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Clarifying and Standardizing the Lévy Flight in Scientific Computing

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Keywords: Lévy flight; stochastic process; scientific computing; reproducibility; standardization; heavy-tailed distributions

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Clarifying and Standardizing the Lévy Flight in Scientific Computing

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Abstract

The Lévy flight is a fundamental stochastic model widely utilized across various domains, including engineering optimization and biological foraging. However, the current literature reveals notable inconsistencies in its definition, parameter interpretation, and algorithmic implementation, which compromise the reliability and reproducibility of empirical findings. To address these issues, this paper proposes a unified framework derived from a systematic review and critical analysis of existing studies. The framework standardizes the core definition, clarifies the roles of key parameters, and establishes a consistent computational paradigm, thereby resolving ambiguities and enhancing the consistency, reliability, and cross-study comparability of Lévy flight applications in scientific computing. Additionally, a targeted analysis examines the influence of the Lévy index, demonstrating how its variation systematically affects both the global scale and local behavior of the flights. This analysis provides a practical guidance for selecting appropriate Lévy index values for specific application contexts. Overall, this work provides a comprehensive and actionable reference to strengthen the methodological rigor and coherence of research involving Lévy flight models.

Keywords: Lévy flight; stochastic process; scientific computing; reproducibility; standardization; heavy-tailed distributions

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MSC: 60G50; 62P35; 68Q10; 68T20

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1. Introduction

The term "Lévy flight" (hereafter abbreviated as LF) was first introduced in 1937 by the renowned French mathematician Paul Lévy (1886–1971) [1]. It describes a type of random walk that shares similarities with Brownian motion (BM). As documented in the literature [1,4–7], LF has been widely adopted as a search strategy [2,3] and applied across various fields, including financial market analysis, engineering optimization, animal foraging simulation, and molecular dynamics.

In contrast to the BM, which performs efficiently in environments where targets are abundant and predictable, the LF proves optimal for "blind search", that is, search without prior information. This characteristic makes it suitable for searching in uncertain environments with higher efficiency, even allowing it to be applied to cases such as identifying divisors of unfactorized odd composite integers, as demonstrated in [8].

Despite its significance, the existing literature lacks a consistent definition, parameter interpretation, and a standardized computational methodology, with some publications containing notable inaccuracies. To address these issues, this paper aims to provide a clear and precise characterization of the LF, clarify the meaning and application of its parameters, and establish a standardized computational approach.

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We begin by systematically reviewing relevant academic publications and online sources, categorizing them by definition, parameter interpretation, and calculation method, and explicitly identifying inconsistencies among them. Drawing upon authoritative mathematical references, we then propose a unified theoretical framework and computational paradigm to resolve prevailing ambiguities. Furthermore, we conduct a preliminary investigation into the influence of the Lévy index on LF behavior to guide researchers and practitioners in designing effective LF-based search strategies for practical applications.

The paper is structured into six sections. Section 1 provides an introduction to the study. Section 2 presents the terminology and abbreviations used throughout the paper. Section 3 offers a comparative analysis of the relevant literature, highlighting identified inconsistencies. Section 4 introduces a unified framework for defining the LF, clarifying its parameters, and normalizing its computing issues. Section 5 examines the influence of the Lévy index on the behavior of the LF. Finally, Section 6 concludes the paper by summarizing the key findings and discussing related future researches.

2. Terms and Abbreviations

The following abbreviations are used throughout this paper:

CF: characteristic function.
DF: distribution function.
CDF: cumulative distribution function.
PDF: probability density function.
LD: Lévy distribution.
LSD: Lévy stable distribution.
HTD: heavy-tailed distribution.
LTD: long-tailed distribution.

It is worth noting that, following the conventions in [9,10], CDF and DF are mathematically equivalent; CDF is simply referred to as DF.

Through a comprehensive analysis of the literature and online sources, this section identifies inconsistencies in the description of the LF, particularly with respect to its definition, parameter interpretation, and computational methodology.

