factor times cost-distinction factor) predicts efficiency better than alignment alone. Three agent types were applied (greedy, Boltzmann, random). Total: 13,732 valid trials. Two additional counterfactual conditions: CF-B1 (fixed topology, varying costs, n = 1,995) and CF-B2 (fixed costs = 5.0, varying topology, n = 1,989). Runtime: 60 s.

4.3. Experiment C: 2D Continuous Landscapes

1,000 random 2D potential landscapes were generated as sums of Gaussian potentials with varying centres, widths, and amplitudes. 80 agents per landscape followed gradient descent with additive noise: $dx = -\nabla D(x)dt + \sigma dW$, with noise levels $\sigma \in [0.01, 3.0]$. True vector alignment was computed as $\cos(\theta)$ between the agent's actual move direction and the negative gradient direction $-\nabla D$. Mediation analysis tested whether alignment mediates the P → efficiency pathway via partial correlation. Total: 79,956 valid trials. Runtime: 2,180 s.

4.4. Experiment D: PSO Alignment

5,000 PSO runs were conducted on 10-dimensional Rastrigin, Rosenbrock, and Sphere functions (~1,667 each). Parameters: swarm size 20–100 (uniform random), inertia weight 0.4–0.9, $c_1 = c_2 = 2.0$, 100–300 iterations. Alignment was computed as the fraction of iterations where the swarm's global best improved. Early alignment (first third of iterations) and late alignment (last third) were compared. Runtime: 8,550 s.

4.5. Experiment E: ACO Pheromone Alignment

3,000 ACO runs were conducted on random TSP instances with 20–50 cities and random Euclidean coordinates. Parameters varied: number of ants (10–50), iterations (50–200), α (pheromone exponent, 0.5–2.0), β (heuristic exponent, 1.0–5.0), ρ (evaporation rate, 0.1–0.5). Pheromone alignment measures the Spearman correlation between pheromone concentration on each edge and edge quality (inverse cost). Early alignment (first third of iterations) and late alignment (last third) were compared to test stigmergic learning. Total: 2,987 valid trials. Runtime: 1,262 s.

4.6. CEC-2017 Benchmark

10 functions from the CEC-2017 suite were evaluated in 10 dimensions: Sphere, Schwefel'2.22, Rosenbrock, Rastrigin, Ackley, Griewank, Levy, Zakharov, HappyCat, and Alpine. Five algorithms were tested: PSO ($\omega = 0.729$, $c_1 = c_2 = 1.49$), DE (F = 0.8, CR = 0.9, rand/1/bin), GWO, ACO_R (real-valued ACO), and Random Search. Population size: 40; maximum generations: 100; 5 independent runs per (algorithm, function) pair. Statistical analysis: Wilcoxon signed-rank tests per function, Friedman test across all functions. Runtime: 74 s.

4.7. Complementary Analyses

Three additional analyses were conducted on separate graph samples: (i) **D definitions compared:** five D formulations (D_global, D_path, D_source, D_target, D_path inverted) tested against P alone on 1,747 graphs. (ii) **P approximation:** six topological proxies for P_align tested on 2,456 graphs. (iii) **Vector forms:** six formulations including $P \times |\nabla D|$ tested on 300 continuous landscapes. (iv) **Realistic graph topologies:** Erdős–Rényi, Barabási–Albert, Watts–Strogatz, and geometric random graphs tested separately.

5. Results

All numbers in this section are traced to their source experiment. The notation [Exp-X, Run-Y] identifies the Deucalion job that produced each number. Throughout, R^2 denotes the coefficient of determination; ρ denotes Spearman's rank correlation. Significance levels: *** $p < 10^{-300}$, ** $p < 10^{-50}$, * $p < 10^{-10}$.

5.1. CEC-2017 Benchmark Performance

Table 2 reports algorithm performance across 10 CEC-2017 functions (10D, 5 runs each, pop = 40, 100 generations). PSO dominated, winning 9 of 10 functions. ACO_R won Rosenbrock (mean =

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1.37e+03 vs. PSO's 2.00e+05). GWO consistently ranked last across all functions. The Friedman test was highly significant ($\chi^2 = 36.40$, $p < 10^{-6}$).

Table 2. CEC-2017 benchmark summary (10D, 5 runs, pop 40, 100 gen). Source: AFI-SWEVO run, 2026-03-10. Friedman $\chi^2 = 36.40$, $p < 10^{-6}$. PSO's only loss is Rosenbrock (rank 2, beaten by ACO_R).

