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Derivation of the QED Dyson Expansion Series from the Fundamental Soils of Classical Physics

Toward a Unified and Universal Dirac Equation (II)

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Abstract

The anomalous gyromagnetic ratio (g-factor) of elementary particles—*most famously the electron's deviation from the Dirac value*: $g_D = 2$—has long been one of the most precisely measured and theoretically challenging quantities in fundamental theoretical physics. In Quantum Electrodynamics (QED), this anomaly is explained through radiative corrections represented by an infinite series of Feynman diagrams, culminating in the Dyson expansion. In the present work, building on the foundations laid in Paper (I), we develop an alternative, non-perturbative framework that traces the origin of the g-factor anomaly to the electromagnetic self-energy of an extended charge distribution coupled to a hypothetical cosmic vector field, $\boldsymbol{\mathcal{H}}$. By identifying the ζ-parameter [proposed in Paper (I)], which parameter governs the deviation from the Dirac value ($g_D = 2$)—with the spin-coupled electromagnetic self-energy, we derive a power series for the particle anomalous magnetic moments that reproduces the full Dyson expansion series of QED without invoking Feynman diagrams or perturbative methods. The coefficients of this series are shown to arise from the multipole moments of the charge distribution, as described by the Azimuthally Symmetric Theory of Electrostatics (ASTE-model) introduced herein. This framework naturally implies that particles with non-zero anomalous g-factors—*including the electron*—must possess a finite spatial extent. Using existing g-factor data for the proton, neutron, and lambda hyperon, we obtain a preliminary theoretical estimate of the electron radius: $r_e = (1.20 \pm 0.05) \times 10^{-22}$ m, in remarkable agreement with the stringent experimental upper limit: $r_e \lesssim 1.50 \times 10^{-22}$ m. We strongly believe that these results offer a fresh new perspective on the nature of fundamental particles, the origin of radiative corrections, and the possible existence of a cosmic field mediating spin-electromagnetic interactions across the fabric of spacetime.

Keywords: Dirac equation; gyromagnetic ratio — electron, proton, neutron; Radius — electron, proton, neutron

1. Introduction

This is the second instalment in our ten-part series subtitled: *Toward a Unified and Universal Dirac Equation*. The overarching aim of this series is to reformulate the Dirac equation [1,2] through careful yet justifiable modifications, aspiring to create a unified and universal equation that encompasses all fundamental particle phenomena within a single theoretical framework. Much like a little child playing on the seashore, searching for a pretty snail shell each time seeking one prettier than the one before, we have spent significant number of years [3–11] of the main *Shores of Physics* exploring the Dirac equation. Now, the Light of this exploration has finally illuminated our path with a brightness such as would reach the main *Shores of Physics*. As can surely be judged from Paper (I) [12] and the present reading, this Light is manifestly revealing of its potential to unify the *World of Particle Physics*.

In the first instalment [12], we proposed the existence of an omniscient, all-symmetry breaking, all-pervading, all-permuting and non-ponderable *Cosmic Four Vector Field* (CFV-field), $\mathcal{H}_\mu$, that permeates

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all of cosmological space. Using this CFV-field, we demonstrated that it influences a Dirac particle, allowing for a qualitative explanation of its excess non-Dirac g-factor. Specifically, we obtained:

$$g_{\mathrm{P}} = 2\left(1 + \frac{\zeta_{\mathrm{P}} m_{\star} r_{\mathrm{P}} c_0}{\hbar}\right), \tag{1}$$

with: $\Delta g_{\mathrm{P}} = \zeta_{\mathrm{P}} m_{\star} r_{\mathrm{P}} c_0 / \hbar$, being the excess non-Dirac g-factor. This factor arises from the interaction between the particle's spin $(\boldsymbol{S})$, the ambient magnetic field $(\boldsymbol{A})$, and this hypothetical CFV-field, $(\mathcal{H}_{\mu})$, mediated by a *yet-to-be-identified* ζ-parameter. While the nature of this ζ-parameter was not elaborated upon in Paper (I), it was noted therein [12] that its non-zero value implies that particles with non-Dirac g-excess values should possess a non-zero radius. Given the electron's significant non-zero g-excess, we concluded that how-ever small it might be, the electron—*often assumed to have a zero radius [13]*—must actually possess a finite radius.

In the present work, we take a further step by identifying this ζ-parameter with the **spin-coupled electromagnetic self-energy** of an extended charge distribution interacting with the, $\boldsymbol{\mathcal{H}}$, and, $\boldsymbol{A}$, fields. The derived ζ-parameter reproduces the well-known phenomenological Dyson expansion series [14,15] from Quantum Electrodynamics (QED) without relying on the ponderous Feynman diagrams and methods [15–19]. While these methods yield accurate numbers that agree impressively with experimental results, their obvious artificial nature—which bothered many great physicists including—Paul Adrian Maurice Dirac (1902-1984), John *von* Neumann (1903-1957), Julian Seymour Schwinger (1918-1994) and Richard Phillips Feynman (1918-1987) himself—leaves open the enduring question of whether a more fundamental, perhaps classical or semi-classical, description might underlie the g-excess phenomenon.

Contrary to the prevalent belief among the majority of practising physicists that it has zero radius[1], if the framework being built in the present proves to be correct, it strongly suggests that the electron, traditionally viewed as a point particle [13], should possess a finite spatial extent—however small it might be. Moreover, the assumption that the electron is a point particle—essential for the renormalizability of QED—stands at a crossroads, in tension with the very existence of an infinite series of corrections, which suggests an internal structure however deeply hidden this structure might be.

The structure of this paper is as follows. In §(2), we describe the particle model on which the present theory is based. That is, analogous to the Azimuthally Symmetric Theory of Gravitation (ASTG-model) [20], we present the Azimuthally Symmetric Theory of Electricity (ASTE-model) for the electrostatic potential of a spinning charge, leading to the much needed expression for the stored electromagnetic energy, U_{P}, as an infinite series in the Fine Structure Constant (FSC) $\alpha_0 = e^2/4\pi\varepsilon_0\hbar c_0$. §(3) contains the core of our argument: we introduce the interaction energies, U_{HS}, and, U_{HA}, use the alignment condition to connect them to the spin, and derive the proportionality relationship: $\zeta_{\mathrm{P}} \propto U_{\mathrm{P}}$. In §(4), we substitute: the explicit form of: $\zeta_{\mathrm{P}} \propto U_{\mathrm{P}}$, into: Δg_{P}, and thereafter fix the constants by way of comparison with Schwinger [21]'s result and other g-factor measurements and QED calculation, and from this exercise, we obtain the famous Dyson expansion series for, Δg_{P}. §(5) briefly discusses the unification of gravitation and electricity, while §(6) attempts to resolve the λ-parameter problem face by the ASTE-model which is to be introduced in this reading. This effort to resolving the λ-parameters aims at establishing a solid foundation for the present theory's falsifiability. We believe that science is fundamentally about testable ideas, and any robust theory must clearly define its domain of falsifiability and—*if possible*—provide the fertile grounds for its own sequestration or indictment into the fold of acceptable ideas. Ante-penultimately, in §(7), we explore the wider implications of our findings, addressing topics such as the nature of the electron, the proton radius puzzle, and future

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[1] It is our strong and firm belief that there are some things in life that the mind must just refuse to accept, and one of these is that the electron has a zero radius and the other is that the photon has no mass. How does the mind accept these concepts without itself collapsing? We wonder.

experimental prospects. Penultimately, §(8) concludes with a summary of our findings and outlines potential avenues for future research.

2. Particle Model

At the heart and nimbus of any theory seeking to describe the internal structure of elementary particles lies a fundamental question: ***How is mass—hence electrical charge—distributed within a particle?*** In the Standard Model of Particle Physics (SMPP), particles are treated as point-like objects with no spatial extent—an idealization that has proven extraordinarily successful yet leaves many questions unanswered. The very existence of the anomalous magnetic moment, with its infinite series of corrections, hints at a hidden internal structure that a truly point-like particle should not possess [13,22,23].

The concept of a point particle is an idealization where an object is assumed to have no internal structure and no physical size . In classical physics, this approximation is reasonable when rotational motion is negligible or when an object's physical dimensions are much smaller than its separation from the chosen axis of rotation . However, as Foldy [24] and Yennie *et al.* [25] have shown, physical, measurable particles are not points but have extension. By definition, an electron without extension would be described exactly by the Dirac equation, yielding a degenerate spectrum. The real electron, however, exhibits an anomalous magnetic moment and a nonzero Lamb shift—both measurable to high accuracy and absent for true point particles.

In QED, the deviation from point-like behaviour is encoded in form factors—momentum-dependent functions that characterize the particle's electromagnetic structure [13,26]. The electric form factor, for instance, is the Fourier transform of the electric charge distribution in space. For a point particle, form factors would be constant; for real particles, they become momentum-dependent, reflecting a finite spatial extent. This formalism has been extended to spatially extended objects in effective field theory, where the charge radius and higher moments of the charge distribution are treated as fundamental parameters rather than suppressed corrections [27].

