Empowerment of Knowledge Rediscovery in the Cultivation of Top Mathematical Talents: Construction and Practice of the Undergraduate-Master-Doctoral Integrated Training Model

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Article

Empowerment of Knowledge Rediscovery in the Cultivation of Top Mathematical Talents: Construction and Practice of the Undergraduate-Master-Doctoral Integrated Training Model

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Abstract

As the core pillar of basic disciplines, the cultivation of top innovative mathematical talents is a key measure to address the country's core technological bottlenecks and support technological self-reliance and self-improvement. In the traditional mathematical talent training model, there are significant connection barriers between undergraduate and postgraduate stages. Problems such as fragmented scientific research training and delayed stimulation of innovative potential seriously restrict the growth of high-level top mathematical talents. Taking the major of Mathematics and Applied Mathematics in Qian Weichang College of Shanghai University as the practice carrier, and in conjunction with the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, this study takes the "knowledge rediscovery" concept as the core educational guidance, and systematically constructs and practices the "3+1+X" undergraduate-master-doctoral integrated training model. This model breaks the traditional linear teaching logic of "definition-theorem-proof-example", guides students to re-experience the generation process of mathematical knowledge as "quasi-researchers", and realizes the seamless connection of knowledge learning, scientific research training and thinking improvement between undergraduate and postgraduate stages through the whole-cycle training link of "foundation consolidation-scientific research connection-flexible further study". After 6 years of practical testing (2019-2025), the model has trained a total of 74 students, with an further study rate of 81%. Students' scientific research capabilities and academic achievements have been significantly improved, and the training model has formed a good demonstration effect of on-campus promotion and off-campus radiation. This paper systematically explains the inherent compatibility between the "knowledge rediscovery" concept and the undergraduate-master-doctoral integrated cultivation of top mathematical talents, details the core design and key implementation measures of the "3+1+X" training model, summarizes the practical effects and core experiences, and puts forward the direction of future in-depth development, providing a replicable and promotable practical paradigm for the integrated cultivation of top innovative talents in mathematics and other basic disciplines in China.

Keywords: knowledge rediscovery; mathematics education; cultivation of top talents; undergraduate-master-doctoral integration; 3+1+X model

1. Introduction

Mathematics is the cornerstone of natural sciences and the core support for major scientific and technological innovations, playing an irreplaceable key role in national strategic fields such as chip research and development, aerospace, artificial intelligence, and biomedicine. The quality of cultivating high-level top innovative mathematical talents is directly related to the improvement of national original innovation capabilities and the construction of core advantages in technological competition, and is also an important talent guarantee for realizing technological self-reliance and self-improvement. Currently, the cultivation of mathematical talents in China's higher education still

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faces many urgent problems to be solved. The traditional training model adopts a segmented approach of "undergraduate focusing on theoretical teaching and postgraduate focusing on scientific research innovation", and there are obvious connection barriers between undergraduate and postgraduate stages, which has become a key bottleneck restricting the growth of top mathematical talents.

Under the traditional training model, mathematics teaching at the undergraduate stage mostly follows the linear indoctrination logic of "definition-theorem-proof-example". Students mainly passively receive knowledge, lacking exploration and in-depth thinking about the generation process of mathematical knowledge. Scientific research training is fragmented and lacks systematicness, making it difficult to form complete scientific research thinking and innovative awareness. However, the postgraduate stage requires students to quickly adapt to the scientific research paradigm and carry out independent research, leading most students to spend a lot of time completing the identity transformation from "knowledge recipients" to "scientific research innovators". The talent growth cycle is prolonged, and the stimulation of innovative potential is delayed, which is difficult to meet the country's urgent demand for high-level top mathematical talents.

To solve the problem of segmentation between academic stages in the cultivation of top mathematical talents, the key lies in building a seamlessly connected training system between undergraduate and postgraduate stages, advancing the cultivation of scientific research thinking and academic methods, and realizing the continuity and progression of knowledge learning and scientific research training. As an educational concept compatible with the characteristics of mathematics, "knowledge rediscovery" aims to guide students to break away from mechanical knowledge memory and re-experience the generation process of mathematical knowledge as "quasi-researchers"—starting from the germination of specific mathematical problems, through historical inspiration, trial-and-error exploration, logical argumentation to law construction, and finally realizing the thinking leap from "passive knowledge reception" to "active knowledge creation". This concept is highly consistent with the training needs of top innovative mathematical talents, providing a new educational perspective and practical path for the undergraduate-master-doctoral integrated cultivation of mathematics, which can effectively break the connection barriers between academic stages, strengthen scientific research empowerment, and promote the rapid growth of top talents.

Based on this, Qian Weichang College of Shanghai University, in conjunction with the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, relying on the advantages of the national pilot college platform and national academic resources, integrated the "knowledge rediscovery" concept into the entire training process, and officially launched the "3+1+X" undergraduate-master-doctoral integrated training practice for the major of Mathematics and Applied Mathematics in 2019. After 6 years of exploration and improvement, it has constructed a whole-cycle training link of "foundation consolidation-scientific research connection-independent further study", forming a complete training system with dual-subject collaboration, integration of courses and scientific research, and digital intelligence quality monitoring, and has achieved remarkable results in breaking academic stage barriers, strengthening scientific research empowerment, and cultivating top talents. Combining 6 years of practical experience, this paper systematically explains the practical path of undergraduate-master-doctoral integrated cultivation of mathematics empowered by the knowledge rediscovery concept, providing reference for the reform of top innovative talent cultivation in basic disciplines in China.

2. Inherent Compatibility between the Knowledge Rediscovery Concept and the Undergraduate-Master-Doctoral Integrated Cultivation of Top Mathematical Talents

2.1. Core Connotation of the Knowledge Rediscovery Concept

Derived from the essential requirements of mathematics education, the core of the knowledge rediscovery concept is not to let students repeatedly discover existing mathematical knowledge, but to guide them to stand in the perspective of knowledge creators, experience the exploration process

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consistent with the generation of mathematical knowledge, and achieve in-depth understanding, active construction and innovative application of mathematical knowledge. In the field of mathematics education, the core connotation of knowledge rediscovery is mainly reflected in three aspects.

