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Article

# Structure of the Direct Electric Current in Conductors

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## Abstract

The structure of the direct electrical current in a conductor is discussed neither in the educational nor scientific literature. This problem is directly related to the boundary condition for the normal component of current density. The generally accepted approach to this problem requires more careful consideration. The following article is devoted to the analysis of the direct current distribution in a conductor. In this paper, we consider two mutually exclusive approaches to explain the structure of direct current in a conductor.

**Keywords:** surface charge distribution; conductors; electric current; electrodynamics

## 1. Introduction

It is generally accepted in physics that normal current density component  $J_n$  at the conductor boundary with a non-conductive medium is zero. An elementary proof of this assumption is based on the continuity equation (law of conservation of electric charge). Owing to the potentiality of the electric field and the proportionality of current density to electrostatic field intensity (for simplicity, we assume a cylindrical conductor), the generated homogeneous electric current is directed along the axis of the conductor.

The flaw in this proof is as follows. The generally accepted boundary condition, which is stated in [1], says that the charges are distributed over the surface of the conductor and create a field that leads to the directed movement of electrons.

Evidently the distribution of charges over the surface of the conductor should be heterogeneous. Field lines originate on positive charges and terminate on negative charges (Fig. 1). Under the generally accepted boundary condition, field lines bypass the charges that are the sources of the field. This does not support the generally accepted approach. Figure 1 depicts that  $J_n$  is not equal zero.

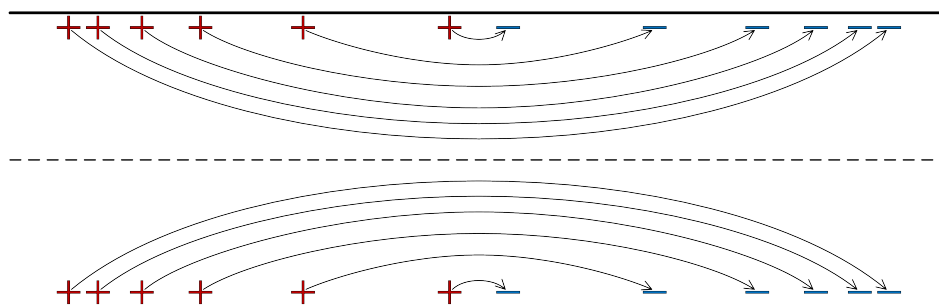


Figure 1. Field lines.

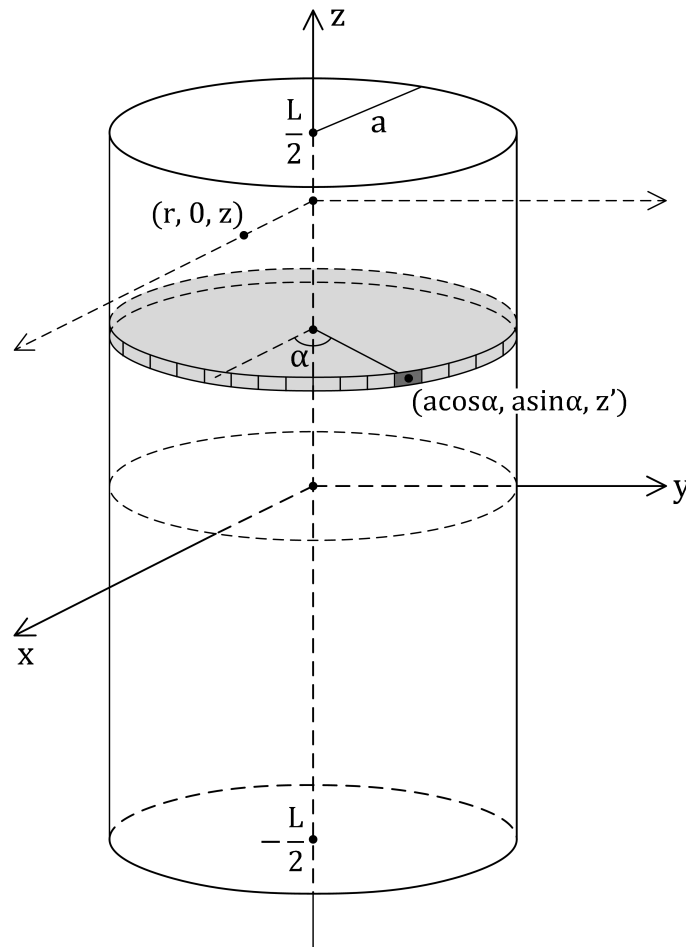
## 2. Field Inside a Cylindrical Conductor