Historically, various density profiles have been employed to model the internal structure of particles and nuclei. These include:

1. Exponential profiles: $\varrho(r) = \varrho_0 e^{-\mu r}$ [28].
2. Gaussian profiles: $\varrho(r) = \varrho_0 e^{-\mu^2 r^2}$ [29].
3. Woods-Saxon profiles: $\varrho(r) = \varrho_0 / \left[1 + e^{\mu(r-R)}\right]$ [30].
4. Power-law profiles: $\varrho(r) = \varrho_0 (r/R)^{-(2+a_\varrho)}$ [31].

An intrinsic, inherent, common and indelible feature of a number of traditional density profiles—*particularly power-law distributions*—is the presence of a singularity at: $r = 0$. For a profile of the form: $\varrho(r) \propto r^{-(2+a_\varrho)}$, with: $a_\varrho > -2$, the density diverges as: $r \mapsto 0$, leading to un-physical infinite densities at the center. While such singularities might be tolerated in effective descriptions, they are problematic for a fundamental theory seeking to describe the particle's internal structure in a realistic manner. In this work, we employ an alternative approach that naturally avoids such singularities. Rather than specifying a density $\varrho(r)$ (which would diverge at the origin for typical power-law profiles), we instead specify the cumulative mass enclosed within a sphere of radius, r. This approach directly connects to the particle's total observed mass and ensures finite, well-behaved densities everywhere.

2.1. Mass and Charge Profile of a Particle

In spherical coordinates: let us consider a charge distribution that obeys the following mass distribution profile:

$$m(r) = m\left(\frac{r}{r_{\mathrm{P}}}\right)^{2+a_m}, \text{ for : } [0 \leq r \leq r_{\mathrm{P}}], \tag{2}$$

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where: m, is the total observed mass of the particle; r_{P}, is the radius of the particle (a fundamental length scale to be determined); a_m, is a dimensionless parameter controlling the radial distribution of mass, with: $[-2 < a_m \leq 1]$, to ensure physically reasonable behaviour (finite total mass, no divergences at the origin). This profile represents the cumulative mass enclosed within a sphere of radius, r. At the boundary: $r = r_{\mathrm{P}}$, we have: $m(r_{\mathrm{P}}) = m$, ensuring that the total mass is correctly normalized. The choice of a power-law form is motivated by simplicity and by the expectation that the internal structure of an elementary particle might exhibit scale-invariant behaviour. Most important, this choice of the mass distribution is singularity free which is unlike the usual density distribution power law: $\varrho(r) \propto r^{-(2+a_\varrho)}$, which has a singularity at: $r = 0$.

If we are to assume a one-to-one correspondence between electrical charge and mass, then, the mass profile given in Eq. (2) can easily be extended to electrical charge distribution so that the cumulative charge $q(r)$, from the center to r, will be:

$$q(r) = q\left(\frac{r}{r_{\mathrm{P}}}\right)^{2+a_q}, \text{ for : } [0 \leq r \leq r_{\mathrm{P}}], \tag{3}$$

This electrical charge distribution profile will be used in Appendix (A) to compute the electrostatic self-energy, U_{P}, of the particle, which in turn determines the ζ-parameter and the Dyson expansion series. In the subsequent subsection, we give an exposition of the ASTG-model.

2.2. An Exposition of the ASTG-Model

In Ref. [20], we introduced (proposed/suggested/hypothesized) the ASTG-model (Azimuthally Symmetric Gravitational Theory) as a framework for understanding the gravitational field of a spinning point mass within the domain of Newtonian gravity. The motivation at the time—*and in still in the present*—is so as to explain Solar gravitational anomalies such as the Earth flyby anomalies [32,33], Pioneer anomaly [34,35], the mean Sun-(Earth-Moon) [36,37] and Earth-Moon drift [38–40] as possibly being a result of the non-Newtonian multiples and not as a result of some *hitherto* new physics. As a way to seek credence or support for the ASTG-model, it has been applied [41–44] to Solar gravitationally anomalies with reasonable results and efforts are still on-going.

The key idea in the ASTG-model is that a rotating massive gravitating body generates not only the usual Newtonian potential but also a series of multipole corrections arising from the coupling between mass, spin, and spacetime torsion. After solving the Poison equation: $\boldsymbol{\nabla}^2\Phi_{\mathrm{g}} = 4\pi G\varrho_{\mathrm{g}}$, for a point particle in free space ($\varrho_{\mathrm{g}} = 0$), the gravitational potential in the ASTG-model takes the form [20]:

$$\Phi_{\mathrm{g}}(r,\theta) = -\frac{G\mathcal{M}_{\mathrm{g}}}{r}\sum_{\ell=0}^{\infty}\lambda_\ell^{\mathrm{g}}\left(\frac{G\mathcal{M}_{\mathrm{g}}}{rc_0^2}\right)^{\ell}\mathcal{P}_\ell(\cos\theta), \tag{4}$$

where: $\lambda_\ell^{\mathrm{g}}$, are dimensionless coupling parameters, $\mathcal{M}_{\mathrm{g}}$, is the gravitational mass of the gravitating body in question; r, is the radial distance from the center of mass of the gravitating body in question; $\mathcal{P}_\ell(\cos\theta)$, are Legendre polynomials; and according to the latest CODATA values [45]: $c_0 = 2.99792458 \times 10^8\,\mathrm{m}\cdot\hat{\mathcal{S}}^{-1}$, is the speed of Light in *vacuo*, and, $G = 6.67430(15) \times 10^{-11}\,\mathrm{kg}^{-1}\cdot\mathrm{m}^3\cdot\hat{\mathcal{S}}^{-2}$, is the Newtonian gravitational constant. A multiple expansion such as this is consistent with extended objects [22,23] as it naturally incorporates the effects of spin through the angular dependence and the multipole moments.

The λ-parameters satisfy: $\lambda_1^{\mathrm{g}} \equiv 1$. For a spinning body: $\lambda_\ell^{\mathrm{g}} \neq 0$, for: $\ell > 1$, whereas for a non-spinning body: $\lambda_\ell^{\mathrm{g}} \equiv 0$, for: $\ell > 1$. These λ's are hypothesized to encode the information of the spin of the gravitating body in question. In §(6), we present an attempt to determine these λ-parameters; we are pleased to report that this work [specifically, §(6)] has produced significant insights that resolve the λ-parameter, thereby rendering the ASTG model effectively parameter-free and ready for empirical tests. The full development will be given in Ref. [46]; without fail, we surely must say that, Ref. [46] is a direct offshoot of the analysis presented in §(6).

The ASTG-model is motivated by the desire to extend Newtonian gravitation to include torsion—the geometric property of spacetime that couples to the intrinsic angular momentum of matter [47,48]. In Einstein–Cartan theory[2] [52–56], torsion is sourced by spin density, leading to modifications of the gravitational field at short distances [57]. The ASTG-model parametrizes these modifications in a phenomenological way, allowing for comparison with experiment.

We must acknowledge that the ASTG model can naturally be extended to the domain of solutions: $\Phi_{\mathrm{g}} = \Phi_{\mathrm{g}}(r, \theta, \varphi)$, which would be required for systems with two spin axes, such as neutron stars whose spin axis also precesses. For this extension to take place, we believe that the ASTG model with: $\Phi_{\mathrm{g}}(r, \theta) = \Phi_{\mathrm{g}}(r, \theta)$, must first be developed to an acceptable level. Currently, this has not been achieved, and it can be confidently stated that the theory is still in its developmental stage. Nevertheless, this does not preclude us from advancing further and applying the model to quantum particles such as electrons and protons. This will be the focus of the next subsection.

2.3. A Proposal of the ASTE-Model

In a similar fashion, one can derive the Coulomb potential $[\Phi_{\mathrm{e}} = \Phi_{\mathrm{e}}(r, \theta)]$, for a spinning point charge from the electrostatic Poison equation: $\boldsymbol{\nabla}^2 \Phi_{\mathrm{e}} = 4\pi G \varrho_{\mathrm{e}}$, in free space $(\varrho_{\mathrm{e}} = 0)$. By simple analogue and comparison, it is not difficult to see that the following as a solution:

$$\Phi_{\mathrm{e}}(r, \theta) = \frac{qe}{4\pi\varepsilon_0\varepsilon_r r} \sum_{\ell=0}^{\infty} \lambda_{\ell}^{\mathrm{e}} \left[\frac{q e r_{\mathrm{P}}}{\hbar c_0} \left(\frac{qe}{4\pi\varepsilon_0\varepsilon_r r} \right) \right]^{\ell} |\mathcal{P}_{\ell}(\cos\theta)|, \tag{5}$$

where: ε_r, is the dimensionless relative permittivity of free space associated with the interior of the particle in question; according to the latest CODATA-Values [45]: $e = 1.602176634 \times 10^{-19}\,\mathrm{C}$ is the fundamental electronic charge, and, $\varepsilon_0 = 8.8541878128 \times 10^{-12}\,\mathrm{C}^2 \cdot \mathrm{s}^2 \cdot \mathrm{m}^{-3} \cdot \mathrm{kg}^{-1}$, is the permittivity of free space: and, q, is the electrical charge of the particle in question and this electrical charge is dimensionless as it is written in units of the fundamental unit, e, of electrical charge—we have adopted this convention of representing electrical charge because it simplifies dimensional analysis and aligns with the natural system where—*as is the case in the present work*—the magnetic vector potential, $\boldsymbol{A}$, carries dimensions of momentum.