Firstly, at the level of teaching logic, it breaks the linear logic of "definition first, theorem follow-up, proof conclusion". Driven by specific mathematical problems, students develop the demand for knowledge in the process of problem exploration, and fully experience the complete exploration process of "problem proposal-attempted solution-trial-and-error reflection-logical argumentation-law refinement", so as to truly understand the essence and internal logic of mathematical knowledge. Secondly, at the level of learning subject, it highlights students' dominant position in exploration, transforming teachers' "teaching" into "guidance". As organizers and guides of knowledge exploration, teachers provide exploration materials and build exploration platforms for students instead of directly giving established conclusions, so as to fully mobilize students' enthusiasm and initiative in exploration. Thirdly, at the level of ability cultivation, it balances the mastery of mathematical knowledge and the cultivation of scientific research thinking. In the process of knowledge rediscovery, students' core abilities such as logical reasoning, abstract modeling, trial-and-error exploration and academic expression are simultaneously trained, laying a solid thinking foundation for scientific research innovation and realizing the synchronous advancement of "knowledge learning" and "ability improvement".

The knowledge rediscovery concept is highly compatible with the abstract, logical and systematic characteristics of mathematics. The formation of mathematical knowledge itself is a process of continuous exploration, demonstration and improvement. From Euclidean geometry to non-Euclidean geometry, from the embryonic form of calculus to the rigorous limit theory, the birth of every mathematical concept and theorem originates from the drive of specific problems and continuous exploration and demonstration. Guiding students to re-experience this process can not only enable them to have a deeper understanding of the essence and internal logic of mathematical knowledge, but also help them form a thinking mode consistent with the laws of mathematics in exploration, which is exactly the core requirement for cultivating top innovative mathematical talents and the core foundation for the knowledge rediscovery concept to empower the cultivation of top mathematical talents [1].

2.2. Theoretical Basis of the Knowledge Rediscovery Concept

The "knowledge rediscovery" concept discussed in this paper was first systematically proposed by the author combining the characteristics of mathematics and the laws of higher education talent training. Its formation is not isolated, but gradually improved on the basis of drawing on relevant domestic and foreign educational theories, absorbing the development laws of mathematics, and combining the actual needs of cultivating top mathematical talents in China, forming a theoretical support system with distinct pertinence and practical guidance, which provides a solid theoretical basis for the construction and practice of the subsequent undergraduate-master-doctoral integrated training model [1].

From the perspective of educational theory, the knowledge rediscovery concept mainly draws on the core ideas of constructivist learning theory and discovery learning theory. Constructivist learning theory emphasizes that learning is a process in which students actively construct knowledge rather than passively receive external information. Through their own exploration, experience and reflection, students gradually form a personalized understanding of knowledge, which is highly consistent with the core demand of the knowledge rediscovery concept of "guiding students to actively explore and construct mathematical knowledge". Discovery learning theory advocates that education should guide students to actively participate in the process of knowledge discovery, master knowledge and improve abilities in exploration, which provides important reference for the practical path of "re-experiencing the knowledge generation process and cultivating scientific research thinking" in the knowledge rediscovery concept. At the same time, this concept deeply absorbs the core ideas of educational mathematics theory proposed by Academician Zhang Jingzhong. In "From Mathematics Education to Educational Mathematics", Academician Zhang Jingzhong clearly puts forward that the core of educational mathematics is to "transform mathematics to make it more suitable

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for teaching and learning", focusing on combining the logical structure of mathematical knowledge with students' cognitive laws, and making abstract mathematical knowledge easier to understand and explore by simplifying and optimizing the presentation of mathematical knowledge [3]. This core idea is highly consistent with the knowledge rediscovery concept, providing important theoretical support for it, making the exploration process of mathematical knowledge more in line with students' learning characteristics, effectively reducing the difficulty of exploration, improving the effectiveness of knowledge exploration, and making the knowledge rediscovery concept more practically operable.

From the perspective of mathematical discipline theory, the logic, systematicness and exploration of mathematical knowledge are the core disciplinary supports of the knowledge rediscovery concept. The development of mathematics itself is a process of continuous exploration, discovery and improvement. The formation of every mathematical concept, theorem and formula has gone through a complete process from problem proposal, trial-and-error exploration to logical argumentation and law refinement. This disciplinary characteristic determines that mathematics education cannot be limited to mechanical knowledge transmission, but should guide students to re-experience this exploration process. Based on this essential characteristic of mathematics, the knowledge rediscovery concept combines the generation laws of mathematical knowledge with the laws of talent training, emphasizing that by guiding students to simulate the process of knowledge discovery, they can deepen their understanding of the essence of mathematical knowledge and cultivate scientific research thinking and innovative abilities that meet the development needs of mathematics [1].

From the perspective of top talent training theory, the core trait of high-level top innovative talents is their ability to conduct independent exploration and independent innovation, and the cultivation of this ability needs to run through the entire process of talent training. In the traditional top talent training model, the cultivation of scientific research thinking and innovative abilities is mostly concentrated in the postgraduate stage, resulting in a long talent growth cycle and delayed stimulation of innovative potential. Based on the core needs of top talent training, the knowledge rediscovery concept advances the cultivation of scientific research thinking to the undergraduate stage, constructing a whole-cycle training link of "basic exploration-scientific research transition-independent innovation", which is in line with the "step-by-step and continuous growth" training law of top talents, provides theoretical support for the seamless connection between undergraduate and postgraduate stages, and also provides a new theoretical perspective for solving the pain points of the traditional training model [4].

2.3. Compatibility with the Undergraduate-Master-Doctoral Integrated Cultivation of Top Mathematical Talents

The core goal of the undergraduate-master-doctoral integrated cultivation of top mathematical talents is to break the barriers between academic stages, realize the continuity and progression of training between undergraduate and postgraduate stages, and cultivate high-level top talents with solid mathematical foundation, systematic scientific research thinking and strong innovative ability. This is highly consistent with the educational demands of the knowledge rediscovery concept, and their compatibility is mainly reflected in three aspects.

Firstly, it meets the "thinking continuity" requirement of integrated cultivation. The undergraduate-master-doctoral integrated cultivation of mathematics is not a simple superposition of academic stages, but a continuous progression of thinking modes. From the exploration of mathematical knowledge at the undergraduate stage to the scientific research innovation of mathematics at the postgraduate stage, the core is the cultivation and deepening of scientific research thinking. The knowledge rediscovery concept guides students to carry out inquiry-based learning as "quasi-researchers" from the undergraduate stage, advancing the cultivation of scientific research thinking, so that students form the scientific research thinking habit of "problem-driven, logical argumentation and trial-and-error reflection" at the undergraduate stage. After entering the postgraduate stage, they can quickly adapt to the scientific research paradigm, realizing a smooth thinking transition from "basic exploration" to "high-level innovation", and effectively solving the problem of thinking connection disruption in traditional training.

