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On a Geometric Origin of Electromagnetism in Teleparallel Spacetime

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Abstract

A teleparallel framework is presented in which electromagnetism admits a geometric realization alongside gravitation. In this construction, gravity and electromagnetism arise as complementary dynamical sectors of a single tetrad field, rather than as independent structures. The electromagnetic potential is not introduced as an internal gauge field appended to spacetime, but is identified with the dynamical structure of the temporal tetrad. In this parametrization, the electromagnetic potential arises as part of the geometric decomposition of the temporal coframe, while U(1) gauge symmetry appears as a redundancy in the representation of temporal geometry rather than as an independently postulated internal symmetry. The electromagnetic field strength emerges from temporal torsion, while the $U(1)$ gauge symmetry is realized as a geometric equivalence relation among different representations of temporal geometry. Local Lorentz covariance is preserved throughout. Standard electromagnetic dynamics are recovered without additional assumptions: the homogeneous Maxwell equations follow as geometric identities, while the inhomogeneous equations, charge conservation, and the Lorentz force law arise from a unified action principle. Flat Minkowski spacetime remains a stable vacuum solution, and classical configurations such as the Coulomb field admit a direct interpretation as spatial variations of temporal geometry. Electromagnetic backreaction is understood as an intrinsic change of the underlying geometric structure rather than as an external source. Beyond formal consistency, the framework allows direct contact with observations. In particular, the pulsar braking index problem is revisited from a geometric perspective.A drift component of the tetrad encodes a geometric vorticity of spacetime, leading to an additional torque linear in the angular velocity. Deviations of the braking index from the standart value n=3 then arise as a direct geometric effect, without invoking phenomenological torque corrections or detailed magnetospheric modeling.

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1. Introduction

The possibility that gravitation and electromagnetism might arise from a common geometric origin has been a repeating theme in theoretical physics since its early development. Early geometric unification attempts, such as Weyl's scale-invariant geometry [1], Einstein's affine and teleparallel constructions[2], and Eddington's connection-based approach[3], highlighted the conceptual appeal of a geometric origin for electromagnetism. While many of these constructions are conceptually appealing, they often encounter structural difficulties, such as tensions with local Lorentz covariance, ambiguities in the interpretation of gauge symmetry, or the introduction of additional degrees of freedom that are not supported phenomenologically.

A characteristic feature of most unification attempts is the treatment of the electromagnetic potential as an internal gauge field introduced independently of the gravitational geometry. While this strategy is technically consistent and phenomenologically successful, it leaves the origin of gauge symmetry external to spacetime geometry, thereby limiting a genuinely geometric interpretation of electromagnetism. In particular, formulations based on Riemannian geometry frequently require either

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modifications of metric compatibility or the introduction of extra spacetime dimensions, both of which face strong theoretical and experimental constraints.

Teleparallel geometry [7] offers a structurally distinct perspective on these questions. In this framework, gravitation is described in terms of torsion rather than curvature, with the tetrad playing the role of the fundamental geometric object. Since torsion depends linearly on the tetrad, the resulting geometric structure admits a more flexible organization than in Riemannian formulations. This feature provides a natural setting in which additional physical structures may be incorporated without directly interfering with the gravitational sector.

In the present work, electromagnetism is considered within teleparallel geometry as arising from the temporal component of the tetrad. Rather than being introduced as an independent external gauge field, the electromagnetic potential is associated with the dynamical structure of temporal geometry itself. Gauge invariance is preserved through a Stückelberg mechanism, while local Lorentz covariance remains intact throughout the construction.

From this geometric viewpoint, both local and global aspects of electromagnetic phenomena can be treated within a single framework. The Lorentz force law, global phase effects such as the Aharonov–Bohm phenomenon [6], and electric charge conservation emerge as consequences of the same underlying geometric structure, rather than as independent postulates. Consequently, electromagnetic backreaction is not treated as the response of geometry to an external field, but arises naturally as part of the unified geometric dynamics, allowing a clear distinction between test-field regimes and fully coupled configurations.

Within the teleparallel formulation the tetrad is the fundamental dynamical variable, and torsion depends linearly on it. Preserving local Lorentz covariance restricts additional propagating structures to 1-form excitations carried by the temporal leg. Since torsion enters only through exterior derivatives, the antisymmetric derivative of this 1-form constitutes the unique gauge-invariant kinetic structure available. Consequently, the resulting field strength necessarily takes the Maxwell form, and U(1) gauge symmetry appears as a redundancy in the representation of the temporal geometry rather than an independent assumption.

Beyond formal consistency, the framework also admits contact with observationally accessible systems. In particular, the observations of pulsars reveal systematic deviations of the braking index [8] from the canonical value $n = 3$ predicted by electromagnetic dipole radiation, thereby reviving the question of with respect to which structure rotation is fundamentally defined.

In this work, the spin-down of pulsars is examined from a perspective in which spacetime is not merely a passive stage but a dynamical and organizing element of rotational motion.

Within the teleparallel framework, spacetime is not merely a passive background but may have an intrinsic local flow and rotational structure. The drift component of the tetrad provides a natural geometric description of this flow, encoding the local rotational properties of spacetime itself.

The rotation of a neutron star cannot therefore be defined independently of this geometric structure. Rather than rotating in an abstract or absolute space, the star evolves within a spacetime that already carries a preferred local orientation and flow. As a result, the angular velocity of the star must be defined relative to this underlying geometric background.

This gives rise to an additional torque of purely geometric origin, which supplements the standard electromagnetic dipole braking. The resulting geometric torque is linear in the angular velocity and thus contributes directly to the rotational evolution of the star.

Consequently, the observed spin-down behavior of pulsars is no longer governed solely by electromagnetic processes. The braking index becomes a direct observational imprint of the interplay between stellar rotation and the geometric properties of spacetime.

The paper is organized as follows. Section 2 introduces the teleparallel framework and the geometric structure of the temporal tetrad. In Section 3, the electromagnetic sector and its gauge structure are derived. Section 4 is devoted to fundamental physical tests and global phase effects. Physical consistency tests such as Coulomb potential and some physical limits are discussed in

Section 5. By the following chapter, the full unified action is given explicitly. Chapter 7 is concerned with pulsars and the braking index problem. The paper concludes with a general assessment of the results and a discussion of possible observational signatures.

2. Mathematical Framework

2.1. The Temporal Tetrad in the Framework

The starting point of the present work is the view that gravitation and electromagnetism do not necessarily originate from independent geometric structures. Rather, the possibility is considered that both interactions arise from different dynamical aspects of a single teleparallel tetrad field. [7]

From this perspective, the commonly drawn distinction between a gravitational tetrad and an electromagnetic tetrad does not reflect a fundamental physical necessity, but should instead be regarded as a consequence of adopting different consistent interpretations of the same underlying geometric structure.

Within the Geometric Drift Vector (GDV) framework [5], the tetrad field is considered only in its gravitational sector. This choice may equivalently be interpreted as selecting a tetrad that is Fermi–Walker transported [11] along the temporal direction, corresponding to a non-rotating local Lorentz frame. In this sense, the absence of temporal torsion is not a gauge artifact but a deliberate restriction to observer frames that exclude the degrees of freedom.

The spatial legs of the tetrad are chosen in a general form suitable for gravitational and cosmological dynamics,

$$e^i = a(t)\,dx^i + W^i(t,\mathbf{x})\,dt, \tag{1}$$

such that gravitation, inertia, and cosmological evolution are encoded in the spatial components of the torsion tensor and in the torsion trace vector Ref. [5]

$$T_\mu = T^\nu{}_{\nu\mu}. \tag{2}$$

Within this restricted setting, the temporal leg of the tetrad is taken in its simplest form,

$$e^0 = c\,dt, \tag{3}$$

not because this choice exhausts all possible temporal structures, but because it deliberately excludes electromagnetic degrees of freedom. As a direct consequence, the temporal torsion vanishes identically, and the theory describes purely gravitational and inertial phenomena encoded entirely within the spatial torsion sector. [4]

2.2. Temporal origin of the electromagnetic mode

In a local and Lorentz–covariant teleparallel framework [7], introducing a single propagating 1–form excitation of the tetrad without additional structure requires that no preferred internal spatial direction be selected. Embedding such an excitation into a spatial tetrad leg e^i necessarily singles out a fixed internal spacelike direction, reducing the local Lorentz group to its stabilizer and thereby introducing a preferred spatial structure. Avoiding this either breaks local Lorentz covariance or entangles the would–be gauge freedom with Lorentz transformations, obstructing a genuine $U(1)$ interpretation.

By contrast, the temporal decomposition

$$e^0 = u + \kappa A \tag{4}$$

admits a representation redundancy $A \to A + d\chi$, $u \to u - \kappa d\chi$ that leaves the tetrad, metric, and torsion invariant. At the lowest derivative order, torsion depends only on exterior derivatives, and the unique local Lorentz–invariant propagating structure built from a single 1–form is $F = dA$, yielding the Maxwell kinetic term. Under these assumptions, electromagnetism can therefore arise consistently only from the temporal coframe.

2.3. Temporal Tetrad and the Geometric Origin of Electromagnetism

2.3.1. Geometric Extension of the Temporal Tetrad

In the present formulation, the restriction adopted in the previous framework is relaxed, and the temporal component of the tetrad is allowed to acquire a more general geometric structure. In this context, the quantities Φ and A_i are not introduced as independent gauge fields. Rather, they parametrize different components of a single geometric object, namely the temporal tetrad one–form e^0. Within this approach, the electromagnetic potential is not viewed as a fundamental field appended to the theory, but instead emerges, at the level of representation, from the decomposition of the temporal geometry into a background structure and a dynamical excitation.

The temporal tetrad is decomposed as

$$e^0 = u + \kappa\, A, \tag{5}$$

where u denotes a gauge–invariant background one–form encoding the physical temporal flow of spacetime. It defines a temporal foliation associated with a given observer congruence and is assumed to be globally integrable,

$$du = 0. \tag{6}$$

This condition fixes a non–twisting time direction and should be understood as a gauge choice at the level of spacetime slicing rather than a physical restriction.

As a consequence, the spacetime metric and all observable quantities remain Lorentz covariant and gauge independent. The electromagnetic field acts on spacetime through torsion rather than through the metric, while the definition of time remains observer dependent and does not affect physical observables.

The one–form $A = A_\mu dx^\mu$ is identified with the electromagnetic potential, while the constant κ ensures that the temporal tetrad carries the correct physical dimensions. The role and physical meaning of κ are discussed. (Please see Appendix I) The coupling κ also controls the backreaction of the electromagnetic energy–momentum on spacetime geometry, consistent with Einstein–Cartan–type [12] structures in which torsion sources curvature.

2.3.2. Temporal Torsion and Gauge Redundancy

In component form, the decomposition reads

$$e^0{}_0 = u_0 + \kappa\, \Phi, \qquad e^0{}_i = u_i + \kappa\, A_i. \tag{7}$$

The temporal torsion two–form is defined by

$$T^0 = de^0, \tag{8}$$

with components

$$T^0{}_{\mu\nu} = \partial_\mu e^0{}_\nu - \partial_\nu e^0{}_\mu. \tag{9}$$

Since teleparallel torsion depends exclusively on exterior derivatives, the background temporal one–form u does not contribute to the torsion on the integrable branch $du = 0$. In this sector, which holds identically in the cosmological rest frame where $u = c\,dt$, the temporal torsion reduces to

$$T^0 = \kappa\, dA. \tag{10}$$

In this sense, u should not be regarded as an independently measurable field; only the combination $e^0 = u + \kappa A$ has geometric and physical meaning. The identification of u with a Fermi–Walker

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transported frame [11] fixes the redundant rotational degrees of freedom of the temporal leg without introducing any preferred–frame effects. Local Lorentz covariance is preserved by construction, since any local Lorentz transformation of the tetrad corresponds merely to a change of observer frame.

For more general situations, such as rotating observers or Kerr–type geometries where $du \neq 0$, the temporal torsion decomposes as

$$T^0 = du + \kappa\, dA. \tag{11}$$

The electromagnetic field remains unambiguously defined by $F = dA$, leaving the Maxwell identity $dF = 0$ unchanged (see Appendix II).

2.3.3. Electromagnetic Field from Temporal Torsion

On the integrable branch, the electromagnetic field strength may be identified directly with temporal torsion according to

$$F_{\mu\nu} \;\equiv\; \frac{1}{\kappa}\, T^0{}_{\mu\nu}, \tag{12}$$

which yields the familiar relation

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{13}$$

The electric and magnetic fields are defined as

$$E_i = F_{0i}, \qquad B^k = \frac{1}{2}\,\epsilon^{kij} F_{ij}. \tag{14}$$

The essential point is that the electromagnetic field strength depends exclusively on the exterior derivative of A. The background temporal structure u specifies the observer–dependent notion of time but has no direct effect on $F_{\mu\nu}$ in this sector. Electromagnetism is therefore not introduced as an independent gauge field appended to the theory, but instead emerges as the dynamical expression of temporal geometry through torsion. (Please see Appendix III)

2.3.4. **Geometric Necessity of the Maxwell Kinetic Term**

In the teleparallel framework adopted here, the tetrad is the fundamental dynamical variable and torsion depends linearly on it. Requiring locality, local Lorentz covariance, and restricting attention to the lowest nontrivial derivative order, severely constrains the admissible dynamical structures in the temporal sector.

Since the electromagnetic degrees of freedom are entirely encoded in a single 1–form excitation of the temporal tetrad, gauge invariance implies that the only admissible dynamical object is the antisymmetric derivative

$$F := dA. \tag{15}$$

At dimension four, local Lorentz covariance admits only two scalar densities constructed from F,

$$F_{\mu\nu} F^{\mu\nu}, \qquad F_{\mu\nu} \tilde{F}^{\mu\nu}. \tag{16}$$

In the Abelian case, the second term is topological and does not contribute to the local equations of motion. The remaining term yields precisely the Maxwell kinetic structure.

Consequently, under the stated assumptions, the Maxwell form is not a freely chosen option but the unique dynamical realization of a propagating electromagnetic sector within teleparallel temporal geometry.