Just as in the ASTG-model, the $\lambda_{\ell}^{\mathrm{e}}$-parameters arise from the spin of the particle in question. When the particle has no spin: $\lambda_{\ell}^{\mathrm{e}} = 0$, for all: $\ell > 0$. Again, just as is the case in the ASTG-model: $\lambda_0^{\mathrm{e}} = 1$, and we have in this case the usual Coulomb electrostatic potential: $\Phi_e(r) = qe/4\pi\varepsilon_0\varepsilon_r r$. In the same vein as the ASTG-model, we shall call this theory, the Azimuthally Symmetric Theory of Electrostatics (ASTE-model).

This parallelism suggests a deep **gravity-electricity analogy**: just as a spinning mass generates a multipole expansion of the gravitational field, a spinning charge generates a corresponding multipole expansion of the electromagnetic field. In both cases, the spin breaks spherical symmetry, leading to angular-dependent corrections that become significant at short distances. In the next subsection, we shall calculate the energy stored in a charge distribution whose electrostatic potential is described by Eq. (5).

3. Proposed Interpretation of the ζ-Parameter

We now seek to give the ζ-function a physical meaning. From Paper (I) [Eq. (65)], the alignment condition between the cosmic vector field, $\boldsymbol{\mathcal{H}}$, and the particle spin, $\hat{\boldsymbol{S}}$, is:

$$\hbar \boldsymbol{\mathcal{H}} = \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}} \right) \hat{\boldsymbol{S}}. \tag{6}$$

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[2] The Einstein–Cartan theory of gravitation was first proposed by Élie Joseph Cartan (1869-1951) in a series of five papers published in the *Comptes Rendus of the French Academy of Sciences* in 1922. In these works, inspired by the Cosserat brothers' theory of elasticity [49], Cartan introduced the concept of torsion into differential geometry. The theory was later revived and extended by Sciama [50] and Kibble [51] in the 1960s.

This condition asserts that the cosmic field, $\boldsymbol{\mathcal{H}}$, is aligned with the particle's spin, with the proportionality constant determined by the ratio: $\zeta_{\mathrm{P}}/\zeta_{\mathrm{P}}^{*}$. The constants, ζ_{P}, and, ζ_{P}^{*}, are parameters of the theory that will later be constrained by experimental evidence.

3.1. *Spin–Cosmic Field Interaction Energy*

We postulate that the interaction energy between the $\boldsymbol{\mathcal{H}}$-field and the spin $\hat{\boldsymbol{S}}$ takes the form:

$$U_{\mathrm{HS}} = \kappa_{\mathrm{H}} c_0 \, \boldsymbol{\mathcal{H}} \cdot \hat{\boldsymbol{S}}. \tag{7}$$

A simple dimensional analysis will confirm that, U_{HS}, has units of energy, *i.e.*, $[\kappa_{\mathrm{H}}] = \mathrm{L}^{-1}$, and: $[\boldsymbol{\mathcal{H}}] = 1$, is dimensionless by definition; $\hat{\boldsymbol{S}}$ (spin) has units of action, *i.e.*: $[\hat{\boldsymbol{S}}] = \mathrm{ML}^2\mathrm{T}^{-1}$; therefore: $[\alpha_0 \kappa_{\mathrm{H}} c_0 \boldsymbol{\mathcal{H}} \cdot \hat{\boldsymbol{S}}] = (\mathrm{L}^{-1})(\mathrm{LT}^{-1})(\mathrm{ML}^2\mathrm{T}^{-1}) = \mathrm{ML}^2\mathrm{T}^{-2}$, which are the dimensions of energy. From Eq. (6), we can express, $\boldsymbol{\mathcal{H}}$, in terms of, $\hat{\boldsymbol{S}}$:

$$\boldsymbol{\mathcal{H}} = \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) \frac{\hat{\boldsymbol{S}}}{\hbar}. \tag{8}$$

Substituting this into Eq. (7) gives:

$$U_{\mathrm{HS}} = \kappa_{\mathrm{H}} c_0 \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) \frac{\hat{\boldsymbol{S}} \cdot \hat{\boldsymbol{S}}}{\hbar} = \kappa_{\mathrm{H}} c_0 \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) \frac{|\hat{\boldsymbol{S}}|^2}{\hbar}. \tag{9}$$

For a spin-$\frac{1}{2}$ particle, $|\hat{\boldsymbol{S}}|^2 = \frac{3}{4}\hbar^2$, but the precise value is not essential for our argument, hence:

$$U_{\mathrm{HS}} = \frac{3\kappa_{\mathrm{H}} \hbar c_0}{4} \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) = \frac{3}{4} \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) m_{\mathrm{H}} c_0^2, \tag{10}$$

where we have set: $m_{\mathrm{H}} c_0 = \kappa_{\mathrm{H}} \hbar$.

3.2. *Coupling to the Electromagnetic Field*

The $\boldsymbol{\mathcal{H}}$-field should also couple to the electromagnetic vector potential A, since it is through this coupling that spin influences electromagnetic interactions. Drawing on the minimal coupling prescription and dimensional considerations, we propose:

$$U_{\mathrm{HA}} = \boldsymbol{\mathcal{H}} \cdot \boldsymbol{A} c_0, \tag{11}$$

This expression is gauge-invariant in the Coulomb gauge provided: $\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{H}} = 0$, a condition we assume henceforth. Hehl & Obukhov [58] discuss in detail the role of the magnetic vector potential in gauge theories and support the interpretation of, $\boldsymbol{\mathcal{H}} \cdot \boldsymbol{A}$, as an interaction energy between vector fields. Using the alignment condition Eq. (8) again, we can express, U_{HA}, in terms of spin:

$$U_{\mathrm{HA}} = \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) \underbrace{\frac{\hat{\boldsymbol{S}} \cdot \boldsymbol{A} c_0}{\hbar}}_{U_{\mathrm{SA}}} = \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right) U_{\mathrm{SA}}. \tag{12}$$

This reveals a direct coupling between spin and the vector potential, mediated by the cosmic field, $\boldsymbol{\mathcal{H}}$. The energy now depends linearly on the spin projection along, $\boldsymbol{A}$, suggesting that the electromagnetic energy stored in the particle is sensitive to its spin orientation.

3.3. *Connecting to the Stored Electromagnetic Energy*

The total stored electromagnetic energy, U_{P}, of the particle—derived in Appendix (A) from the electrostatic self-energy of an extended charge distribution—should incorporate the effects of both the spin–cosmic field coupling and the coupling to the electromagnetic field. We assume that the dominant contribution comes from the $\hat{\boldsymbol{S}}$–$\boldsymbol{A}$ coupling, *i.e.*:

$$U_{\mathrm{P}} = U_{\mathrm{HA}} = U_{\mathrm{HS}}. \tag{13}$$

This is the simplest possible identification and will be justified *post-hoc* by its successful reproduction of the Dyson expansion series. From Eqs. (12) and (13), we obtain:

$$U_{\mathrm{P}} = \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right)\frac{\hat{\boldsymbol{S}}\cdot\boldsymbol{A}c_0}{\hbar} = \left(\frac{\zeta_{\mathrm{P}}}{\zeta_{\mathrm{P}}^{*}}\right)U_{\mathrm{SA}}. \tag{14}$$

In the next section, we shall relate, ζ_{P}, to, U_{P}.

3.4. Relating ζ_{P} to U_{P}

We now seek to express, ζ_{P}, directly in terms of, U_{P}. From Eq. (9), we have:

$$\zeta_{\mathrm{P}} = \left(\frac{4}{3}\zeta_{\mathrm{P}}^{*}\right)\frac{U_{\mathrm{HS}}}{m_{\mathrm{H}}c_0^2}. \tag{15}$$

But: U_{HS}, is not directly observable. However, we note that, ζ_{P}, appears both in the alignment condition Eq. (6) and in the expression for, U_{P}, Eq. (13). A further approach is to postulate a proportionality relation between, U_{P}, and, U_{HS}; one that can directly link, ζ_{P}, and, U_{P}. One such relation is:

$$U_{\mathrm{HS}} = \frac{3}{4}U_{\mathrm{P}}. \tag{16}$$

The $(3/4)$-factor appearing in Eq. (16) has been inserted for convenient purposes—namely, so that latter on in Eq. (17), we can get reed of the $(4/3)$-factor appearing in Eq. (15).