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Secondly, it meets the "scientific research progression" requirement of integrated cultivation. Mathematical scientific research innovation is not a castle in the air, but is built on a solid knowledge foundation and systematic scientific research training. The training process guided by the knowledge rediscovery concept is a process of continuous progression of scientific research training: in the first 3 years of undergraduate study, basic scientific research enlightenment is carried out through exploration modules in courses, enabling students to master basic exploration methods and argumentation logic; in the 4th year of undergraduate study, scientific research connection is realized by participating in real scientific research projects, allowing students to move from classroom exploration to actual scientific research; in the postgraduate stage, independent scientific research innovation is carried out on the basis of previous training, forming a complete scientific research training link of "basic enlightenment-scientific research transition-independent innovation", which is in line with the growth law of top innovative talents and realizes the gradual improvement of scientific research ability.

Thirdly, it meets the "talent individuality" requirement of integrated cultivation. The growth of top innovative mathematical talents has significant individual differences. Different students have different academic interests, research potential and career plans, and integrated cultivation needs to take into account the personalized development of talents. The knowledge rediscovery concept emphasizes student-centeredness, respects students' dominant position in exploration, and encourages them to carry out personalized exploration according to their own interests; the integrated training model designed based on this concept can provide students with personalized training programs and diverse further study paths, meeting the academic development and career planning needs of different students, realizing the top training goal of "teaching students in accordance with their aptitude", and allowing each student's research potential to be fully exerted.

3. Core Design of the "3+1+X" Undergraduate-Master-Doctoral Integrated Training Model Empowered by Knowledge Rediscovery

Taking the major of Mathematics and Applied Mathematics in Qian Weichang College of Shanghai University as the practice carrier, and in conjunction with the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, the "3+1+X" undergraduate-master-doctoral integrated training model is systematically constructed by integrating the "knowledge rediscovery" concept into the entire training process. Among them, "3" refers to the first 3 years of undergraduate study, focusing on foundation consolidation and thinking cultivation; "1" refers to the 4th year of undergraduate study, serving as a bridge period for academic stage connection, focusing on scientific research connection and transition; "X" refers to the flexible further study stage, providing three diverse further study paths (combined master-doctoral program, direct doctoral program, and master's program) according to students' academic performance and career plans. The entire training model takes the knowledge rediscovery concept as the core and dual-subject collaboration as the support, realizing the seamless connection of knowledge learning, scientific research training and thinking improvement between undergraduate and postgraduate stages, and building a whole-cycle training link for top mathematical talents of "foundation consolidation-scientific research connection-independent further study", giving full play to the empowering role of the knowledge rediscovery concept in cultivating top talents.

3.1. "3": First 3 Years of Undergraduate Study—Foundation Consolidation and Thinking Cultivation

The first 3 years of undergraduate study are a critical stage for building mathematical foundation and enlightening scientific research thinking. Relying on the high-quality teachers and teaching resources of Qian Weichang College, and supported by the "knowledge rediscovery" concept of the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, the mathematics teaching and training path is reconstructed, breaking the traditional linear indoctrination teaching model, and fully integrating the exploration logic of knowledge rediscovery into the entire process of curriculum teaching and scientific research enlightenment, realizing the dual goals of

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"foundation consolidation + thinking cultivation" and laying a solid foundation for the growth of top talents.

In the design of core curriculum modules, guided by knowledge rediscovery, the core mathematics courses are systematically reconstructed and optimized, and national first-class courses such as Linear Algebra and Mathematical Analysis are offered, with the exploration module of "theorem tracing-independent derivation-application expansion" deeply embedded in the entire curriculum teaching process. For example, around the core knowledge point of "matrix diagonalization", a full-process exploration task of "problem proposal-attempted derivation-counterexample verification-law refinement" is designed: first, driven by the specific problem of "how to simplify matrix operations", students' demand for knowledge of matrix diagonalization is stimulated; then students are guided to try to derive the conditions for matrix diagonalization by combining the learned knowledge of matrix eigenvalues and eigenvectors; then through counterexample verification, students realize that "not all matrices can be diagonalized", and then extract the necessary and sufficient conditions for matrix diagonalization; finally, students are guided to apply the knowledge of matrix diagonalization to practical problems such as linear transformation and differential equation solving, realizing the rediscovery and reapplication of knowledge [2].

At the same time, the Ruijietong AI intelligent learning platform is matched to provide students with personalized practice and feedback services. According to students' exploration status, targeted learning resources are accurately pushed to help students solidly master mathematical foundations and develop independent exploration learning habits. In the scientific research enlightenment link, courses such as "Introduction to Scientific Research Methods" and "Frontier Lectures on Mathematics" are offered from the 2nd academic year, inviting cross-school postgraduate tutors from the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education to give lectures, allowing students to understand the basic methods, academic norms and frontier directions of mathematical scientific research in advance and establish correct scientific research awareness; in the 3rd academic year, the college, in conjunction with the committee, sets up mathematical "scientific research interest groups" to carry out basic exploration by connecting with scientific research projects of member units of the committee. Students carry out simple mathematical exploration projects in groups, systematically learning basic scientific research skills such as literature retrieval, data analysis, logical argumentation and academic report, and implanting mathematical scientific research thinking in advance. In addition, strengthen the training of students' logical argumentation, academic writing and interdisciplinary collaboration abilities, and offer supporting courses such as academic writing and mathematical modeling, which are in line with the committee's "undergraduate-basic-research" whole-chain training concept, laying a solid ability foundation for students to connect with postgraduate scientific research tasks.

3.2. "1": 4th Year of Undergraduate Study—Scientific Research Connection and Transition

As a "bridge period" between undergraduate and postgraduate stages, the core goal of the 4th year of undergraduate study is to realize a smooth transition from knowledge learning and basic exploration at the undergraduate stage to scientific research innovation and independent research at the postgraduate stage. Qian Weichang College and the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education jointly design a collaborative connection mechanism to achieve seamless connection between undergraduate and postgraduate stages from three dimensions: courses, scientific research and tutors, helping students quickly adapt to the training requirements of the postgraduate stage, reducing the adaptation cost of further study, and giving full play to the connection and empowerment role of the knowledge rediscovery concept.