(Please see Appendix X)

3. Electromagnetic Sector from Temporal Torsion

In the present framework, electromagnetism is not introduced as an independent internal gauge field, but is instead understood as emerging from the temporal sector of teleparallel tetrad geometry. The fundamental dynamical object is the tetrad field itself, while physical observables are encoded in

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its associated torsion. In particular, the electromagnetic field strength is identified with the temporal component of torsion, rather than with the temporal tetrad one form directly.

This viewpoint implies that the temporal tetrad admits a certain representational freedom without affecting physical observables. Since torsion depends only on derivatives of the tetrad, different representations of the temporal tetrad that differ by an additive exact one form correspond to the same physical configuration. As a result, electromagnetic gauge freedom appears as an intrinsic feature of the temporal tetrad geometry, rather than as an externally imposed internal symmetry.

Since neither the temporal background u nor the potential A carry direct physical meaning, and since teleparallel torsion depends exclusively on exterior derivatives, the antisymmetric derivative dA represents the only admissible propagating and gauge-invariant structure. This uniquely fixes the electromagnetic sector to the Maxwell form.

3.1. Gauge Freedom and Temporal Reparametrization

Within this geometric setting, electromagnetic gauge transformations are defined as transformations of the dynamical potential,

$$A \longrightarrow A + d\chi, \tag{17}$$

under which the electromagnetic field strength remains invariant,

$$F \longrightarrow F. \tag{18}$$

Consequently, all observable electromagnetic quantities depend exclusively on $F_{\mu\nu}$ and are insensitive to the particular representative chosen for the potential.

Geometrically, this freedom reflects the fact that the temporal tetrad e^0 may be shifted by an exact one–form without modifying its associated torsion. Explicitly, the transformation (Please see Appendix IV)

$$e^0 \longrightarrow e^0 + \kappa\, d\chi \tag{19}$$

Concretely, under

$$A \to A + d\chi, \qquad u \to u - \kappa\, d\chi,$$

the temporal tetrad is strictly invariant,

$$e^0 \to e^0,$$

and therefore the spacetime metric

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$

is manifestly gauge invariant. The invariance $T^0 = de^0$ under $d\chi$ then follows automatically and does not require treating e^0 itself as gauge-variant.

The Stückelberg field χ therefore does not introduce new propagating degrees of freedom; it parametrizes the representation freedom in the split (u, A), while all observables depend only on $F = dA$.

The split into (u, A) is observer/representation dependent, while $F = dA$ and all observables remain representation independent.

It is crucial that the transformation $(A, u) \mapsto (A + d\chi,\ u - \kappa d\chi)$ is not a spacetime diffeomorphism and not a local Lorentz transformation. It is a redundancy of the *decomposition* $e^0 = u + \kappa A$: different pairs (u, A) represent the same geometric one-form e^0 and therefore the same metric and torsion.

In an explicit coordinate representation, writing

$$A = \Phi\, dt + A_i\, dx^i, \tag{20}$$

the temporal tetrad takes the form

$$e^0 = u + \kappa\big(\Phi\, dt + A_i\, dx^i\big). \tag{21}$$

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Under a gauge transformation $A \to A + d\chi$, the components transform as

$$\Phi \;\to\; \Phi + \partial_t \chi, \qquad A_i \;\to\; A_i + \partial_i \chi, \tag{22}$$

while the temporal torsion $T^0{}_{\mu\nu}$ and the field strength $F_{\mu\nu}$ remain invariant.

It is important to note that electromagnetic gauge transformations act on spacetime one forms, whereas local Lorentz transformations act on the internal indices of the tetrad. These symmetries operate on distinct geometric structures and are therefore fully compatible.

3.2. Homogeneous Maxwell Equations

In the present approach, the electromagnetic field strength is derived directly from the temporal torsion of the tetrad. Assuming an integrable background temporal flow, $du = 0$, the field strength may be written as

$$F = dA. \tag{23}$$

As an immediate consequence, the homogeneous Maxwell equations follow identically,

$$dF = d(dA) = 0. \tag{24}$$

In component form, this relation reproduces

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0. \tag{25}$$

These equations therefore reflect the geometric definition of the field strength, rather than independent dynamical content.

The fact that the homogeneous Maxwell equations arise as geometric identities is not a shortcoming but a structural feature of the teleparallel formulation.

It reflects that no magnetic monopoles or additional constraints are introduced by hand.

3.3. Inhomogeneous Maxwell Equations from the Action Principle

The dynamical content of the electromagnetic field and its coupling to sources arise from a unified action principle. Implementing the geometric decomposition

$$e^0 = u + \kappa(A + d\varphi), \tag{26}$$

as a constraint, the total action is written as

$$S = S_{\text{grav}}[e] + S_{\text{EM}}[A, e] + S_{\text{constraint}}[e^0, A, \varphi, J]. \tag{27}$$

The electromagnetic action is taken in the standard Maxwell form,

$$S_{\text{EM}}[A, e] = -\frac{1}{4} \int d^4x\, |e|\, F_{\mu\nu} F^{\mu\nu}. \tag{28}$$

The constraint action is given by

$$S_{\text{constraint}} = \int d^4x\, |e|\, J^\mu \Big(e^0{}_\mu - u_\mu - \kappa\big(A_\mu + \partial_\mu \varphi\big) \Big), \tag{29}$$

where J^μ is identified with the electromagnetic current density and φ is a Stückelberg scalar ensuring gauge invariance.

The Stückelberg scalar φ is introduced to preserve $U(1)$ gauge invariance of the geometric constraint, ensuring that the identification of the electromagnetic potential with the temporal tetrad does not represent a fixed gauge choice but a physically redundant description of the same underlying geometry. It does not introduce an independent dynamical degree of freedom and carries no physical excitations.

(Please see Appendix IX)

Variation with respect to A_μ yields the inhomogeneous Maxwell equations,

$$\nabla_\nu F^{\nu\mu} = \kappa\, J^\mu, \tag{30}$$

The appearance of κ reflects the chosen normalization of the current within the geometric constraint and does not modify the universal form of Maxwell dynamics, which remains invariant under rescalings of κ. While variation with respect to φ implies current conservation,

$$\nabla_\mu J^\mu = 0. \tag{31}$$

Finally, variation with respect to J^μ recovers the geometric relation linking the electromagnetic potential to the temporal tetrad, confirming that the electromagnetic sector is naturally contained within the temporal geometry.

(Please see Appendix VI)

4. Fundamental Physical Tests and Conservation Laws

In the preceding sections, a unified teleparallel framework has been established in which electromagnetism is shown to possess a geometric origin associated with the temporal structure of spacetime. This framework preserves local Lorentz symmetry while reproducing the standard Maxwell dynamics in full. No new geometric assumptions or additional structures are introduced in the present section. The purpose here is to examine whether the proposed theory successfully passes a set of fundamental physical tests.

To this end, the motion of charged test particles is analysed, the Lorentz force law is derived from the unified action principle, and the conservation laws implied by the theory are discussed. The goal is to demonstrate that this geometric interpretation of electromagnetism reproduces all experimentally established electromagnetic phenomena without invoking preferred reference frames or introducing additional gauge structures.

4.1. Test Particle Motion in the Unified Geometry

The physical analysis begins with the motion of a classical charged test particle propagating within the unified teleparallel geometry defined in the preceding sections. All geometric quantities entering this analysis are restricted to the tetrad-based structures already introduced.

4.1.1. Action for a Charged Test Particle

Electromagnetism is not introduced as an independent internal gauge field but is encoded in the temporal leg of the tetrad through the decomposition

$$e^0 = u + \kappa\,(A + d\varphi), \tag{32}$$

where u denotes the temporal flow one–form and φ is a Stückelberg scalar ensuring representation invariance.

The action for a point particle of rest mass m and electric charge q moving along a worldline $x^\mu(\tau)$ is written in the standard form

$$S_p = -m \int d\tau \sqrt{-\,\eta_{ab}\,(e^a{}_\mu\, \dot{x}^\mu)\,(e^b{}_\nu\, \dot{x}^\nu)}\; +\; q \int d\tau\, \dot{x}^\mu A_\mu, \tag{33}$$

where $\dot{x}^\mu = dx^\mu/d\tau$. Although the second term appears as a conventional minimal coupling, it should be emphasized that A_μ is not treated here as an independent dynamical field. Rather, it represents the one–form component arising from the geometric parametrization of the temporal tetrad.

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By using the decomposition, the interaction term may be equivalently rewritten as

$$q \int d\tau \, \dot{x}^\mu A_\mu = \frac{q}{\kappa} \int d\tau \, (e^0{}_\mu - u_\mu) \, \dot{x}^\mu \; - \; q \int d\tau \, \frac{d\varphi}{d\tau}, \tag{34}$$

where the last term is a total derivative and does not contribute to the equations of motion. Up to this boundary term, the particle action can therefore be expressed entirely in terms of tetrad variables as

$$S_p = -m \int ds \; + \; \frac{q}{\kappa} \int (e^0 - u), \tag{35}$$

making explicit that both gravitational and electromagnetic couplings are encoded within the same geometric structure.

The action is invariant under worldline reparameterizations $\tau \to \tau'(\tau)$. Gauge invariance follows from the redundancy of the decomposition , since under

$$A_\mu \to A_\mu + \partial_\mu \chi, \qquad \varphi \to \varphi - \chi, \tag{36}$$

the tetrad e^0 and hence the action remain invariant up to a total derivative. Phase–sensitive phenomena, such as the Aharonov–Bohm effect[6], are therefore consistently described within the present framework.

4.1.2. Equation of Motion

Variation of the action with respect to the worldline $x^\mu(\tau)$ yields the particle equation of motion. Introducing the proper time via

$$ds^2 = \eta_{ab} \, e^a{}_\mu e^b{}_\nu \, dx^\mu dx^\nu, \tag{37}$$

and defining the four–velocity

$$u^\mu := \frac{dx^\mu}{ds}, \tag{38}$$

the resulting equation can be written in covariant form as

$$m \, u^\nu \nabla_\nu u^\mu = q \, F^\mu{}_\nu u^\nu, \tag{39}$$

where ∇_ν denotes the Levi–Civita covariant derivative associated with the metric induced by the tetrad.

The field strength (Please see Appendix V)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{40}$$

is not introduced as an independent object but is identified with the antisymmetric derivative of the electromagnetic component encoded in the temporal tetrad. Equation is thus recognized as the standard relativistic Lorentz force law, recovered here as the matter–sector representation of the underlying teleparallel geometry.

From this perspective, gravitational effects arise from the spacetime geometry defined by the full tetrad, while electromagnetic interactions are associated with the temporal torsion sector of the same geometric framework. The exact recovery of the Lorentz force demonstrates that the unified teleparallel construction reproduces the experimentally established dynamics of charged particles without introducing preferred frames or violating local Lorentz symmetry.

4.2. Aharonov–Bohm Effect and Global Geometric Structure

The Aharonov–Bohm effect [6] provides a striking example suggesting that electromagnetic interactions can not always be understood solely in terms of local forces. In the classical picture, a charged particle is expected to be affected only in regions where electric or magnetic fields are present;

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if the fields vanish, no force acts on the particle. In quantum mechanics, however, a particle is described not merely as a point tracing a definite path, but as a wave carrying a phase. As a result, even when the local fields vanish, the global structure of the electromagnetic potential may leave observable traces in the particle's behaviour.

This feature appears most clearly in the quantum phase accumulated along a closed path. For a particle of electric charge q, the phase difference in the semiclassical approximation is given by

$$\Delta\theta = \frac{q}{\hbar} \oint_{\gamma} A_{\mu}\, dx^{\mu}, \tag{41}$$

where γ denotes a closed spacetime curve. The crucial point is that while the Lorentz force depends only on the electromagnetic field strength $F_{\mu\nu}$, the phase depends directly on the potential A_{μ}. For this reason, a measurable phase shift may arise even in situations where $F_{\mu\nu} = 0$.

In the standard interpretation, this phenomenon is often attributed to the global properties of the potential and to the topological structure of spacetime. Although the relation $F = dA$ holds locally, the potential need not be globally expressible as a single valued exact form.

The unified teleparallel framework proposed in this work places this observation within a more direct geometric setting. Here, the electromagnetic potential A_{μ} does not appear as an independent internal field, but is related to the geometric structure of the temporal tetrad. In particular, under the decomposition

$$e^{0} = u + \kappa A, \tag{42}$$

the closed–loop integral of the potential can be related directly to the global properties of the temporal geometry. From this relation one obtains

$$\oint_{\gamma} A = \frac{1}{\kappa} \oint_{\gamma} e^{0} - \frac{1}{\kappa} \oint_{\gamma} u. \tag{43}$$

In situations such as a cosmological rest frame, the background temporal flow may be taken as $u = c\, dt$. For a closed curve γ, this implies

$$\oint_{\gamma} u = 0, \tag{44}$$

and the expression simplifies to

$$\oint_{\gamma} A = \frac{1}{\kappa} \oint_{\gamma} e^{0}. \tag{45}$$

(Please see Appendix VII) This result suggests that the Aharonov–Bohm phase [6] is not merely a technical quantity tied to a particular gauge choice. Rather, the phase is sensitive to the global organization of the temporal direction along the closed path traced by the particle. In other words, when a particle completes a closed loop in space, the temporal structure need not return to its initial configuration, and this mismatch manifests itself as a quantum phase.

The local vanishing of the field strength,

$$F = dA = 0, \tag{46}$$

indicates that no force acts on the particle along its trajectory; however, it does not preclude physical effects arising from the global arrangement of the temporal geometry. Quantum mechanics does not introduce a new interaction in this context, but rather probes the global geometric structure of spacetime through the accumulated phase.

From this perspective, the Aharonov–Bohm effect [6] should not be regarded as a mere additional consistency test of the theory. Instead, it appears as a natural consequence of the geometric origin of the electromagnetic potential.

Local dynamics are governed by the Lorentz force, while global phase effects reflect the same underlying geometry when considered along closed paths.

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5. Full Unified Action

5.1. Motivation for a Unified Action

The present work is based on the Geometric Drift Vector (GDV) framework, in which gravitation, inertia, and large scale cosmic dynamics are understood as manifestations of the kinematical structure of spacetime. Within this approach, the torsion trace plays a central role: its temporal component measures local departures from the global spacetime flow and provides an effective geometric description of galactic dynamics and cosmic expansion without introducing additional matter sectors.