2.1. Inconsistencies in Defining the LF

All LF-related publications, including those posted on the webpage such as [11–13], categorize the LF as a type of random walk, which is systematically introduced in Frank Spitzer's early graduate mathematical textbook [14]. By analogy with physical motion, a random walk is a stochastic process that models the random motion of a particle (or random walker) in space. As described in [15] the particle's trajectory is characterized by random increments or jumps occurring at discrete time intervals. Mathematically, for independent and identically distributed (*iid*) random variables (steps) $X_1, X_2, ...,$ and X_n, the random walk S_n is defined by

$$S_n = \sum_{i=1}^{n} X_i = S_{n-1} + X_n. \quad (1)$$

The distribution law of the random steps dominates the characteristics of the random walk. The random steps following the Bernoulli distribution produce a simple random walk (SRW), while those following the Gaussian distribution produce the BM. However, literature has not yet given a universal and unified description for the distribution of LF steps. Table 1 presents several selected publications. Seen in the table, descriptions for the distribution of LF steps vary across these publications, covering the LD, LSD, LTD, HTD, and power-law tail.

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Table 1. Distribution of LF Steps Described in Reviewed Publications

Reference	Description for the Distribution of LF steps
[3,13]	A power-law tail
[6]	Lévy stable distribution, Lévy distribution
[7]	Stable heavy-tailed distribution
[12,17,21,24]	A heavy-tailed distribution
[16,20,22]	A Lévy distribution
[18]	Lévy tailed probability distribution
[19]	Lévy tailed probability distribution, Lévy distribution
[25]	A long-tailed distribution

According to [23], the LD represents a specific instance within the class of LSDs. As documented in [26–29], the LSD constitutes a subclass of LTDs, which in turn belong to the broader family of HTDs. The class of HTDs includes various distributions, such as the Pareto, log-logistic, and Weibull distributions. LTDs encompass a wider range of distributions that extend beyond the scope of LSDs. Specifically, LSDs form a proper subset of LTDs, and LTDs themselves constitute a proper subset of HTDs. Figure 1 provides a schematic illustration of the hierarchical relationships among the LD, LSD, LTD, and HTD.

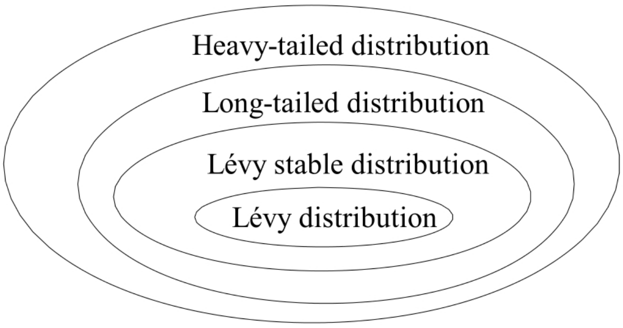

Figure 1. Relationships among the LD, LSD, LTD, and HTD.

On the other hand, the LSD exhibits the power-law tail [33], a characteristic inherited from the HTD. However, as demonstrated in [45–47], not all HTDs conform to the power-law distribution. These relationships are illustrated in Figure 2.

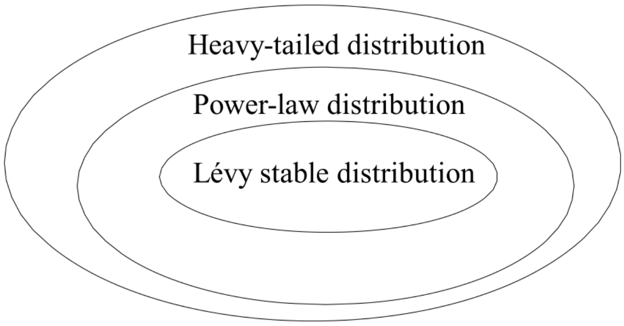

Figure 2. Relationships among HTD, LSD, and the power-law distribution.

The relationships in Figures 1 and 2, in conjunction with the descriptions in Table 1, indicate inconsistencies in the current definition of the LF. Therefore, it is imperative to refine and further enhance its conceptual clarity.

