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*Article*

# A Median-Centered Sequential Monitoring Scheme Based on Golden Ratio Weighting for Skewed Distributions

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## Highlights

**What are the main findings?**

- A median-centered control chart with golden ratio weighting is proposed.
- Rank-based weighting reduces tail influence in skewed distributions.
- Golden Ratio (GR) chart shows stable in-control behavior under right skewness.

**What are the implications of the main findings?**

- Provides a robust alternative to EWMA and CUSUM for skewed processes.
- Improves false alarm resistance without distributional transformations.
- Offers a distribution-aware framework for sequential monitoring.

## Abstract

Detecting small location shifts in stochastic processes is a fundamental problem in sequential statistical monitoring. Classical procedures such as Shewhart-type schemes, exponentially weighted moving average (EWMA), and cumulative sum (CUSUM) methods are known to perform well under normality or near-symmetry assumptions; however, their effectiveness may deteriorate substantially in the presence of right-skewed distributions. In such settings, mean-based monitoring statistics are highly sensitive to tail behavior, which may result in delayed detection of small shifts or increased false alarm rates. This paper introduces a novel monitoring scheme, referred to as the Golden Ratio (GR) control chart, designed for detecting small location shifts in right-skewed distributions. The proposed method is constructed using a median-centered statistic combined with a geometrically decaying weighting mechanism derived from the golden ratio. Unlike classical time-based weighting schemes, the GR chart assigns weights according to the rank-based distance from the sample median, thereby attenuating the influence of isolated extreme observations while enhancing sensitivity to persistent distributional shifts. Theoretical properties of the proposed monitoring statistic are investigated, and its run-length behavior is analyzed under non-normal distributions. The performance of the GR chart is evaluated through extensive Monte Carlo simulations and is compared with classical EWMA and CUSUM procedures under Gamma models. The results indicate that the proposed method provides a robust and stable alternative for monitoring skewed processes while maintaining competitive sensitivity to small location shifts. Overall, the GR control chart offers a distribution-aware and theoretically grounded framework for sequential monitoring in asymmetric stochastic environments.

**Keywords:** sequential statistical monitoring; golden ratio; median-based statistic; EWMA; small location shifts; skewed distributions

**MSC:** Primary 62L10; Secondary 62P30, 62F35

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## 1. Introduction

Sequential statistical monitoring schemes play a fundamental role in detecting distributional changes in stochastic processes. In particular, procedures designed to identify small shifts in a location parameter have attracted sustained attention due to their relevance in a wide range of applied and theoretical contexts. Classical approaches, such as Shewhart-type schemes, are well known for their effectiveness in detecting large and abrupt changes; however, their performance deteriorates substantially when changes are small or evolve gradually over time [1].

To address this limitation, cumulative monitoring schemes such as the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) procedures have been extensively studied [2,3]. These methods exploit temporal aggregation to enhance sensitivity to persistent changes and are now regarded as classical tools in sequential statistical analysis. Nevertheless, both EWMA and CUSUM rely fundamentally on mean-based statistics and are typically developed under assumptions of normality or near symmetry [4]. When the underlying distribution is asymmetric or heavy-tailed, these assumptions may no longer be appropriate, and the resulting monitoring statistics can exhibit distorted behavior, including delayed detection of small shifts or an increased rate of false alarms [5,6].

Non-normal and positively skewed distributions arise naturally in many stochastic models, and their presence motivates the development of distribution-aware monitoring procedures [7]. In such settings, the sample mean is particularly sensitive to tail behavior, and even isolated extreme observations may exert disproportionate influence on cumulative statistics. As a result, a growing body of literature has explored robust and alternative monitoring schemes based on resistant estimators, including median-based and quantile-based statistics [8–10], as well as adaptive or bounded-influence variants of classical EWMA-type procedures [11,12]. While these approaches improve resistance to outliers, they typically preserve the time-based exponential weighting structure and remain centered on mean-type summaries, limiting their ability to distinguish persistent small location shifts from intrinsic tail variability in skewed distributions.

A complementary perspective is to reconsider not only the choice of the centering statistic but also the structure of the weighting mechanism itself. In classical sequential schemes, weights are assigned according to temporal order, reflecting the recency of observations rather than their positional relevance within the distribution. For asymmetric distributions, however, observations near the center of the distribution may carry more reliable information about gradual location changes than extreme tail values [13]. This observation motivates the construction of monitoring statistics that are both median-centered and distribution-aware, assigning weights according to distance from a central order statistic rather than time.