In the original formulation of the GDV framework, the temporal tetrad leg was chosen to be fixed and aligned with the cosmic time direction. This assumption isolates the gravitational sector and suppresses any further structure associated with the temporal geometry. Teleparallel geometry, however, does not require such a restriction. The temporal tetrad may be treated as a general one-form, opening the possibility that additional physical content resides in its geometric structure.

Allowing this degree of freedom suggests a natural extension of the GDV framework. In the present construction, the electromagnetic potential is not introduced as an independent field. Instead, it is identified with a specific geometric component of the temporal tetrad. In this way, the framework is extended from a purely gravitational description to a unified geometric setting in which gravitation and electromagnetism arise from different aspects of the same teleparallel structure.

The unified tetrad may be written schematically as

$$e^0 = u + \kappa\big(A + d\varphi\big), \qquad e^i = a(t)\,dx^i + W^i(t,\mathbf{x})\,dt, \tag{47}$$

where u denotes the background temporal flow, W^i the geometric drift vector, A_μ the electromagnetic potential, and φ a Stückelberg scalar ensuring $\mathrm{U}(1)$ gauge invariance. Within this picture, electromagnetism and gravitation do not inhabit separate geometric sectors, but emerge from different components of a single tetrad field.

5.2. Unified Action

Once a common tetrad origin for gravitation and electromagnetism has been identified, it is natural to formulate a single variational principle encompassing both interactions. The guiding requirement is that the resulting action reproduce the established physical theories in appropriate limits, while preserving the characteristic geometric structure of the GDV framework.

Accordingly, we consider the unified action

$$S = \int d^4x\, e \left[\frac{1}{2\kappa}\,\mathcal{L}_{\text{grav}} - \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\text{m}} + \mathcal{L}_{\text{constraint}}\right], \tag{48}$$

where $e = \det(e^a{}_\mu)$ denotes the tetrad determinant.

The gravitational sector is described by the Gravitation Lagrangian

$$\mathcal{L}_{\text{grav}} = T + \beta\big(T^\mu u_\mu\big)^2, \tag{49}$$

with T the torsion scalar constructed from the Weitzenböck connection [4], T^μ the torsion vector, and u_μ the normalized temporal background one–form introduced previously. The dimensionless parameter β controls the coupling between torsion and the preferred temporal direction.

Although the electromagnetic sector is written explicitly in the standard Maxwell form for clarity, its appearance is geometrically motivated by the temporal component of the torsion tensor.

(Please see Appendix XI)

The electromagnetic sector is given by the standard Maxwell term

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}, \tag{50}$$

where $F_{\mu\nu}$ is the field strength associated with the electromagnetic potential.

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The matter contribution $\mathcal{L}_{\mathrm{m}}$ describes additional non-gravitational fields and is assumed to couple minimally to the tetrad.

Finally, the term $\mathcal{L}_{\mathrm{constraint}}$ enforces the geometric identification between the temporal tetrad component and the electromagnetic potential, ensuring the consistency of the unified description.

In the comoving frame defined by the temporal flow u^μ, the additional term reduces to the square of the temporal component of the torsion trace. This quantity characterizes the local divergence of the geometric drift and the expansion tendency of spacetime. Its appearance in the action reflects the direct physical influence of the underlying kinematical structure.

The individual contributions to the action admit the following interpretation:

The torsion scalar T represents the teleparallel analogue of the gravitational kinetic term and, in the GDV regime, encodes the gradient energy associated with the drift structure.

The electromagnetic sector is described by the standard field strength $F_{\mu\nu}$. Its homogeneous equations follow identically from the geometric definition, reflecting the torsional origin of the electromagnetic field within the present framework.

The constraint term enforces the geometric identification of the electromagnetic potential within the temporal tetrad while preserving gauge invariance and avoiding the introduction of additional degrees of freedom.

5.3. Physical Limits and Consistency

The unified framework reproduces established physical regimes through controlled geometric limits.

Gravitational regime. In the absence of electromagnetic excitations, the action reduces to the GDV-modified teleparallel description of gravitation. In this limit, the torsion–trace contribution governs galactic and cosmological dynamics without invoking independent dark matter or dark energy components. In this case, please check the Ref. [5].

Electromagnetic regime. When gravitational effects are negligible, the theory reduces smoothly to standard Maxwell electrodynamics in flat spacetime. Electric charge conservation emerges as a Noether identity associated with the geometric redundancy of the temporal sector.

Newtonian limit and equivalence principle. The theory satisfies the Weak Equivalence Principle [9]. Free fall corresponds to adaptation to the spacetime flow, and observers comoving with this flow experience no physical acceleration, despite the presence of torsion.

By taking the tetrad, rather than the metric, as the fundamental geometric object, the present framework avoids several structural difficulties encountered in earlier metric-based unified field theories. Gravitation and electromagnetism appear as distinct yet inseparable aspects of a single teleparallel geometry, yielding a coherent and testable unified description compatible with known physics.

6. Geometric Interpretation of the Braking Index

One of the long-standing observational puzzles in pulsar astrophysics is the systematic deviation of the measured braking index from the canonical value $n = 3$ predicted by pure electromagnetic dipole radiation. While the standard model assumes that the spin-down torque is entirely of electromagnetic origin, observationally determined values typically satisfy $n < 3$, and in some cases even $n > 3$, suggesting the presence of additional physical contributions to the rotational dynamics.

6.0.1. Standart Spin Down Law

In this section, it is shown that within the teleparallel framework developed in this work, the braking index anomaly admits a natural geometric interpretation. The deviation from $n = 3$ does not indicate a breakdown of Maxwell electrodynamics, but rather reflects the presence of an additional torque associated with the vorticity of spacetime drift encoded in the tetrad.

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The rotational evolution of a pulsar is conventionally described by

$$I\dot{\Omega} = \tau_{\rm EM}, \tag{51}$$

where I is the stellar moment of inertia and Ω the angular frequency. For electromagnetic dipole radiation in vacuum, the torque takes the form

$$\tau_{\rm EM} = -K_3\Omega^3, \tag{52}$$

with

$$K_3 = \frac{B_p^2 R^6 \sin^2\chi}{12Ic^3}, \tag{53}$$

where B_p is the polar magnetic field strength, R the stellar radius, and χ the magnetic obliquity. This leads to the well–known prediction $n = 3$ for the braking index,

$$n \equiv \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2}. \tag{54}$$

6.0.2. The Classical Spin-Down Law

In the teleparallel formulation adopted here, spacetime geometry is described by a tetrad containing a nontrivial drift (or shift) component,

$$e^i = a(t)\,dx^i + W^i\,dt. \tag{55}$$

The spatial drift field W^i carries physical information beyond coordinate freedom. In particular, its antisymmetric spatial derivative,

$$B_{[ij]} = \frac{1}{2}(\partial_i W_j - \partial_j W_i), \tag{56}$$

encodes the vorticity of the spacetime drift.

Within teleparallel geometry, this quantity is directly related to the antisymmetric components of the contortion tensor,

$$K_{ij0} \propto B_{[ij]}, \tag{57}$$

which are responsible for vortex-like and Coriolis-type inertial effects. It is therefore natural to associate a geometric angular velocity

$$!_W \equiv \frac{1}{2}\nabla\times\mathbf{W} \tag{58}$$

with the local rotational properties of spacetime itself.

6.0.3. Geometric drift torque as the leading dissipative coupling

The stellar rotation must be defined relative to the local rotational structure of the drift field, encoded by the geometric angular velocity ω_W. Accordingly, the only rotationally invariant scalar controlling dissipation is the relative angular velocity

$$\Delta\Omega := \Omega - \omega_W.$$

In the absence of detailed microphysics, the appropriate description is an effective (linear-response) theory. The leading dissipative contribution is therefore fixed by symmetry and derivative counting via a Rayleigh dissipation function of lowest order,

$$\mathcal{R} = \frac{1}{2}\,I\,\Gamma\,(\Omega-\omega_W)^2,$$

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where Γ has dimensions of an inverse time and characterizes the rate at which the stellar rotation relaxes toward the geometric drift. The associated torque is obtained from

$$\tau_{\rm GDV} = -\frac{\partial \mathcal{R}}{\partial \Omega} = -I\Gamma(\Omega - \omega_W).$$

This form is the unique linear dissipative coupling consistent with rotational symmetry: it vanishes when $\Omega = \omega_W$ (co-rotation with the geometric flow) and provides the leading correction to the electromagnetic dipole torque. Higher-order terms such as $(\Omega - \omega_W)^3$ or nonlocal memory kernels are subleading in the effective expansion and are not required at the minimal level.

The appearance of the linear structure is fixed by geometry and symmetry, while the numerical value of Γ encodes unresolved microphysical dissipation and is treated as an effective parameter.

The relaxation rate Γ does not represent a new fundamental coupling constant. Its appearance reflects the effective description of dissipative alignment between stellar rotation and the geometric drift. While the existence of a relative angular velocity $(\Omega - \omega_W)$ and the linear structure of the associated torque are fixed by geometry and symmetry, the numerical value of Γ encodes unresolved microphysical dissipation and coarse–graining effects. In this sense, Γ plays a role analogous to transport coefficients such as viscosity or conductivity in hydrodynamic effective theories: it is not introduced as an independent dynamical ingredient, but parametrizes the rate at which the system relaxes toward the geometrically preferred configuration.

6.0.4. Total Spin Down Law

The rotational dynamics of the neutron star must then be formulated relative to this geometric drift. The simplest rotationally invariant dissipative coupling between the stellar angular velocity and the spacetime vorticity leads to an additional geometric torque,

$$\tau_{\rm GDV} = -I\Gamma(\Omega - \omega_W), \tag{59}$$

where Γ is a characteristic alignment rate determined by the strength of the geometric drift.

The total spin–down equation becomes

$$\dot{\Omega} = -K_3\Omega^3 - \Gamma(\Omega - \omega_W). \tag{60}$$

In the regime $\omega_W \ll \Omega$, this reduces to

$$\dot{\Omega} \simeq -(K_3\Omega^3 + \Gamma\Omega). \tag{61}$$

In this limit, the braking index is readily computed, (Please see Appendix VIII)

$$n = \frac{3K_3\Omega^2 + \Gamma}{K_3\Omega^2 + \Gamma}. \tag{62}$$

Solving for the relative strength of the geometric contribution yields

$$\frac{\Gamma}{K_3\Omega^2} = \frac{3-n}{n-1}. \tag{63}$$

This result shows that the braking index directly measures the ratio between the geometric drift torque and the electromagnetic dipole torque. Values $n < 3$ correspond to a non–negligible geometric contribution, while $n \to 3$ is recovered when the drift vorticity vanishes.

6.1. Observational Tests: Crab and Vela Pulsars

The geometric interpretation of the braking index can be directly tested using well–studied pulsars for which precise timing measurements are available. In this subsection, the implications of

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the proposed model using the Crab and Vela pulsars is illustrated as representative examples. No additional free parameters are introduced beyond those already appearing in the spin–down equation.

The data is used from The Australia Telescope National Facility Pulsar Catalogue. [10]

The Crab pulsar (PSR B0531+21) is one of the youngest and best–characterized rotation–powered pulsars. Its measured braking index is

$$n_{\rm Crab} \simeq 2.5, \tag{64}$$

significantly below the canonical dipole value. The angular frequency and its derivative are approximately

$$\Omega \simeq 190\,{\rm s}^{-1}, \qquad \dot{\Omega} \simeq -2.4\times 10^{-9}\,{\rm s}^{-2}. \tag{65}$$

Using the relation

$$\frac{\Gamma}{K_3\Omega^2} = \frac{3-n}{n-1}, \tag{66}$$

the Crab braking index implies

$$\frac{\Gamma}{K_3\Omega^2} \simeq 0.3. \tag{67}$$

This indicates that the geometric drift contribution constitutes a moderate but non–negligible fraction of the total spin–down torque. The corresponding geometric alignment timescale, $\tau_\Gamma \sim \Gamma^{-1}$, is of order 10^4 years, comparable to the characteristic age of the system.

The Vela pulsar (PSR B0833−45) represents a more evolved system and exhibits a much lower braking index,

$$n_{\rm Vela} \simeq 1.4. \tag{68}$$

With an angular frequency

$$\Omega \simeq 70\,{\rm s}^{-1}, \tag{69}$$

the inferred ratio becomes

$$\frac{\Gamma}{K_3\Omega^2} \simeq 4. \tag{70}$$

In this case, the geometric drift torque dominates over the electromagnetic dipole contribution.

The associated alignment timescale remains of order 10^4-10^5 years, consistent with the evolutionary stage of the pulsar. This behavior naturally explains why the braking index of Vela deviates more strongly from the dipole prediction than that of the Crab pulsar.

The present analysis assumes slowly varying Γ and ω_W. However, abrupt changes in the braking index observed following pulsar glitches may be naturally accommodated within the same framework by allowing for transient variations of the geometric drift. In this interpretation, glitch recovery reflects the relaxation of the local spacetime drift toward a new quasi–stationary configuration.

7. Conclusion

In this work, a teleparallel-based framework has been presented in which electromagnetism and gravitation can be treated within a single geometric structure. The formulation developed here does not require the electromagnetic potential to be introduced as an independent internal gauge field appended to the theory. Instead, the analysis shows that it may be represented as arising from the geometric structure of the temporal tetrad. Within this perspective, the electromagnetic field can be understood as a dynamical expression of the temporal organization of spacetime.

Historically, attempts at unified field theories have encountered serious difficulties, including the loss of gauge symmetry, violations of local Lorentz covariance, or the emergence of phenomenologically excluded effects. In particular, Weyl's [1]conformal approach led to path-dependent lengths, while many later constructions required the introduction of extra spacetime dimensions or additional degrees of freedom. The teleparallel framework presented here is constructed so as to avoid such pathologies: the electromagnetic sector is neither associated with metric scale transformations nor does it require any modification of the fundamental symmetries of spacetime.

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A key structural feature of teleparallel geometry is that torsion depends linearly on the tetrad. This property makes it possible for the homogeneous Maxwell equations to arise as purely geometric identities, without the need for additional assumptions. At the same time, electromagnetic gauge invariance is preserved through a Stückelberg mechanism. The resulting $U(1)$ symmetry does not appear as a spacetime transformation, but rather as an equivalence relation associated with the freedom in representing the temporal geometry. In this way, local Lorentz symmetry and gauge symmetry coexist consistently, each acting on a distinct geometric level.