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2.2. Inconsistencies in Describing the PDF of the LD

For the random variable **X** that follows the LD with parameters μ and σ, denoted by $X \sim Levy(\mu, \sigma)$, the description of **X**'s PDF varies in different literature. Reference [23], cited in [30], provides its formula with a function $L_{\mu,\sigma}(x)$ by

$$L_{\mu,\sigma}(x) = \sqrt{\frac{\sigma}{2\pi}} \frac{e^{-\frac{\sigma}{2(x-\mu)}}}{(x-\mu)^{3/2}}. \tag{2}$$

Formula (2) also appears in Section 3.5 of reference [16], Section 3.3 of reference [20], Section 11.3 of reference [21], and on the webpage [31]. As illustrated in Table 2, the meanings and scopes of two parameters, μ and σ, are inconsistent across these publications.

Table 2. Meanings and scopes of μ and σ in describing the LD from reviewed publications

Reference	μ's meaning	μ's scope	σ's meaning	σ's scope
[16]	Not declared	Not assigned	Scale parameter	Not assigned
[20]	Minimal step	$0 < \mu < x < \infty$	Scale parameter	Not assigned
[21]	Location parameter	Not assigned	Scale parameter	Not assigned
[31]	Location parameter	$\mu \leq x$	Scale parameter	$\sigma > 0$

Notably, a potential ambiguity arises in Section 3.5 of [16]. The condition on μ is $\mu \leq x$ in Eq. (3.53) but $0 < \mu < x$ in Eq. (3.61). Moreover, Eq.(3.53) is introduced using the term "Lévy probability distribution," while Eq.(3.61) is described as "a simple version of Lévy distribution." Per the conventions in [9,10], both "Lévy probability distribution" and "Lévy distribution" should refer to the DF (or CDF). However, they are referred to the PDF in [16].

2.3. Inconsistencies in Describing the CF of the LSD

According to [33], which cites the earlier work [34], most PDFs for the LSD lack of closed-form expressions. This compels practitioners to utilize the CF to describe the LSD. For a given random variable **X** with continuous CDF $F(x)$, its CF is defined by

$$\varphi_X(t) = E[e^{itX}] = \int_{-\infty}^{\infty} e^{itx} dF(x), \tag{3}$$

where x is real, i is the imaginary unit such that $i^2 = -1$, and $E[\cdot]$ denotes the expected value.

If **X** follows the LSD, it is denoted by $X \sim S_\alpha(\sigma, \beta, \mu)$ and its CF, $\varphi_0(x)$, is most popularly given by

$$\ln \varphi_0(x) = \begin{cases} -\sigma^\alpha |x|^\alpha \{1 - i\beta sign(x) \tan \frac{\pi\alpha}{2}\} + i\mu x, \alpha \neq 1 \\ -\sigma|x|\{1 - i\beta sign(x) \frac{2}{\pi} \ln |x|\} + i\mu x, \alpha = 1 \end{cases}, \tag{4}$$

where α is the Lévy index, β is the skew parameter, σ is the scale parameter, and μ is a location (shift) parameter.

For the sake of computation, it is often helpful to use a different parameterization:

$$\ln \varphi_0(x) = \begin{cases} -\sigma^\alpha |x|^\alpha \{1 + i\beta sign(x) \tan \frac{\pi\alpha}{2}((\sigma|x|)^{1-\alpha} - 1)\} + i\mu_0 x, \alpha \neq 1 \\ -\sigma|x|(1 + i\beta sign(x) \frac{2}{\pi} \ln(\sigma|x|)) + i\mu_0 x, \alpha = 1 \end{cases}. \tag{5}$$

The location parameters μ and μ_0 in (4) and (5) are related by

$$\begin{cases} \mu = \mu_0 - \beta\sigma \tan \frac{\pi\alpha}{2}, \alpha \neq 1 \\ \mu = \mu_0 - \beta\sigma \frac{2}{\pi} \ln \sigma, \alpha = 1 \end{cases}. \tag{6}$$