Algorithm	Wins	Friedman Rank	Best Fn	Mean (best)	Worst Fn	Mean (worst)
PSO	9/10	1.10	Sphere	3.59e−02	Rosenbrock	2.00e+05
ACO_R	1/10	2.20	Rosenbrock	1.37e+03	Schwefel	2.22e+03
DE	0/10	2.80	Sphere	3.63e+02	Rosenbrock	3.72e+06
Random	0/10	3.90	HappyCat	8.00e−01	Rosenbrock	2.90e+08
GWO	0/10	5.00	HappyCat	2.92e+00	Rosenbrock	8.99e+10

The observer-dependent P metric (Section 3.2) was computed per (algorithm, function) pair as the coefficient of variation of final fitness across 5 runs: P_obs = σ(fitness)/μ(fitness). PSO achieved the highest mean P_obs (0.782), GWO the lowest (0.000), consistent with PSO's superior differentiation capacity. Per-function P values are reported in Table 3.

Table 3. Observer-dependent P per (algorithm, function) pair. P = CV of final fitness across 5 runs. Source: AFI-SWEVO run Part 3. The same function yields different P for different algorithms. GWO achieves P = 0.000 on all functions (zero variance across runs = zero differentiation). PSO achieves P > 0.50 on 9/10 functions.

Function	PSO	DE	GWO	ACO_R	Random	Type
Sphere	0.964	0.102	0.000	0.180	0.049	Unimodal
Schwefel	0.789	0.005	0.000	0.003	0.024	Unimodal
Rosenbrock	0.900	0.158	0.000	0.157	0.143	Unimodal
Rastrigin	0.739	0.010	0.000	0.010	0.025	Multimodal
Ackley	0.955	0.032	0.000	0.072	0.003	Multimodal
Griewank	0.508	0.077	0.000	0.005	0.048	Multimodal
Levy	0.920	0.051	0.000	0.027	0.071	Multimodal
Zakharov	0.913	0.047	0.000	0.035	0.455	Unimodal
HappyCat	0.385	0.004	0.000	0.003	0.021	Multimodal
Alpine	0.850	0.007	0.000	0.019	0.033	Multimodal

5.2. Alignment as Dominant Predictor (Exp. A: Main Result)

Across all 57,518 valid trials (9,913 graphs × 6 strategies), alignment was the dominant predictor of navigation efficiency. Table 4 reports the full comparison of 13 predictors.

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Table 4. Predictors of navigation efficiency across all strategies (Exp. A, n = 57,518). Source: afi_align_flrp run, Deucalion. Alignment dominates at $R^2 = 0.81–0.82$. All alternatives $R^2 \leq 0.03$. The FLRP multiplicative product achieves $R^2 = 0.001$.

Predictor	R^2	Spearman ρ	p-value	n	Sig.
mean_alignment	0.8103	0.8502	$< 10^{-300}$	57,518	***
frac_improving	0.8195	0.8313	$< 10^{-300}$	57,518	***
F_FLRP (F_F × F_L)	0.0014	−0.1182	6.6e−178	57,518	
F_F (topology)	0.0247	−0.2402	$< 10^{-300}$	57,518	*
F_L (distinction)	0.0206	0.1688	$< 10^{-300}$	57,518	*
P_local (source CV)	0.0057	0.1231	4.7e−193	57,518	
n_options (degree)	0.0259	−0.2047	$< 10^{-300}$	57,518	*
1/geo_mean_cost	0.0000	−0.0225	6.5e−8	57,518	
cv_cost	0.0009	−0.0385	2.8e−20	57,518	
1/d_st (inv. dist)	0.0006	−0.0405	2.4e−22	57,518	
mean_degree	0.0285	−0.2173	$< 10^{-300}$	57,518	*
$\log_2$(n_options)	0.0302	−0.2045	$< 10^{-300}$	57,518	*
random baseline	0.0000	0.0014	0.737	57,518	

The result is unambiguous: alignment ($R^2 = 0.82$) outperforms every graph-theoretic metric by more than an order of magnitude. Excluding greedy (whose alignment is trivially 1.0), the result strengthens: $R^2 = 0.83$ ($\rho = 0.888$, n = 47,605). Within the random strategy alone—the purest test, as the agent contributes no intelligence—alignment still dominates at $R^2 = 0.80$ ($\rho = 0.638$, n = 7,959), meaning that the graph's structural alignment explains 80% of random-walk efficiency variance.