Within the present framework, electromagnetism is not interpreted as a new force or as an auxiliary field added to the formalism, but as a sector associated with the dynamical structure of the temporal tetrad. This viewpoint allows static configurations such as the Coulomb field to be represented without invoking spacetime curvature, as particular spatial variations of the temporal geometry. Similarly, in this study the classical electromagnetic theory is corrected however, by using the global phase phenomena such as the Aharonov–Bohm effect [6] can be understood as consequences of the same underlying geometric structure accumulated along closed paths. In this case, it can be underlined that the electromagnetic potential A_μ appears not as a gauge quantity, but as a geometric manifestation of the underlying spacetime structure. Moreover, this geometric tetrad-based formulation suggests a natural starting point for future investigations in which quantum aspects may be incorporated, raising the possibility that certain quantum phenomena could admit a geometric origin within an extended version of the present framework.

In this sense, the potential is no longer a purely abstract mathematical object, but a natural component of the geometric organization of the temporal tetrad. The physical content of the electromagnetic field therefore does not arise from a structure externally attached to spacetime, but from the local and global organization of time. This perspective reframes the long-standing discussion of the physical status of the electromagnetic potential without invoking ontological claims, by returning it to its geometric origin.

A central structural outcome of the present framework is that the electromagnetic sector can be consistently accommodated only through the temporal component of the tetrad. This is not a matter of arbitrary choice, but follows from basic geometric and symmetry requirements. Associating a propagating 1–form with a spatial tetrad leg would necessarily introduce preferred spatial directions, thereby spoiling local Lorentz covariance, isotropy, and observer independence. By contrast, the temporal tetrad component already defines a covariant flow structure common to all observers and naturally supports kinematical quantities such as acceleration and vorticity.

A 1–form excitation carried by this temporal leg does not select a physical direction, but rather represents a redundancy in the description of temporal geometry. As a result, the emergence of a gauge potential, its antisymmetric field strength, and the associated gauge symmetry arise naturally from the exterior calculus of the temporal sector. In this sense, the geometric origin of electromagnetism identified here is both minimal and structurally enforced within the teleparallel tetrad framework.

The physical consistency of the theory has been examined through standard tests and through applications to observationally sensitive systems. Flat Minkowski spacetime is preserved as a natural and stable vacuum solution, and the Coulomb field is recovered as an electromagnetic configuration without requiring spacetime curvature.

The pulsar braking index problem offers a direct observational probe of the physical mechanisms governing rotational evolution. Within the classical electromagnetic dipole model, the spin–down torque scales as Ω^3, leading unavoidably to the prediction $n = 3$. High–precision timing observations, however, demonstrate that many pulsars systematically deviate from this value, indicating that electromagnetic dipole braking alone does not provide a complete description of pulsar spin–down.

In this work, these deviations are traced to the geometric structure of spacetime within a teleparallel formulation of gravity. A nontrivial drift component of the tetrad naturally defines a geometric vorticity encoding local rotational properties of spacetime. The rotation of the neutron star is therefore not independent of this structure, but must be defined relative to it. This coupling generates an

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additional geometric torque, linear in the angular velocity, which supplements the electromagnetic dipole contribution.

Within this framework, the braking index ceases to be a purely electromagnetic quantity and instead becomes a direct observational measure of the interplay between rotation and spacetime geometry. Young and rapidly rotating pulsars naturally approach the electromagnetic limit $n \to 3$, while older and slower systems exhibit $n < 3$ as geometric effects become increasingly important.

In this sense, the geometric drift vector (GDV) provides a minimal and geometrically motivated resolution of the pulsar braking index problem, without the introduction of additional phenomenological parameters.

The present construction does not fall within the scope of the Coleman Mandula and Weinberg Witten no-go theorems. This is because the unification achieved here does not rely on a group-theoretic mixing of spacetime and internal symmetries, but instead on a purely geometric identification. Since the electromagnetic gauge symmetry does not appear as a new spacetime generator, the assumptions underlying these theorems are not satisfied within the present framework.

The teleparallel framework can be understood as providing a clear, controlled, and observationally accessible setting in which this role of temporal geometry can be systematically explored.

Data Availability Statement: No new data were generated or analyzed in this study. All observational values used for illustration are taken from publicly available literature sources.

Acknowledgments: AI-assisted tools were used solely for formatting. All scientific content, derivations, and interpretations are the responsibility of the author.

Appendix A. Definition and Physical Meaning of the Coupling Constant κ

In this appendix, the mathematical necessity, dimensional structure, and physical interpretation of the constant κ are explained in detail.

Appendix A.1. Extension of the temporal tetrad

The fundamental assumption of the theory is that the temporal tetrad can be written in the form

$$e^0 = u + \kappa A, \tag{A71}$$

where u represents the background temporal flow and $A = A_\mu dx^\mu$ denotes the electromagnetic potential. The one-form e^0 is a geometric object with dimensions of length, whereas the electromagnetic potential A_μ possesses different physical dimensions. Consequently, the coexistence of these two quantities within a single expression necessitates the presence of a conversion factor that reconciles their dimensions. This factor is identified as the coupling constant κ.

At this stage, κ is not an arbitrary coupling parameter but a necessary conversion coefficient that allows the electromagnetic potential to be consistently embedded into temporal geometry.

Appendix A.2. Torsional origin of the electromagnetic field

As a consequence of this definition of the temporal tetrad, the temporal torsion tensor

$$T^0{}_{\mu\nu} = \partial_\mu e^0{}_\nu - \partial_\nu e^0{}_\mu \tag{A72}$$

is directly related to the electromagnetic field strength via

$$F_{\mu\nu} = \frac{1}{\kappa} T^0{}_{\mu\nu}. \tag{A73}$$

Appendix A.3. Dimensional analysis of κ

While the temporal tetrad e^0 carries dimensions of length, the electromagnetic potential has dimensions of energy per unit charge. Accordingly, the coupling constant κ must have dimensions

$$[\kappa] = \frac{\text{length}}{[A]}. \tag{A74}$$

This dimensional structure makes explicit that κ serves as a scale transforming electromagnetic quantities into geometric ones.

Appendix A.3.1. Relation of κ to the Planck scale

When natural physical scales are taken into account, the most natural and consistent expression for κ is

$$\kappa = \frac{e}{m_P c}, \tag{A75}$$

where e denotes the elementary electric charge and

$$m_P = \sqrt{\frac{\hbar c}{G}} \tag{A76}$$

is the Planck mass.

It is worth emphasizing that the appearance of the Planck mass in the definition of κ is not accidental.

In a unified geometric framework, κ sets the scale at which electromagnetic excitations of the temporal tetrad become comparable to purely gravitational torsional modes.

The ratio $\kappa = e/(m_P c)$ therefore reflects a geometric unification limit: electromagnetic fields correspond to extremely weak deformations of spacetime geometry when measured against the natural gravitational scale set by m_P. In this sense, κ characterizes the separation between electromagnetic and gravitational sectors rather than a phenomenological tuning parameter.

Numerically, $\kappa \sim 10^{-20}$. Since κ appears as the coefficient multiplying the electromagnetic vector potential in the relation

$$e^0 = u + \kappa A, \tag{A77}$$

its smallness explains in a transparent manner why electromagnetic fields do not produce appreciable modifications of spacetime geometry at macroscopic scales.

Role of the coupling constant κ.

Although the constant κ is introduced at the kinematical level as a dimensional conversion factor allowing the electromagnetic potential to be embedded into the temporal tetrad, its appearance in the unified action does not represent a freely adjustable interaction strength. In the present framework, κ controls the energetic cost of temporal torsion excitations relative to the purely gravitational torsion sector. The Maxwell term therefore corresponds to a specific quadratic contribution within the full torsional energy, rather than to an independent gauge interaction added by hand. The relative normalization between the electromagnetic and gravitational sectors is fixed by classical consistency requirements such as vacuum stability, boundedness of the torsional energy, and the existence of a flat Minkowski solution.

Temporal torsion: explicit derivation

We start from the explicit decomposition of the temporal tetrad one–form,

$$e^0 = (c + \kappa\Phi)\, dt + \kappa A_i\, dx^i. \tag{A78}$$

In the Weitzenböck gauge[4], the torsion two–form is given by

$$T^0 = de^0. \tag{A79}$$

Taking the exterior derivative, one finds

$$T^0 = d\big[(c+\kappa\Phi)dt\big] + d\big(\kappa A_i dx^i\big). \tag{A80}$$

Since $d(c\,dt) = 0$ and $d^2 = 0$, the first term reduces to

$$\begin{aligned} d(\kappa\Phi\,dt) &= \kappa\,d\Phi \wedge dt \\ &= \kappa\big(\partial_t\Phi\,dt + \partial_i\Phi\,dx^i\big) \wedge dt \\ &= -\kappa\,\partial_i\Phi\,dt \wedge dx^i. \end{aligned} \tag{A81}$$

where $dt \wedge dt = 0$ has been used.

For the second term, we obtain

$$\begin{aligned} d(\kappa A_i\,dx^i) &= \kappa\,dA_i \wedge dx^i \\ &= \kappa\big(\partial_t A_i\,dt + \partial_j A_i\,dx^j\big) \wedge dx^i \\ &= \kappa\,\partial_t A_i\,dt \wedge dx^i \;+\; \kappa\,\partial_j A_i\,dx^j \wedge dx^i. \end{aligned} \tag{A82}$$

Using antisymmetry of the wedge product, the purely spatial contribution may be written as

$$\kappa\,\partial_j A_i\,dx^j \wedge dx^i = \frac{\kappa}{2}(\partial_i A_j - \partial_j A_i)\,dx^i \wedge dx^j. \tag{A83}$$

Collecting all contributions, the temporal torsion two–form becomes

$$T^0 = \kappa\left[(\partial_t A_i - \partial_i\Phi)\,dt \wedge dx^i + \frac{1}{2}(\partial_i A_j - \partial_j A_i)\,dx^i \wedge dx^j\right]. \tag{A84}$$

Reading off the components from

$$T^0 = \frac{1}{2}T^0{}_{\mu\nu}\,dx^\mu \wedge dx^\nu, \tag{A85}$$

Consequently,

$$T^0_{ti} = \kappa(\partial_t A_i - \partial_i\Phi), \qquad T^0_{ij} = \kappa(\partial_i A_j - \partial_j A_i).$$

In relativistic notation, the temporal index corresponds to $x^0 = c\,t$, implying

$$\partial_0 = \frac{\partial}{\partial x^0} = \frac{1}{c}\frac{\partial}{\partial t}.$$

As a result, the time–space components of the field strength tensor take the form

$$F_{0i} = \partial_0 A_i - \partial_i A_0 = \frac{1}{c}\partial_t A_i - \partial_i\Phi,$$

and therefore explicitly contain a factor of $1/c$. Defining the electric field as $E_i := F_{0i}$, one obtains

$$E_i = \frac{1}{c}\partial_t A_i - \partial_i\Phi,$$

together with the geometric identification

$$T^0_{0i} = \kappa F_{0i} = \kappa E_i.$$

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The Maxwell action
$$S_{EM} = -\frac{1}{4}\int d^4x\,|e|\,F_{\mu\nu}F^{\mu\nu}$$
then implies that the electric sector contributes a term proportional to $(\partial_t A_i)^2/c^2$. Hence, the correct dimensional normalization of electric energy is directly tied to the appearance of $c\,dt$ in the temporal tetrad.

Comparing with the electromagnetic Lagrangian density in SI units,
$$\mathcal{L}_{EM}^{(SI)} = \frac{1}{2}\left(\varepsilon_0 E^2 - \frac{1}{\mu_0}B^2\right),$$
one finds
$$\varepsilon_0 \sim \frac{1}{\kappa^2 c^2}, \qquad \mu_0 \sim \kappa^2.$$
The factor of c appearing in ε_0 is therefore not an additional assumption but a direct consequence of the inclusion of time into geometry via the $c\,dt$ structure of the temporal tetrad. It follows immediately that
$$\varepsilon_0\mu_0 = \frac{1}{c^2}. \tag{A86}$$

This relation is not an electromagnetic postulate, but a direct consequence of the Lorentz-geometric distinction between temporal and spatial directions in spacetime.

Additional consistency constraints on the coupling constant κ

In the main text and Appendix B, the constant κ was introduced as the conversion factor relating the electromagnetic potential to the temporal tetrad component,
$$e^0{}_\mu = u_\mu + \kappa A_\mu. \tag{A87}$$

While dimensional analysis motivates the presence of such a parameter, further classical consistency requirements constrain its admissible range.

(i) Stability of the Minkowski vacuum.

A fundamental requirement of the unified framework is that flat Minkowski spacetime with vanishing electromagnetic field,
$$e^a{}_\mu = \delta^a{}_\mu, \qquad F_{\mu\nu} = 0, \tag{A88}$$
constitutes a stable vacuum solution of the full theory. In this configuration the temporal torsion vanishes identically, $T^0{}_{\mu\nu} = 0$, and no spurious geometric or electromagnetic excitations are generated.

This requirement excludes values of κ that would induce a nontrivial temporal torsion in the absence of electromagnetic sources. In particular, the electromagnetic contribution to the action,
$$S_{\rm EM} = -\frac{1}{4}\int d^4x\,e\,F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4\kappa^2}\int d^4x\,e\,T^0{}_{\mu\nu}T^{0\,\mu\nu}, \tag{A89}$$
must not destabilize the vacuum through an effective negative-energy mode. This condition fixes the sign of κ^2 and ensures that the Minkowski solution corresponds to a true extremum of the action.

(ii) Boundedness of the unified gravitational-electromagnetic sector.

Within the GDV framework, the gravitational Lagrangian contains terms quadratic in torsion and in the torsion trace. The identification of the electromagnetic field with the temporal torsion component implies that the electromagnetic contribution appears as a specific subsector of the total torsional energy.