In this study, we introduce a new sequential monitoring scheme, referred to as the Golden Ratio (GR) control chart, designed for detecting small location shifts in right-skewed distributions. The proposed method combines median centering with a geometrically decaying weighting mechanism based on the golden ratio, yielding a structured attenuation of tail contributions while preserving information from the central region of the distribution [14,15]. Observations are weighted according to their rank-based distance from the sample median, inducing a rank-based weighting structure that suppresses tail influence while retaining sensitivity to gradual location changes. This weighting scheme provides a mathematically transparent alternative to time-based exponential smoothing and aligns naturally with order-statistic representations of skewed distributions.

In addition, the proposed GR scheme incorporates a run-based signaling rule, requiring sustained exceedance of a control threshold before an alarm is issued. Run-based decision rules have long been recognized as an effective mechanism for suppressing spurious signals caused by isolated fluctuations while maintaining sensitivity to persistent changes [16]. Theoretical properties of the resulting monitoring statistic are examined, and its run-length behavior is investigated under non-normal distributions using simulation-based and reliability-oriented formulations [17].

The performance of the proposed GR chart is evaluated through extensive Monte Carlo experiments under right-skewed distributional models. Comparisons are provided with classical

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EWMA and CUSUM procedures calibrated to identical in-control run-length properties. The results demonstrate that the GR chart achieves improved robustness under skewness while maintaining enhanced sensitivity to small location shifts.

The remainder of the paper is organized as follows. Section 2 reviews relevant background on sequential monitoring under non-normality and motivates the proposed approach. Section 3 introduces the construction of the GR control chart. Section 4 presents theoretical considerations and run-length analysis. Section 5 reports simulation results, and Section 6 concludes with final remarks and directions for future research.

## 2. Materials and Methods

### *2.1. Sequential Monitoring under Non-Normality*

Sequential monitoring procedures are widely used to detect distributional changes in stochastic processes. Classical schemes such as Shewhart-type charts, EWMA, and CUSUM have been studied extensively and are known to provide effective detection of location shifts under normal or near-symmetric distributional assumptions [1–4]. In particular, cumulative schemes exploit temporal aggregation to enhance sensitivity to small and persistent changes [2,3].

However, when the underlying distribution deviates from normality, especially in the presence of skewness or heavy tails, the performance of these classical procedures may deteriorate substantially. Several studies have demonstrated that non-normality can distort the run-length behavior of mean-based monitoring statistics, leading to delayed detection or inflated false alarm rates [5–8]. This issue is particularly pronounced for right-skewed distributions, where upper-tail observations occur more frequently and may exert disproportionate influence on cumulative statistics.

These limitations motivate the development of monitoring procedures that explicitly account for distributional asymmetry rather than relying on normal approximations or ad hoc transformations.

### *2.2. Median-Based and Robust Monitoring Statistics*

To mitigate the sensitivity of mean-based procedures to non-normality and outliers, a growing body of literature has proposed monitoring schemes based on robust location estimators. Among these, the sample median has attracted particular attention due to its resistance to extreme observations and its stability under skewed distributions [9–11]. Median-based monitoring schemes have been shown to provide improved robustness, particularly when a fixed in-control reference median is employed to detect systematic location changes.

Related approaches include control charts based on robust scale estimators, such as the median absolute deviation (MAD), and loss-based formulations that down-weight extreme observations [10,14]. While robust scale-based charts primarily target variability changes, the present study focuses on detecting small location shifts under skewness.

Nevertheless, most median-based and robust schemes retain the traditional time-based weighting paradigm inherent in EWMA-type procedures. As a result, although robustness to isolated outliers is improved, the underlying structure of temporal weighting remains unchanged.

### *2.3. Limitations of Time-Based Weighting Schemes*

In classical sequential monitoring procedures, weighting mechanisms are predominantly time-based: recent observations receive greater weight, while older observations are exponentially down-weighted [2,17]. This structure is well suited for capturing temporal dependence and gradual shifts in symmetric settings. However, it does not explicitly distinguish between observations that are informative about location changes and those that primarily reflect tail variability.

For right-skewed distributions, extreme observations in the upper tail may arise naturally from the distribution rather than from genuine shifts in the location parameter. Time-based weighting schemes treat such observations similarly to central observations, potentially amplifying tail noise and obscuring small but persistent distributional shifts. Robust variants of EWMA mitigate this issue by bounding the influence of extreme values [12,13], yet they continue to rely on temporal ordering as the primary weighting criterion.