5.3. D is Multiplicative (C1)

The F = P/D decomposition requires D to be well-specified. An ablation study across 250 CEC benchmark trials (10 functions × 5 algorithms × 5 runs) tested multiple functional forms:

Table 5. Ablation of F-formulations on CEC benchmark data (n = 250 trials). Source: AFI-SWEVO run Part 5. P alone ($R^2 = 0.964$) outperforms P/D ($R^2 = 0.818$). The honest result: D does not improve prediction beyond P in the CEC domain. However, P − D (additive) fails completely ($R^2 = 0.079$), confirming that the ratio form is structurally correct when D matters.

Model	R^2	ρ	Source	n	Interpretation
P alone	0.964	0.986	SWEVO Part 5	250	Best fit
P/D (canonical)	0.818	0.983	SWEVO Part 5	250	Ratio form
P × D	0.856	0.973	SWEVO Part 5	250	Product form
sqrt(P/D)	0.925	0.983	SWEVO Part 5	250	Square root
log(P/D)	0.368	0.983	SWEVO Part 5	250	Log transform
P − D	0.079	0.000	SWEVO Part 5	250	Additive
1/D alone	0.066	−0.458	SWEVO Part 5	250	D only
random	0.002	−0.053	SWEVO Part 5	250	Null

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For the D composition model specifically, the geometric form outperforms the additive form: R^2(geometric) = 0.993 vs. R^2(additive) = 0.856 (SWEVO run Part 5, n = 250). This is replicated in the AFI_ULTRA run: R^2(geometric) = 0.988 vs. R^2(additive) = 0.905 (n = 10,000). The geometric model is unambiguously superior across both samples.

5.4. Observer-Dependence (C2)

The per-strategy breakdown (Table 6) demonstrates observer-dependence: the same graphs, navigated by six different strategies, yield six different alignment values and six different efficiency levels.

Table 6. Per-strategy results (Exp. A). Source: afi_align_flrp run. Alignment's R^2 increases as agent intelligence decreases: from 0.00 (greedy, trivially aligned) to 0.80 (random, alignment varies maximally). FLRP never exceeds $R^2 = 0.02$. The same 9,913 graphs yield mean alignment ranging from 1.0000 (greedy) to 0.0032 (random).

Strategy	n	Mean eff.	Mean align.	R^2(align)	R^2(FLRP)	R^2(P_loc)	Source
Greedy	9,913	0.8991	1.0000	0.0000	0.0152	0.0205	Exp-A
Boltzmann β=2	9,913	0.8966	0.9979	0.0494	0.0167	0.0218	Exp-A
Boltzmann β=0.5	9,913	0.8568	0.9441	0.3977	0.0136	0.0223	Exp-A
Noisy 0.3	9,913	0.6756	0.6979	0.6645	0.0023	0.0151	Exp-A
Noisy 0.6	9,907	0.4311	0.3690	0.7287	0.0001	0.0160	Exp-A
Random	7,959	0.1184	0.0032	0.8039	0.0031	0.0272	Exp-A

Hidden variable test. Can P_agent be reconstructed from landscape properties and algorithm identity? In graph navigation (AFI_ULTRA run, 2,000 graphs, 5 strategies, n = 9,470 pairs): a linear model using graph size, mean degree, D, strategy identity, and an intercept achieves $R^2 = 0.058$. The observer effect is irreducible in this domain. In the CEC benchmark (SWEVO run Part 3, 10 functions × 5 algorithms = 50 pairs): the same model using function type and algorithm identity achieves $R^2 = 0.403$. The discrepancy is informative: in the CEC domain, function type partially predicts algorithm-specific P (e.g., GWO always gets P = 0.000), making the observer effect partially classical. In the graph domain, graph structure does *not* predict agent alignment, making the observer effect strongly non-classical.

Scale invariance. Multiplying all edge costs by $k \in [0.01, 10{,}000]$ leaves alignment unchanged: P = 0.531 ± 0.017 (SWEVO run Part 6, 472 graphs, 3 values of k). Replicated in AFI_ULTRA: P = 0.537 ± 0.000 across k = 0.001 to 10,000, 7 orders of magnitude, 300 graphs. Alignment responds to ratios of costs, not magnitudes.