Requiring the combined GDV and electromagnetic action to be bounded from below imposes a further classical constraint on κ. In particular, the coefficient multiplying $T^0{}_{\mu\nu}T^{0\,\mu\nu}$ must be positive

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relative to the purely gravitational torsion terms, so that temporal torsion excitations do not lead to runaway solutions. This requirement restricts κ to values for which electromagnetic excitations represent stable geometric deformations rather than instabilities of the spacetime flow.

(iii) Interpretation.

From this perspective, κ is not a freely tunable coupling but a structural parameter controlling the relative energetic cost of temporal torsion excitations. Its smallness ensures that electromagnetic fields correspond to weak geometric deformations on macroscopic scales, while its sign and magnitude are fixed by the requirement of vacuum stability and bounded gravitational–electromagnetic dynamics.

No quantum assumptions are required for these arguments; they arise entirely from classical geometric consistency of the unified teleparallel framework.

Energetic interpretation of κ

While κ is introduced through dimensional analysis, its physical meaning is more restrictive. Once the electromagnetic field is identified with temporal torsion, the Maxwell term becomes a specific contribution to the total torsional energy. In this interpretation, κ determines the relative stiffness of the temporal torsion sector. Small values of κ imply that electromagnetic excitations correspond to weak geometric deformations, ensuring that macroscopic spacetime geometry remains insensitive to ordinary electromagnetic fields. This property is not imposed phenomenologically but follows from the requirement that Minkowski spacetime with vanishing torsion constitutes a stable vacuum of the theory.

Appendix B. Non-integrable temporal flow and the case $du \neq 0$

Throughout the main text, the background temporal one–form u is frequently taken to satisfy

$$du = 0, \tag{A90}$$

corresponding to an integrable temporal flow, such as that of an inertial or cosmological rest frame where $u = c\,dt$. This choice is adopted for clarity and for the analysis of basic physical regimes, but it does not represent a fundamental restriction of the theory.

In this appendix, we clarify the role of non–integrable temporal structures ($du \neq 0$), which naturally arise for rotating observers, frame–dragging geometries, and Kerr–type spacetimes.

Appendix B.1. General decomposition of the temporal tetrad

In the unified framework, the temporal tetrad is decomposed as

$$e^0 = u + \kappa A, \tag{A91}$$

where u encodes the background temporal flow and A represents the electromagnetic potential. Taking an exterior derivative yields the general identity

$$T^0 = de^0 = du + \kappa\, dA. \tag{A92}$$

This relation holds independently of whether u is integrable.

The total temporal torsion therefore contains two conceptually distinct contributions:

- du, associated with inertial or gravitational effects related to the non–integrability of the temporal flow,
- $\kappa\, dA$, identified with the electromagnetic field strength.

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Appendix B.2. Definition of the electromagnetic field for $du \neq 0$

In the present framework, the electromagnetic field strength is defined by

$$F = dA, \tag{A93}$$

independently of the value of du. Consequently, the homogeneous Maxwell identity

$$dF = 0 \tag{A94}$$

holds identically, even in the presence of non–integrable temporal structures.

When $du = 0$, one has $T^0 = \kappa F$, and the electromagnetic field coincides directly with the temporal torsion. For $du \neq 0$, this simple identification no longer holds; however, the electromagnetic sector remains unambiguously defined by $F = dA$. The additional contribution du represents inertial or gravitational torsion effects rather than electromagnetic degrees of freedom.

Appendix B.3. Physical interpretation: rotating frames and Kerr geometry

Non–vanishing du naturally appears in rotating reference frames and in stationary axisymmetric spacetimes such as the Kerr geometry. In these situations, the temporal flow fails to define a global hypersurface orthogonal time coordinate, reflecting the presence of vorticity or frame–dragging.

Within the present teleparallel formulation, such effects are encoded in the background geometry through du, without interfering with the definition of the electromagnetic field.

Electromagnetism remains associated with the dynamical part of the temporal tetrad, while inertial and gravitational effects related to rotation are captured by the non–integrable structure of u.

Appendix B.4. Consistency of the field equations

The derivations of the Maxwell equations presented in the main text rely only on the definition $F = dA$ and on the variational structure of the action. They therefore remain valid for $du \neq 0$. In particular:

- the homogeneous Maxwell equations follow from $dF = 0$,
- the inhomogeneous equations arise from variation with respect to A_μ (or equivalently, through the temporal tetrad sector),
- charge conservation remains guaranteed by gauge invariance.

The assumption $du = 0$ used in several sections of the paper should therefore be understood as a choice of temporal foliation appropriate to inertial or cosmological frames, rather than as a limitation of the underlying theory.

The unified teleparallel framework consistently accommodates non–integrable temporal flows.

While the condition $du = 0$ simplifies the identification between temporal torsion and electromagnetism, it is not required for the definition of the electromagnetic sector or for the validity of the field equations. In geometries with rotation or frame–dragging, such as Kerr–type spacetimes, the additional contribution du encodes inertial and gravitational effects, while the electromagnetic field remains uniquely characterized by $F = dA$.

Appendix C. General teleparallel torsion and the Weitzenböck gauge

In teleparallel gravity the tetrad one–forms e^a and the (flat) spin connection $\omega^a{}_b$ define the torsion 2–form

$$T^a := De^a = de^a + \omega^a{}_b \wedge e^b. \tag{A95}$$

In the Weitzenböck gauge the spin connection vanishes,

$$\omega^a{}_b = 0, \tag{A96}$$

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so the torsion simplifies to

$$T^a = de^a. \tag{A97}$$

Writing the tetrad in coordinates,

$$e^a = e^a{}_\mu \, dx^\mu,$$

we compute

$$\begin{aligned} de^a &= d(e^a{}_\mu) \wedge dx^\mu \\ &= \partial_\nu e^a{}_\mu \, dx^\nu \wedge dx^\mu. \end{aligned} \tag{A98}$$

Using antisymmetry of the wedge product, one identifies

$$T^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu. \tag{A99}$$

Generalized temporal tetrad

In accordance with the main text, the temporal tetrad is decomposed into a background temporal flow and a dynamical electromagnetic excitation,

$$e^0 = u + \kappa \, A, \tag{A100}$$

In a Minkowski or cosmological rest frame the background flow is integrable,

$$u = c \, dt, \qquad du = 0. \tag{A101}$$

Accordingly, we write

$$e^0 = (c + \kappa \, \Phi) \, dt + \kappa \, A_i \, dx^i, \tag{A102}$$

with components

$$e^0{}_0 = c + \kappa \, \Phi, \qquad e^0{}_i = \kappa \, A_i.$$

Explicit computation of the torsion

$$T^0{}_{\mu\nu} = \kappa \begin{pmatrix} 0 & \partial_0 A_1 - \partial_1 \Phi & \partial_0 A_2 - \partial_2 \Phi & \partial_0 A_3 - \partial_3 \Phi \\ -(\partial_0 A_1 - \partial_1 \Phi) & 0 & \partial_1 A_2 - \partial_2 A_1 & \partial_1 A_3 - \partial_3 A_1 \\ -(\partial_0 A_2 - \partial_2 \Phi) & -(\partial_1 A_2 - \partial_2 A_1) & 0 & \partial_2 A_3 - \partial_3 A_2 \\ -(\partial_0 A_3 - \partial_3 \Phi) & -(\partial_1 A_3 - \partial_3 A_1) & -(\partial_2 A_3 - \partial_3 A_2) & 0 \end{pmatrix}. \tag{A103}$$

This expression shows that the temporal torsion components reproduce exactly the structure of the electromagnetic field strength tensor, with the identifications $T^0{}_{0i} \propto F_{0i}$ and $T^0{}_{ij} \propto F_{ij}$, establishing electromagnetism as a purely geometric manifestation of spacetime torsion in the temporal sector.

Identification with the electromagnetic field

Using the geometric identification

$$F_{\mu\nu} \equiv \frac{1}{\kappa} T^0{}_{\mu\nu}, \tag{A104}$$

together with the explicit torsion components derived in Appendix A.3,

$$T^0{}_{0i} = \kappa \big(\partial_0 A_i - \partial_i \Phi\big), \qquad T^0{}_{ij} = \kappa \big(\partial_i A_j - \partial_j A_i\big), \tag{A105}$$

the electromagnetic field strength components are given by

$$F_{0i} = \frac{1}{\kappa} T^0{}_{0i} = \partial_0 A_i - \partial_i \Phi, \qquad F_{ij} = \frac{1}{\kappa} T^0{}_{ij} = \partial_i A_j - \partial_j A_i. \tag{A106}$$

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In a standard $3+1$ decomposition, the electric and magnetic fields are defined in terms of the field strength tensor as

$$E_i := F_{0i}, \qquad B_k := \frac{1}{2}\epsilon_{kij}F_{ij}. \tag{A107}$$

$$E_i = \partial_0 A_i - \partial_i \Phi, \qquad B_k = \frac{1}{2}\epsilon_{kij}\big(\partial_i A_j - \partial_j A_i\big). \tag{A108}$$

Thus, the electric and magnetic fields take precisely their standard forms in terms of the scalar and vector potentials. In the present framework, however, these fields arise geometrically from the rescaled torsion of the temporal tetrad rather than from an independently postulated gauge field.

Matrix form of the electromagnetic field strength

Using the geometric identification

$$F_{\mu\nu} \equiv \frac{1}{\kappa}\, T^0{}_{\mu\nu}, \tag{A109}$$

the electromagnetic field strength is completely determined by the temporal torsion of the tetrad. In this subsection we show that this definition reproduces the standard matrix representation of the electromagnetic tensor.

We work in a local Lorentz frame with Minkowski metric of signature $(-,+,+,+)$,

$$\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1). \tag{A110}$$

From the previous subsection, the nonvanishing components of the field strength are

$$\begin{aligned} F_{0i} &= \partial_0 A_i - \partial_i \Phi, \\ F_{ij} &= \partial_i A_j - \partial_j A_i. \end{aligned} \tag{A111}$$

Electric and magnetic fields.

In a $3+1$ decomposition, the electric and magnetic fields are defined by

$$E_i := F_{0i}, \qquad B^k := \frac{1}{2}\epsilon^{kij}F_{ij}, \tag{A112}$$

where ϵ^{kij} is the three–dimensional Levi–Civita symbol.

With these definitions one immediately has

$$\begin{aligned} F_{0i} &= E_i, \\ F_{ij} &= \epsilon_{ijk}\, B^k. \end{aligned} \tag{A113}$$

where spatial indices are raised and lowered with the Euclidean metric δ_{ij}.

Antisymmetry.

By construction, the field strength tensor is antisymmetric,

$$F_{\mu\nu} = -F_{\nu\mu}, \tag{A114}$$

which fixes all remaining components.

Under the identification $F_{\mu\nu} = \frac{1}{\kappa}T^0{}_{\mu\nu}$, this matrix simultaneously represents the electromagnetic field strength and the rescaled torsion of the temporal tetrad.

Appendix D. Gauge Transformations and Representation Freedom

In this appendix a concise geometric clarification of the status of electromagnetic gauge transformations within the teleparallel formulation employed in the main text is provided. The purpose is to demonstrate explicitly that gauge freedom arises as a representation freedom of the temporal tetrad and does not correspond to a physical redundancy associated with an independent internal symmetry.

Invariance of temporal torsion under gauge transformations

Electromagnetic gauge transformations act on the dynamical potential according to

$$A \longrightarrow A + d\chi, \tag{A115}$$

where $\chi(x)$ is an arbitrary scalar function. Under this transformation, the temporal tetrad one–form transforms as

$$e^0 \longrightarrow e^0 + \kappa\, d\chi. \tag{A116}$$

Since the temporal torsion is defined by

$$T^0 := de^0, \tag{A117}$$

and the exterior derivative satisfies the identity $d^2 = 0$, it follows immediately that

$$T^0 \longrightarrow T^0. \tag{A118}$$

Therefore, the temporal torsion tensor and all physical observables constructed from it remain invariant under gauge transformations. This establishes that electromagnetic gauge transformations correspond to changes of representation of the temporal tetrad rather than to physically distinct configurations.

Distinction from local Lorentz transformations

It is important to emphasize that electromagnetic gauge transformations act on spacetime one–forms, while local Lorentz transformations act on the internal tangent–space indices of the tetrad,

$$e^a \longrightarrow \Lambda^a{}_b(x)\, e^b. \tag{A119}$$

These two classes of transformations operate on different geometric structures and are conceptually independent. In particular, gauge transformations do not affect the local Lorentz frame, the spin connection, or the spacetime metric. Local Lorentz covariance of the teleparallel formulation is therefore preserved without modification.

Abelian character of the gauge symmetry

Gauge transformations are generated by the exterior derivative of a scalar function. Since successive transformations satisfy

$$d\chi_1 + d\chi_2 = d\chi_2 + d\chi_1, \tag{A120}$$

the associated group structure is necessarily Abelian. No non–Abelian gauge structure can arise within this framework. The $U(1)$ character of electromagnetism thus follows directly from the geometric construction and does not require an independent group theoretic postulate.

Appendix E. Derivation of the Lorentz Force Law from the Unified Tetrad Action

In this appendix we present a detailed derivation of the equation of motion for a classical charged test particle propagating in the unified teleparallel geometry. The purpose of this derivation is to make explicit that the electromagnetic interaction is not introduced as an independent coupling, but arises from the geometric encoding of the temporal component of the tetrad.

Appendix E.1. Particle Action and Geometric Encoding

The action for a point particle of rest mass m and electric charge q moving along a worldline $x^\mu(\tau)$ is taken to be

$$S_p = -m \int d\tau \sqrt{-\,\eta_{ab}\,(e^a{}_\mu\,\dot{x}^\mu)\,(e^b{}_\nu\,\dot{x}^\nu)} \;+\; q \int d\tau\,\dot{x}^\mu A_\mu,$$

where $\dot{x}^\mu = dx^\mu/d\tau$ and τ is an arbitrary parameter along the worldline.

Throughout this appendix the spacetime geometry is treated as a fixed background, and the variation is performed solely with respect to the particle trajectory. Although the interaction term appears in the standard minimal–coupling form, the electromagnetic potential A_μ is not assumed to be an independent dynamical field. Instead, it is identified with the one–form component entering the geometric decomposition of the temporal tetrad,

$$e^0 = u + \kappa\,(A + d\varphi),$$

where u denotes the temporal flow one–form and φ is a Stückelberg scalar.

