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Local Updates of Thevenin Equivalents in Linear Networks via Rank-One Perturbations (Sherman–Morrison)

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Abstract

We derive an exact, practical method to update Thévenin parameters (open-circuit voltage and equivalent resistance) of a linear network under a *single internal branch* modification (open/short/resistance change), without recomputing the full nodal solution from scratch. The change is modeled as a *rank-one* perturbation of the nodal admittance matrix, and the Sherman–Morrison identity yields closed-form port updates in terms of three physically interpretable scalars: local self-coupling, port–branch coupling, and state projection across the modified b ranch. We discuss limiting cases (open and short), include a brief note on complex admittances (phasors/Laplace), and provide a reproducible Python check.

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1. Introduction

Thévenin and Norton equivalents replace a linear network seen from a port by a compact model that preserves external behavior [6,7]. In teaching, design, and verification, one often asks: *how does the port equivalent change if an internal branch is opened, shorted, or its resistance is adjusted?*

Instead of fully re-solving the network for each local change, this work shows that modifying a single branch induces a *rank-one* update of the nodal admittance matrix. The underlying algebraic identities (Sherman–Morrison/Woodbury) are classical [1–3]; our contribution is a port-oriented presentation that directly yields updated Thévenin parameters, with clear interpretations and physically consistent limiting cases. We also relate the approach to Schur/Kron reduction perspectives commonly used in network theory [4,5].

2. Nodal Model and Thévenin Parameters

Consider a linear (DC resistive) network with n non-reference nodes and ground as reference. Its nodal formulation is

$$\mathbf{Y}\,\mathbf{v} = \mathbf{i}_{\mathrm{ext}}, \tag{1}$$

where $\mathbf{Y} \in \mathbb{R}^{n\times n}$ is the nodal admittance matrix, $\mathbf{v} \in \mathbb{R}^{n}$ is the nodal-voltage vector, and $\mathbf{i}_{\mathrm{ext}} \in \mathbb{R}^{n}$ stacks equivalent external current injections due to independent sources.

Let the port be between nodes a and b (both non-ground). Define the selector

$$\mathbf{s}_{ab} = \mathbf{e}_a - \mathbf{e}_b. \tag{2}$$

Then the open-circuit port voltage is

$$V_{oc} = \mathbf{s}_{ab}^{\mathsf{T}}\mathbf{v}. \tag{3}$$

With independent sources turned off, the Thévenin resistance between a and b can be obtained by injecting 1 A from a to b:

$$\mathbf{Y}\,\mathbf{v}^{(test)} = \mathbf{s}_{ab}, \qquad R_{th} = \mathbf{s}_{ab}^{\mathsf{T}}\mathbf{v}^{(test)}. \tag{4}$$

If $\mathbf{Y}$ is invertible and $\mathbf{Z} = \mathbf{Y}^{-1}$, then

$$R_{th} = \mathbf{s}_{ab}^{\mathsf{T}}\mathbf{Z}\,\mathbf{s}_{ab}. \tag{5}$$

Remark 1. *Equation* (5) *matches the* effective resistance *between a and b in resistive networks. Ground anchoring (Dirichlet condition) avoids singularity due to undefined absolute potential.*

3. Local Change: Modifying a Single Branch

Consider an internal branch between nodes m and n with original conductance g and new conductance g'. Define

$$\Delta g = g' - g, \qquad \mathbf{u} = \mathbf{e}_m - \mathbf{e}_n. \tag{6}$$

At the nodal-matrix level the modification is

$$\mathbf{Y}' = \mathbf{Y} + \Delta g\,\mathbf{u}\mathbf{u}^{\mathsf{T}}. \tag{7}$$

3.1. Sherman–Morrison

Lemma 1 (Sherman–Morrison, rank-one case)**.** *Let $\mathbf{Y}$ be invertible and $\mathbf{Z} = \mathbf{Y}^{-1}$. If $1 + \Delta g\,\mathbf{u}^{\mathsf{T}}\mathbf{Z}\mathbf{u} \neq 0$, then*

$$\mathbf{Z}' = (\mathbf{Y}')^{-1} = \mathbf{Z} - \frac{\Delta g\,\mathbf{Z}\mathbf{u}\mathbf{u}^{\mathsf{T}}\mathbf{Z}}{1 + \Delta g\,\mathbf{u}^{\mathsf{T}}\mathbf{Z}\mathbf{u}}. \tag{8}$$

Remark 2. *The denominator $1 + \Delta g\,\mathbf{u}^{\mathsf{T}}\mathbf{Z}\mathbf{u}$ vanishes only if the perturbation induces a singular configuration in this representation (e.g., loss of anchoring/disconnection).*

4. Main Results: Updates for R_{th} and V_{oc}

Define three scalars (dependent on port (a, b) and branch (m, n)):

$$\alpha = \mathbf{u}^{\mathsf{T}}\mathbf{Z}\mathbf{u}, \qquad \beta = \mathbf{s}_{ab}^{\mathsf{T}}\mathbf{Z}\mathbf{u}, \qquad \gamma = \mathbf{u}^{\mathsf{T}}\mathbf{v}. \tag{9}$$

Interpretation:

- α: local self-coupling of branch (m, n) (related to effective resistance between m and n).
- β: port–branch coupling (how strongly the port “sees” the internal change).
- γ: base-state voltage across the branch (m, n).