The normal component of the current density being equal to zero is possible only in the absence of near-surface current, the value of which decreases with approaching to the middle of cylindrical conductor. Surface charges that create the field are also moving, but the resultant surface current is negligible.

What is this near-surface current? The existence of a normal component of the current density near the surface of the conductor leads to the appearance of an uncompensated charge, which causes the appearance of a near-surface relaxation current.

The relaxation time constant, as stated in [2], is  $\tau = \varepsilon/\sigma = \varepsilon_0 \varepsilon_r/\sigma$ , where  $\sigma$  is conductivity. Toward the middle of the conductor, the surface current decreases and the volume current increases, reaching a maximum. To describe this process, we must use a more complicated boundary condition:  $dI_s/dz = 2\pi a J_n$  instead of  $J_n = 0$  (where  $I_s$  is the value of near-surface current,  $z$  is parallel to the axis of the cylinder coordinate, and  $a$  is the radius of the cylindrical conductor). Note that there is no such distribution in the surface charge density for the cylindrical conductor, which leads to the creation of a uniform field inside it.

Next, we obtain the potential  $\varphi$  of the electric field inside the cylindrical conductor at point  $(r, \theta, z)$  created by charge  $\sigma^*(z') a da dz'$  located at point  $(a \cos \alpha, a \sin \alpha, z')$  (Fig. 2).



**Figure 2.** Calculation of electric potential in a cylindrical conductor.

The surface charge has an axisymmetric distribution:

$$d^2 \varphi = \frac{1}{4\pi\varepsilon_0} \frac{\tau^*(z') a d\alpha dz'}{2\pi a \sqrt{(a \cos \alpha - r)^2 + a^2 \sin^2 \alpha + (z' - z)^2}}, \quad (1)$$

where surface charge density  $\sigma^*(z')$  is expressed using linear density  $\tau^*(z')$ :

$$\sigma^*(z') = \frac{\tau^*(z')}{2\pi a}. \quad (2)$$

After a series of simple transformations, Eq. (1) can be reduced to the following form:

$$d^2\varphi = \frac{1}{8\pi^2\epsilon_0 a} \frac{\tau^*(z') dx dz'}{\sqrt{1+p^2}\sqrt{1-2\rho x+\rho^2}} \quad (3)$$

where  $p = (z' - z)/a$ ,  $\rho = t/\sqrt{1+p^2}$ ,  $t = r/a$ , and  $x = \cos\alpha/\sqrt{1+p^2}$ . The expression  $\sqrt{1-2\rho x+\rho^2}$  can be expanded into a series using the Legendre polynomials:

$$\frac{1}{\sqrt{1-2\rho x+\rho^2}} = \sum_{k=0}^{\infty} \rho^k P_k(x) = P_0(x) + \rho P_1(x) + \rho^2 P_2(x) + \dots, \quad (4)$$

where  $P_0(x) = 1$ ,  $P_1(x) = x$ ,  $P_2(x) = \frac{1}{2}(3x^2 - 1)$ . Integrating Eq. (3) over  $\alpha$  in the range from 0 to  $2\pi$ , we obtain

$$d\varphi \approx \frac{1}{8\pi\epsilon_0 a} \frac{\tau^*(z') dz' \left(2 + \frac{t^2}{2(1+p^2)} \left(\frac{3}{1+p^2} - 2\right)\right)}{\sqrt{1+p^2}}. \quad (5)$$