In terms of curriculum connection, formulate mutual recognition rules for cross-stage mathematics courses, introduce 3-4 core postgraduate mathematics courses (such as Lie Groups and Lie Algebras, Advanced Numerical Analysis, Modern Algebraic Topology, etc.) relying on the committee's "golden course matrix" resources, and advance the core postgraduate courses to the 4th year of undergraduate study; the college improves the credit recognition system, and the credits of postgraduate courses taken

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by students in the 4th year of undergraduate study can be directly recognized as postgraduate credits, reducing repetitive learning and realizing the continuity and in-depth extension of mathematical knowledge learning. At the same time, optimize the teaching of cross-stage courses, always guided by the knowledge rediscovery concept, adopt inquiry-based and seminar-based teaching methods, guide students to deeply explore the essence and application of high-level mathematical knowledge, and lay a solid knowledge foundation for postgraduate scientific research innovation.

In terms of scientific research connection, relying on the national resource linkage advantages of the committee, promote students to enter the mathematical education alliance research teams jointly established by more than 50 universities across the country to participate in real mathematical scientific research projects; Qian Weichang College fully opens interdisciplinary scientific research platforms and mathematical scientific research laboratories, providing students with sufficient scientific research venues, experimental equipment, project data and other supports, and supporting students to participate in sub-project research of national and provincial-level mathematical scientific research projects. By participating in real scientific research projects, students realize the transformation from "classroom exploration" to "real mathematical scientific research", systematically learn the design, implementation and summary methods of scientific research projects in scientific research practice, master the basic norms of literature review and scientific research paper writing, accumulate valuable scientific research experience, and make full preparations for carrying out independent scientific research innovation in the postgraduate stage, realizing a smooth transition of scientific research ability.

In terms of tutor connection, implement the early intervention system of postgraduate tutors. From the 4th year of undergraduate study, the supervisor of the student's postgraduate stage will participate in the academic and scientific research guidance of the undergraduate stage in advance, and jointly formulate a personalized connection training plan for the student with the undergraduate tutor, clarifying the research direction, scientific research goals and learning plan of the student's further study stage. The early intervention of tutors enables postgraduate tutors to timely understand students' mathematical foundation, academic interests and research potential, realizing the seamless connection of training programs; at the same time, students can be familiar with the research direction and guidance style of postgraduate tutors in advance, quickly establish scientific research collaboration relations, and directly carry out scientific research work after entering the postgraduate stage, effectively avoiding the gap period of scientific research connection.

3.3. "X": Flexible Further Study Stage—Combined Master-Doctoral Program/Direct Doctoral Program/Master's Program

The "X" stage is a flexible further study period, with the core of respecting the differences in students' academic development and career plans and providing diverse and adaptive further study paths. Under the collaborative guidance of Qian Weichang College and the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, three different further study paths (combined master-doctoral program, direct doctoral program, and master's program) are designed for students according to their academic performance, research potential and interests during the undergraduate stage, realizing the top training goal of "teaching students in accordance with their aptitude", enabling different types of mathematical talents to obtain adaptive training and development, and giving full play to the empowering role of the knowledge rediscovery concept in the personalized development of talents.

The direct doctoral program is oriented to students with outstanding academic potential and clear research aspirations for the doctoral stage. Relying on the committee's resource advantages, it connects with top tutors in the field of mathematics from top overseas universities such as Yale University and National University of Singapore, as well as domestic double first-class universities such as Tsinghua University and Peking University. The college establishes a direct doctoral cooperation mechanism with target universities, and determines the student's doctoral stage mathematical research topic as early as the 4th year of undergraduate study, allowing students to directly carry out continuous

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scientific research work after entering the doctoral stage, and cultivating high-level mathematical academic talents with original innovation capabilities. The focus of the direct doctoral program is to strengthen students' independent scientific research capabilities. Relying on the supervisor's scientific research projects and platform resources, students are guided to carry out cutting-edge and original mathematical research, cultivating their academic vision and original innovation capabilities.

The combined master-doctoral program (X=4-5 years) is oriented to students with solid academic foundation and strong research potential. Students complete the core course learning and scientific research accumulation of the postgraduate stage in the first 1-2 years of the master's stage, and directly transfer to the doctoral stage after passing the combined master-doctoral assessment jointly organized by the college and the committee. The combined master-doctoral program focuses on the continuity of scientific research training. The research content of the master's stage is closely connected with the research direction of the doctoral stage, allowing students to complete the deepening of scientific research methods and the accumulation of research foundation in the master's stage, and carry out more in-depth and systematic original research on the basis of the master's stage research in the doctoral stage, avoiding the interruption of scientific research and improving the efficiency of scientific research innovation.

The master's program (X=2-3 years) is oriented to students who focus on mathematical application research and have the willingness to integrate production, education and research. The training focus is to strengthen students' mathematical application capabilities and technology transformation capabilities, encouraging them to participate in enterprise cooperation projects and horizontal scientific research topics, and deeply integrating mathematical knowledge with application fields such as artificial intelligence, aerospace and biomedicine, cultivating compound mathematical talents with solid mathematical foundation and strong application capabilities. In the master's stage, students are equipped with a dual-tutor system of "university tutor + enterprise tutor". The university tutor is responsible for the guidance of mathematical theories and methods, and the enterprise tutor is responsible for the guidance of application scenarios and technology transformation, realizing the deep integration of mathematical theories and application practices, and transforming the achievements of knowledge rediscovery into practical application value.

4. Key Implementation Measures of the Undergraduate-Master-Doctoral Integrated Cultivation Empowered by Knowledge Rediscovery

4.1. Cross-Stage and Cross-School Mathematics Tutor Collaboration Mechanism

Relying on the on-campus teachers of Qian Weichang College and the cross-school expert resources of the committee, a joint tutor group of "committee mathematics experts + on-campus mathematics tutors + postgraduate mathematics tutors" is established. Each student is equipped with 3 instructors in the field of mathematics, who are responsible for concept guidance, basic training and scientific research guidance respectively, forming a cross-stage and cross-school tutor collaborative training system, providing professional guidance and support for the whole-cycle training of students, and realizing the all-round empowerment of the knowledge rediscovery concept.

Committee mathematics experts are mainly composed of well-known professors and national-level teaching masters in the field of mathematics education in China, undertaking the responsibilities of guiding the "knowledge rediscovery" concept and academic frontier guidance. They provide students with cutting-edge research trends and academic research method guidance in the field of mathematics, participate in the assessment of important scientific research nodes of students, and provide professional suggestions for students' choice of research direction; on-campus mathematics tutors are mainly composed of high-quality teachers from Qian Weichang College, undertaking the responsibilities of basic teaching and academic guidance in the undergraduate stage, integrating the knowledge rediscovery concept into curriculum teaching and basic exploration guidance, solidifying students' mathematical foundation and cultivating their independent exploration learning habits; postgraduate mathematics tutors are mainly composed of experts in the field of mathematics scientific

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research inside and outside the university, participating in student training from the 4th year of undergraduate study, undertaking the responsibilities of scientific research guidance and further study stage training, guiding students to participate in real scientific research projects, formulating postgraduate scientific research training programs, and guiding students to carry out independent scientific research innovation.