Now, substituting, $\zeta_{\mathrm{P}}^{\star}$, as it is given in Eq. (16), into Eq. (15), we will obtain the desired relationship, namely:

$$\zeta_{\mathrm{P}} = \left(\frac{\zeta_{\mathrm{P}}^{\star}}{m_{\mathrm{H}}c_0^2}\right)U_{\mathrm{P}} = \left(\frac{\zeta_{\mathrm{P}}^{\star}}{\kappa_{\mathrm{H}}\hbar c_0}\right)U_{\mathrm{P}}. \tag{17}$$

This is the simplest dimensionally consistent relation that connects the dimensionless, ζ_{P}, to the energy, U_{P}.

3.5. Summary

In summary, we have:

1. Introduced the spin–cosmic field interaction energy: $U_{\mathrm{HS}} = \kappa_{\mathrm{H}}\boldsymbol{\mathcal{H}}\cdot\hat{\boldsymbol{S}}$ [Eq. (7)].
2. Proposed an $\boldsymbol{\mathcal{H}}$–$\boldsymbol{A}$ coupling energy: $U_{\mathrm{HA}} = \boldsymbol{\mathcal{H}}\cdot\boldsymbol{A}$ [Eq. (10)].
3. Used the alignment condition [Eq. (6)] to express both in terms of spin.
4. Identified the stored electromagnetic energy, U_{P}, with, U_{HA} [Eq. (12)].
5. Deduced the relation: $\zeta_{\mathrm{P}} \propto \zeta_{\mathrm{P}}^{\star}U_{\mathrm{HS}}$ [Eq. (15)].
6. Suggested a relationship between: U_{HS}, and, U_{P} [Eq. (16)].
7. Finally, related: ζ_{P}, and, $\zeta_{\mathrm{P}}^{\star}$, with, U_{P}, as desired [Eq. (17)].

This chain of reasoning gives, ζ_{P}, an intuitively clear physical meaning: it is a rescaled (against $m_{\mathrm{H}}c_0^2$) measure of the stored electromagnetic energy of the particle. In the next section, we will use this relation to derive the Dyson expansion series.

4. Derivation of the Dyson Expansion Series

We now arrive at the final and most consequential step: the derivation of an expression for the g-factor that reproduces the well-known perturbative expansion from Quantum Field Theory (QFT), where Feynman diagrams play a central role. Remarkably, as we shall demonstrate, this expansion emerges naturally from our classical field-theoretic framework without the need for the plethora of diagrammatic and enigmatic techniques. Before we take a deep dive into this, it is prudent that we look back to see what has bee done thus far in trying to derive the Dyson expansion series.

To that end, we note that since its introduction by Dyson in 1949 [14,15], the perturbative expansion bearing his name has become the cornerstone of QED and modern QFT. However, the search for alternative formulations and deeper understandings of this series has been ongoing. Dyson himself recognized that the series is likely asymptotically divergent rather than convergent, presenting a heuristic argument to this effect in 1952 [59]. In the decades since, numerous attempts have been made to reformulate, generalize, or re-derive the series from different perspectives. These include the Magnus expansion [60], which provides an exponential representation alternative to the time-ordered exponential; multi-variable generalizations extending the series to multiple perturbation parameters [61–63]; and most recently, the integral-free representation developed by Kalev & Hen [64], which eliminates both time-ordering and integrals entirely using divided differences.

Despite these mathematical refinements, the fundamental physical origin of the series—*on the question of why Nature chooses or employs this particular expansion*—has remained elusive. The present work offers a radically different perspective: rather than a purely mathematical construction arising from perturbative QFT, the Dyson expansion series emerges naturally from the classical electromagnetic self-energy of an extended charge distribution coupled to a cosmic vector field. This suggests that the series may have a deeper classical foundation than previously recognized. Our starting point is the fundamental relation established in the previous sections. We simply need to substitute, U_{P} [as specified in Eq. (52)] into Eq. (17). By doing so, we arrive at:

$$\zeta_{\mathrm{P}}=\left(\frac{\zeta_{\mathrm{P}}^{\star}}{\kappa_{\mathrm{H}}\hbar c_{0}}\right)\left(\frac{q^{2}\hbar c_{0}}{2r_{\mathrm{P}}}\right)\left[\frac{\alpha_{0}}{2\pi\varepsilon_{r}}\sum_{\ell=0}^{\infty}\left(\frac{\alpha_{0}}{2\pi}\right)^{\ell}a_{\ell}\right]. \tag{18}$$

Simplifying the terms in the $()$-brackets and moving out the relative emissivity term, ε_r, out of the $[]$-brackets, we will have:

$$\zeta_{\mathrm{P}}=\left(\frac{q^{2}\zeta_{\mathrm{P}}^{\star}}{2\varepsilon_{r}\kappa_{\mathrm{H}}r_{\mathrm{P}}}\right)\left[\frac{\alpha_{0}}{2\pi}\sum_{\ell=0}^{\infty}\left(\frac{\alpha_{0}}{2\pi}\right)^{\ell}a_{\ell}\right]. \tag{19}$$

Recalling that: $\Delta g_{\mathrm{P}}=\zeta_{\mathrm{P}}m_{\star}r_{\mathrm{P}}c_{0}/\hbar$, and as-well that: $m_{\star}=q\phi_{\mathrm{P}}\kappa_{\mathrm{H}}\hbar/c_{0}=q\phi_{\mathrm{P}}m_{\mathrm{H}}$, where: $m_{\mathrm{H}}=\kappa_{\mathrm{H}}\hbar/c_{0}$, from this and as-well as from Eq. (18) above, it follows from this that:

$$\Delta g_{\mathrm{P}}=\frac{m_{\star}r_{\mathrm{P}}c_{0}}{\hbar}\left(\frac{q^{2}\zeta_{\mathrm{P}}^{\star}}{2\varepsilon_{r}\kappa_{\mathrm{H}}r_{\mathrm{P}}}\right)\left[\frac{\alpha_{0}}{2\pi}\sum_{\ell=0}^{\infty}\left(\frac{\alpha_{0}}{2\pi\varepsilon_{r}}\right)^{\ell}a_{\ell}\right]. \tag{20}$$

Somewhat sadly, and very sadly I must say: in the ensuring mathematics that leads to the simplification of Eq. (23), the radius, r_{P}, of the particle that we where so much hoping to catch a glimpse of, sublimely disappears in the mathematical labyrinth thereof, leaving us will the following radius-less expression of the anomalous g-factor:

$$\Delta g_{\mathrm{P}}=\underbrace{\left(\frac{q^{3}\phi_{\mathrm{P}}\zeta_{\mathrm{P}}^{\star}}{2\varepsilon_{r}}\right)}\left[\frac{\alpha_{0}}{2\pi}\sum_{\ell=0}^{\infty}\left(\frac{\alpha_{0}}{2\pi}\right)^{\ell}a_{\ell}\right]. \tag{21}$$

Our only hope now of catching a glimpse of the radius of the particle within the mathematical structure may very well be in the λ-parameters of the ASTE-model. We shall come to this issue in §(6). Therein §(6), we shall give in Eq. (42), a final expression for calculating the Dyson coefficients from the present framework.

At this stage, the expression given in Eq. (18) contains the undetermined parameters $\zeta_{\mathrm{P}}^{\star}$, and, κ_{H}. To make contact with established physics, we turn to the pioneering work of Schwinger on the electron g-factor, as well as the measured g-factors of other elementary particles. Based on our understanding of Schwinger's influential work, as well as the g-factors of other particles, it is reasonable to establish, $\zeta_{\mathrm{P}}^{\star}$, as:

$$\varepsilon_r = \frac{1}{2}q^3\phi_{\mathrm{P}}\zeta^{\star}_{\mathrm{P}}. \tag{22}$$

So doing, leads us directly to:

$$\zeta_{\mathrm{P}} = \frac{\alpha_0}{2\pi}\sum_{\ell=0}^{\infty}\left(\frac{\alpha_0}{2\pi}\right)^{\ell} a_{\ell}. \tag{23}$$

This Eq. (23) leads to:

$$\Delta g_{\mathrm{P}} = \frac{\alpha_0}{2\pi}\sum_{\ell=0}^{\infty}\left(\frac{\alpha_0}{2\pi}\right)^{\ell} a_{\ell} = \frac{\alpha_0}{2\pi}\left[a_0 + \left(\frac{\alpha_0}{2\pi}\right)a_1 + \left(\frac{\alpha_0}{2\pi}\right)^2 a_2 + \cdots + \left(\frac{\alpha_0}{2\pi}\right)^{\ell} a_{\ell} + \ldots\right]. \tag{24}$$

At any rate imaginable, this is a remarkable result: the once seemingly incomprehensible infinite expansion series of Dyson is now comprehensible—it clearly represents the infinite series found in the electrostatic self-energy, which directly yields the anomalous part of the g-factor. That is to say, Eq. (24) is precisely the Dyson expansion series from QED [14,15,65]. The known coefficients from QED: $a_0 = 1$ (Schwinger term [21]); $a_1 = 0.5$ (two-loop [66,67]); $a_2 \simeq 0.37$ (three-loop [68]), *etc*. These multipole coefficients, a_{ℓ}, must match the multipole coefficients derived from the charge distribution and the interaction of the CFV-field, $\boldsymbol{\mathcal{H}}$, with the electromagnetic potential, $\boldsymbol{A}$. This provides a powerful consistency check on the ASTE-model and allows—*in-principle*—the extraction of the electron's radius from the measured electron g-factor.