Assuming that  $\tau^*(z') = \beta z'$  and integrating over  $z'$  from  $-L/2$  to  $+L/2$ , we obtain

$$\begin{aligned} \varphi(r, z) \approx & \frac{\beta}{4\pi\epsilon_0} \left\{ a \left( \sqrt{1+g_-^2} - \sqrt{1+g_+^2} \right) + z \ln \frac{\sqrt{1+g_-^2} + g_-}{\sqrt{1+g_+^2} - g_+} \right. \\ & - \left[ \frac{a}{2} \left( \frac{1}{\sqrt{1+g_+^2}} - \frac{1}{\sqrt{1+g_-^2}} \right) + \frac{a}{4} \left( \frac{1}{(1+g_-^2)^{\frac{3}{2}}} - \frac{1}{(1+g_+^2)^{\frac{3}{2}}} \right) \right. \\ & \left. \left. - \frac{z}{4} \left( \frac{g_-}{(1+g_-^2)^{\frac{3}{2}}} - \frac{g_+}{(1+g_+^2)^{\frac{3}{2}}} \right) \right] t^2 \right\}, \quad (6) \end{aligned}$$

where  $g_{\pm} = (L \pm 2z)/2a$ .

Using Eq. (6), it is possible to obtain expressions for the longitudinal  $E_z(r, z) = -\partial\varphi/\partial z$  and transverse  $E_r(r, z) = -\partial\varphi/\partial r$  components of the electric field intensity in the conductor, as well as the approximate value of  $\beta$ . In particular, away from the ends of the conductor,

$$E_z(0, z) \approx \frac{\beta}{4\pi\epsilon_0} \left( 2 - \ln \frac{L^2 - 4z^2}{a^2} + \frac{8z}{L^2 - 4z^2} \right), \quad (7)$$

$$E_r(r, z) \approx -\frac{\beta z(3L^2 - 4z^2)}{\pi\epsilon_0(L^2 - 4z^2)^2} r, \quad (8)$$

$$\beta \approx \frac{2\epsilon_0 I_{tot}}{\sigma a^2 \left( \ln \frac{L}{a} - 1 \right)}, \quad (9)$$

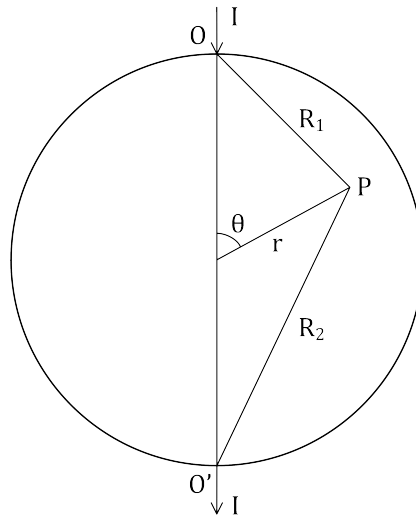
where  $I_{tot}$  is the total current flowing through the conductor. The results obtained here suggest that the generally accepted boundary condition  $J_n = 0$  needs to be revised.

### 3. Field Inside a Conducting Ball

Let us determine the distribution of the potential of the electric field in a conducting ball. Current  $I$  enters through one pole (point  $O$ ) and exits through the opposite pole (point  $O'$ ) (Fig. 3).

The solution to a similar problem is given in [3]. However, the boundary condition used to solve this problem was  $\partial\varphi/\partial r|_{r=a} = 0$ , where  $a$  is the radius of the ball. Surface current leads to the following boundary condition:

$$\frac{\partial\varphi}{\partial r} \Big|_{r=a} \sin\theta + \frac{1}{a} \frac{\partial\varphi}{\partial\theta} \Big|_{r=a} \cos\theta = 0. \quad (10)$$