Establish a regular communication mechanism for joint tutors, jointly hosting 1 online + offline joint tutor meeting every semester to synchronize students' mathematics learning, scientific research progress, ability improvement and other situations, conduct collective discussions on problems arising in the student training process, and dynamically adjust personalized training programs to ensure the seamless connection of training requirements among different academic stages and different tutors. At the same time, establish a cross-school and cross-stage resource sharing mechanism. Qian Weichang College opens on-campus resources such as mathematical scientific research laboratories, project data and scientific research platforms, and opens cross-school library resources, scientific research project resources, academic exchange resources and other resources relying on the committee platform, supporting students to carry out continuous mathematical research across stages and universities, so that students can fully utilize the high-quality resources of both subjects and comprehensively improve the training quality.

4.2. Integration of Mathematics Courses and Scientific Research

The core of the undergraduate-master-doctoral integrated cultivation of mathematics guided by the knowledge rediscovery concept is to realize the deep integration and mutual promotion of curriculum learning and scientific research training, breaking the traditional training model of "separation of curriculum learning and scientific research training", and constructing a three-level curriculum system of "undergraduate core mathematics courses-postgraduate advanced mathematics courses-mathematical scientific research special courses", realizing the integrated design of courses and scientific research. Curriculum learning lays the foundation for scientific research training, and scientific research training provides application scenarios for curriculum learning, forming a virtuous cycle of "promoting research through learning and deepening learning through research", and giving full play to the integration and empowerment role of the knowledge rediscovery concept.

In the construction of the three-level curriculum system, undergraduate core mathematics courses focus on basic mathematical knowledge, integrate exploration modules of knowledge rediscovery, focus on consolidating mathematical foundations and cultivating exploration thinking, and provide basic mathematical theories and method support for scientific research training; postgraduate advanced mathematics courses focus on high-level mathematical knowledge, connect with cutting-edge scientific research directions, focus on the in-depth extension and application of mathematical knowledge, and provide more systematic and cutting-edge mathematical theory support for scientific research innovation; mathematical scientific research special courses are closely combined with scientific research projects, offering personalized scientific research special courses (such as "Algebraic Number Theory Research Special Course", "Computational Mathematics Research Special Course", etc.) according to students' research directions and scientific research projects, taught by postgraduate tutors, focusing on the cultivation of scientific research methods, academic norms and research skills, and realizing the precise connection between curriculum learning and scientific research training. The content of the three-level curriculum system is integrated with the high-quality resources of the committee's "golden course matrix", and cross-school experts organized by the committee review and optimize the curriculum content to ensure the scientificity, frontier and connection of the curriculum content, and strengthen the continuity and in-depth extension of mathematical knowledge.

In the advanced design of scientific research training, the mathematical "small exploration" at the undergraduate stage is closely connected with the "large project" at the postgraduate stage, forming a complete scientific research chain of "basic mathematical exploration-in-depth mathematical research-achievement output". In the first 3 years of undergraduate study, basic mathematical exploration is carried out through exploration modules in courses and scientific research interest groups, allowing

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students to complete basic scientific research achievements such as "course papers" and "exploration reports"; in the 4th year of undergraduate study, participate in real scientific research projects to complete phased scientific research achievements such as "scientific research mid-term reports" and "sub-project research reports"; in the postgraduate stage, carry out independent scientific research innovation on the basis of previous research, completing high-level scientific research achievements such as master's/doctoral dissertations and academic paper publications. The entire scientific research training process always practices the continuous innovation concept of knowledge rediscovery, allowing students to experience the rediscovery and re-innovation of knowledge at each scientific research stage, and gradually improving their scientific research capabilities and innovative abilities.

4.3. Digital Intelligence Support and Quality Monitoring

Relying on modern information technology, a digital intelligence training support and quality monitoring system is constructed. Relying on the Ruijietong AI intelligent learning platform and postgraduate management system, a whole-cycle growth data file of students is established to realize the whole-process tracking, dynamic assessment and quality monitoring of the student training process, ensuring the quality of integrated cultivation, making the training process guided by the knowledge rediscovery concept more precise and effective, and providing strong guarantee for the cultivation of top talents.

In terms of digital intelligence support, the Ruijietong AI intelligent learning platform provides personalized learning support for undergraduate curriculum teaching. Through big data analysis of students' curriculum learning and exploration task completion status, it accurately grasps students' knowledge mastery and exploration ability level, pushing personalized learning resources and exploration tasks for students, and realizing personalized teaching of "teaching students in accordance with their aptitude"; the postgraduate management system provides digital support for postgraduate scientific research training, realizing the digital management of scientific research projects, academic achievements and training processes, facilitating tutors and students to track scientific research progress in real time and adjust scientific research plans in a timely manner. At the same time, relying on the online academic exchange platform of both subjects, students are provided with cross-school and cross-stage academic exchange opportunities, inviting experts in the field of mathematics at home and abroad to give online academic lectures and scientific research guidance, broadening students' academic horizons and allowing them to timely understand the cutting-edge research trends in the field of mathematics.

In terms of quality monitoring, a "phased assessment" mechanism is established to build a closed-loop management system for training quality. According to different training stages, differentiated assessment goals and content are set to ensure the effective realization of training goals at each stage. At the end of the 3rd academic year, a basic ability assessment is carried out. On-campus tutors and committee experts jointly assess students' mathematical foundation, exploration ability and scientific research enlightenment status, and those who pass the assessment enter the scientific research connection stage of the 4th year of undergraduate study; at the end of the 4th academic year, an assessment of scientific research connection effectiveness is carried out. The joint tutor group assesses students' postgraduate course learning status, scientific research project participation and scientific research ability improvement, and those who pass the assessment enter the corresponding further study path according to their academic performance; in the further study stage, mid-term assessment, combined master-doctoral assessment, dissertation proposal assessment and other links are set up according to different training paths, which are organized and assessed by experts jointly by the college and the committee to ensure the quality of postgraduate training. At the same time, establish a feedback and optimization mechanism for training quality. According to the results of phased assessments, timely summarize the problems existing in the training process, and dynamically optimize the training program, curriculum system and teaching methods to ensure the continuous improvement of integrated training quality.