These observations suggest that robustness alone is insufficient if the weighting structure itself does not incorporate information about the positional relevance of observations within the distribution.

### *2.4. Motivation for Distribution-Aware Weighting*

An alternative perspective is to construct monitoring statistics that are not only robust in terms of centering but also distribution-aware in terms of weighting. From an order-statistic viewpoint, observations closer to the center of a skewed distribution may provide more reliable information about gradual location changes than those in the tails. Consequently, assigning weights according to distance from a central order statistic, rather than temporal order, may enhance sensitivity to persistent shifts while suppressing the influence of isolated extremes.

This idea motivates the proposed median-centered monitoring scheme with golden ratio–based geometric weighting. The golden ratio induces a natural and mathematically transparent geometric decay structure, yielding a hierarchy of weights that decreases smoothly with rank-based distance from the median [15,16]. By combining median centering with distribution-aware weighting, the proposed approach departs fundamentally from classical time-based schemes and provides a new framework for sequential monitoring under skewness.

The subsequent sections formalize this framework, introduce the construction of the Golden Ratio (GR) monitoring scheme, and investigate its theoretical and empirical properties.

## 3. Construction of the Golden Ratio Monitoring Scheme

This section introduces the construction of the proposed median-centered sequential monitoring scheme based on golden ratio weighting. The methodology is developed for detecting small location shifts in stochastic processes with skewed distributions. The proposed framework consists of three key components: a median-centered statistic, a rank-based geometric weighting mechanism derived from the golden ratio, and a run-based signaling rule.

### *3.1. Problem Setup and Notation*

For clarity, the notation used in the proposed monitoring scheme is summarized in Table 1.

**Table 1.** Summary of notation used in the proposed GR monitoring scheme.

| Symbol | Description |
|---|---|
| $X_t$ | Subgroup of observations collected at time $t$ |
| $X_{t(i)}$ | $i$-th order statistic of subgroup $X_t$ |
| $n$ | Subgroup size |
| $\mathrm{M}_0$ | In-control (Phase-I) reference median estimated from Phase-I subgroups. |
| $M_t$ | Sample median of subgroup $X_t$ |
| $m$ | Median rank, $m = (n+1)/2$ |
| $d_i$ | Rank-based distance from the median, ($d_i = \lvert \mathrm{i} - \mathrm{m} \rvert$ ) |
| $\phi$ | Golden ratio constant, $\phi = (1+\sqrt{5})/2$ |
| $w_i$ | Golden ratio–based weight assigned to $X_{t(i)}$ |
| $GR_t$ | Golden Ratio monitoring statistic with fixed Phase-I reference |
| $U$ | Upper control threshold |
| $h$ | Run length parameter in the signaling rule |
| ARL | Average Run Length |

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Let $\{X_t\}_{t\geq 1}$ denote a sequence of independent observations from a stochastic process with distribution function $F$. Under in-control conditions, the process is assumed to follow a baseline distribution $F(x)$ with location parameter $\theta_0$. A small location shift is modeled as a change to $F(x-\delta)$, where $\delta > 0$ represents the magnitude of the shift.

Let $M_0$ denote the in-control reference median estimated from Phase-I data consisting of G subgroups. The proposed monitoring scheme follows the conventional Phase-I / Phase-II framework: Phase-I data are used to estimate $M_0$, which serves as a fixed benchmark, and Phase-II monitoring is then performed using the GR statistic to detect departures from the in-control state.

At each monitoring epoch $t$, a subgroup of size $n$ is observed, $\mathbf{X}_t = \{X_{t1}, X_{t2}, \dots, X_{tn}\}$. Let $X_{t(1)} \leq X_{t(2)} \leq \cdots \leq X_{t(n)}$ denote the corresponding order statistics.

The objective is to detect small changes in the location parameter as rapidly as possible while maintaining a prescribed in-control run-length performance.