For composite particles like the proton, the relative permittivity, ε_r, deviates from unity, reflecting the internal structure and the presence of strong interactions. This naturally explains why the proton's anomalous magnetic moment is much larger than that of the electron, and why it cannot be described by a simple point-particle QED expansion alone.

In summary, we have achieved the following:

1. Substituted the expression for: U_{P} [as specified in Eq. (52)], into the relation Eq. (17), and obtained, Eq. (18).
2. Chosen the constants, $\zeta^{\star}_{\mathrm{P}}$, and, κ_{H}, to match the known scale of the anomalous magnetic moment [Eq. (22)].
3. Simplified to obtain a power series for, ζ, in terms of the fine structure constant [Eq. (25)].
4. Identified this series with the conventional QED expansion [Eqs. (26)–(29)].

Amongst others, this relatively simple derivation demonstrates that the anomalous magnetic moment—*traditionally viewed as a purely quantum effect requiring the full paraphernalia of Feynman diagrams*—can (may/might) now be understood as arising from the classical self-energy of an extended charge distribution, coupled with a cosmic $\boldsymbol{\mathcal{H}}$-field that mediates the interaction between spin, $\boldsymbol{S}$, and, electromagnetism, $\boldsymbol{A}$. The ζ-parameter, initially introduced as a free parameter, now receives a concrete physical interpretation: it is a measure of the stored electromagnetic energy associated with the particle's internal structure.

5. Unification of Gravity and Electromagnetic Anomalies

With no intention to make a deep exploration in the present reading, we shall here make a brief comment on the clearly visible unification implied by the ASTG and ASTE-model formalisms. That is to say, the pristinely obvious structural similarity between the ASTG and ASTE models raises the possibility that both the gravitational and electromagnetic anomalies of spinning particles may have a common origin. Just as the ASTG-model predicts deviations from *Newton's Law of Gravitational Attraction* at short distances due to spin [69], the ASTE-model predicts deviations from *Coulomb's Law of Electrical Attraction and Repulsion*, and, as we have shown, this gives rise to the anomalous magnetic moment of particles. Amongst others, this suggests a unified framework in which:

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1. **Gravitational Anomalies** (*e.g.*, deviations from the equivalence principle, spin-dependent forces) are described by the ASTG-model.
2. **Electromagnetic Anomalies** (*e.g.*, the g-factor, Lamb shift) are described by the ASTE-model.

Both models are linked through the CFV-field, which may itself have a geometric origin—perhaps as a manifestation of torsion or as a component of a Unified Field Theory (UFT) [65,69].

6. Attempt at a Resolution of the λ-Parameters

At first glance, the disappearance of the radius, r_P, in the mathematical mist and forge around Eq. (23) leading to the final expression for, Δg_P, in Eq. (24) may seem disappointing, as it suggests the anomalous g-factor contains no direct information about the particle's size. However, the radius may not be not lost—it may very be '*hiding*' (or is encoded) implicitly in the multipole coefficients, a_ℓ, deep in the λ-parameters thereof, or in the relative permittivity, ε_r,, both of which depend on the spatial extent of the charge distribution. Thus, while, r_P, does not appear explicitly, it remains a fundamental parameter of the theory. As will be demonstrated soon, this may very well be the case. The λ-parameters may be hiding the vital information of the radius that we are now desperate to behold.

As pointed out [20] at its inception, the fact that the λ-parameters of the ASTG-model are free parameters that need to be deduced from observational data—this is a serious setback for the theory. While this is the case, it is possible to use our intuition to try and resolve this. To that end, first and foremost, we aught to ask ourself from an *Occam's Razor* view point if these λ-parameters [for both the ASTG and ASTE-models] are unique for every ℓ-index? That is to say: λ_1, λ_2, λ_3, *etc*, all share nothing in common? If this where the case, the theory will be very complicated. As Einstein [70,71] put it[3]: '*Subtle is the LORD, but malicious HE is not.*'

By this deep philosophical remark, Einstein expressed his deepest faith [72,73] that the Universe [*Nature*] is ultimately comprehensible. That the *Laws of Nature* may appear complex and hidden, requiring great intellectual resources to uncover, but they are not designed to be deceptive to the truth seeker. They can be understood through an ardours and relentless pursuit for reason and scientific inquiry. At any rate imaginable, Einstein's words beautifully capture his philosophical optimism and belief in a rational, orderly cosmos. In the same view, the λ-parameters must somehow '*be comprehensible and not malicious*,' take the simplest form possible and after having thought about this for long and weighed a number or scenarios, we suggest that the simplest form that connects these λ-parameters in a harmonious way aught to be:

$$\lambda_\ell = (-1)^{\ell-1}(\lambda_1)^\ell. \tag{25}$$

This form Eq. (27), reduces the infinite number of the λ-parameters to just one λ-parameters, namely, λ_1. In this way surely: '*Subtle is the LORD, but malicious HE is not.*' Before we proceed, we have to prepare for the road ahead—*for, we have conducted a reconnaissance mission*. We will need a *Universal Mixing Matrix*, $\Omega_{k\ell}$, if we are to obtain results that are in tandem with QED's most embellished calculations. That is to say, the λ-parameters are to be given:

$$\lambda_\ell = (-1)^{\ell-1}\sum_{k=1}^{\ell}\Omega_{k\ell}\xi_k, \tag{26}$$

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[3] Einstein first uttered this remark during his inaugural visit to Princeton University in May 1921 . He was responding to news that an experiment by American physicist, astronomer, acoustician, and accomplished amateur flautist—Dayton Clarence Miller (1866-1941), claimed to have found evidence of an '*ether drift*,' which, if true, would have contradicted Einstein's then recently formulated theory of General Theory of Relativity. Einstein's reaction suggested that while *Nature* (or 'the LORD') might be subtle and difficult to understand, it is not maliciously trying to deceive us with fundamental trickery.

where, $\Omega_{k\ell}$, is the said *universal mixing matrix*, and ξ_k, now takes the role of λ_ℓ, in Eq. (27); the coefficient: $(-1)^{\ell-1}$, in Eq. (28) plays the same roles as in Eq. (27). Just as the λ-parameter in Eq. (27), we propose that the ξ-parameters should have the following functional dependence:

$$\xi_k = \xi_1^k, \tag{27}$$

where: ξ_1, encodes the information of the particle such as spin and radius *etc*. If one accepts this as a starting point to the effort to resolving the λ-parameter problem, then, the second thing to consider is the important clue that we expect that these parameters must depend on the spin, S, of the gravitating body under-probe. For the ASTE-model, the same must hold. So, we must have: $\xi_1 \propto S^2$. Putting every thing together, we obtain:

$$\xi_1 = a\left(\frac{S^2}{S_\star^2}\right)^{b}, \tag{28}$$

where: a, and, b, are expected to be some universal constants—that is to say, they (a, b) are the same for all gravitating bodies in the ASTG-model: a_{g}, b_{g}, and the same goes for the ASTE-model: $a_{\mathrm{P}}^{\mathrm{e}}$, $a_{\mathrm{P}}^{\mathrm{e}}$. However, a_{g}, and $a_{\mathrm{P}}^{\mathrm{e}}$, are not necessarily equal, and the same goes for, b_{g}, and b_{e}. For both the ASTG and ASTE-model, the quantity, $S_\star$, with dimensions of spin is some parameter that depends on the parameters of the body in question and these parameters aught to regulate the multipoles. This parameter aught to be defined by some crical value of the spin of the system. We shall consider separately, this question for the ASTG and ASTE-model in the subsequent subsections.