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5. Implementation Effects of the Undergraduate-Master-Doctoral Integrated Cultivation Empowered by Knowledge Rediscovery

Since the launch of the "3+1+X" undergraduate-master-doctoral integrated training model for the major of Mathematics and Applied Mathematics in Qian Weichang College of Shanghai University in 2019, after 6 years of practical testing (as of June 2025), a total of 3 batches of 74 students have been trained, among which 60 have successfully entered the "1+X" further study stage, with an further study rate of 81%. Remarkable results have been achieved in improving students' scientific research capabilities, ensuring further study quality, and demonstrating the training model, fully verifying the feasibility and effectiveness of the undergraduate-master-doctoral integrated cultivation of top mathematical talents empowered by the knowledge rediscovery concept, and providing a successful practical sample for the cultivation of top innovative mathematical talents.

5.1. Significant Improvement of Students' Scientific Research Capabilities, Excellent Academic Achievements and Further Study Quality

Under the whole-cycle scientific research training guided by the knowledge rediscovery concept, students have formed systematic scientific research thinking and solid scientific research capabilities since the undergraduate stage, and their academic achievement output capabilities and scientific research innovation capabilities have been significantly improved. In the past 5 years, undergraduates participating in the integrated cultivation have published a total of 12 SCI-indexed academic papers, among which the paper published by Ren Baiying as the first author was selected as an ESI highly cited paper, achieving a major breakthrough in high-level academic paper publication by undergraduates; students have participated in national scientific research projects 56 times, winning 71 awards in high-level discipline competitions such as the "American College Student Mathematical Modeling Competition" and the "National College Student Mathematics Competition", with the number and quality of competition awards ranking among the top in the university.

Students' scientific research innovation capabilities have been highly recognized by universities at home and abroad. Among the 60 students entering the "X" further study stage, 13 directly pursued doctoral degrees in top overseas universities such as Yale University and National University of Singapore, with an overseas further study rate of 21%; 38 entered domestic double first-class universities such as Tsinghua University, Peking University and Fudan University for further study, accounting for 63%. The further study institutions are all top universities in the field of mathematics, and the further study quality is significantly better than that of the traditional training model. At the same time, the training model has realized the continuity of scientific research. 80% of students' mathematical research directions in the further study stage are consistent with the scientific research projects connected in the 4th year of undergraduate study, effectively avoiding the interruption of scientific research and greatly improving the efficiency of scientific research innovation.

Zi-Han Gao is a benchmark case of dual-subject collaborative training. During the undergraduate period, the student joined the research group jointly established by the college and the committee, and deeply participated in the core research on linear algebra and matrix theory guided by "knowledge rediscovery", completing solid scientific research accumulation in the undergraduate stage; after being admitted to a top domestic university for direct doctoral study, she continued to focus on this research direction, publishing an ESI highly cited paper in the field of mathematics as the first author in the first year of direct doctoral study alone, and subsequently publishing [4] more SCI-indexed papers and winning the National Scholarship. The rapid output of his scientific research achievements fully reflects the scientific research connection advantage of the integrated training model guided by the knowledge rediscovery concept, demonstrating the empowering role of knowledge rediscovery in the scientific research capabilities of top talents.

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5.2. Significant Demonstration Effect of the Training Model, Forming a Good Pattern of On-Campus Promotion and Off-Campus Radiation

After 6 years of practice and improvement, the "3+1+X" undergraduate-master-doctoral integrated training model has formed a replicable and promotable training program suitable for the major of Mathematics and Applied Mathematics, and has formed a good promotion effect in Shanghai University, being learned from by 5 colleges including the School of Science, School of Computer Engineering and Science, and School of Artificial Intelligence of the university. Combining their own professional characteristics, each college integrates the knowledge rediscovery concept with the cultivation of top innovative talents in their own majors, constructing an integrated training model with professional characteristics, forming an integrated training pattern of "led by Qian Weichang College and linked by multiple colleges", effectively promoting the overall improvement of the quality of cultivating top innovative talents in basic disciplines and interdisciplinary disciplines of Shanghai University.

Nationwide, relying on the national academic platform of the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, the core design and practical experience of this training model have been widely promoted. Many universities in the central and western regions and local universities across the country have come to Shanghai University for exchange and study, introducing the core design concept of the "3+1+X" training model, and constructing an integrated training model suitable for their own mathematics majors in combination with their own school-running positioning and teacher resources. Through holding national educational mathematics seminars and top talent training forums, the committee shares and promotes the practical experience of Shanghai University, providing practical reference for the integrated cultivation of top innovative talents in mathematics and other basic disciplines in China, and forming a good off-campus radiation effect.

5.3. Positive Feedback from Teachers and Students, Wide Recognition of the Training Model

From the perspective of student feedback, the recognition of the training model by students participating in the integrated cultivation is extremely high. 95% of students believe that measures such as "taking postgraduate courses in advance during the undergraduate stage" and "participating in real scientific research projects at an early stage" have effectively reduced their adaptation costs in the further study stage, enabling them to quickly integrate into postgraduate study and scientific research; 90% of students recognize the personalized training programs formulated by the joint tutor group, believing that the personalized training programs can fully meet their own academic interests and development needs, allowing their mathematical strengths and research potential to be fully exerted. Students generally express that the inquiry-based learning guided by the knowledge rediscovery concept has freed them from mechanical knowledge memory, enabling them to have a deeper understanding of the essence of mathematical knowledge, and at the same time forming systematic scientific research thinking in the process of scientific research exploration, laying a solid foundation for their subsequent academic development.

From the perspective of tutor feedback, 96% of postgraduate tutors indicate that students under the integrated training model have significantly better scientific research adaptability and academic norm awareness than those trained under the traditional model. After entering the postgraduate stage, they can quickly enter the scientific research role and carry out independent scientific research work, with the average time to enter the scientific research role shortened by about half a year. Tutors generally believe that the scientific research enlightenment and connection in the undergraduate stage guided by the knowledge rediscovery concept enable students to master basic scientific research methods and academic norms in the undergraduate stage, effectively avoiding the initial gap period of scientific research in the postgraduate stage and improving the training efficiency of the postgraduate stage; the cross-stage and cross-school joint tutor collaboration mechanism realizes the integration of training resources and the connection of training requirements, making the cultivation of top talents more systematic and professional.