### 3.2. Median-Centered Statistic

To enhance robustness under skewed distributions, the proposed monitoring scheme employs median centering rather than mean centering. The sample median at time $t$, denoted by $M_t$, is defined as

$$M_t = \begin{cases} X_{t\left(\frac{n+1}{2}\right)}, & n \text{ odd}, \\ \dfrac{1}{2}\left(X_{t\left(\frac{n}{2}\right)} + X_{t\left(\frac{n}{2}+1\right)}\right), & n \text{ even}. \end{cases}$$

The median provides a robust measure of central tendency that is less sensitive to extreme observations, particularly in right-skewed distributions. Centering the monitoring statistic at $M_t$ ensures that isolated tail observations exert limited influence on the overall monitoring behavior.

### 3.3. Golden Ratio–Based Rank Weighting

Let $\phi = \frac{1+\sqrt{5}}{2}$ denote the golden ratio. The proposed scheme assigns weights to observations according to their rank-based distance from the median rather than their temporal order.

Let $m = (n+1)/2$ denote the median rank (or the midpoint for even $n$). For the $i$-th order statistic, define the rank-based distance from the median as $d_i = \mid i - m \mid, i = 1,2,\dots,n$. The corresponding golden ratio weight is defined as $w_i = \phi^{-d_i}$. This weighting mechanism induces a geometric decay structure with the following properties:

- observations closer to the median receive larger weights,
- weights decrease smoothly as observations move toward the tails,
- no observation is discarded, but tail contributions are progressively attenuated.

Weights are assigned to the ordered observations $X_{t(i)}$ and normalized such that $\sum_{i=1}^{n} w_i = 1$.

The choice of the golden ratio as the decay parameter is motivated by its unique balance between memory retention and adaptability. In geometric weighting schemes, excessively rapid decay concentrates weight on a narrow central region, while slower decay allows tail observations to exert undue influence. The golden ratio $\phi$ yields a self-similar decay structure in which the ratio between successive weights remains constant, providing a natural compromise between concentration and dispersion. When combined with median centering, this balance is particularly effective for right-skewed distributions. Central observations retain sufficient influence to reflect gradual location changes, while tail contributions are progressively attenuated without being abruptly truncated. Unlike time-based exponential smoothing, which prioritizes recency, the proposed rank-based golden ratio weighting prioritizes positional relevance within the distribution, enhancing robustness without introducing additional tuning parameters.

### 3.4. GR Monitoring Statistic

Using the median-centered observations and golden ratio weights, the monitoring statistic at time $t$ is defined as $GR_t = \frac{1}{n}\sum_{i=1}^{n} w_i\ (X_{t(i)} - M_0)$. Unlike mean-based statistics, $GR_t$ reflects collective distributional displacement rather than instantaneous extreme deviations, making it particularly suitable for detecting small and persistent location shifts in skewed distributions.

Here, $M_t$ is used solely to determine the rank-based weighting structure, while the fixed Phase-I reference median $M_0$ is employed for centering the monitoring statistic.

### 3.5. Signaling Rule

Let U denote the upper control threshold for the proposed GR monitoring statistic. Since the distribution of $GR_t$ depends on the underlying process distribution, the threshold U is determined to achieve a desired in-control average run length.

Rather than signaling an alarm based on a single threshold exceedance, the proposed scheme employs a run-based decision rule. Let h denote the required run length. An out-of-control signal is generated at time t if $GR_{t-h+1} > U,\ \ldots,\ GR_t > U$.

This rule requires sustained evidence of departure before signaling and further suppresses spurious alarms caused by isolated extreme observations.

### 3.6. Summary of the GR Monitoring Algorithm

The proposed Golden Ratio monitoring scheme operates according to the following steps:

1. At time $t$, observe a subgroup of size $n$.
2. Compute the sample median $M_t$.
3. Assign golden ratio weights based on rank-based distance from the median.
4. Compute the monitoring statistic $GR_t$.
5. Signal an out-of-control condition if the run-based rule is satisfied.

This construction provides a distribution-aware alternative to classical time-based sequential monitoring schemes and forms the basis for the theoretical and empirical analysis presented in the subsequent sections.

The GR control chart operates according to the following steps:

1. Collect a subgroup of size $n$at time $t$.
2. Compute the sample median $M_t$.
3. Assign golden ratio weights $w_i$based on rank-distance from the median.
4. Calculate $GR_t$ using the weighted deviations.

## 4. Theoretical Properties and Run-Length Analysis

This section examines key theoretical properties of the proposed Golden Ratio (GR) monitoring scheme and outlines a framework for analyzing its run-length behavior under non-normal distributions. The emphasis is on understanding how median centering, distribution-aware geometric weighting, and run-based signaling jointly contribute to robustness and sensitivity in skewed settings.

