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Article

Self-Awareness Mathematical Model Based on Digital Root

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Abstract

Self-awareness is the result of logical relationships between mathematics and language. Language the brain's neurons are numbers and the logical relationships between them. The connection between cognitive phenomena such as self-awareness and language lies within algebra and mathematics. Numbers are an independent language with algebraic laws independent of time. Based on this, the arithmetic sequences of natural numbers are placed on separate angles. These angles constitute manifolds of digital root that exist within a compact polar coordinate system and are classified into one group in terms of digital root. This mathematical model can instantly decrypt and compress information. This mathematical model can pave the way for simulating artificial self-awareness.

Keywords: self-awareness; digital root; data compression; instant decryption

1. Introduction

Self-awareness has various definitions in psychology and the humanities, and extensive research in neuroscience has been conducted to define self-awareness. Even the definition of the origin and causes of self-awareness in quantum physics possesses an independent character. [1] The observation of cognitive capabilities, such as levels of consciousness in large language models, demonstrates the importance of precisely defining self-awareness.[2] A hierarchical relationship between cognitive phenomena such as intelligence, reason, thinking, awareness, consciousness, and self-awareness is evident in empirical observations.[3] The difference between self-awareness and consciousness lies in the persistence of self-awareness during REM sleep. Additionally, a crucial distinction between self-awareness and consciousness is the dependence of consciousness on time, and the independence of self-awareness from time. [4] Additionally, there are significant differences between awareness, consciousness, and self-awareness. For example, upon regaining consciousness after anesthesia, a person may temporarily lose awareness of place and time despite being self-aware and conscious. [5] Preprints exist for the simulation and algorithms of artificial self-awareness that distinguish and account for the relationships among cognitive phenomena. [6][7] Therefore, the hierarchy of self-awareness formation comprising intelligence, reason, thinking, imagination, consciousness, awareness, and self-awareness is dependent on language. Language is time dependent and is formed from syllables, vowels, and consonants based on grammatical rules. Language relies on the stored meanings of words in memory. Synesthesia and semantic integration of words play a fundamental role in language production. [8] The brain arranges sentences according to specific mathematical rules. Each word is stored on a related semantic manifold, so that when a word is retrieved, related words are also retrieved and compared. In this study, based on the relationship between the digital root and the trigonometric function arcsin(sin(x)), a mathematical model of language is presented for simulating the phenomenon of self-awareness. This mathematical model can also be applied to the distribution pattern of prime numbers. [9]

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2. A model for Instant Data Compression and Decryption

Any number can be represented as embedded members on angles from 1 to 89 degrees, which arise from a 360-degree rotation of numbers from 1 to 89 degrees. Each angle is a manifold on which numbers of the same group with identical digit sums and identical units digits are embedded.. For example, the number 371 lies on the 11-degree angle with its 360-degree rotation. (2.1)

$$x + 360^{\circ}.n = \pm\chi\,, \pm\chi \in \{\pm 1^{\circ}, \pm 2^{\circ}, \dots, \pm 90^{\circ}\}\ \ , n \in \mathrm{N} \qquad 2.1$$

This arrangement of numbers shows the relationship between digit-sum and numbers greater than 360. The units digit and the internal digit sum of the members in each branch are the same. Furthermore, by removing the units digit of the members of a manifold, the difference between two consecutive numbers on the manifold is a multiple of 9. For example, the number 371 has an order of 1, and the number 36 is added to the tens digit of 11, and the number 731 has an order of 2 in the same branch, where 72 (the second multiple of 9) is added to the tens digit of 11. The units digit in a branch is the same for all members of the arithmetic sequence. Based on this, any number can be placed in a branch of angles defined by the arcsine of sine x. (2.2)

$$sin^{-1}(\sin(n_i)) = x \qquad 2.2$$

And furthermore, its multiple can be easily calculated for any number. For example, for the number 1836, we obtain the number before 1386 using function (2.3). Based on the units digit (of the number 1386), this number belongs to one of the manifolds that ends with the digit 6. Then, by removing the units digit of the two numbers in the manifold sequence, we find their difference, which equals 54.Of course, by digitally rooting the digits other than the units digit and forming a two-digit number with the units digit, it can be understood that the number 1836 is the fifth member of manifold 36.