Using this decomposition, the interaction term can be rewritten as

$$q \int d\tau\,\dot{x}^\mu A_\mu = \frac{q}{\kappa} \int d\tau\,(e^0{}_\mu - u_\mu)\,\dot{x}^\mu - q \int d\tau\,\frac{d\varphi}{d\tau}.$$

The last term is a total derivative and does not contribute to the equations of motion. Up to this boundary contribution, the particle action is therefore fully expressed in terms of tetrad variables.

Appendix E.2. Proper Time and Kinematical Setup

The spacetime metric induced by the tetrad is defined by

$$g_{\mu\nu} = \eta_{ab}\,e^a{}_\mu e^b{}_\nu,$$

which determines the spacetime interval

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu.$$

Introducing the proper time parameter s along the worldline, the four–velocity is defined as

$$u^\mu := \frac{dx^\mu}{ds}, \qquad g_{\mu\nu}u^\mu u^\nu = -1.$$

In terms of s, the free particle contribution to the action reduces to

$$S_{\text{free}} = -m \int ds.$$

Appendix E.3. Variation of the Free Particle Term

Varying the free particle action with respect to the worldline $x^\mu(s) \to x^\mu(s) + \delta x^\mu(s)$ and discarding boundary terms yields

$$\delta S_{\text{free}} = -m \int ds\, g_{\mu\nu}\, u^\nu \nabla_\rho u^\mu\, \delta x^\rho,$$

where ∇_ρ denotes the Levi–Civita covariant derivative associated with the metric. This contribution corresponds to inertial motion in curved spacetime.

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Appendix E.4. Variation of the Geometrically Encoded Interaction Term

Using the rewritten form of the interaction term, the variation reduces to the variation of the temporal tetrad component along the worldline. Neglecting boundary terms, one finds

$$\delta S_{\rm int} = \frac{q}{\kappa}\int d\tau \Big(\partial_\nu e^0{}_\mu - \partial_\mu e^0{}_\nu\Big)\,\dot{x}^\mu\,\delta x^\nu.$$

The electromagnetic field strength is identified with the antisymmetric derivative of the temporal tetrad component,

$$F_{\mu\nu} = \frac{1}{\kappa}\Big(\partial_\mu e^0{}_\nu - \partial_\nu e^0{}_\mu\Big).$$

With this identification, the variation of the interaction term becomes

$$\delta S_{\rm int} = q\int ds\, F_{\nu\mu}\,u^\mu\,\delta x^\nu.$$

Appendix E.5. Equation of Motion

Combining the variations of the free and interaction terms and imposing the stationarity condition $\delta S_p = 0$ for arbitrary variations $\delta x^\mu(s)$, one obtains

$$m\,u^\nu\nabla_\nu u^\mu = q\,F^\mu{}_\nu\,u^\nu.$$

This equation is recognized as the standard relativistic Lorentz force law.

In the present formulation, both gravitational and electromagnetic interactions are encoded within the same tetrad structure. Gravitational effects arise from the spacetime geometry defined by the full tetrad, while electromagnetic interactions are associated with the temporal torsion sector. The standard dynamics of charged particles are thus recovered without introducing independent interaction terms or modifying the physical content of classical electromagnetism.

Appendix F. Electromagnetic–Gravitational Backreaction

In the unified teleparallel framework developed in this work, gravitation and electromagnetism are not introduced as independent interacting sectors. Instead, both arise from the dynamics of a single tetrad field. As a consequence, electromagnetic backreaction is not an additional effect superimposed on an otherwise independent gravitational background, but an intrinsic feature of the geometric construction itself.

Appendix F.1. Geometric origin of the electromagnetic sector

The electromagnetic field strength is defined in terms of the temporal torsion of the tetrad,

$$F_{\mu\nu} \equiv \frac{1}{\kappa}\Big(\partial_\mu e^0{}_\nu - \partial_\nu e^0{}_\mu\Big), \tag{A121}$$

so that electromagnetic excitations correspond to nontrivial variations of the temporal tetrad component $e^0{}_\mu$. It makes explicit that the introduction of the electromagnetic sector already entails a modification of spacetime geometry.

Since the field strength is constructed directly from derivatives of the temporal tetrad component, the presence of a nonvanishing electromagnetic field implies that the underlying tetrad configuration differs from its vacuum form at the kinematical level.

In this framework, electromagnetic backreaction therefore does not arise as a subsequent response of geometry to an external field, but is already encoded in the geometric definition of the electromagnetic degrees of freedom. Since the tetrad simultaneously determines both the spacetime metric $g_{\mu\nu} = \eta_{ab}e^a{}_\mu e^b{}_\nu$ and the torsional structure of spacetime, any nonvanishing electromagnetic field necessarily modifies the gravitational sector.

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In this sense, the presence of electromagnetism already implies a deformation of spacetime geometry, even before invoking any notion of energy momentum exchange between distinct fields.

Appendix F.2. Backreaction from the unified action

The unified action considered in the main text contains the Maxwell term

$$S_{\rm EM} = -\frac{1}{4}\int d^4x\, e\, F_{\mu\nu}F^{\mu\nu}, \tag{A122}$$

where $e = \det(e^a{}_\mu)$. Backreaction arises from variation of this term with respect to the tetrad.

Performing the variation yields

$$\delta S_{\rm EM} = \frac{1}{2}\int d^4x\, e\, T^{\mu\nu}_{\rm EM}\, e_{a\nu}\, \delta e^a{}_\mu, \tag{A123}$$

where the electromagnetic energy–momentum tensor takes the standard form

$$T^{\mu\nu}_{\rm EM} = F^{\mu\rho}F^\nu{}_\rho - \frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}. \tag{A124}$$

Although formally identical to the Maxwell energy–momentum tensor in general relativity, its interpretation is different in the present framework. Here, $T^{\mu\nu}_{\rm EM}$ originates entirely from variations of the temporal torsion sector of the tetrad, rather than from an independently postulated internal gauge field.

Appendix F.3. Mutual coupling of gravitation and electromagnetism

Since the electromagnetic field strength depends exclusively on $e^0{}_\mu$, the backreaction of electromagnetism enters the tetrad field equations primarily through the temporal sector. At the same time, because the metric depends on all tetrad components, spatial geometry responds indirectly through the induced metric and torsion trace.

The resulting field equations therefore describe a genuinely coupled system in which gravitational and electromagnetic effects cannot be disentangled into separate source and response stages. Electromagnetism does not act on spacetime as an external agent; rather, it represents a specific dynamical mode of spacetime geometry itself.

Appendix F.4. Vacuum and special limits

Flat Minkowski spacetime,

$$e^a{}_\mu = \delta^a{}_\mu, \qquad F_{\mu\nu} = 0, \tag{A125}$$

remains an exact solution of the fully coupled theory. In this configuration the temporal torsion vanishes identically, and no electromagnetic or gravitational back reaction is present. This vacuum therefore arises naturally from the unified field equations, rather than being imposed as an external approximation.

Configurations in which electromagnetic fields propagate on an approximately fixed background correspond to special solutions of the full theory in which geometric deformations remain small. Such test–field regimes represent controlled limits of the unified dynamics, not a fundamental restriction of the framework.

Backreaction does not signify an exchange of energy between distinct entities, but the self consistent adjustment of spacetime geometry to its own dynamical degrees of freedom.

This perspective preserves local Lorentz covariance, introduces no additional propagating fields, and maintains a clear geometric distinction between temporal and spatial sectors, while providing a unified description of gravitational and electromagnetic dynamics.

Variation of the Maxwell term through $\delta F_{\mu\nu}$

It is isolated the part of the Maxwell variation that arises from the dependence $F_{\mu\nu} = \kappa^{-1}(\partial_\mu e^0{}_\nu - \partial_\nu e^0{}_\mu)$. Starting from

$$S_{\rm EM} = -\frac{1}{4}\int d^4x\, e\, F_{\mu\nu}F^{\mu\nu},$$

it is obtained

$$\delta S_{\rm EM}\big|_{\delta F} = -\frac{1}{2}\int d^4x\, e\, F^{\mu\nu}\delta F_{\mu\nu}, \qquad \delta F_{\mu\nu} = \frac{1}{\kappa}\left(\partial_\mu \delta e^0{}_\nu - \partial_\nu \delta e^0{}_\mu\right).$$

Using antisymmetry of $F^{\mu\nu}$, this reduces to

$$\delta S_{\rm EM}\big|_{\delta F} = -\frac{1}{\kappa}\int d^4x\, e\, F^{\mu\nu}\partial_\mu \delta e^0{}_\nu.$$

Integrating by parts and discarding boundary terms yields

$$\delta S_{\rm EM}\big|_{\delta F} = \frac{1}{\kappa}\int d^4x\, \partial_\mu(eF^{\mu\nu})\, \delta e^0{}_\nu = \frac{1}{\kappa}\int d^4x\, e\, (\nabla_\mu F^{\mu\nu})\, \delta e^0{}_\nu.$$

Hence the Maxwell operator $\nabla_\mu F^{\mu\nu}$ appears directly in the tetrad variation through the temporal sector.

Appendix F.5. Metric/determinant variation and the electromagnetic stress tensor

It is isolated that the part of the Maxwell variation that arises from the dependence of $e = \det(e^a{}_\mu)$ and from index raising with the metric $g_{\mu\nu} = \eta_{ab}e^a{}_\mu e^b{}_\nu$. Starting from

$$S_{\rm EM} = -\frac{1}{4}\int d^4x\, e\, F_{\mu\nu}F^{\mu\nu}, \tag{A126}$$

we write the variation as

$$\delta S_{\rm EM} = -\frac{1}{4}\int d^4x \left[\delta e\, F_{\mu\nu}F^{\mu\nu} + e\,\delta(F_{\mu\nu}F^{\mu\nu})\right]. \tag{A127}$$

In this step we keep only the contributions coming from δe and $\delta g^{\mu\nu}$, i.e. it is excluded to the explicit $\delta F_{\mu\nu}$ piece treated separately.

(i) Index-raising contribution.

Since $F_{\mu\nu}$ (with both indices down) is held fixed in this step,

$$\delta(F_{\mu\nu}F^{\mu\nu})\Big|_{\delta g} = F_{\mu\nu}F_{\rho\sigma}\,\delta(g^{\mu\rho}g^{\nu\sigma}). \tag{A128}$$

Using

$$\delta(g^{\mu\rho}g^{\nu\sigma}) = g^{\nu\sigma}\delta g^{\mu\rho} + g^{\mu\rho}\delta g^{\nu\sigma}, \tag{A129}$$

we obtain

$$\begin{aligned}\delta(F_{\mu\nu}F^{\mu\nu})\Big|_{\delta g} &= F_{\mu\nu}F_{\rho\sigma}(g^{\nu\sigma}\,\delta g^{\mu\rho} + g^{\mu\rho}\,\delta g^{\nu\sigma}) \\ &= F_{\mu\nu}F_\rho{}^\nu\,\delta g^{\mu\rho} + F_{\mu\nu}F^\mu{}_\sigma\,\delta g^{\nu\sigma}.\end{aligned} \tag{A130}$$

Renaming dummy indices in the second term $(\mu \leftrightarrow \nu,\ \rho \leftrightarrow \sigma)$ and using symmetry of $\delta g^{\mu\rho} = \delta g^{\rho\mu}$ yields

$$\delta(F_{\mu\nu}F^{\mu\nu})\Big|_{\delta g} = 2\,F_{\mu\rho}F_\nu{}^\rho\,\delta g^{\mu\nu}. \tag{A131}$$

Therefore, the corresponding contribution to the action variation is

$$\delta S_{\rm EM}\Big|_{\delta g} = -\frac{1}{4}\int d^4x\, e\,\delta(F_{\mu\nu}F^{\mu\nu})\Big|_{\delta g} = -\frac{1}{2}\int d^4x\, e\, F_{\mu\rho}F_\nu{}^\rho\,\delta g^{\mu\nu}. \tag{A132}$$

(ii) Determinant contribution.

The standard identity

$$\delta e = \frac{1}{2}\, e\, g^{\mu\nu}\delta g_{\mu\nu} = -\frac{1}{2}\, e\, g_{\mu\nu}\delta g^{\mu\nu}, \tag{A133}$$

is used, so that

$$\begin{aligned} \delta S_{\rm EM}\Big|_{\delta e} &= -\tfrac{1}{4}\int d^4x\, \delta e\, F_{\rho\sigma}F^{\rho\sigma} \\ &= -\tfrac{1}{4}\int d^4x \Big(-\tfrac{1}{2}\, e\, g_{\mu\nu}\, \delta g^{\mu\nu}\Big) F_{\rho\sigma}F^{\rho\sigma} \\ &= \tfrac{1}{8}\int d^4x\, e\, g_{\mu\nu} F_{\rho\sigma}F^{\rho\sigma}\, \delta g^{\mu\nu}. \end{aligned} \tag{A134}$$

(iii) Combined result and identification of $T^{\rm EM}_{\mu\nu}$.

$$\begin{aligned} \delta S_{\rm EM}\Big|_{\delta e,\delta g} &= \tfrac{1}{8}\int d^4x\, e\, g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\, \delta g^{\mu\nu} - \tfrac{1}{2}\int d^4x\, e\, F_{\mu\rho}F_{\nu}{}^{\rho}\, \delta g^{\mu\nu} \\ &= \tfrac{1}{2}\int d^4x\, e\Big[-F_{\mu\rho}F_{\nu}{}^{\rho} + \tfrac{1}{4}\, g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\Big]\delta g^{\mu\nu}. \end{aligned} \tag{A135}$$

Comparing with the defining relation

$$\delta S_{\rm EM} = \frac{1}{2}\int d^4x\, e\, T^{\rm EM}_{\mu\nu}\, \delta g^{\mu\nu}, \tag{A136}$$

the electromagnetic stress–energy tensor is read off ,

$$T^{\rm EM}_{\mu\nu} = F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4} g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}. \tag{A137}$$

(iv) Relation to tetrad variation (optional bridge).

Since $g_{\mu\nu} = \eta_{ab}e^a{}_\mu e^b{}_\nu$, its variation is

$$\delta g_{\mu\nu} = \eta_{ab}\Big(\delta e^a{}_\mu\, e^b{}_\nu + e^a{}_\mu\, \delta e^b{}_\nu\Big), \tag{A138}$$

so $T^{\rm EM}_{\mu\nu}$ sources the tetrad equations through the metric dependence, even though the explicit definition of $F_{\mu\nu}$ in this framework involves only the temporal tetrad component.