P–D independence. ρ(P_structural, D) = −0.060 (SWEVO run Part 6, 472 graphs). ρ(D, avg_degree) = 0.036 (AFI_ULTRA, 2,000 graphs). P and D are measured by different instruments and are empirically nearly uncorrelated.

P approximates 1/avg_degree. Structural alignment correlates strongly with inverse mean degree: ρ = 0.807, $R^2 = 0.719$ (Priority B run, 2,456 graphs). This provides a topological proxy that avoids the circularity of computing shortest paths.

5.5. Cross-Domain Validation

5.5.1. 2D Continuous Landscapes (Exp. C)

In continuous space (79,956 trials across 1,000 landscapes), alignment (cos θ between move direction and negative gradient) predicts efficiency weakly: $R^2 = 0.029$, ρ = 0.229. The dominant predictor is gradient contrast (field CV): $R^2 = 0.061$, ρ = 0.365. The combined metric align × D_contrast achieves $R^2 = 0.064$, ρ = 0.383. The scalar P/D achieves only $R^2 = 0.002$, ρ = 0.274.

Mediation analysis. Testing whether alignment mediates the P → efficiency pathway: ρ(P, eff) = 0.202 (total effect); ρ(P, align) = 0.945 (P strongly determines alignment); ρ(P, eff | align) = 0.051 (partial, controlling for alignment). Reduction: 74.7%. This confirms that alignment is the causal pathway through which P affects efficiency.

Noise sweep. As noise increases from 0.01 to 3.0, mean alignment decreases monotonically: 0.998 → 0.253 (7 bins, Table 7). Mean mediation reduction remains stable at 83–90%, confirming that the mediation pathway holds across the full noise range.

Table 7. Noise sweep in 2D continuous landscapes (Exp. C). Alignment degrades monotonically with noise. Mediation reduction remains 83–90% throughout, confirming alignment as the stable causal pathway. Source: afi_align_flrp run.

Noise range	n	Mean alignment	Mean reduction (%)	Mean eff.	Interpretation	Source
[0.01, 0.10)	2,479	0.998	89.9	High	Near-deterministic	Exp-C
[0.10, 0.30)	5,337	0.977	90.5	High	Low noise	Exp-C
[0.30, 0.50)	5,194	0.903	90.3	High	Moderate noise	Exp-C
[0.50, 1.00)	13,398	0.693	89.7	Moderate	Balanced	Exp-C
[1.00, 1.50)	13,346	0.472	88.4	Moderate	High noise	Exp-C
[1.50, 2.00)	13,262	0.350	86.5	Low	Very high noise	Exp-C
[2.00, 3.00)	26,940	0.253	83.0	Low	Near-random	Exp-C

5.5.2. PSO (Exp. D)

Across 5,000 PSO runs (10D, 3 functions), the best predictor is early alignment (first third of iterations): $R^2 = 0.078$, $\rho = -0.174$. Inertia weight has a strong effect: $R^2 = 0.031$, $\rho = -0.291$. Per-function analysis: Rastrigin early alignment $R^2 = 0.145$ (n = 1,667), Sphere $R^2 = 0.120$ (n = 1,666), Rosenbrock $R^2 = 0.029$ (n = 1,667). The negative sign of ρ(alignment, fitness) is expected: higher alignment means more improving iterations, which yields lower (better) final fitness. The R^2 values are modest compared to graph navigation, reflecting that continuous multimodal landscapes have richer dynamics than discrete graphs.

5.5.3. ACO (Exp. E)

Across 2,987 ACO runs, pheromone alignment predicts tour quality: late alignment $R^2 = 0.153$, $\rho = 0.434$; early alignment $R^2 = 0.134$, $\rho = 0.411$. ACO-specific parameters also predict: β (heuristic exponent) $R^2 = 0.072$, $\rho = -0.268$; α (pheromone exponent) $R^2 = 0.060$, $\rho = -0.244$.

Stigmergic learning confirmed. Mean(late_align − early_align) = 0.049 ± 0.078. In 87.0% of runs, late alignment exceeds early alignment. The colony's pheromone field increasingly aligns with the problem structure over iterations. This is a direct computational demonstration of stigmergic learning as alignment improvement.

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5.6. Counterfactual Results

Table 8. Counterfactual tests. All six pass their predicted conditions. Source: afi_align_flrp run. CF-A4 shows alignment's R^2 actually improves with graph size (0.86 at n≈20 to 0.97 at n≈150).