4.1. Thévenin Resistance Update

Theorem 1 (Exact update of R_{th})**.** *Under Equation* (7) *and the assumptions of Lemma* 1*, the Thévenin resistance between a and b updates as*

$$R'_{th} = R_{th} - \frac{\Delta g\,\beta^2}{1 + \Delta g\,\alpha}, \qquad R_{th} = \mathbf{s}_{ab}^{\mathsf{T}}\mathbf{Z}\mathbf{s}_{ab}. \tag{10}$$

4.2. Open-Circuit Voltage Update

Theorem 2 (Exact update of V_{oc})**.** *Under Equation* (7) *and the assumptions of Lemma* 1*, the open-circuit port voltage updates as*

$$V'_{oc} = V_{oc} - \frac{\Delta g\,\beta\,\gamma}{1 + \Delta g\,\alpha}, \qquad V_{oc} = \mathbf{s}_{ab}^{\mathsf{T}}\mathbf{v}. \tag{11}$$

4.3. Limiting Cases

Corollary 1 (Opening the branch). *If the branch is removed ($g' = 0$), then $\Delta g = -g$ and Equations (10) and (11) give*

$$R'_{th} = R_{th} + \frac{g\,\beta^2}{1 - g\,\alpha}, \qquad V'_{oc} = V_{oc} + \frac{g\,\beta\,\gamma}{1 - g\,\alpha}. \tag{12}$$

Corollary 2 (Shorting the branch). *If $g' \to \infty$, then $\Delta g \to +\infty$ and taking the limit in Equations (10) and (11) yields*

$$\lim_{g'\to\infty} R'_{th} = R_{th} - \frac{\beta^2}{\alpha}, \qquad \lim_{g'\to\infty} V'_{oc} = V_{oc} - \frac{\beta\,\gamma}{\alpha}. \tag{13}$$

5. Problem Schematic

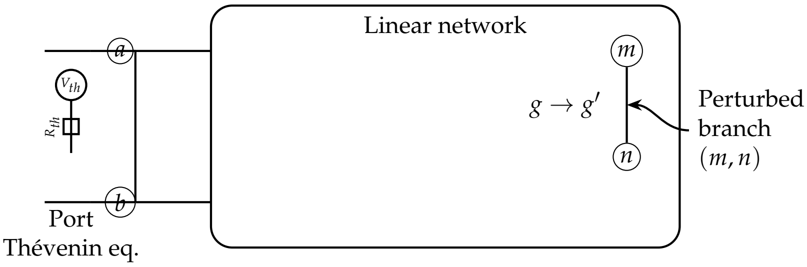

Figure 1. Port (a,b) and the desired Thévenin equivalent. The network undergoes a local change on an internal branch (m,n).

6. Discussion and Applications

6.1. What the Formulation Enables

Equations Equations (10) and (11) answer, in closed form, questions of the type: *given a port equivalent, how do R_{th} and V_{oc} change if a specific internal branch is opened, shorted, or revalued?* The full effect of the perturbation is compressed into three scalars (α, β, γ).

6.2. Computational Remarks

Although the formulas are expressed using $\mathbf{Z} = \mathbf{Y}^{-1}$, practical computation does not require forming $\mathbf{Z}$. One may reuse a sparse factorization of $\mathbf{Y}$ and obtain α, β, R_{th} via a small number of solves and scalar products, which is advantageous in repeated "what-if" studies or parametric sweeps [3].

6.3. Typical Use Cases

- **Contingency/topology analysis:** fast updates under line/branch outages or switching events.
- **Topology optimization and design:** evaluate the impact of local conductance changes on a given port.
- **Teaching and sensitivity:** formalizes "set a branch to infinity (open) or zero (short)" and quantifies the effect.

Method	Typical cost	Exactness	Scope
Full re-solve	high (rebuild/solve)	exact	general
Rank-one update (this work)	low (solves + scalars)	exact	single-branch

Table 1. Conceptual comparison of approaches for local changes.