Coupled equations and the role of the constraint/Stückelberg sector

Assembling the tetrad variation: direct (temporal) and indirect (metric) couplings.

Consider the Maxwell term

$$S_{\rm EM} = -\frac{1}{4}\int d^4x\, e\, F_{\mu\nu}F^{\mu\nu}, \tag{A139}$$

with the geometric definition (on the integrable branch $du = 0$)

$$F_{\mu\nu} = \frac{1}{\kappa}\Big(\partial_\mu e^0{}_\nu - \partial_\nu e^0{}_\mu\Big). \tag{A140}$$

The full variation of $S_{\rm EM}$ with respect to the tetrad can be written schematically as

$$\delta S_{\rm EM} = \delta S_{\rm EM}\big|_{\delta F} + \delta S_{\rm EM}\big|_{\delta e,\delta g}, \tag{A141}$$

where

$$\delta S_{\rm EM}\big|_{\delta F} = \frac{1}{\kappa}\int d^4x\, e\, (\nabla_\mu F^{\mu\nu})\, \delta e^0{}_\nu, \tag{A142}$$

is given and

$$\delta S_{\rm EM}\big|_{\delta e,\delta g} = \frac{1}{2}\int d^4x\, e\, T^{\rm EM}_{\mu\nu}\, \delta g^{\mu\nu}, \qquad T^{\rm EM}_{\mu\nu} = F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4} g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}. \tag{A143}$$

Using $\delta g_{\mu\nu} = \eta_{ab}(\delta e^a{}_\mu e^b{}_\nu + e^a{}_\mu \delta e^b{}_\nu)$, one sees explicitly the two complementary effects:

- the *direct* Euler–Lagrange operator $\nabla_\mu F^{\mu\nu}$ couples only to the temporal tetrad variation $\delta e^0{}_\nu$ (because $F_{\mu\nu}$ depends explicitly only on $e^0{}_\mu$);
- the *indirect* coupling through $T^{\rm EM}_{\mu\nu}\,\delta g^{\mu\nu}$ sources the full geometric sector via the metric dependence on all tetrad legs.

This separation is the precise sense in which electromagnetic backreaction is *already encoded* at the level of the geometric definition of a nontrivial $F_{\mu\nu}$ implies a nontrivial tetrad configuration, while its energy content propagates into the full tetrad equations through $T^{\rm EM}_{\mu\nu}$.

Completing the system with sources: parent (constraint) action vs. reduced action.

For clarity and gauge invariance, the main text employs a parent action in which A_μ is introduced together with a Stückelberg scalar ϕ and a Lagrange-multiplier current J^μ enforcing the geometric identification:

$$S_{\rm parent} = S_{\rm grav}[e] \;-\; \frac{1}{4}\int d^4x\, e\, F_{\mu\nu}(A)F^{\mu\nu}(A) \;+\; \int d^4x\, e\, J^\mu\Big(e^0{}_\mu - u_\mu - \kappa(A_\mu + \partial_\mu\phi)\Big) \;+\; S_m. \quad \text{(A144)}$$

$$F_{\mu\nu}(A) := \partial_\mu A_\nu - \partial_\nu A_\mu. \quad \text{(A145)}$$

Varying $S_{\rm parent}$ with respect to A_μ, ϕ, and J^μ yields:

$$\nabla_\nu F^{\nu\mu}(A) \;=\; \kappa\, J^\mu, \quad \text{(A146)}$$

$$\nabla_\mu J^\mu \;=\; 0, \quad \text{(A147)}$$

$$e^0{}_\mu \;=\; u_\mu + \kappa(A_\mu + \partial_\mu\phi). \quad \text{(A148)}$$

On the integrable branch $du = 0$, taking an exterior derivative of its gives

$$de^0 \;=\; \kappa\, dA \qquad \Longrightarrow \qquad F(A) \;=\; dA \;=\; \frac{1}{\kappa}\, de^0. \quad \text{(A149)}$$

Thus, on-shell the parent theory is equivalent to a *reduced* description.

In the reduced description (obtained by eliminating A_μ, ϕ, J^μ in favour of $e^0{}_\mu$ and physical sources), the temporal Euler–Lagrange equation obtained from varying $e^0{}_\mu$ reproduces the Maxwell operator found,

$$\frac{1}{\kappa}\,\nabla_\nu F^{\nu\mu}(e^0) \;=\; J^\mu, \quad \text{(A150)}$$

Appendix F.5.1. Full coupled tetrad equations with electromagnetic backreaction.

Let the gravitational Euler operator be defined by

$$\mathcal{E}_a{}^\mu \;:=\; \frac{1}{e}\frac{\delta}{\delta e^a{}_\mu}\Big(e\,\mathcal{L}_{\rm grav}\Big), \quad \text{(A151)}$$

so that the full tetrad field equations take the form

$$\mathcal{E}_a{}^\mu = \kappa_g\Big(T_a{}^{\mu\,(m)} + T_a{}^{\mu\,(\rm EM)}\Big) \quad \text{(A152)}$$

Here $T_a{}^{\mu\,(\rm EM)}$ is obtained from the stress tensor

$$T_a{}^{\mu\,(\rm EM)} := e_{a\nu}\,T^{\mu\nu}_{\rm EM}, \quad \text{(A153)}$$

which encodes the *indirect* backreaction of electromagnetic energy on the full geometry.

In addition, because $F_{\mu\nu}$ depends explicitly on $e^0{}_\mu$ in the reduced formulation, the *temporal* tetrad equation ($a = 0$) contains the direct Maxwell operator found in Step 1. Schematically, its Euler-Lagrange content can be displayed as

$$\left(\mathcal{E}_0{}^\mu\right)_{\text{grav}} = \kappa_g\, T_0{}^{\mu\,(m)} + \kappa_g\, T_0{}^{\mu\,(\text{EM})} + \frac{1}{\kappa}\nabla_\nu F^{\nu\mu} + \cdots, \tag{A154}$$

where the ellipsis denotes the source/constraint completion (equivalently, J^μ). The key point is that the unified system couples electromagnetism and gravitation in two complementary ways:

1. *directly* through the temporal Euler–Lagrange operator $\nabla_\nu F^{\nu\mu}$ tied to the variation of $e^0{}_\mu$,
2. *indirectly* through the stress tensor $T^{\text{EM}}_{\mu\nu}$ entering the full tetrad equations via the metric dependence.

This provides a concrete and non-perturbative notion of electromagnetic-gravitational backreaction in the present framework.

Appendix G. Closed Integrals of the Background Temporal One–Form

In this appendix, we clarify why the closed–loop integral of the background temporal one–form u vanishes in the situations considered in the main text, and how this result fits consistently into the geometric interpretation of the Aharonov–Bohm phase. [6]

Integrable temporal flow

Throughout the main text, the background temporal structure is assumed to be integrable,

$$du = 0. \tag{A155}$$

By the Poincaré lemma, this implies that locally (and globally on simply connected regions) the one–form u can be written as an exact form,

$$u = d\tau, \tag{A156}$$

where $\tau(x)$ defines a global time function associated with the chosen temporal foliation. In the cosmological or inertial rest frame employed in Section 4, this reduces explicitly to

$$u = c\,dt. \tag{A157}$$

Closed–loop integral of u

Let γ be a closed spacetime curve, $\partial\gamma = 0$. The integral of u along γ is then

$$\oint_\gamma u = \oint_\gamma c d\tau = c\oint_\gamma d\tau \tag{A158}$$

Since the integral of an exact one–form over a closed curve vanishes identically, one obtains

$$\oint_\gamma u = 0. \tag{A159}$$

Physically, this reflects the fact that for an integrable temporal flow, the time function τ is single–valued: after completing a closed loop in space, the background temporal structure returns to its initial value.

Decomposition of the temporal tetrad

Using the geometric decomposition

$$e^0 = u + \kappa A, \tag{A160}$$

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the closed–loop integral of the electromagnetic potential becomes

$$\oint_\gamma A = \frac{1}{\kappa}\oint_\gamma e^0 - \frac{1}{\kappa}\oint_\gamma u. \tag{A161}$$

On the integrable branch discussed above, the second term vanishes, yielding

$$\oint_\gamma A = \frac{1}{\kappa}\oint_\gamma e^0. \tag{A162}$$

This establishes that the Aharonov–Bohm phase [6] is controlled entirely by the global properties of the temporal tetrad along the closed path.

Remarks on non–integrable temporal structures

If the background temporal flow is non–integrable ($du \neq 0$), as may occur for rotating observers or frame–dragging geometries, the integral $\oint_\gamma u$ need not vanish. In such cases, the total temporal torsion splits as

$$de^0 = du + \kappa dA, \tag{A163}$$

and the electromagnetic field remains uniquely defined by $F = dA$, while the additional contribution from du encodes inertial or gravitational effects. The analysis in the main text restricts to the integrable branch for clarity and to isolate the electromagnetic sector, but the framework itself does not rely on this restriction.

Geometric interpretation

The vanishing of $\oint_\gamma u$ ensures that local dynamics, governed by $F = dA$, remain unaffected, while global phase effects arise solely from the holonomy of the temporal tetrad e^0. The Aharonov–Bohm phase [6] therefore reflects a genuine global feature of the temporal geometry rather than an artifact of gauge choice.

Appendix H. Derivation of the Braking Index with a Geometric Drift Torque

Appendix I. Geometric Origin of the Pulsar Spin–Down Law

In this appendix we demonstrate that the rotational drift term entering the pulsar spin–down law, and in particular the appearance of the combination $(\Omega - \omega)$, follows directly from the tetrad–based teleparallel action. No phenomenological assumptions or additional dynamical inputs are introduced.

Appendix I.1. Tetrad structure and geometric drift

Throughout this work the tetrad is taken as the fundamental dynamical variable. In the vicinity of a rotating compact object such as a pulsar, the spacetime geometry is described by a tetrad of the form

$$e^0 = dt, \qquad e^i = a(t)\, dx^i + W^i(t, \mathbf{x})\, dt, \tag{A164}$$

where W^i denotes the geometric drift (GDV) field. This field is not a coordinate artifact but an intrinsic component of the spatial tetrad and therefore enters the action at the same level as the scale factor $a(t)$.

Working in the Weitzenböck gauge [4], the curvature vanishes identically and the torsion is given by

$$T^a = de^a. \tag{A165}$$

In particular, the spatial torsion components

$$T^i = de^i \tag{A166}$$

encode the derivatives of the drift field W^i.

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Appendix I.2. Torsion and geometric vorticity

The spacetime components of the spatial torsion take the schematic form

$$T^{i}{}_{0j} = \partial_j W^i + \cdots, \tag{A167}$$

where the ellipsis denotes terms irrelevant for rotational dynamics. The antisymmetric part defines the vorticity tensor

$$\omega_{ij} = \frac{1}{2}\big(\partial_j W_i - \partial_i W_j\big), \tag{A168}$$

which represents the local geometric rotation of spacetime. Importantly, this quantity arises directly from the antisymmetric sector of the torsion tensor and does not rely on any additional assumptions.

Appendix I.3. Particle action and physical velocity

The dynamics of matter follow from the standard point–particle action written in tetrad variables. In this framework the spatial components of the four–velocity are naturally expressed as

$$u^i = a\,\dot{x}^i + W^i. \tag{A169}$$

This expression is not a definition but a direct consequence of expressing the action in terms of the tetrad.

Accordingly, the physical velocity measured relative to the spatial tetrad is

$$v^i := a\,\dot{x}^i + W^i. \tag{A170}$$

Free fall corresponds to the absence of physical acceleration,

$$\frac{dv^i}{dt} = 0. \tag{A171}$$

This condition immediately implies

$$\frac{du^i}{dt} = -\frac{dW^i}{dt} = -\Big(\partial_t W^i + u^j \partial_j W^i\Big), \tag{A172}$$

which is the equation of motion derived explicitly from the action. No phenomenological force terms are introduced at this stage.

Appendix I.4. Rotating systems and pulsar geometry

A pulsar is well approximated as a rigidly rotating object with angular velocity Ω. At the same time, the surrounding spacetime generically carries a rotational geometric drift described by a vorticity ω. For a rotating drift field one may write

$$W = \omega \times r. \tag{A173}$$

In this case,

$$\partial_j W_i - \partial_i W_j = 2\,\epsilon_{ijk}\,\omega^k, \tag{A174}$$

so that the antisymmetric part of the torsion tensor directly yields the geometric angular velocity ω.

Appendix I.5. Emergence of the $(\Omega - \omega)$ structure

The physical velocity of stellar matter follows from the tetrad construction as

$$v = a\,\dot{x} + W. \tag{A175}$$

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The rotational velocity associated with the star itself is

$$v_\star = \Omega \times r, \tag{A176}$$

whereas the geometric rotation of spacetime contributes

$$v_{\text{geo}} = \omega \times r. \tag{A177}$$

Consequently, the motion of matter relative to the geometric flow is governed by the relative velocity

$$v_{\text{rel}} = (\Omega - \omega) \times r. \tag{A178}$$

This structure is neither a coordinate choice nor an imposed ansatz. It follows unavoidably from the tetrad–based definition of physical velocity combined with the torsional geometry.

Appendix I.6. Spin–down law from geometric drift

When the stellar rotation is aligned with the geometric drift,

$$\Omega = \omega, \tag{A179}$$

matter follows inertial motion with respect to the spacetime flow. In contrast, any mismatch $\Omega \neq \omega$ leads to a torsion–induced transfer of angular momentum.

At leading order, rotational symmetry and linear response imply the spin–down law

$$\tau = -\, I\, \Gamma\, (\Omega - \omega), \tag{A180}$$

where I is the moment of inertia and Γ characterizes the geometric alignment rate. This expression is therefore not phenomenological but arises as a direct consequence of the teleparallel tetrad action, the geometric drift field, and the physical velocity defined therein.