**Figure 3.** Calculation of electric potential in a conducting ball.

The use of this boundary condition significantly affects the determination of the electric potential, which at an arbitrary point  $P$  inside the ball can be represented as

$$\begin{aligned}\varphi(r, \theta) &= \frac{I}{2\pi\sigma} \left( \frac{1}{R_1} - \frac{1}{R_2} + \sum_{n=1}^{\infty} C_n \left(\frac{r}{a}\right)^n P_n(\cos \theta) \right) \\ &= \frac{I}{2\pi\sigma} \left( \frac{1}{\sqrt{a^2 - 2ar \cos \theta + r^2}} - \frac{1}{\sqrt{a^2 + 2ar \cos \theta + r^2}} + \sum_{n=1}^{\infty} C_n \left(\frac{r}{a}\right)^n P_n(\cos \theta) \right).\end{aligned}\quad (11)$$

Using the transformations associated with the expansion in Legendre polynomials, we obtain

$$\begin{aligned}\varphi(r, \theta) &= \frac{I}{2\pi\sigma} \left( \frac{1}{a} \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n P_n(\cos \theta) - \frac{1}{a} \sum_{n=0}^{\infty} (-1)^n \left(\frac{r}{a}\right)^n P_n(\cos \theta) + \sum_{n=1}^{\infty} C_n \left(\frac{r}{a}\right)^n P_n(\cos \theta) \right) \\ &= \frac{I}{2\pi\sigma} \left( \sum_{n=0}^{\infty} \left(\frac{2}{a} + C_{2n+1}\right) \left(\frac{r}{a}\right)^{2n+1} P_{2n+1}(\cos \theta) + \sum_{n=1}^{\infty} C_{2n} \left(\frac{r}{a}\right)^{2n} P_{2n}(\cos \theta) \right).\end{aligned}\quad (12)$$

Substituting Eq. (12) into Eq. (10), we obtain an expansion in terms of  $\sin(n\theta)$ . If we equate the coefficients of  $\sin(n\theta)$  to zero, we obtain a system of equations for calculating  $C_n$ . In the third approximation, we obtain

$$C_2 = 0, \quad C_3 = -\frac{2}{a}, \quad (13)$$

and in the fifth approximation,

$$C_2 = C_4 = 0, \quad C_3 = C_5 = -\frac{2}{a}. \quad (14)$$

Furthermore,

$$\varphi(r, \theta) = \frac{Ir \cos \theta}{\pi a^2 \sigma}, \quad E_r = -\frac{I \cos \theta}{\pi a^2 \sigma}, \quad \text{and} \quad E_\theta = \frac{I \sin \theta}{\pi a^2 \sigma}, \quad (15)$$

which correspond to a uniform field inside the ball with intensity

$$E = \frac{I}{\pi a^2 \sigma}. \quad (16)$$

It is easy to demonstrate that

$$C_1 = 0. \quad (17)$$

Summarizing the results of the coefficient  $C_i$  calculations, we get

$$C_{2n+1} = -\frac{2}{a}, \quad \text{where } n = 1, 2, 3, \dots \quad (18)$$

and

$$C_{2n} = 0, \quad \text{where } n = 1, 2, 3, \dots \quad (19)$$

for the values of the electric field intensity obtained in Eqs. (15) and (16).

It is interesting to note that the bulk current will be equal to

$$I_v = I \sin^2 \theta, \quad (20)$$

and the near-surface current will be

$$I_s = I \cos^2 \theta. \quad (21)$$

#### 4. Conclusion

The inhomogeneous distribution of the surface charge leads to the conclusion that the normal component of the current density is nonzero. Near-surface and bulk currents appear in the conductor, changing their magnitude along the conductor axis. The near-surface current has a relaxing nature. In the case of a cylindrical conductor, an approximate calculation of the current density and electric field intensity (far from its ends and near its axis) has been done. An exact expression is obtained for the bulk and near-surface currents in a conducting ball. It is demonstrated that the electric field inside the ball is uniform.

#### References

1. N. D. Papalexi, *The Course of Physics* (1942), Vol. 2.
2. W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison-Wesley, 1962), 2nd edition.
3. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (1984), 2nd edition.

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