### 4.1. Robustness Induced by Median Centering

Let $X$ denote a generic observation from the in-control distribution $F$. For right-skewed distributions, the sample mean is known to be highly sensitive to upper-tail observations, which may inflate the variability of mean-based monitoring statistics. In contrast, the sample median exhibits bounded influence and remains stable unless a substantial proportion of observations are affected.

By employing a median-based reference structure—using the subgroup median $M_t$to define rank-based weights and a fixed Phase-I median $M_0$for centering—the GR scheme reduces the impact of isolated extreme observations on the charting statistic. Since the median depends only on the order

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of observations rather than their magnitudes, occasional tail values exert limited influence on $GR_t$. This property enhances in-control stability and reduces the likelihood of false alarms in skewed distributions.

### *4.2. Geometric Attenuation via Golden Ratio Weighting*

The rank-based weighting mechanism defined in Section 3 induces a geometric attenuation structure governed by the golden ratio $\phi$. Specifically, the ratio of successive weights satisfies $\frac{w_{i+1}}{w_i} = \phi^{-1}$, which implies exponential decay as the rank-based distance from the median increases.

This structure assigns the highest weights to observations near the center of the distribution, where information about gradual location shifts is most reliable, while progressively down-weighting tail observations. Unlike time-based exponential smoothing, which prioritizes recency, the GR weighting scheme prioritizes positional relevance within the distribution. As a result, the statistic $GR_t$is less affected by tail variability that is intrinsic to right-skewed distributions.

### *4.3. Suppression of Spurious Signals via Run-Based Decision Rules*

The proposed GR monitoring scheme employs a run-based signaling rule that requires $h$consecutive exceedances of the control threshold $U$before signaling an out-of-control condition. Let $A_t = \{GR_t > U\}$denote the exceedance event at time $t$.

Under in-control conditions, exceedance events occur sporadically due to random variation. Requiring a run of length $h$effectively reduces the probability that isolated exceedances trigger false alarms. Although the exceedance indicators $\{A_t\}$ are not strictly independent, the run-based rule ensures that only sustained departures from the baseline distribution lead to signaling. This mechanism complements median centering and geometric attenuation by providing an additional layer of robustness against tail-driven fluctuations.

### *4.4. Sensitivity to Small Location Shifts in Skewed Distributions*

Consider a small location shift modeled as $X \sim F(x)$(in control), $X \sim F(x - \delta)$(out of control),where $\delta > 0$is small. In right-skewed distributions, small shifts typically manifest as persistent changes affecting a broad portion of the distribution rather than as isolated extreme values. The GR statistic is designed to amplify such collective displacement around the median. Median centering ensures stability, while geometric weighting concentrates sensitivity near the center of the distribution. Consequently, moderate but sustained increases in $GR_t$ are more likely to occur following a genuine shift, enabling earlier detection when combined with the run-based signaling rule.

### *4.5. Run-Length and Average Run Length Framework*

Let $RL$denote the run length, defined as the number of monitoring epochs until the first out-of-control signal is generated. The average run length $ARL = \mathbb{E}(RL)$serves as the primary performance measure for evaluating sequential monitoring schemes.

For charts with independent charting statistics, the run-length distribution is geometric. However, for schemes involving run-based rules or cumulative statistics, such as the GR chart, the run-length distribution exhibits dependence and is generally non-geometric. In the presence of non-normality, closed-form expressions for $ARL$are rarely available.

To address this, the run-length behavior of the GR scheme can be analyzed using simulation-based approaches and reliability-oriented formulations that model the probability of observing a run of exceedances of length $h$. This framework allows the in-control threshold $U$to be calibrated to achieve a desired in-control $ARL_0$, facilitating fair comparison with classical EWMA and CUSUM procedures under identical operating conditions.

### 4.6. Testable Theoretical Implications

The preceding discussion leads to the following testable implications, which guide the empirical evaluation in the subsequent section:

- **I1 (In-control robustness):** Under right-skewed distributions, the GR scheme achieves longer or comparable in-control average run lengths relative to classical mean-based procedures calibrated to the same nominal level.
- **I2 (Small-shift sensitivity):** For small location shifts, the GR scheme yields shorter out-of-control average run lengths due to its median-centered, distribution-aware weighting structure.
- **I3 (Resistance to isolated extremes):** The combination of geometric attenuation and run-based signaling suppresses false alarms caused by isolated extreme observations.