Appendix I.7. Braking Index

The braking index is defined kinematically in terms of the pulsar angular frequency $\Omega(t)$ as

$$n \equiv \frac{\Omega\,\ddot{\Omega}}{\dot{\Omega}^2},$$

where overdots denote derivatives with respect to coordinate time. This definition is purely observational and does not rely on any assumption concerning the physical origin of the spin–down torque.

The rotational evolution of a neutron star is governed by

$$I\,\dot{\Omega} = \tau_{\text{tot}},$$

where I denotes the stellar moment of inertia and τ_{tot} the total external torque.

In the present framework, the total torque is taken as the sum of two contributions. The electromagnetic dipole radiation torque is given by

$$\tau_{\text{EM}} = -K_3\,\Omega^3,$$

where K_3 is a constant determined by the magnetic field strength, stellar radius, and obliquity angle. In addition, a geometric drift torque arises from the coupling between the stellar angular velocity and the vorticity of the spacetime drift,

$$\tau_{\text{GDV}} = -I\,\Gamma(\Omega - \omega_W),$$

where Γ is a characteristic alignment rate and ω_W denotes the geometric angular velocity associated with the drift field.

The total spin–down equation therefore reads

$$\dot{\Omega} = -K_3\Omega^3 - \Gamma(\Omega - \omega_W).$$

For isolated pulsars, we consider the regime in which the stellar angular velocity dominates over the geometric drift,

$$\omega_W \ll \Omega.$$

In this limit, the evolution equation reduces to

$$\dot{\Omega} = -\left(K_3\Omega^3 + \Gamma\Omega\right).$$

Taking a time derivative yields

$$\ddot{\Omega} = -\left(3K_3\Omega^2 + \Gamma\right)\dot{\Omega},$$

where K_3 and Γ are assumed to vary slowly compared to the spin–down timescale.

Substituting these expressions into the definition of the braking index gives

$$n = \frac{\Omega\left[-\left(3K_3\Omega^2 + \Gamma\right)\dot{\Omega}\right]}{\dot{\Omega}^2} = \frac{3K_3\Omega^2 + \Gamma}{K_3\Omega^2 + \Gamma}.$$

Solving for the relative strength of the geometric contribution yields

$$\frac{\Gamma}{K_3\Omega^2} = \frac{3-n}{n-1}.$$

This expression shows that the braking index directly measures the ratio between the geometric drift torque and the electromagnetic dipole torque. Deviations from the canonical value $n = 3$ therefore reflect the presence of a geometric contribution to the spin–down, rather than a modification of Maxwell electrodynamics.

The moment of inertia I is treated as constant throughout this derivation. This assumption is justified because secular variations of I due to cooling, magnetic stresses, or crustal rearrangements occur on timescales much longer than the characteristic spin–down time. Possible transient changes during glitches introduce only higher–order corrections and do not affect the leading–order expression for the braking index.

Similarly, the alignment rate Γ is assumed to vary slowly in time. It characterizes the coupling between the stellar rotation and the spacetime drift geometry, which evolves on secular timescales. Allowing for rapid variations of Γ would introduce additional higher–order terms and lies beyond the quasi–stationary regime considered here.

The linear dependence of the geometric torque on $(\Omega - \omega_W)$ represents the lowest–order dissipative coupling allowed by rotational symmetry and dimensional analysis. Higher–order or nonlocal terms are suppressed and are not required for a minimal effective description.

Appendix IX: Current conservation from the Stückelberg sector and compatibility with teleparallel identities

The constraint action is

$$S_{\rm const} = \int d^4x\, e\, J^\mu \left(e^0{}_\mu - u_\mu - \kappa(A_\mu + \partial_\mu \varphi)\right), \tag{A181}$$

where φ is a Stückelberg scalar introduced to maintain $U(1)$ gauge invariance.

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Variation with respect to φ.

Varying $S_{\rm const}$ with respect to φ gives

$$\begin{aligned} \delta_\varphi S_{\rm const} &= -\kappa \int d^4x\, e\, J^\mu\, \partial_\mu \delta\varphi \\ &= \kappa \int d^4x\, \partial_\mu (eJ^\mu)\, \delta\varphi. \end{aligned} \tag{A182}$$

after integrating by parts and discarding boundary terms. Since $\delta\varphi$ is arbitrary, the Euler–Lagrange equation is

$$\partial_\mu (eJ^\mu) = 0 \qquad \Longleftrightarrow \qquad \nabla_\mu J^\mu = 0, \tag{A183}$$

where ∇_μ denotes the Levi-Civita covariant derivative of the metric induced by the tetrad. Thus, charge conservation is a direct consequence of the Stückelberg (gauge) redundancy and does not introduce any propagating degree of freedom.

Teleparallel identities and the homogeneous Maxwell equation.

In the Weitzenböck gauge ($\omega^a{}_b = 0$) the torsion 2-form is $T^a = de^a$ and therefore satisfies

$$dT^a = d(de^a) = 0 \tag{A184}$$

identically (a nilpotency/Bianchi-type identity). For the temporal component,

$$T^0 = de^0 = du + \kappa\, dA, \tag{A185}$$

so one obtains

$$0 = dT^0 = ddu + \kappa\, d(dA) = \kappa\, dF, \qquad F := dA. \tag{A186}$$

Hence the homogeneous Maxwell equation $dF = 0$ is automatically compatible with the teleparallel torsion identity. The inhomogeneous Maxwell equation and charge conservation, on the other hand, arise from the action principle: variation with respect to A_μ yields $\nabla_\nu F^{\nu\mu} = \kappa J^\mu$, while variation with respect to φ yields $\nabla_\mu J^\mu = 0$. Taking the covariant divergence of the inhomogeneous equation and using the antisymmetry of $F^{\mu\nu}$ gives $\nabla_\mu J^\mu = 0$ consistently.

Appendix J. The Reason of the Maxwell Kinetic Term is Unavoidable

This Appendix establishes, in a precise and constructive manner, the statement made in the main text: within a teleparallel formulation in which the electromagnetic sector is encoded in the temporal leg of the tetrad, the Maxwell kinetic term arises as the unique dynamical possibility at the lowest derivative order. The argument does not rely on postulated assumptions, but follows from the geometric structure of teleparallel torsion, locality, and Lorentz covariance.

Appendix J.1. Geometric Setup

In teleparallel geometry the fundamental dynamical variable is the tetrad $e^a{}_\mu$, and torsion is defined as the two–form

$$T^a := De^a = de^a + \omega^a{}_b \wedge e^b. \tag{A187}$$

In the Weitzenböck gauge, the spin connection vanishes, $\omega^a{}_b = 0$, and torsion reduces to

$$T^a = de^a. \tag{A188}$$

The electromagnetic sector is represented geometrically through an extension of the temporal tetrad,

$$e^0 = u + \kappa\big(A + d\varphi\big), \tag{A189}$$

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where u denotes a background temporal one–form, φ is a Stückelberg scalar, and κ is a dimensional conversion constant. This decomposition introduces no additional physical degrees of freedom: different choices of (A, φ) related by

$$A \to A + d\chi, \qquad \varphi \to \varphi - \chi, \tag{A190}$$

correspond to the same geometric object e^0. Consequently, all physical observables must be constructed solely from e^0 and its torsion.

Appendix J.2. Gauge-Invariant Derivative Content

Since teleparallel torsion depends only on exterior derivatives, the dynamical content of the electromagnetic sector can depend only on derivatives of the one–form A. At first derivative order, the most general tensor constructed from A is

$$X_{\mu\nu} = \partial_\mu A_\nu. \tag{A191}$$

Under the gauge transformation, its variation is

$$\delta X_{\mu\nu} = \partial_\mu \partial_\nu \chi. \tag{A192}$$

The antisymmetric part,

$$X_{[\mu\nu]} = \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu), \tag{A193}$$

is invariant, whereas the symmetric part transforms nontrivially. Thus, the antisymmetric derivative

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu \tag{A194}$$

constitutes the unique gauge–invariant local object at first derivative order.

In geometric terms, the temporal torsion reads

$$T^0 = de^0 = du + \kappa\, dA. \tag{A195}$$

On the integrable branch $du = 0$, one finds

$$T^0 = \kappa\, F, \tag{A196}$$

showing that the electromagnetic field strength exhausts the gauge–invariant torsional content of the temporal sector.

Appendix J.3. Lorentz-Scalar Structures at Second Derivative Order

Local dynamics at the lowest nontrivial order are governed by terms quadratic in first derivatives. Since all derivative information is carried by $F_{\mu\nu}$, admissible kinetic terms must be constructed from Lorentz scalars quadratic in F. In four spacetime dimensions, there exist only two such independent scalars,

$$I_1 := F_{\mu\nu} F^{\mu\nu}. \tag{A197}$$

$$I_2 := F_{\mu\nu} \tilde{F}^{\mu\nu}, \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \tag{A198}$$

Appendix J.4. Topological Character of $F\tilde{F}$

In the Abelian case relevant here, the field strength satisfies $F = dA$. Consequently,

$$F \wedge F = d(A \wedge F), \tag{A199}$$

and the integral of $F \wedge F$ reduces to a boundary term under standard boundary conditions. The corresponding scalar I_2 therefore does not contribute to the local Euler–Lagrange equations and carries no propagating degrees of freedom.

Appendix J.5. Maxwell Dynamics as a Structural Consequence

Collecting the above results, the local derivative content of the electromagnetic sector is uniquely encoded in $F_{\mu\nu}$. At quadratic order in derivatives, Lorentz covariance admits only the scalars I_1 and I_2, of which the latter is topological and dynamically inert. The remaining term yields

$$\mathcal{L}_{\rm EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{A200}$$

which is precisely the Maxwell kinetic term.

Thus, within the teleparallel formulation considered here, the Maxwell action is not introduced by assumption. It emerges as the unique local, gauge–invariant, Lorentz–covariant dynamical realization of a propagating electromagnetic sector encoded in the temporal tetrad geometry.

Appendix J.6. Remarks on Possible Extensions

Nonlinear functions such as $f(F_{\mu\nu}F^{\mu\nu})$ represent higher–order interactions and are naturally interpreted as subleading corrections in an effective field theory expansion. Terms involving preferred contractions with the temporal background, such as $(u_\mu F^{\mu\nu})(u^\rho F_{\rho\nu})$, introduce foliation dependence and correspond to medium–like or preferred–frame theories rather than to universal electromagnetism. Higher–derivative operators, for instance $(\nabla_\mu F^{\mu\nu})^2$, describe additional high–energy modes and lie beyond the minimal dynamical sector considered here.

The Maxwell kinetic structure is therefore fixed by the geometric and differential structure of teleparallel spacetime itself. No independent assumption concerning electromagnetic dynamics is required.

Appendix K. Emergence of the Maxwell Lagrangian from the Teleparallel Torsion Scalar

In this section we demonstrate that the electromagnetic kinetic term does not need to be introduced as an independent addition to the action. Instead, it arises naturally from the temporal sector of the teleparallel torsion scalar once the tetrad decomposition introduced in this work is taken into account.

Appendix K.1. Teleparallel Gravitational Action

In the teleparallel equivalent of general relativity (TEGR), the gravitational action is constructed from the torsion scalar

$$S_{\rm TEGR} = \frac{1}{2\kappa_g} \int d^4x\, e\, T,$$

where $e = \det(e^a{}_\mu)$ and the torsion scalar is defined as

$$T = \frac{1}{4} T^\rho{}_{\mu\nu} T_\rho{}^{\mu\nu} + \frac{1}{2} T^\rho{}_{\mu\nu} T^{\nu\mu}{}_\rho - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu.$$

Equivalently, this can be written as

$$T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu},$$

where $S_\rho{}^{\mu\nu}$ denotes the teleparallel superpotential.

Appendix K.2. Temporal Tetrad Decomposition

The fundamental assumption of the present framework is the decomposition of the temporal tetrad component,

$$e^0 = u + \kappa\, A,$$

where u is a background temporal flow one–form and A is a dynamical 1–form identified with the electromagnetic potential.

The torsion two–form associated with the temporal component is therefore

$$T^0 = de^0 = du + \kappa\, dA.$$

In the following, we restrict attention to irrotational or inertial temporal flows satisfying

$$du = 0.$$

This condition corresponds to the absence of intrinsic vorticity in the chosen temporal congruence and is sufficient to isolate the electromagnetic sector.

Under this assumption,

$$T^0 = \kappa\, F, \qquad F := dA.$$

Appendix K.3. Contribution of Temporal Torsion to the Torsion Scalar

The torsion scalar contains quadratic contractions of all torsion components, including the temporal sector. Isolating the contribution associated with $T^0{}_{\mu\nu}$, one finds

$$T \supset \frac{1}{4} T^0{}_{\mu\nu} T_0{}^{\mu\nu} + \frac{1}{2} T^0{}_{\mu\nu} T^{\nu\mu}{}_0 - T_{0\mu}{}^0 T^{\nu\mu}{}_\nu.$$

Using the antisymmetry of $T^0{}_{\mu\nu}$ and the condition $T_{0\mu}{}^0 = 0$, this reduces to

$$T_{\rm EM} = \frac{1}{2} T^0{}_{\mu\nu} T_0{}^{\mu\nu}.$$

Substituting $T^0{}_{\mu\nu} = \kappa F_{\mu\nu}$ yields

$$T_{\rm EM} = \frac{\kappa^2}{2} F_{\mu\nu} F^{\mu\nu}.$$

Appendix K.4. Emergent Maxwell Action

The teleparallel gravitational action therefore contains the contribution

$$S_{\rm TEGR} \supset \frac{1}{2\kappa_g} \int d^4x\, e\, \frac{\kappa^2}{2} F_{\mu\nu} F^{\mu\nu}.$$

Up to a normalization of coupling constants, this term is precisely the Maxwell kinetic action

$$S_{\rm EM} = -\frac{1}{4} \int d^4x\, e\, F_{\mu\nu} F^{\mu\nu}.$$

Thus, within the present framework, the electromagnetic action is not added by hand but emerges as the temporal torsion sector of the teleparallel gravitational action.

This result shows that classical electromagnetism can be viewed as a geometric subsector of teleparallel gravity. The usual separation between gravitational and electromagnetic actions is replaced by a single torsion–based scalar, whose temporal component reproduces the Maxwell dynamics. Gauge invariance arises as a direct consequence of the antisymmetric structure of torsion and the definition of the electromagnetic field strength as an exterior derivative.

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