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6. Core Experiences and Future Prospects of the Undergraduate-Master-Doctoral Integrated Cultivation Empowered by Knowledge Rediscovery

6.1. Core Experiences

After 6 years of practical exploration, the undergraduate-master-doctoral integrated cultivation of the major of Mathematics and Applied Mathematics in Qian Weichang College of Shanghai University has achieved remarkable results, accumulating three core experiences, which provide practical reference for the application of the knowledge rediscovery concept in the integrated cultivation of top innovative talents in basic disciplines, and also provide strong support for further exerting the empowering role of knowledge rediscovery [1]. It should be noted that comparing with the training models of top mathematical talents in European and American countries can better highlight the characteristics and advantages of China's undergraduate-master-doctoral integrated model. In terms of European countries, the training of top mathematical talents is mostly based on the "elite education" model. For example, German universities focus on the in-depth cultivation of undergraduate mathematical foundations, with a high degree of autonomy in course selection and research directions, and the connection between undergraduate and postgraduate stages relies more on the independent matching of students and tutors, lacking systematic institutional design. British top universities such as Oxford and Cambridge adopt a tutorial system for undergraduate training, emphasizing small-class discussions and independent thinking, but the scientific research training is relatively fragmented in the undergraduate stage, and the transition to postgraduate scientific research relies more on the sudden adaptation of students. In American universities, the training of top mathematical talents is characterized by "broad foundation + early scientific research involvement". Undergraduates can participate in scientific research projects led by professors in the early stage, but the training process is highly market-oriented, and the connection between undergraduate and postgraduate stages is relatively loose, often affected by factors such as funding and project cooperation, resulting in insufficient continuity of training. Compared with these international models, China's "3+1+X" undergraduate-master-doctoral integrated training model empowered by the knowledge rediscovery concept has distinct characteristics: first, it emphasizes systematic institutional connection, breaking the barriers between academic stages through the "3-year foundation consolidation + 1-year connection transition + X-year flexible further study" whole-cycle design, which is different from the loose connection between stages in European and American models; second, it takes the knowledge rediscovery concept as the consistent educational guidance, realizing the organic integration of curriculum learning and scientific research training throughout the whole process, making up for the lack of systematic scientific research guidance in the undergraduate stage of European models and the lack of consistent educational concept support in American models; third, it relies on dual-subject collaborative resources, integrating the advantages of national academic platforms and university practice carriers, which is different from the relatively single resource support of European and American universities relying on their own school resources. These characteristics make China's model not only meet the demand for top mathematical talents in China's national strategic development, but also provide a new practical reference for the integrated cultivation of top talents in other countries, especially developing countries.

Firstly, dual-subject collaboration is the key support for the implementation of the concept. The Professional Committee of Educational Mathematics of the Chinese Society of Higher Education and Qian Weichang College of Shanghai University form a two-wheel drive of "concept guidance + practice implementation". The committee gives play to the advantages of a national academic platform, providing core concept guidance of "knowledge rediscovery", cross-school high-quality resource linkage and expert think tank support for the training model, solving the problems of "how to guide the training concept and how to integrate high-quality resources"; relying on the positioning of a national pilot college, Qian Weichang College provides practical carriers, high-quality teacher teams, scientific research platforms and funding guarantees for the training model, solving the problems of "how to implement the training concept and how to carry out the training process". The two parties divide labor and cooperate closely, forming a complete closed loop of "concept guidance-

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resource integration-practice implementation", ensuring that the knowledge rediscovery concept is truly implemented into the entire training process, and providing solid dual-subject support for the efficient advancement of integrated cultivation.

Secondly, the deep integration of the concept and the major is the core guarantee of training quality. The precise connection between the "knowledge rediscovery" educational concept and the disciplinary characteristics of the major of Mathematics and Applied Mathematics is the core guarantee for the effectiveness of integrated cultivation and the key to exerting the empowering role of the concept. The abstract, logical and systematic characteristics of mathematics determine that the cultivation of talents cannot be limited to mechanical knowledge memory, but needs to guide students to understand the essence and generation process of knowledge. The knowledge rediscovery concept is exactly in line with the training needs of mathematics. By integrating the concept into the entire process of curriculum teaching, scientific research training and academic stage connection, the training path of mathematical talents is reconstructed. While students re-experience the knowledge generation process, they form a thinking mode and scientific research ability in line with the laws of mathematics. Integrated cultivation is not a simple superposition of academic stages, but through the deep integration of concepts and majors, realizing the thinking progression of students from "basic mathematical exploration" to "high-level mathematical innovation", and truly solving the pain points of "segmentation between academic stages and lagging scientific research" in the traditional training model.

Thirdly, the construction of a multi-collaborative mechanism is an important guarantee for the sustainable development of the training model. The construction of multi-collaborative mechanisms such as cross-school and cross-stage tutor collaboration, integration of courses and scientific research, and digital intelligence quality monitoring is the key to ensuring the sustainable and efficient development of the training model and the effective empowerment of knowledge rediscovery. The cross-school and cross-stage tutor collaboration mechanism realizes the integration of teacher resources and the connection of training guidance, allowing students to obtain the most professional academic and scientific research guidance; the integration mechanism of courses and scientific research realizes the mutual promotion of knowledge learning and scientific research training, forming a virtuous cycle of "promoting research through learning and deepening learning through research"; the digital intelligence quality monitoring mechanism realizes the whole-process tracking and dynamic optimization of the training process, ensuring the closed-loop management of training quality. The construction of multi-collaborative mechanisms not only guarantees the efficient integration of dual-subject resources, but also realizes the continuity and precision of the whole training cycle, providing stable mechanism support for the growth of top innovative mathematical talents.

6.2. Future Prospects

Facing the country's urgent demand for high-level top innovative mathematical talents, combined with 6 years of practical experience, Shanghai University will continue to take the knowledge rediscovery concept as the core, rely on the advantages of dual-subject collaboration, and continuously optimize the "3+1+X" undergraduate-master-doctoral integrated training model from three aspects: deepening the breadth of collaboration, strengthening international cooperation, and improving the promotion mechanism, promoting the quality of cultivating top innovative mathematical talents to a new level, and further exerting the empowering role of knowledge rediscovery.