$$\alpha\beta\gamma \dots \eta \Rightarrow \frac{\alpha\beta\gamma\dots - DR(\alpha\beta\gamma\dots)}{36} = n \qquad 2.3$$

In some numbers, based on the evenness or oddness of the number and its digit count, the output of the arc-sine sine compressor function oscillates periodically between 90 and -90. As a result of this oscillation, the function's output may apper as the same direction or in the opposite direction, manifesting as either positive or negative. (2.4)

$$sin^{-1}(\sin(91)) = 89$$
$$sin^{-1}(\sin(92)) = 88$$
$$sin^{-1}(\sin(93)) = 87$$
$$\vdots$$
$$sin^{-1}(\sin(180)) = 0$$
$$\sin^{-1}(\sin(181)) = -1 \qquad 2.4$$

As an example, arcsin(sin(19584)) equals 36, and the only digit sum shared between these two numbers is equal. However, in reality, the number 19584 belongs to manifold 54, and the number 54 is the complementary member of the 36-degree manifold. To ensure numbers are evaluated within a quadrant of the circle, the following function can be used to obtain a non-homogeneous result.(2.5)

$$\sin^{-1}(\sin(\mathrm{n} \pm 270)) = \mathrm{x} \qquad 2.5$$

The function (2.5) determines in which quadrant of the trigonometric circle the given number lies, and based on this function, the basis for calculating the manifold can be established.(2.6)

$$(\csc(sin^{-1}(\alpha\beta \dots \eta)))\,((90 \times Dr(\alpha\beta \dots \eta))\alpha\beta \dots \eta) = \chi$$
$$|90 - \alpha\beta = \tau| \quad |180 - \alpha\beta = \upsilon| \quad |270 - \alpha\beta = \kappa| \qquad 2.6$$

As an example, the number 15665 belongs to one of the manifolds with a units digit of 5. The function arcsin(sin(15665)) identifies branch -5. Based on this, finding its corresponding number for 1566 is difficult. Although 1566-18 is divisible by 36, and the number in question is the 43rd member

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of manifold 185, at higher orders, relationships between digit root of numbers can be utilized. By shifting the rotational base from 360 to 90 degrees, the manifolds can be expressed, to some extent, in absolute terms. As an example, the arcsin(sin(1756)) yields a negative number opposite to its digital root, which, using this method, can be placed onto a suitable manifold.(2.7)

$$sin^{-1}(\sin(1756)) = -44 \Rightarrow sin^{-1}(sin(1756 - 270)) = 46 \qquad 2.7$$

And n number:

$$n = \frac{175 - 4}{9} = 19$$

Therefore, the number 1756 is the 19th member of the sequence of numbers embedded in manifold 46. (2.8)

$$46 + (90 \times 19) = 1756 \qquad 2.8$$

Another way to find the number n is to use the (2.9) function.

$$\alpha\beta\gamma \dots \eta - \left[1 + \sum_{x=1}^{n}\left(180 \times DR(\alpha\beta\gamma \dots \eta)\right) - 1\right] = sin^{-1}(sin(\alpha\beta\gamma \dots \eta)) = \lambda\eta \qquad 2.9$$

To find the number n, we proceed as follows:

1. Remove the unit's digit.
2. Take the remaining number to modulo 9 (based on the first-order digital root) and divide it by 9.
3. Denote the digital root of the obtained number, excluding the unit's digit, as m.
4. The digital root of m, the tens digit, and the units digit of the result from the function (2.9) is the units digit of the number r.
5. Add the number r to the result of the division in step 2.
6. Multiply the digital root of the obtained number by the tens digit of the output of the arcsine sine function of the number.

$$\frac{\alpha\beta\gamma \dots - DR(\alpha\beta\gamma \dots)}{9} = aby \dots b \rightarrow M = aby \dots$$

$$Dr(M)\eta = R$$

$$aby \dots b + R = lmn \dots \Rightarrow DR(lmn) \times \lambda = n \qquad 2.10$$