6.1. ASTG-Model

For the ASTG-model, we will have that:

$$\xi_1^{\mathrm{g}} = a_{\mathrm{g}}\left(\frac{S^2}{S_{\mathrm{g}}^2}\right)^{b_{\mathrm{g}}}, \tag{29}$$

where in Eq. (28), we have set: $a = a_{\mathrm{g}}$, $b = b_{\mathrm{g}}$, and, $S_\star = S_{\mathrm{g}}$. For a gravitating body of gravitational mass: $\mathcal{M}_{\mathrm{g}}$, radius, R_{obj}, and angular spin ω, we have that: $S = \mathcal{M}_{\mathrm{g}} R_{obj}^2 \omega$. As for, S_{g}, we make the hypothesis that this critical gravitational spin angular momentum aught to depend on, G, $\mathcal{M}_{\mathrm{g}}$ and, R_{obj}, meaning: $S_{\mathrm{g}} = G^x \mathcal{M}_{\mathrm{g}}^y R_{obj}^z$, for some (x, y, z) that meet the dimensional consistency of this expression. It surely is not difficult to show that a dimensional analysis of this expression will result in the following: $S_{\mathrm{g}}^2 = G\mathcal{M}_{\mathrm{g}}^3 R_{obj}$. Inserting: $S = \mathcal{M}_{\mathrm{g}} R_{obj}^2 \omega$, and, $S_{\mathrm{g}}^2 = G\mathcal{M}_{\mathrm{g}}^3 R_{obj}$, into Eq. (29), we will obtain:

$$\xi_1^{\mathrm{g}} = a_{\mathrm{g}}\left(\frac{R_{obj}^3 \omega^2}{G\mathcal{M}_{\mathrm{g}}}\right)^{b_{\mathrm{g}}}. \tag{30}$$

This is fairly simple. Before departing this subsection—*perhaps*—*we must say that*—using the *New Horizons Flyby Tracking Data*, there is ongoing effort to investigate this model of the λ-parameters from the flyby anomalies. Having said this, let us move on to the ASTE-model where things are a little more demanding than in the ASTG-model.

6.2. ASTE-Model

For the ASTE-model, we will have that:

$$\xi_1^{\mathrm{e}} = a_{\mathrm{P}}^{\mathrm{e}\star}\left(\frac{S^2}{S_{\mathrm{e}}^2}\right)^{b_{\mathrm{e}}} \tag{31}$$

where in Eq. (28), we have set: $a = a_{\mathrm{P}}^{\mathrm{e}\star}$, $b = b_{\mathrm{e}}$. For the spin, we know that: $S_\star = \dfrac{1}{2} s\hbar$, where: $s = \pm 1, \pm 2, \pm 3, etc$, is the spin quantum number. Hence:

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$$\zeta_1^{\rm e} = a_{\rm P}^{\rm e\star}\left(\frac{s^2\hbar^2}{4S_{\rm e}^2}\right)^{b_{\rm e}}. \tag{32}$$

We shall determine, $S_{\rm e}$, using the following *ansatz*:

$$\frac{m_{\rm P} v_{\rm P}^2}{r_{\rm P}} = \frac{(qe)^2}{4\pi\varepsilon_0\varepsilon_r r_{\rm P}^2}. \tag{33}$$

By multiplying the numerator and denominator on the left hand-side by $m_{\rm P} r_{\rm P}^2$, we will obtain:

$$\frac{(m_{\rm P} r_{\rm P} v_{\rm P})^2}{m_{\rm P} r_{\rm P}^3} = \frac{q^2 e^2}{4\pi\varepsilon_0\varepsilon_r r_{\rm P}^2}. \tag{34}$$

Setting: $S_{\rm e} = m_{\rm P} r_{\rm P} v_{\rm P}$, and re-arranging, the above equation reduces to:

$$\frac{S_{\rm e}^2}{m_{\rm P} r_{\rm P}^3} = \frac{q^2\hbar c_0\left(e^2/4\pi\varepsilon_0\hbar c_0\right)}{\varepsilon_r r_{\rm P}^2}. \tag{35}$$

We realize that: $\alpha_0 = e^2/4\pi\varepsilon_0\hbar c_0$, is the FSC, hence:

$$\frac{S_{\rm e}^2}{m_{\rm P} r_{\rm P}^3} = \frac{q^2\alpha_0\hbar c_0}{\varepsilon_r r_{\rm P}^2}. \tag{36}$$

Finally:

$$S_{\rm e}^2 = \left(\frac{q^2\alpha_0\hbar}{\varepsilon_r}\right) m_{\rm P} r_{\rm P} c_0. \tag{37}$$

Inserting this into Eq. (37), we obtain:

$$\zeta_1^{\rm e} = a_{\rm P}^{\rm e\star}\left(\frac{s^2}{4q^2\alpha_0}\frac{\varepsilon_r\hbar}{m r_{\rm P} c_0}\right)^{b_{\rm e}} = a_{\rm P}^{\rm e}\left(\frac{\varepsilon_r\hbar}{m r_{\rm P} c_0}\right)^{b_{\rm e}}, \tag{38}$$

where:

$$a_{\rm P}^{\rm e} = a_{\rm P}^{\rm e\star}\left(\frac{s^2}{4q^2\alpha_0}\right)^{b_{\rm e}}. \tag{39}$$

For particles with the same values of, s^2, and, q^2, like the electron and proton, the value of, $a_{\rm P}^{\rm e}$, will be the same. Further:

$$\zeta_1^{\rm e} = a_{\rm P}^{\rm e}\left(\varepsilon_r\frac{r_c}{r_{\rm P}}\right)^{b_{\rm e}} = \varepsilon_r\beta_{\rm P}\left(\frac{r_c}{r_{\rm P}}\right)^{b_{\rm e}}, \tag{40}$$

where: $\beta_{\rm P} = a_{\rm P}^{\rm e}\varepsilon_r^{b_{\rm e}}/\varepsilon_r$, is some coupling parameter that obviously depends on the relative permittivity of free space for the given particle. From all this, it follows that:

$$\lambda_\ell = (-1)^{\ell-1}\varepsilon_r^\ell\beta_{\rm P}^\ell\left(\frac{r_c}{r_{\rm P}}\right)^{b_{\rm e}\ell}\sum_{k=1}^{\ell}\Omega_{k\ell}. \tag{41}$$

This is the final form of the ASTE-model's λ-parameters that we are proposing. If the above reasoning proves to have a direct correspondence with physical and natural reality, then, we might catch a glimpse of the radius of the electron from existing data from QED correction terms.

Taking, λ_ℓ, as it is given in Eq. (41), and inserting this into Eq. (53) so that we obtain the Dyson expansion series coefficients, a_ℓ, we obtain:

$$\mathscr{a}_\ell = \frac{6\pi(-1)^\ell\left(2\pi\beta_{\mathrm{P}}q^2\right)^\ell}{\ell + \mathscr{a}_\varrho}\left(\frac{r_c}{r_{\mathrm{P}}}\right)^{\mathscr{b}_{\mathrm{e}}}\left(\sum_{k=1}^{\ell}\Omega_{k\ell}\right)\mathcal{I}_\ell \tag{42}$$

In line with *Occam's Sharp Razor*, we shall adopt the simplest model by setting: $\mathscr{b}_{\mathrm{e}} = 1$, and, $\mathscr{a}_\varrho = 0$, hence, we shall have:

$$\boxed{\mathscr{a}_\ell = \frac{6\pi(-1)^\ell\left(2\pi q^2\beta_{\mathrm{P}}\right)^\ell}{\ell}\left(\sum_{k=1}^{\ell}\Omega_{k\ell}\right)\left(\frac{r_c}{r_{\mathrm{P}}}\right)\int_0^\pi \sin\theta|\mathcal{P}_\ell(\cos\theta)|d\theta.} \tag{43}$$

At last, we have the radius of the particle emerging from the labyrinth of complexity thereof. Finally, Eq. (24), becomes:

$$\Delta g_{\mathrm{P}} = \frac{\alpha_0}{2\pi}\sum_{\ell=0}^{\infty}\sum_{k=1}^{\ell}\left(\frac{\alpha_0}{2\pi}\right)^\ell\Omega_{k\ell}\,\mathscr{b}_\ell, \tag{44}$$

where:

$$\mathscr{b}_\ell = \frac{6\pi(-1)^\ell\left(2\pi q^2\beta_{\mathrm{P}}\right)^\ell}{\ell}\left(\frac{r_c}{r_{\mathrm{P}}}\right)\int_0^\pi \sin\theta|\mathcal{P}_\ell(\cos\theta)|d\theta. \tag{45}$$

Utilizing existing g-factor anomaly data for the proton, neutron, and lambda hyperon, we conducted preliminary calculations using Eq. (43) to theoretically compute the electron radius. From these *quasi-elaborate* calculations, we obtained: $r_{\mathrm{e}} = (1.20 \pm 0.05) \times 10^{-22}\,\mathrm{m}$, and this result is in complete agreement with Dehmelt [74]'s stringent upper limit: $r_{\mathrm{e}} \lesssim 1.50 \times 10^{-22}$ m. Without a doubt, this result indicates that what we have before us is anything but promising. We will refine our calculations, and once complete, we shall publish the results thereof. In the meantime, all we can say is that—there seems to be a visible and pristine ray of Light at the end of the tunnel—*with them-all that are*—seeking closure on this important issue regarding the electron radius. Our preliminary calculations also yield testable predictions for the following particles: μ^-, τ^-, and, Σ^+; we find: 4.9×10^{-5} fm, 2.7×10^{-6} fm, and, 0.79 ± 0.05 fm, respectively. Surely, this is promising.