7. Implementation Without Forming an Inverse

Let $\mathbf{s} = \mathbf{s}_{ab}$ and $\mathbf{u} = \mathbf{e}_m - \mathbf{e}_n$. Then

$$\alpha = \mathbf{u}^{\mathsf{T}}\mathbf{x}_u, \quad \text{where } \mathbf{Y}\mathbf{x}_u = \mathbf{u}, \qquad \beta = \mathbf{s}^{\mathsf{T}}\mathbf{x}_u,$$

and $\gamma = \mathbf{u}^\top \mathbf{v}$ from the base state Equation (1). Also, $R_{th} = \mathbf{s}^\top \mathbf{x}_s$ where $\mathbf{Y}\mathbf{x}_s = \mathbf{s}$. With a reused sparse factorization, these operations are efficient [3].

8. Reproducible Python Check

The script below builds a random grounded resistive network, computes (V_{oc}, R_{th}) at a port, modifies a single branch, and compares: (i) full recomputation vs (ii) the update Equations (10) and (11). Agreement up to floating-point rounding validates the identity.

```python
import numpy as np

def build_Y(n, edges, g_to_ground=None):
    Y = np.zeros((n, n), dtype=float)
    for (m, k, g) in edges:
        Y[m, m] += g
        Y[k, k] += g
        Y[m, k] -= g
        Y[k, m] -= g
    if g_to_ground is not None:
        Y += np.diag(g_to_ground)
    return Y

def unit_vec(n, idx):
    e = np.zeros(n)
    e[idx] = 1.0
    return e

def th_port_full(Y, i_ext, a, b):
    Z = np.linalg.inv(Y)
    v = Z @ i_ext
    s = unit_vec(len(i_ext), a) - unit_vec(len(i_ext), b)
    Voc = float(s @ v)
    Rth = float(s @ (Z @ s))
    return Voc, Rth, Z, v

def update_from_base(Z, v, a, b, m, n, Delta_g):
    s = unit_vec(Z.shape[0], a) - unit_vec(Z.shape[0], b)
    u = unit_vec(Z.shape[0], m) - unit_vec(Z.shape[0], n)
    alpha = float(u @ (Z @ u))
    beta  = float(s @ (Z @ u))
    gamma = float(u @ v)
    Rth   = float(s @ (Z @ s))
    Voc   = float(s @ v)
    den = 1.0 + Delta_g * alpha
    if abs(den) < 1e-12:
        raise ValueError("Denominator ~0: degenerate/singular perturbation.")
    Rth_new = Rth - (Delta_g * beta**2) / den
    Voc_new = Voc - (Delta_g * beta * gamma) / den
    return Voc, Rth, Voc_new, Rth_new

np.random.seed(2)

n = 10
g_ground = 0.5 + np.random.rand(n)

edges = []
```

```
for _ in range(18):
    m, k = np.random.choice(n, size=2, replace=False)
    g = 0.2 + 2.0*np.random.rand()
    edges.append((m, k, g))

Y = build_Y(n, edges, g_ground)

# external injections
i_ext = np.random.randn(n)

# port and modified branch
a, b = 1, 7
m, k, g_old = edges[3]
g_new = 0.0               # open branch
Delta_g = g_new - g_old

Voc, Rth, Z, v = th_port_full(Y, i_ext, a, b)
_, _, Voc_upd, Rth_upd = update_from_base(Z, v, a, b, m, k, Delta_g)

edges_mod = edges.copy()
edges_mod[3] = (m, k, g_new)
Y2 = build_Y(n, edges_mod, g_ground)
Voc2, Rth2, _, _ = th_port_full(Y2, i_ext, a, b)

print("Port (a,b) =", (a,b), "Modified branch (m,n) =", (m,k))
print("Base:   Voc=%.6f  Rth=%.6f" % (Voc, Rth))
print("Update: Voc=%.6f  Rth=%.6f" % (Voc_upd, Rth_upd))
print("Full:   Voc=%.6f  Rth=%.6f" % (Voc2, Rth2))
print("Error:  dVoc=%.3e  dR=%.3e" % (abs(Voc2-Voc_upd), abs(Rth2-Rth_upd)))
```

Listing 1: Numerical verification of the Sherman–Morrison port update.

9. Brief Note on Phasors and Laplace

While we presented the derivation for DC resistive networks for didactic clarity, the algebraic structure remains valid for linear time-invariant networks in the phasor or Laplace domain: replace real conductances by complex admittances $g \mapsto Y(j\omega)$ (or $Y(s)$) and apply the same identities, provided the resulting linear system is invertible at the frequency (or s) of interest.

10. Conclusions

We derived closed-form expressions to update (i) Thévenin resistance and (ii) open-circuit port voltage under a single internal-branch modification, modeled as a rank-one update of the nodal admittance matrix. The port impact is captured by three interpretable scalars (α, β, γ), enabling efficient exact "what-if" analyses without full recomputation.

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