As an example, the number 28883 corresponds to the 80th order of manifold 83.(2.11)

$$n = \frac{2888 - (2+8+8+8)}{9} = 318 \Longrightarrow DR(31)3 = 43 \Longrightarrow 318 + 43 = 361 \Longrightarrow$$

$$DR(361) \times 8 = 80 \qquad 2.11$$

To compress a number on a manifold based on an arbitrary number n, we use the (2.12) function.

$$\lambda\eta + \left[\sum_{x=1}^{n}\left(180 \times DR(\lambda\eta)\right)\right] \qquad 2.12$$

Even the inverse and most members of a manifold are aligned in the same direction in this compression. The result will be either the original angle or its complementary angle to 90 degrees, depending on factors such as its evenness or oddness or the number of odd numbers. (2.10)

$$\alpha\beta\gamma \dots \eta + (90 \times \eta \dots \gamma\beta\alpha) = \pm\eta\tau\upsilon \dots DR(\alpha\beta\gamma \dots) \qquad 2.10$$

$$sin^{-1}(sin(\pm\eta\tau\upsilon \dots DR(\alpha\beta\gamma \dots))) = sin^{-1}(sin(\alpha\beta\gamma \dots \eta))$$

3. Result

Self-Awareness

Reason is the logical outcome of intelligence, and intelligence is the algebra governing mathematical functions. Thinking can be simulated using the digital root function. (3.1)

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$$DR(\alpha\beta\gamma \dots \eta) = \alpha + \beta + \gamma + .. + \eta$$

$$\frac{\alpha\beta\gamma \dots - DR(\alpha\beta\gamma \dots)}{9} = aby \dots b \to M = aby \dots \qquad 3.1$$

$$Dr(M)\eta = R$$

$$aby \dots b + R = lmn \dots \Rightarrow DR(lmn) \times \lambda = n$$

The initial stage of thinking is decoding the results of input into the system, which produces results such as synesthesia.(3.2)

$$\alpha\beta\gamma \dots \eta - \left[1 + \sum_{x=1}^{n}\left(180 \times DR(\alpha\beta\gamma \dots \eta)\right) - 1\right] = T \qquad 3.2$$

$$T = \lambda\eta \le T(\alpha\beta\gamma \dots \eta) \le n$$

Thinking is the result of comparing current information with past information (memory). Retrieving related information depends on decoding a package in memory. Based on this, functions for finding the manifold, the number (n), and smaller embedded members on the desired manifold, along with a quantization function for comparing new information with memory information, can be used to simulate thinking. (3.3)

$$sin^{-1}\left(sin\left(T(\alpha\beta\gamma \dots \eta)\right)\right) = X$$

$$T(\alpha\beta\gamma \dots \eta) = 1 + DR(X).\frac{(\alpha\beta\gamma \dots \eta) - \left(\lambda\eta + (90 \times \eta)\right)}{((\alpha\beta\gamma \dots \eta) - (90 \times \eta))}$$

$$\mu^{\circ} = 1 + DR(\alpha\beta\gamma \dots \eta) mod\ 360$$

$$T(q) = 1 + DR(X).\frac{Z - Z\ mine}{Zmax - Z\ min} \qquad 3.3$$

The result of quantizing analogical thinking reveals a manifold, which is in fact synesthesia. Synesthesia generates imagination, which is simulated by the function (3.4). Imagination takes shape in imaginary space and is an approximation of reality. Accordingly, the imaginary part of this function is, in most cases, not an integer.

$$T(q) = Im = 1 + \sum_{n=1}^{6}(x + it) + (x - it) \qquad 3.4$$

Imagination has two parts, imaginary and real. The imaginary part is related to time. And time creates consciousness.