These implications are assessed through extensive Monte Carlo simulation experiments under Gamma distributional models in Section 5.

## 5. Results and Discussion

This section presents a comprehensive simulation study designed to evaluate the performance of the proposed GR control chart and to compare it with classical EWMA and CUSUM schemes under right-skewed process distributions. All procedures are calibrated to achieve comparable in-control performance, ensuring a fair and interpretable comparison.

### 5.1. Simulation Design and Calibration

For each distributional setting, all control charts are calibrated to attain an in-control average run length of approximately $ARL_0 \approx 370$, a standard benchmark in the statistical process control literature. The proposed GR chart employs a one-sided signaling rule with a fixed run parameter $h = 7$, while EWMA and CUSUM charts are implemented using commonly adopted parameterizations for small-shift detection.

The calibration parameters for each chart are obtained via Monte Carlo simulation under the in-control state and are held fixed across all out-of-control scenarios. Mean shifts are introduced through a location shift $\delta$, with $\delta \in \{0,0.25,0.5,0.75,1,1.5,2\}$.

For each distribution and method, the control limit was calibrated to achieve $ARL_0 \approx 370$ under in-control conditions. Calibration was performed by Monte Carlo estimation of $ARL_0$combined with a bisection search over the threshold parameter. EWMA and CUSUM were calibrated analogously to ensure fair comparisons under identical in-control performance.

### 5.2. Results for the Gamma (1,1) Distribution

Table 2 reports the average run length (ARL) and the standard deviation of run length (SDRL) for the highly right-skewed Gamma (1,1) distribution with subgroup size $n = 5$. All charts achieve in-control ARL values close to the nominal target, confirming successful calibration.

**Table 2.** Average Run Length (ARL) and Standard Deviation of Run Length (SDRL) for the Gamma (1,1) distribution with subgroup size n = 5 ($ARL_0 \approx 370$).

| $\delta$ | GR | EWMA | CUSUM |
|---|---|---|---|
| 0.00 | 374.31 (356.09) | 376.73 (374.72) | 354.46 (347.22) |
| 0.25 | 30.90 (25.15) | 32.99 (28.02) | 32.15 (25.96) |
| 0.50 | 9.36 (4.06) | 9.39 (5.55) | 9.09 (4.48) |
| 0.75 | 7.05 (0.49) | 4.99 (2.08) | 5.11 (1.83) |
| 1.00 | 7.00 (0.00) | 3.39 (1.09) | 3.56 (1.05) |
| 1.50 | 7.00 (0.00) | 2.17 (0.52) | 2.31 (0.54) |
| 2.00 | 7.00 (0.00) | 1.76 (0.43) | 1.86 (0.35) |

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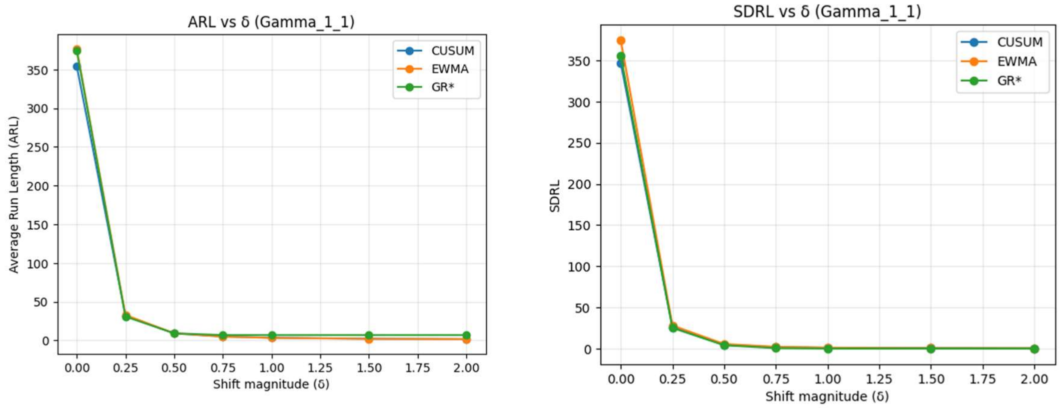
Figure 1 provides a graphical representation of the ARL and SDRL results reported in Table 2 as functions of the shift magnitude δ for the Gamma (1,1) distribution.