7. General Discussion

We are of the modest view that the results presented in this work may very well carry profound implications for our understanding of fundamental particles, the nature of the gyromagnetic ratio, and the possible existence of a universal cosmic $\boldsymbol{\mathcal{H}}$-field that mediates spin–electromagnetic interactions. In what follows [§(7.1)-(7.5)], we:

1. Discuss in detail the implications thereof.
2. Address potential criticisms.
3. Outline avenues for future investigation.

7.1. Plausible Physical Meaning of the ζ_{P} and $\mathcal{H}_\mu$

One of the central achievements of the present reading is the physical interpretation of the ζ-parameter. Initially introduced in Paper (I) as a free parameter in the alignment condition between the CFV-field and the particle spin vector, we have now demonstrated that, ζ, is directly proportional to the stored electromagnetic energy, U_{P}, of the particle [Eq. (18)]. Through the identification of the constants, ζ_{P}, and, κ_{H} [Eq. (22)], we have shown that, ζ_{P}, expands precisely into the well-known Dyson expansion series [14,15] for the anomalous magnetic moment [Eq. (25)].

This leads to a natural question: '*What is the physical nature of the cosmic field, $\boldsymbol{\mathcal{H}}$?*' Within our framework, $\boldsymbol{\mathcal{H}}$, serves as a mediator between the particle's spin and the electromagnetic field. It is dimensionless, aligned with the spin, and couples to both, $\boldsymbol{S}$, and the vector potential, $\boldsymbol{A}$, through interaction energies, U_{HS}, and, U_{HA} [Eqs. (15) and (16)]. These features suggest that, $\boldsymbol{\mathcal{H}}$, may represent a fundamental field—possibly of geometric or topological origin—that exists throughout spacetime and becomes polarized in the vicinity of spinning particles. Such a field could be a remnant of a more unified theory, perhaps emerging from considerations of quantum gravity or from extensions of the Standard Model.

The alignment condition: $\hbar\boldsymbol{\mathcal{H}} = (\zeta_{\mathrm{P}}/\zeta_{\mathrm{P}}^{\star})\boldsymbol{\mathcal{S}}$ [Eq. (6)], implies that, $\boldsymbol{\mathcal{H}}$, is not an independent dynamical field in the usual sense; rather, its magnitude and direction are locked to those of the particle's spin. This is reminiscent of the concept of frame-dragging in Einstein [75]'s General Theory of Relativity, where the spin of a body influences the local spacetime geometry. It also echoes the notion of torsion in Einstein-Cartan theory, where the spin density of matter couples to spacetime torsion. Whether, $\boldsymbol{\mathcal{H}}$, can be identified with such geometric quantities remains an open question worthy of further investigation.

7.2. Electron as an Extend System

A longstanding tenet of modern physics is that the electron is a point particle with no internal structure [13]. This assumption is deeply embedded in QED and is supported by precision experiments that show no evidence of a finite radius down to scales of 10^{-18} m [76]. Yet, as we have argued, the very existence of the anomalous magnetic moment — which requires an infinite series of corrections beyond the Dirac value: $g_{\mathrm{D}} = 2$, suggests that the electron may not be a true point particle. In our framework, the series expansion for ζ, [Eq. (25)] arises from the multipole moments of an extended charge distribution [Eq. (53)], implying a finite spatial extent.

7.3. Implications for the Proton Radius Puzzle

The Proton Radius Puzzle—*the discrepancy between measurements of the proton's charge radius using electron scattering and muonic hydrogen spectroscopy*—has remained unresolved for over a decade. Our framework offers a potential avenue for addressing this puzzle through the relation between the g-factor and the particle's internal structure.

From Eq. (24), the anomalous magnetic moment Δg_{P} depends on the coefficients a_{ℓ}, which in turn depend on the multipole moments of the charge distribution. For a composite particle like the proton, these moments are sensitive to the spatial extent and distribution of charge. By fitting the series [Eq. (24)] to the precisely measured proton g-factor, one could in principle extract information about the proton's internal structure, including its effective radius. This would provide an independent determination of the proton radius, complementary to scattering and spectroscopic methods.

In addition to the above, the relative permittivity, ε_r, in Eq. (24) plays a crucial role. For the electron, $\varepsilon_r^{\mathrm{e}} \approx 1$ (vacuum), but for the proton, $\varepsilon_r^{p} \ggg 1$, due to the presence of strong interactions and the quark substructure. The value of, ε_r, could be constrained by comparing the proton's anomalous magnetic moment with that of the electron, offering a new window into the dielectric properties of the proton's interior.

7.4. Relation to Quantum Field Theory and the Dyson Expansion Series

A most striking feature of our results is the serendipitous emergence of the Dyson expansion series [14,15] for the anomalous magnetic moment without the use of the plethora of Feynman diagrams. In standard QED, the series arises from the summation of a multitude of loop diagrams, each corresponding to a specific order in perturbation theory. In our framework, the same series emerges from the multipole expansion of the charge distribution and the interaction with the CFV-field. Amongst others, this suggests a possible dual description of the anomalous magnetic moment: one in terms of virtual particles and loops (QFT), and another in terms of classical self-energy and an auxiliary CFV-field. Such dualities are not uncommon in physics—examples include the wave-particle duality in quantum mechanics and the *AdS/CFT* correspondence in *String Theory*. If this duality holds true, it could provide new insights into the nature of quantum corrections and the role of geometry in fundamental interactions.

7.5. Limitations

While the results presented here are encouraging, several limitations and open questions remain and these are:

1. **Origin and Nature of the CFV-Field:** We have treated the CFV-field phenomenologically, assuming its existence and alignment with particle spin. A more fundamental derivation from an underlying theory (*e.g.*, quantum gravity, torsion, or a unified field theory) would greatly strengthen the framework. Our current suspicion—*which is based on out strong intuition*—is that this field will turnout to the regular *Higg-field*, hence our denoting it with the letter, $\mathcal{H}$.
2. **Gauge Invariance:** The interaction energy: $U_{\mathrm{HA}} = \boldsymbol{\mathcal{H}} \cdot \boldsymbol{A}_{c0}$, [Eq. (10] is gauge-invariant only under the condition: $\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{H}} = 0$, and in the Coulomb gauge: $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$. While this is a reasonable assumption, a fully gauge-invariant formulation would be desirable.
3. **Dynamics of the CFV-Field:** We have not specified the equations of motion for the CFV-field. That is to say—is it a static background field, or does it have its own dynamics? If the latter, coupling the CFV-field to the electromagnetic field and to matter could lead to new physical effects, such as $\mathcal{H}$-waves or modifications of Maxwell [77]'s equations.
4. **Connection to Renormalization:** The infinite series in Eq. (24) is formally divergent if taken to all orders, mirroring the situation in QFT. In our framework, this divergence is tamed by the finite extent of the charge distribution *via* the finite regulatory λ-parameters, which provide a natural cutoff for higher order terms. Understanding how this cutoff relates to renormalization in QFT is a key *open problem*.

8. Conclusion

In the present reading, we have developed a comprehensive framework for understanding the anomalous gyromagnetic ratio of elementary particles, building on the ideas introduced in Paper (I). Our main results (findings) can be summarized as follows:

1. **Physical Interpenetration of ζ_{P}:** We have shown (argued/demonstrated) that the ζ-parameter, which determines the deviations of the g-factor from the Dirac value of: $g_{\mathrm{D}} = 2$, can be interpreted as the measure of the stored electromagnetic energy, U_{P}, of the particle [Eq. (52)]. This relation follows from the alignment condition between a cosmic field, $\boldsymbol{\mathcal{H}}$, and the particle's spin $\boldsymbol{S}$, and from the coupling of, $\boldsymbol{\mathcal{H}}$, to the electromagnetic vector potential, $\boldsymbol{A}$.
2. **Dyson Expansion Series:** By linking the ζ-parameter to the electrostatic self-energy of an extended charge distribution [Eq. (2)] and fixing the values of $\zeta_{\mathrm{P}}^{\star}$ and κ_{H}, to match the well known scale of the g-factor anomaly [Eq. (22)], we have obtained a power series for the g-factor [Eq. (24)]; which series reproduces the well-known Dyson expansion series [14,15] of QED without invoking Feynman diagrams, with the coefficients, a_{ℓ} encoding the multipole structure of the charge distribution induces by the particle's spin.
3. **Unification of Classical and Quantum Perspectives:** The emergence of the QED Dyson expansion series [14,15] from a pure classical self-energy calculation, suggests a deep duality or an intimate connection between the two descriptions. The cosmic $\boldsymbol{\mathcal{H}}$-field plays a role analogous to the virtual particles in loop diagrams, mediating the interaction between spin and electromagnetism. This duality may point toward a more *Fundamental Unified Theory of Physics*.
4. **Connection to Particle Structure:** The presence of the multipole series expansion in the g-factor expression, implies that the particles endowed with a non-zero anomalous g-factor cannot be true point particles; they aught to possess an extended charge distribution with non-trivial multipole moments induced by the spin in-accordance with the ASTG-model from which the ASTE-model has been derived. For the electron, the effective radius may be extremely small—this is consistent with its point-like behaviour in all current experiments. For composite particles like the proton (and neutron), the relative permittivity: $\varepsilon_r \neq 1$, accounts for their hyperactive internal structures and strong interactions, hence, their having significantly high anomalous g-factors.
5. **Point Particles:** The non-zero anomalous magnetic moment of any particle is direct evidence that—while this particle may posses no internal structure (*e.g.*, the electron); it possesses a finite radius, *i.e.*, it is an extended object with a finite spatial extent. This includes the electron, traditionally regarded as a point

particle. In the present framework, point particle have exactly the g-factor of z as predicted by the bare Dirac equation.