Determined by the four parameters: α, β, μ, and σ, $\varphi_0(x)$ is also expressed in some literature (e.g. [1]) with a function $p_{\alpha,\beta}(x, \mu, \sigma)$ in the form

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$$p_{\alpha,\beta}(x,\mu,\sigma) = \exp\{i\mu x - \sigma^{\alpha}|x|^{\alpha}(1 - i\beta\frac{x}{|x|}\omega(x,\alpha))\}, \tag{7}$$

where i is the imaginary unit and

$$\omega(x,\alpha) = \begin{cases} \tan\frac{\pi\alpha}{2} & if \alpha \neq 1, 0 < \alpha < 2, \\ -\frac{2}{\pi}\ln|x| & if \alpha = 1. \end{cases}.$$

In the case $\mu = 0$ and $\beta = 0$, which is the most popular case, it follows

$$p_{\alpha,0}(x,0,\sigma) = e^{-\sigma^{\alpha}|x|^{\alpha}} = e^{-\lambda|x|^{\alpha}}, \tag{8}$$

where $\lambda = \sigma^{\alpha}$ is also called a scale parameter because σ is a scale parameter.

Notably, the PDF expressed by formula (2) is derived by taking $\alpha = 0.5$ and $\beta = 1$ in formula (5), as detailed in reference [23].

Formulas (4) and (7) can be found in [1,7,23,34–37]. However, the scopes of the four parameters differ across the listed literature, as summarized in Table 3. Interestingly, [1] is inconsistent with [34], despite citing the latter.

Table 3. Scopes of α, β, μ, and σ in the CF from the reviewed references

Reference	α	β	μ	σ
[1], citing [34]	$\alpha \in [0,2]$	$\beta \in [-1,1]$	Not mentioned	$\sigma > 0$
[7]	Not mentioned	Not mentioned	Not mentioned	Not mentioned
[23,34–37]	$\alpha \in (0,2]$	$\beta \in [-1,1]$	$\mu \in (-\infty,\infty)$	$\sigma > 0$

2.4. Inconsistencies in Describing the Power-law Distribution

As seen in [1,20], the PDF of an LSD has the asymptotic property given by

$$\lim_{x\to\infty} L_{\mu,\sigma}(x) = \frac{\lambda\sin(\frac{\pi\alpha}{2})\Gamma(1+\alpha)}{\pi|x|^{1+\alpha}}, \tag{9}$$

where α is the Lévy index and $\Gamma(\cdot)$ is the Gamma function of integer variable z , calculated by

$$\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt. \tag{10}$$

This indicates that large random steps adhere to the power-law distribution, which is approximately described by

$$L_{\alpha}(x) \approx x^{-(1+\alpha)}, \tag{11}$$

where x is the random step with large values.

Formula (11) appears in [5,6,17,19,23,30,32,40]. However, the interpretation and scope of α are inconsistent in these references, as listed in Table 4.

Table 4. Meaning and scope of α in (11) vary in literature

Reference	α's meaning	α's scope
[5,19]	Not mentioned	$0 < \alpha \leq 2$
[6,40]	Shape parameter	$0 < \alpha < 2$
[17]	Type of the LF	$1 \leq \alpha < 2$
[23]	Index of stability	$0 < \alpha < 2$
[25]	Not mentioned	$0 < \alpha < 2$
[30]	Power-law index	$1 < \alpha < 2$
[32]	Levy index	$0 < \alpha \leq 2$

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Notably, reference [42] introduces a distinct representation by

$$L(x_j) = |x_j|^{1-\alpha}, \tag{12}$$

where x_j represents the length of the flight, and $1 < \alpha \leq 2$ denotes the power-law exponent. This is obviously different from those in Table 4, revealing another inconsistency.