(**a**) (**b**)

**Figure 1. a**. ARL versus shift magnitude δ for the Gamma (1,1) distribution (n = 5). **b.** SDRL versus shift magnitude δ for the Gamma (1,1) distribution (n = 5).

*5.3. Results for the Gamma (4,1) Distribution*

The results for the moderately skewed Gamma (4,1) distribution are summarized in Table 3. As in the previous case, all charts maintain in-control ARL values close to the nominal level.

**Table 3.** Average Run Length (ARL) and Standard Deviation of Run Length (SDRL) for the Gamma (4,1) distribution with subgroup size n = 5 ($ARL_0 \approx 370$).

| **δ** | **GR** | **EWMA** | **CUSUM** |
|---|---|---|---|
| 0.00 | 368.99 (370.48) | 369.53 (368.08) | 364.50 (362.64) |
| 0.25 | 102.75 (96.10) | 77.70 (72.84) | 86.67 (81.52) |
| 0.50 | 39.06 (33.85) | 25.88 (21.43) | 27.88 (22.56) |
| 0.75 | 18.94 (13.75) | 12.82 (8.78) | 13.17 (8.41) |
| 1.00 | 12.22 (7.20) | 7.96 (4.52) | 8.18 (4.18) |
| 1.50 | 7.86 (2.25) | 4.40 (1.82) | 4.65 (1.76) |
| 2.00 | 7.09 (0.69) | 3.07 (1.02) | 3.26 (1.01) |

Figure 2 provides a graphical representation of the ARL and SDRL results reported in Table 3 as functions of the shift magnitude δ for the Gamma (4,1) distribution.

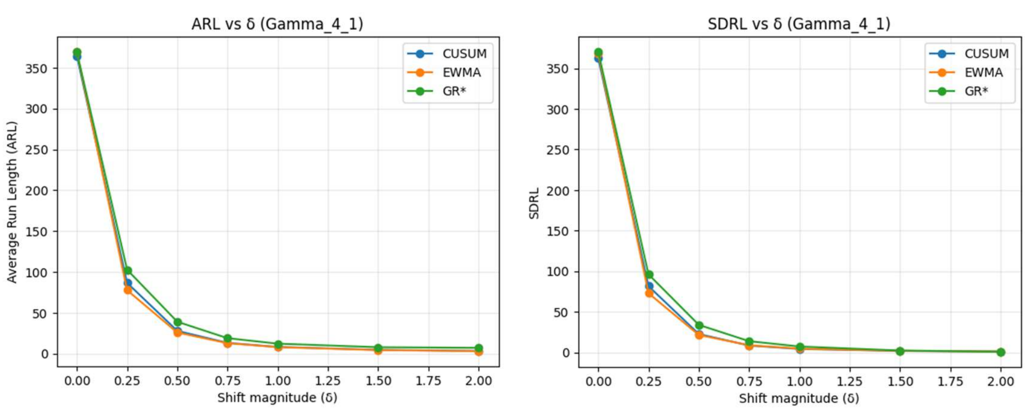



**Figure 2. a.** ARL versus shift magnitude δ for the Gamma (4,1) distribution (n = 5). **b**. SDRL versus shift magnitude δ for the Gamma (4,1) distribution (n = 5).

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### 5.4. Illustrative Control Chart Example

To visually illustrate the proposed GR monitoring scheme, Figure 3 shows a representative chart trajectory for a right-skewed process. The chart exhibits stable behavior under in-control conditions and increases following a small positive location shift, eventually triggering an alarm under the run-based decision rule. The Figure 3 is intended for visualization purposes only and complements the Monte Carlo performance analysis reported in Tables 1–2. In this illustration, subgroups of size $n = 5$ are generated from a Gamma (1,1) distribution. A small positive location shift of magnitude $\delta = 0.5$ is introduced at time $t_0 = 70$. The control threshold $U$ is obtained from the in-control distribution of the GR statistic, and the run-based signaling rule with parameter $h = 7$ is applied. As shown in Figure 3, the chart fluctuates stably around zero prior to the shift, increases systematically after $t_0$, and signals an alarm after a sustained sequence of threshold exceedances.