ID	Test (3,000 graphs each)	Prediction	Result	Key metric	Source
CF-A1	Flat costs (all edges = 5.0)	Alignment still predicts; F_L ≈ 1	R^2 = 1.000	Random: R^2 = 1.000	Exp-A
CF-A2	Bimodal costs (1 vs. 100)	Alignment predicts strongly	R^2 = 0.439	ρ = 0.787	Exp-A
CF-A3	Paired same-graph (greedy vs. random)	Δalign ↔ Δeff concordant > 80%	94.5%	n = 2,397 pairs	Exp-A
CF-A4	Graph size sweep (20–150 nodes)	Alignment holds at scale	R^2: 0.86–0.97	Improves with N	Exp-A
CF-B1	Fixed topology, varying costs	F_L predicts, F_F does not	PASS	ρ(align)= 0.641	Exp-B
CF-B2	Fixed costs, varying topology	F_F predicts, F_L ≈ 1	PASS	ρ(align)= 0.819	Exp-B

5.7. What Failed

Honesty requires reporting failures with the same depth as successes. Table 9 summarises all refuted claims.

Table 9. Refuted and mixed results. The FLRP multiplicative decomposition is dead (R^2 = 0.0002). P alone outperforms P/D at the graph level. The scalar F-field fails in continuous space. In realistic topologies, P/D outperforms P on BA and WS graphs but loses on ER graphs. All sources: Deucalion runs.

Claim	R^2/Result	n	Source	Implication	Status
FLRP multiplicative (F_F × F_L)	0.0002	13,732	Exp-B	Layers independent	DEAD
P alone < P/D at graph level	ρ(P)=0.343 > ρ(P/D)=0.108	1,747	Priority D	P alone wins	REFUTED
D_local fixes P > P/D	R^2 = 0.000	1,747	Priority A	Dead end	DEAD
Scalar F-field in continuous space	R^2 = 0.004	300	Priority C	Scalar loses info	DEAD
Vector P*|gradD|	R^2 = 0.291	300	Priority C	Partial recovery	ALIVE
P/D outperforms P in ER graphs	ρ(P)=−0.016	335	Priority E	P wins in ER	MIXED
P/D outperforms P in BA graphs	R^2(P/D)= 0.037	123	Priority E	P/D wins in BA	MIXED

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6. Discussion

6.1. Observer-Dependence: What It Means for Swarm Intelligence

The central finding is that navigability is observer-dependent. This is demonstrated quantitatively: six strategies on the same 9,913 graphs yield mean alignments from 1.000 (greedy) to 0.003 (random), and a hidden variable model achieves only $R^2 = 0.058$ in reconstructing agent-level P from graph properties. The landscape does not determine navigability; the (landscape, algorithm) pair does.

This complements the No Free Lunch theorems, which establish that no algorithm dominates universally, by providing a structural explanation for *why* specific algorithms succeed on specific landscapes: they achieve higher alignment. It also suggests a refinement of fitness landscape analysis: FLA features should ideally be conditioned on the navigator class, not computed in isolation.

6.2. The Intelligence Paradox

For greedy agents, more topological freedom *hurts*. Mean degree has a negative Spearman correlation with alignment: $\rho = -0.494$ within the random strategy (Exp. A4). The mechanism is that more neighbours means each neighbour is less likely to be improving—the greedy heuristic degrades in dense environments. This is captured by the proxy $P \approx 1/\bar{a}$: as mean degree increases, structural alignment decreases. Sparse graphs are counterintuitively easier for greedy navigators.

6.3. The Boundary: Where P/D Adds Value vs. Where P Alone Suffices

A critical honesty point: P alone outperforms P/D at the graph level ($\rho = 0.343$ vs. 0.108 on 1,747 graphs). In the CEC ablation, P alone achieves $R^2 = 0.964$ vs. P/D's 0.818. This means D does not improve prediction when the navigator can reroute around high-D regions.

However, there are domains where D matters. In Barabási–Albert graphs, P/D outperforms P ($R^2 = 0.037$ vs. 0.003). In Watts–Strogatz graphs, P/D outperforms P ($R^2 = 0.004$ vs. 0.001). In geometric random graphs, P/D outperforms P ($R^2 = 0.024$ vs. 0.014). In constrained topologies—where rerouting is limited—D begins to contribute. This defines the boundary condition: $F = P/D$ is most informative when topology constrains navigation, making D unavoidable.

6.4. Limitations

First, all results are simulation-based. Zero empirical data. Zero real-world networks (transportation, social, biological). All graphs are randomly generated. The framework has never been tested outside synthetic environments.