2.5. Inconsistencies in Describing the Computation of the LF

Mantegna's algorithm, derived from the power-law distribution (see Eq. (4) in [38]), was originally proposed for numerical simulation of the Lévy stable symmetrical stochastic process with Lévy index $\alpha \in [0.3, 1.99]$. Given that the LD inherits the behavior of the LSD, as described in Figure 1, the algorithm has been adopted for computing the LF in various literature since its introduction. By the algorithm, the large LF step s is calculated with two random variables u and v by

$$s = \frac{u}{v^{1/\alpha}}. \tag{13}$$

The random variables u and v admit the normal distributions, namely,

$$u \sim N(0, \sigma_u^2), \quad v \sim N(0, \sigma_v^2), \tag{14}$$

with $\sigma_v = 1$ and

$$\sigma_u = \left\{ \frac{\Gamma(1+\alpha)\sin(\pi\alpha/2)}{2^{(\alpha-1)/2}\alpha\Gamma((1+\alpha)/2)} \right\}^{1/\alpha}. \tag{15}$$

Formulas (11), (13), (14), and (15) are each found in one or more of the references [8,16,20,30,39–43]. However, as summarized in Table 5, the meaning and scope of the parameter α vary across these references.

Table 5. α's meaning and scope vary across references

Reference	α's meaning	α's scope
[8]	Levy index	$0 < \alpha \leq 2$
[16,20]	Not mentioned	$0 < \alpha \leq 2$
[30]	Power-law index	$1 < \alpha < 2$
[39]	Lévy distribution index	$0 < \alpha \leq 2$
[40]	Shape parameter	$0 < \alpha < 2$
[41]*	Not mentioned	Not assigned
[42]*	Power-law exponent	$1 < \alpha \leq 2$
[43]*	A constant	$\alpha = 1.5$

Notably, references [41–43] do not explicitly present formula (11), leaving the power-law distribution to be inferred. Furthermore, reference [30] proposes a formula similar to, yet distinct from, (15) for calculating σ_u, while reference [42] employs $s = 0.05 \times u/v^{1/\alpha}$ instead of (13). These discrepancies highlight inconsistencies in the LF computation.

3. A Unified Framework

In light of the many inconsistencies outlined in the previous section, this section first conducts a comprehensive analysis of these discrepancies, then proposes a unified framework for the description and computation of the LF.

3.1. Analysis of the Existing Inconsistencies

The inconsistencies listed in Table 1 arise from determining a suitable distribution the LF follows. The inconsistencies shown in Table 4 mainly concern α's meaning and scope. The inconsistencies

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illustrated in Table 3 involve the scopes of the four parameters—α, β, μ, and σ. The inconsistencies exhibited in Table 2 are associated with the meanings and scopes of the parameters μ and σ. The inconsistencies revealed in Table 5 mainly concern the meaning and scope of α, as well as the formulation for computing the LF. By synthesizing all available information, particularly from references [1,23,33,37], it can be conclusively determined that α represents the Levy index, β denotes the skewness parameter, μ corresponds to the shift (location) parameter, and σ signifies the scale parameter throughout the description and analysis of the LF, regardless of whether they are explicitly stated or implied. Consequently, the four parameters in (2), (4), (5), (11), and (13) maintain consistent meanings and scopes both explicitly and implicitly. Additionally, it is important to note that the PDF described in (2) applies specifically to the case where $\alpha = 0.5$ and $\beta = 1$ are implicitly evaluated, as highlighted in reference [23]. Furthermore, reference [1] emphasizes that α and β play dominant roles, whereas μ and σ play a lesser role.

Based on these fundamental insights, corroborated by Figures 1 and 2, the following conclusions can be drawn:

1. By using the LSD, the inconsistencies associated with defining the LF can be validly addressed. The LSD can also concurrently satisfies the requirements for describing the CF, modeling the power law, and computing the LF.
2. By restricting $\alpha \in (0,2)$, $\beta \in [-1,1]$, $\mu \in (-\infty, \infty)$, and $\sigma \in (0, \infty)$, a pure LF can be ensured, thereby excluding the BM due to BM's well-established and independent mathematical model.

3.2. A Unified Description

Summarized from the previous analysis and conclusions, here present a unified description of the LF.