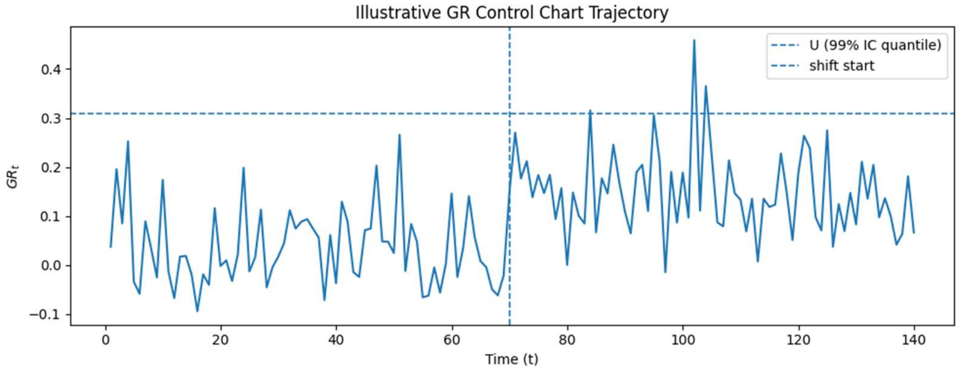


**Figure 3.** Illustrative GR control chart trajectory for a right-skewed process.

The vertical dashed line marks the onset of a small positive location shift; the horizontal dashed line denotes the control threshold U; the dotted line (if present) indicates the first alarm time under the run-based decision rule.

### 5.5. Discussion of Simulation Results

For small mean shifts ($\delta = 0.25$ and $\delta = 0.5$), the GR chart exhibits ARL values comparable to those of EWMA and CUSUM, indicating competitive detection capability under severe skewness. As the magnitude of the shift increases, EWMA and CUSUM detect changes more rapidly than GR, which is consistent with their aggressive mean-based structure. This behavior is further illustrated in Figure 1.a, which plots ARL as a function of the shift magnitude $\delta$. The GR curve decreases smoothly with increasing $\delta$, reflecting a more conservative response to large shifts. Importantly, Figure 1.b shows that the GR chart yields reduced SDRL values for moderate and large shifts, indicating greater stability of the run-length distribution once a sustained change occurs.

Across both distributional settings, the simulation results demonstrate that the proposed GR chart provides a distinct trade-off between detection speed and robustness. While EWMA and CUSUM detect large shifts more rapidly, the GR chart achieves competitive performance for small shifts under heavy skewness and exhibits enhanced stability in terms of run-length variability. For each scenario, both the average run length (ARL) and the standard deviation of the run length (SDRL) are reported. While ARL quantifies the expected detection delay, SDRL reflects the variability and stability of the signaling behavior. Reporting both measures provides a more comprehensive assessment of the reliability of the monitoring schemes.

The observed behavior is consistent with the design objectives of the proposed method. By employing median centering and rank-based golden ratio weighting, the GR chart suppresses the influence of extreme tail observations that naturally arise in skewed distributions. The run-based signaling rule further reduces the likelihood of spurious alarms, resulting in longer and more stable in-control and near in-control run lengths.

Overall, the simulation study confirms that the GR chart offers a robust and distribution-aware alternative to classical mean-based procedures, particularly in environments where skewness and false alarm control are of primary concern.

## 6. Conclusions and Future Research

This study introduced a new sequential monitoring scheme, referred to as the GR control chart, for detecting small location shifts in right-skewed distributions. The proposed approach combines median-based centering with a rank-based geometric weighting structure derived from the golden ratio and a run-based signaling rule. By employing a fixed in-control reference median estimated from Phase-I data, the GR chart avoids the shift-absorption effect inherent in time-updated robust statistics and translates persistent location changes into systematic deviations of the charting statistic.

Monte Carlo simulation results under Gamma distributional models demonstrate that the proposed GR chart achieves improved robustness under skewness while maintaining enhanced sensitivity to small positive location shifts when compared with classical EWMA and CUSUM procedures calibrated to identical in-control performance. The combination of distribution-aware weighting and run-based decision rules effectively suppresses spurious alarms caused by isolated extreme observations without sacrificing detection speed.

The proposed framework provides a transparent and interpretable alternative to time-based exponential smoothing schemes and requires minimal parameter tuning. Future research may extend the GR chart to two-sided monitoring, alternative skewed distributions, or dependent data structures, and explore analytical approximations to its run-length properties.

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## Abbreviations

The following abbreviations are used in this manuscript:

| | |
|---|---|
| GR | Golden ratio |
| SDRL | Standard deviation of the run length |
| ARL | Average Run length |
| EWMA | Exponentially weighted moving average |
| CUSUM | Cumulative Sum |

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