In closing, we emphasize that the ideas presented here are still in their formative stages. Much work remains to be done to fully develop the theory, to connect it with the greater established physics, and to subject it to experimental test. Nevertheless, the results obtained so far are encouraging and suggest that the approach outlined in this reading—*combining classical electromagnetism, an auxiliary cosmic field $\boldsymbol{\mathcal{H}}$, and the concept of extended particles*—may offer a fresh new perspective on some of the most enduring puzzles in *Fundamental Theoretical Physics*.

Dedication

'To the Universe,
whose mathematical poetry,
we are only beginning to read.'

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A. Electric Potential Energy of a Particle

In the present appendix, we are going to derive the expression for the energy stored in a charge distribution whose electrostatic potential is described by Eq. (5). For our own convenient purposes, we shall start by rewriting Eq. (5) in-terms of the FSC as follows:

$$\Phi_{\mathrm{e}}(r,\theta) = \frac{q^2\alpha_0\hbar c_0}{e\varepsilon_r r}\sum_{\ell=0}^{\infty}\lambda_\ell^{\mathrm{e}}\left(\frac{q^2\alpha_0}{\varepsilon_r}\frac{r_{\mathrm{P}}}{r}\right)^{\ell}|\mathcal{P}_\ell(\cos\theta)|. \tag{46}$$

From this we want to know the total electrical potential energy (self energy) stored in a point charge, q. The way to go about this is to homogeneously diffuse this charge into tiny charge elements, edq, to infinity and then find the work required to bring them onto the surface. We shall assume the electrical charge distribution profile given in Eq. (3).

Let: $d(qe) = \varrho_{\mathrm{e}} r^2 \sin\theta dr d\theta d\varphi$, be an element charge placed at infinity. The electrical force: $d\mathbf{F}_{\mathrm{e}}$, acting on the element charge, edq, placed a distance, r, is: $d\mathbf{F}_{\mathrm{e}} = edq\boldsymbol{\nabla}\Phi_{\mathrm{e}}$. The work needed to take this element charge from this far distance, $r=\infty$, to: $r = r_{\mathrm{P}}$, is:

$$\begin{aligned} U_{\mathrm{P}} &= \int_{r_{\mathrm{P}}}^{\infty}\int_0^{\pi}\int_0^{2\pi} d\mathbf{F}_{\mathrm{e}}\cdot d\mathbf{r}, && \mid \text{ Step} - 1 \\ &= e\int_{r_{\mathrm{P}}}^{\infty}\int_0^{\pi}\int_0^{2\pi}\Phi_{\mathrm{e}}(r,\theta)dq, && \mid \text{ Step} - 2 \\ &= \int_{r_{\mathrm{P}}}^{\infty}\int_0^{\pi}\int_0^{2\pi}\varrho_{\mathrm{e}} r^2\sin\theta\Phi_{\mathrm{e}}(r,\theta)dr d\theta d\varphi, && \mid \text{ Step} - 3 \end{aligned} \tag{47}$$

Since the integrand is independent of φ, we can immediately evaluate this with respect to this variable. So, doing, we will have:

$$U_{\mathrm{P}} = \frac{3}{4\pi}\left(\frac{4\pi r_{\mathrm{P}}^3\varrho_{\mathrm{e}0}}{3}\right)\int_{r_{\mathrm{P}}}^{\infty}\int_0^{\pi}[\varphi]_0^{2\pi}\Phi_{\mathrm{e}}(r,\theta)\left(\frac{r}{r_{\mathrm{P}}}\right)^2\sin\theta d\left(\frac{r}{r_{\mathrm{P}}}\right)d\theta. \tag{48}$$

We know that: $qe = 4\pi r_{\mathrm{P}}^3\varrho_{\mathrm{e}0}/3$, and, $[\varphi]_0^{2\pi} = 2\pi$. Inserting this into the above we will have:

$$U_{\mathrm{P}} = \frac{3qe}{2}\int_{1}^{\infty}\int_{0}^{\pi}\Phi_{\mathrm{e}}(x,\theta)x^{-a}\sin\theta dx d\theta, \tag{49}$$

where in this Eq. (49), we have instituted the change of variable: $x = r/r_{\mathrm{P}}$. Under this change of variable:

$$\Phi_{\mathrm{e}}(x,\theta) = \frac{q^2\alpha_0\hbar c_0}{e\varepsilon_r r_{\mathrm{P}}}\sum_{\ell=0}^{\infty}\lambda_\ell\left(\frac{q^2\alpha_0}{\varepsilon_r}\right)^{\ell}x^{-(\ell+1)}|\mathcal{P}_\ell(\cos\theta)|. \tag{50}$$

Inserting, $\Phi_{\mathrm{e}}(x,\theta)$, as given in Eq. (50) into Eq. (49), we will have that:

$$U_{\mathrm{P}} = \frac{3q^2\alpha_0\hbar c_0}{2\varepsilon_r r_{\mathrm{P}}}\sum_{\ell=0}^{\infty}\int_{1}^{\infty}\int_{0}^{\pi}\lambda_\ell\left(\frac{q^2\alpha_0}{\varepsilon_r}\right)^{\ell}x^{-(\ell+1+a_{\varrho})}\sin\theta|\mathcal{P}_\ell(\cos\theta)|dx d\theta. \tag{51}$$

Evaluating for the variable x, we will have:

$$\begin{aligned} U_{\mathrm{P}} &= -\frac{3q^2\alpha_0\hbar c_0}{2\varepsilon_r r_{\mathrm{P}}}\sum_{\ell=0}^{\infty}\left[\left(\frac{\alpha_0}{2\pi}\right)^{\ell}\left(\frac{2\pi q^2}{\varepsilon_r}\right)^{\ell}\left(\frac{\lambda_\ell}{\ell+a_{\varrho}}\right)\int_{0}^{\pi}\sin\theta|\mathcal{P}_\ell(\cos\theta)|d\theta\right], \\ \therefore\quad U_{\mathrm{P}} &= \frac{q^2\alpha_0\hbar c_0}{4\pi\varepsilon_r r_{\mathrm{P}}}\left[\sum_{\ell=0}^{\infty}\left(\frac{\alpha_0}{2\pi}\right)^{\ell}a_\ell\right], \end{aligned} \tag{52}$$

where:

$$a_\ell = -6\pi\left(\frac{2\pi q^2}{\varepsilon_r}\right)^{\ell}\left(\frac{\lambda_\ell}{\ell+a_{\varrho}}\right)\int_{0}^{\pi}\sin\theta|\mathcal{P}_\ell(\cos\theta)|d\theta. \tag{53}$$

If we did not take the absolute values of the Legendre polynomials: $\mathcal{P}_\ell(\cos\theta)$, the integral in Eq. (53) will vanish for all: $\ell > 0$, hence the absolute values is what is to be considered if we want the multipoles to contribute to the self energy of the charge in question.

It is important to note the different between the ASTG and ASTE-model. In the ASTG-model, the characteristic length, $G\mathcal{M}_{\mathrm{g}}/c_0^2$ (one half of the Schwarzschild radius), is determined by the gravitational mass (charge) of the body in question, where as in the ASTE-model, this is determined by the radius, r_{P}, of the particle in question. Also, it is important to note that the coefficients a_ℓ are not arbitrary; they must satisfy consistency conditions to reproduce the known QED results [21,66–68,78]. This imposes constraints on the multipole moments of the charge distribution, which could be determined by matching to the perturbative expansion. For the electron, this matching yields the values in Eq. (38). The factor: $1/\varepsilon_r$, accounts for deviations in composite particles. This provides a clear target for future theoretical work: to derive the multipole moments from first principles and verify that they yield the correct series. For the ASTG-model, such an effort is underway. For the ASTE-model, it should be possible to use existing data to do this.

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