1) The LF is a random walk whose step lengths follow the Lévy stable distribution, characterized by four parameters: α, β, μ, and σ. Specifically, $\alpha \in (0,2)$ denotes the Lévy index, $\beta \in [-1,1]$ represents the skew parameter, $\sigma \in (0, \infty)$ is the scale parameter, and $\mu \in (-\infty, \infty)$ serves as the location (or shift) parameter.

2) The CF, $\varphi_{\alpha,\beta,\mu,\sigma}(x)$, of the LF is given by

$$\varphi_{\alpha,\beta,\mu,\sigma}(x) = \exp\{i\mu t - \sigma^{\alpha}|t|^{\alpha}(1 - i\beta\frac{t}{|t|}\omega(t,\alpha))\}, \tag{16}$$

where i is the imaginary unit, and

$$\omega(t,\alpha) = \begin{cases} \tan\frac{\pi\alpha}{2} & if \alpha \neq 1, \\ -\frac{2}{\pi}\ln|t| & if \alpha = 1. \end{cases}.$$

When $\mu = 0$ and $\beta = 0$, the CF turns to be

$$\varphi_{\alpha,\lambda}(k) = e^{-\lambda|k|^{\alpha}}, \tag{17}$$

where $\lambda = \sigma^{\alpha} \in (0, \infty)$ is a scale parameter.

3) The PDF, $L_{\alpha,\beta,\mu,\sigma}(x)$, of the LF is calculated through the CF $\varphi_{\alpha,\beta,\mu,\sigma}(x)$ by

$$L_{\alpha,\beta,\mu,\sigma}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi_{\alpha,\beta,\mu,\sigma}(t)e^{itx}dt. \tag{18}$$

In the case of $\alpha = 0.5$ and $\beta = 1$, the PDF is given by

$$L_{\mu,\sigma}(x) = \sqrt{\frac{\sigma}{2\pi}}\frac{e^{-\frac{\sigma}{2(x-\mu)}}}{(x-\mu)^{3/2}}, -\infty < \mu < x < \infty. \tag{19}$$

In the other case, there is no analytic form for the PDF.

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4) The large random steps of the LF adhere to the power-law distribution, which is approximately described by

$$L_{\alpha}(x) \approx x^{-(1+\alpha)}, \tag{20}$$

where x is the random step with large values.

5) Mantegna's algorithm, expressed in (13), (14), and (15), is currently a standard algorithm to calculate the large steps of the LF under the condition $\alpha \in [0.3, 1.99]$.

4. Influence of the Lévy index α on the LF Behavior

The LF movement alternates between frequent short-distance jumps and occasional long-distance jumps [7]. Both [1,38] noted that the Lévy index α can influence the LF movement; however, neither study pursued further exploration, except that [38] discussed its impact on computational efficiency of the LF itself. Through extensive numerical simulations, we have observed that a large value of α preserves the characteristics of the LF movement, whereas a small α value results in feature resembling that of the simple random walk. This behavior can be seen in Table 6, where each row from top to bottom corresponds to α's values of 0.25, 0.3, 0.5, 0.75,1.0, 1.25, 1.5, 1.75, and 1.99, respectively. Furthermore, it is evident that a higher α value leads to a reduced spatial extent of the LF.

Table 6. α's Influence on the LF Behavior(to be continued)

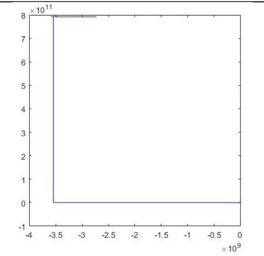
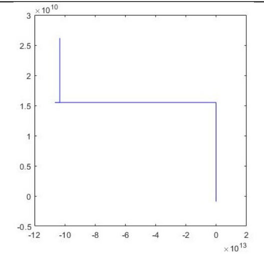
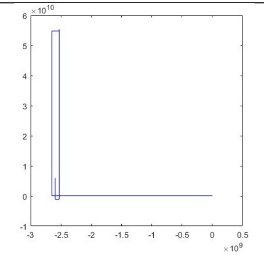
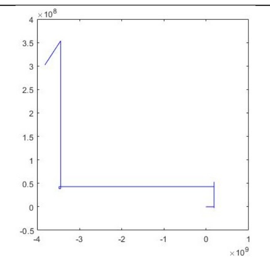
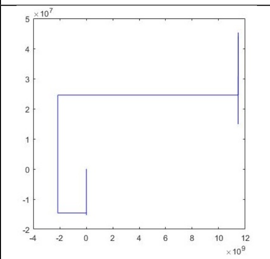
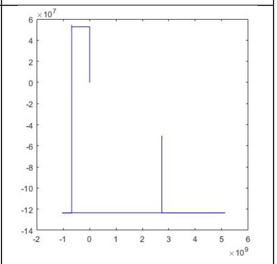
α	Movement 1	Movement 2	Movement 3
0.25			
0.3			