Firstly, deepen the breadth of collaboration and expand interdisciplinary training directions in response to national strategic needs. Relying on the platform advantages of the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, further expand the scope of cross-school collaboration, unite more mathematical resources from universities and research institutes, and build a broader alliance for cultivating top mathematical talents. At the same time, combined with the national strategic needs in fields such as chip research and development, aerospace, artificial intelligence and biomedicine, add interdisciplinary training directions such as "Mathematics + Artificial Intelligence", "Mathematics + Aerospace" and "Mathematics + Biomedicine", integrate the knowledge rediscovery concept into the entire process of interdisciplinary cultivation, cultivate compound top

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innovative mathematical talents with solid mathematical foundation and strong interdisciplinary application capabilities, and better serve national strategic needs.

Secondly, strengthen international cooperation and enhance students' international academic competitiveness. Relying on the committee's international academic resources, connect with mathematics majors of top overseas universities, establish an international joint undergraduate-master-doctoral integrated training mechanism, introduce high-quality overseas mathematics education resources, scientific research platforms and tutor resources, and carry out joint curriculum teaching, joint scientific research guidance and joint degree training. Provide students with more overseas academic exchange, joint training and scientific research internship opportunities, allowing them to contact the cutting-edge research trends in the international mathematics field, cultivate their international academic vision and cross-cultural scientific research collaboration capabilities, enhance their international academic competitiveness, and cultivate high-level top innovative mathematical talents with international influence.

Thirdly, improve the promotion mechanism and provide more practical references for the cultivation of top talents in basic disciplines. Jointly with the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, systematically summarize the practical experience of the "3+1+X" undergraduate-master-doctoral integrated cultivation, compile the "Operation Guide for the Undergraduate-Master-Doctoral Integrated Cultivation of the Major of Mathematics and Applied Mathematics", and systematically and standardizedly sort out the experience in training concepts, training models, implementation measures and quality monitoring, providing a replicable and promotable practical plan for mathematics majors in more universities across the country. At the same time, through holding national top talent training workshops and seminars, and carrying out paired assistance with other universities, promote the practical experience of Shanghai University to more local universities and universities in the central and western regions, promote the overall improvement of the quality of cultivating top innovative talents in mathematics and other basic disciplines in China, and cultivate more high-level top innovative talents in basic disciplines for the country.

7. Conclusion

The cultivation of top innovative mathematical talents is a key measure to support national technological self-reliance and self-improvement and solve the "bottleneck" technical problems. Breaking the barriers between academic stages in the traditional training model and building a seamlessly connected training system between undergraduate and postgraduate stages is the core path to improve the quality of cultivating top innovative mathematical talents [4]. As an educational concept compatible with the characteristics of mathematics, the knowledge rediscovery concept provides a new perspective and path for the undergraduate-master-doctoral integrated cultivation of mathematics, and its empowering role in cultivating top talents runs through the entire process of foundation consolidation, scientific research improvement and personalized development.

Taking the major of Mathematics and Applied Mathematics in Qian Weichang College of Shanghai University as the practice carrier, and in conjunction with the Professional Committee of Educational Mathematics of the Chinese Society of Higher Education, this study has systematically constructed and practiced the "3+1+X" undergraduate-master-doctoral integrated training model with the "knowledge rediscovery" concept as the core. Through 6 years of practice and exploration, the model has effectively broken the connection barriers between undergraduate and postgraduate stages in the traditional training model, solved the pain points of fragmented scientific research training and delayed stimulation of innovative potential, and realized the seamless connection of knowledge learning, scientific research training and thinking improvement between different academic stages. The practical results show that this model not only significantly improves students' scientific research capabilities and academic achievements, but also forms a good demonstration effect of on-campus promotion and

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off-campus radiation, providing a replicable and promotable practical paradigm for the cultivation of top innovative talents in mathematics and other basic disciplines in China.

The research practice of this paper fully verifies that the "knowledge rediscovery" concept has inherent compatibility with the undergraduate-master-doctoral integrated cultivation of top mathematical talents. The core connotation of knowledge rediscovery—breaking linear teaching logic, highlighting students' dominant position, and balancing knowledge mastery and ability cultivation—is highly consistent with the core requirements of integrated cultivation for thinking continuity, scientific research progression and talent individuality. By integrating this concept into the entire training process, we can guide students to re-experience the generation process of mathematical knowledge as "quasi-researchers", help them form systematic scientific research thinking from the undergraduate stage, and realize a smooth transition from "knowledge recipients" to "scientific research innovators", which is an important key to improving the quality of cultivating top mathematical talents.

The "3+1+X" undergraduate-master-doctoral integrated training model constructed in this study, with dual-subject collaboration as the support, and cross-stage and cross-school tutor collaboration, integration of courses and scientific research, and digital intelligence quality monitoring as the key implementation measures, forms a complete training system of "concept guidance-resource integration-practice implementation-quality assurance". The "3-year foundation consolidation" lays a solid mathematical foundation and scientific research thinking for students; the "1-year scientific research connection" realizes the smooth transition between undergraduate and postgraduate stages; the "X-year flexible further study" meets the personalized development needs of different students, forming a whole-cycle training link that conforms to the growth law of top mathematical talents. This model not only enriches the practice of undergraduate-master-doctoral integrated cultivation in basic disciplines, but also provides a new path for the reform of top talent training in mathematics education.

It should be noted that this study still has certain limitations. On the one hand, the practice carrier of this model is mainly the major of Mathematics and Applied Mathematics in Qian Weichang College of Shanghai University, which has certain advantages in platform resources and teacher teams. The applicability of the model in local universities and universities with relatively weak resources needs to be further tested and optimized. On the other hand, the research on the long-term growth effect of students under the integrated training model is still in the preliminary stage. The follow-up needs to track and study the academic development, scientific research achievements and career development of trained students for a long time to further verify the long-term effectiveness of the model.

In the future, we will take the limitations of this study as the direction of in-depth research, and further optimize and improve the "3+1+X" integrated training model. On the one hand, we will further expand the scope of practice, strengthen cooperation with more local universities and universities in the central and western regions, adjust and optimize the training program according to the actual school-running conditions of different universities, and enhance the applicability and replicability of the model. On the other hand, we will establish a long-term tracking mechanism for students' growth, systematically sort out the long-term effect of the model on students' academic development and scientific research innovation, and continuously enrich and improve the theoretical and practical system of undergraduate-master-doctoral integrated cultivation empowered by the knowledge rediscovery concept. At the same time, we will further deepen interdisciplinary collaboration and international cooperation, integrate more high-quality resources, cultivate more high-level top innovative mathematical talents with solid foundation, strong innovation and international vision, and provide stronger talent support for the country's technological self-reliance and self-improvement and the development of basic disciplines.

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