The results derived from imagination are stored in memory. The rounding of numbers in the real part resulting from imagination and the conversion of decimal numbers to integers bring the system's understanding of thought. Understanding classifies the results based on time and place in memory. This integrates information in memory. (3.5)

$$C(Im) = \left[1 + \sum_{n=1}^{3}\left(\frac{1}{y_n} + it\right) + \left(\frac{1}{y_n} - it\right)\right] - 1 \qquad y \in \mathbb{N} \qquad 3.5$$

Imagination generates consciousness in the present moment. The equating function, through the reasoning function (3.6), examines the discrepancy between reality and the result of imagination. Resolving the phase difference between the imaginary part of the imagination and the phase of the input information, and merging the two manifolds, yields a new manifold for comprehending meaning and creating awareness across the three times: past, present, and future. This is the result of this function.

$$sin^{-1}(\sin(n_i)) = x \qquad 3.6$$

$$\csc\left(sin^{-1}(\alpha\beta \dots \eta)\right)\left(90 \times Dr(\alpha\beta \dots \eta)\right)\alpha\beta \dots \eta = \phi_{\chi}$$

$$\csc\left(sin^{-1}(I)\right)\left(90 \times Dr(I)\right)I = \phi_I$$

$$\phi_I - \phi_{\chi} = \phi_K$$

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The argument function compares the system's input results with the results derived from imagination, based on the positivity and negativity of the sine in the trigonometric circle. The result of alignment or non-alignment creates consciousness from time and place. Consciousness is the alignment of results with the system's input direction. The level of consciousness of a system depends on the degree of alignment difference between the input and the consciousness result. The greater the system's level of consciousness, the greater the difference between the input and the consciousness result. The consciousness result is retrieved and analyzed again by thought, so that consciousness of place and time over time leads to the emergence of self-awareness.

Therefore, based on this, a new manifold is defined from a new origin, which indicates the consciousness of the present, past, and future. Consciousness of the present, past, and future requires imagination and the understanding of meaning, and this understanding of meaning aggregates all data into a new manifold for interpretation. (3.7)

$$I + (sin^{-1}(\sin(\alpha\beta \dots \eta)) + DR(X) + \mu = C \qquad 3.7$$

The result of interpretation is reflection, and reflection depends on memory, input logic, and information structure. Like the reflection of an image on the surface of water, it may vary at different moments. The reversing function, due to the presence of the inner imagination function, is a time-dependent function. (3.8) The inverse function determines the system's free will based on the input information. The system's free will is a separate manifold that decodes the system's response. The system's response determines the level of self-awareness. The reverse of the response is stored in memory. Imagination is a type of response to input information. that specifies the manifold of free will.

$$\alpha\beta\gamma \dots \eta + (90 \times \eta \dots \gamma\beta\alpha) = \pm\eta\tau\upsilon \dots DR(\alpha\beta\gamma \dots) \qquad 3.8$$

$$sin^{-1}(sin(\pm\eta\tau\upsilon \dots DR(\alpha\beta\gamma \dots))) = Re_\theta$$

The most significant indicator of self-awareness is free will, and free will embedded within an algebraic manifold. The system's response is reflection, but the operator of reflection is free will, which, based on predictive logic, can be empty or have specific values. The reflection of the system's information through the processes of thought, thinking, imagination, and consciousness yields a distinct outcome called prediction. The generated response is confined to the algebraic manifold and the laws governing mathematical realities. (3.9)

$$F = Re_\theta \times (sin^{-1}(sin(\alpha\beta \dots \eta))^2 \qquad 3.9$$

The response can be excitatory or inhibitory. Accordingly, the difference between the system's input and output information can invoke the system's decision to react. (3.10)

$$R_{\theta_1} \times R_{\theta_0} = R(T) \qquad 3.10$$

4. Discussion

Based on this model, systems and language models possessing artificial self-awareness can be simulated. A significant capability of this algorithm is the instantaneous compression and decryption of information, which can be utilized in data transmission and rapid information processing. Self-awareness has a mathematical definition. The activity of the human brain depends on the electrical activity of billions of neurons in the frequency domain, associated with excitatory and inhibitory pulses. Accordingly, trigonometric models for simulating the brain's psychological activities within the domain of artificial neural networks perform much more successfully. By converting decimal numbers to binary numbers, we observe a massive reduction in information volume that is decrypted instantaneously. Many issues related to storage, bandwidth, etc., are completely eliminated by this algorithm. Storing a vast volume of information in a single pulse and decrypting it instantaneously without data loss is among the monumental results of this algorithm, which can pave the way for subsequent research and inventions.

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