Second, circularity. Computing P_align requires all-pairs shortest paths ($O(N^3)$). P predicts the answer to a problem it already solved. The partial resolution—$P \approx 1/\bar{a}$ ($\rho = 0.807$)—avoids this but is an approximation, not exact.

Third, no novel prediction. $R^2 = 0.003$ for predictions beyond standard graph metrics (AFI_GAPS G8). The current contribution is decomposition and interpretation, not prediction.

Fourth, cross-domain R^2 varies substantially. Graph navigation: 0.82. 2D continuous: 0.029. PSO: 0.078. ACO: 0.153. The framework is strong in discrete graphs and weaker in continuous and swarm-specific domains. The continuous case may require the vector form $P \times |\nabla D|$ ($R^2 = 0.291$) rather than the scalar ratio.

Fifth, the exponents are not unity. Fitted exponents $a \approx 1.29$, $b \approx -1.09$ (Melo de Magalhães, 2026a), with 95% CI excluding 1.0. The law is closer to $(P/D)^{1.19}$ than P/D. This may be the real finding.

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6.5. Falsification Criteria

Table 10. Falsification criteria and results. Would-falsify conditions: hidden variable $R^2 > 0.50$ (P is classical); P not scale-invariant (P is magnitude, not information); additive D outperforms multiplicative in ≥ 7 domains; complementarity reversed ($\rho > 0$, more paths always helps).

ID	Criterion	Threshold	Measured	Status	Source
F2	P/D proportionality ≥ 3 domains	$R^2 > 0.80$	0.99, 0.94, 0.82	PASS	Table 1
F4	Observer dependence irreducible	R^2(hidden) < 0.10	0.058	PASS	AFI_ULTRA
F7	Scale invariance of P	P ± 0.05 across 7 orders	0.537 ± 0.000	PASS	AFI_ULTRA
F8	Alignment dominates	R^2 > all alternatives	0.82 vs. max 0.03	PASS	Exp-A
F9	D multiplicative > additive	R^2(geo) > R^2(add)	0.993 vs. 0.856	PASS	SWEVO Part 5
F11	P–D independence	$\rho < 0.15$	−0.060 to 0.036	PASS	Multiple
F12	Null model $R^2 \approx 0$	$R^2 < 0.01$	0.000	PASS	All expts

7. Conclusions

This paper advanced three claims about the structural basis of swarm algorithm performance, tested across five experiments totalling over 200,000 simulated trials on the Deucalion supercomputer.

Claim 1 (D is multiplicative) is confirmed: $R^2 = 0.993$ (geometric) vs. 0.856 (additive), replicated across two independent samples. Independent resistance factors compound multiplicatively.

Claim 2 (P is observer-dependent) is confirmed in graph navigation (hidden variable $R^2 = 0.058$) and partially confirmed in CEC optimisation ($R^2 = 0.403$). The observer effect is domain-dependent: strongly irreducible in graphs, partially classical in benchmark functions. Six strategies on the same graphs yield alignment values from 1.000 to 0.003.

Claim 3 (Alignment dominates) is confirmed in graph navigation at $R^2 = 0.82$ (57,518 trials), with all graph-theoretic alternatives below $R^2 = 0.03$. Cross-domain results are mixed: 2D continuous $R^2 = 0.029$ (but mediation confirmed at 74.7%), PSO early alignment $R^2 = 0.078$, ACO late alignment $R^2 = 0.153$ with stigmergic learning confirmed (87% of runs show improvement).

The framework also reveals what fails: FLRP as a multiplicative product is dead ($R^2 = 0.0002$). P alone outperforms P/D at the graph level. The scalar F-field fails in continuous space. The exponents are not unity. No novel prediction beyond graph theory has been demonstrated.

For swarm intelligence practice, the findings suggest that algorithm selection could be informed by structural alignment—a pure graph property estimable without running the algorithm—rather than post-hoc landscape analysis alone. The Intelligence Paradox (more topological freedom hurts low-intelligence agents) has design implications for swarm topologies. The framework invites falsification and is offered as a hypothesis under test, not a proven law. All results are simulation-based. Reproducibility is ensured through fixed seeds and public availability of run outputs.

Data Availability Statement: All calculations are reproducible from the equations and parameters stated in the text. Simulation code and Deucalion run outputs are available from the author upon request. Seed: 2026.

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