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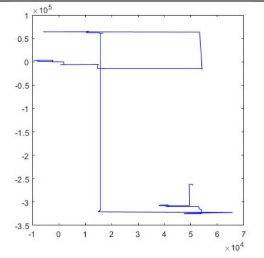
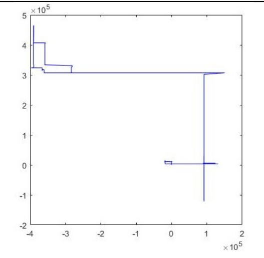
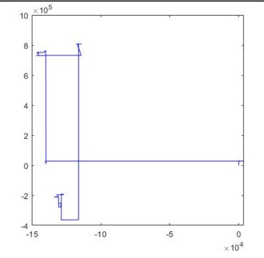
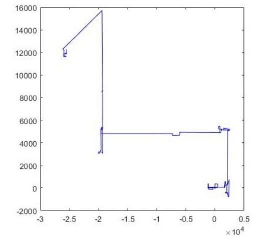
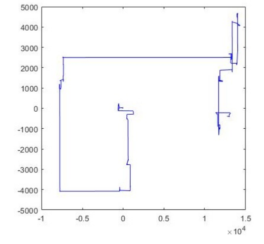
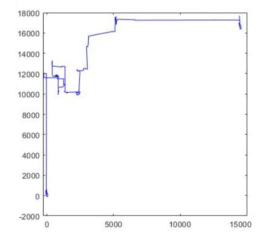
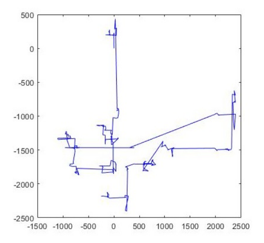
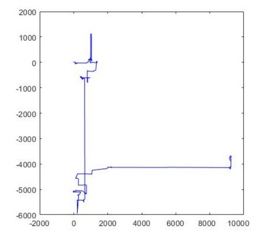
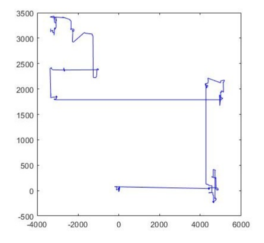
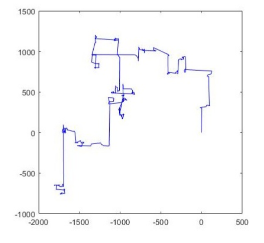
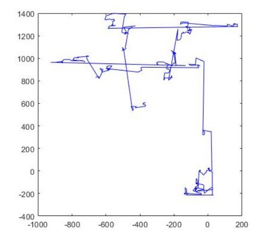
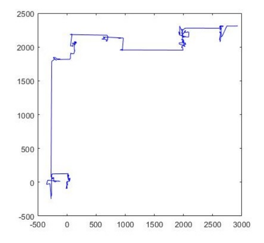
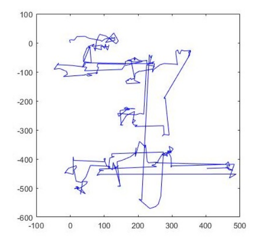
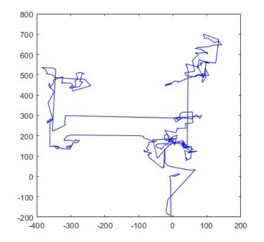
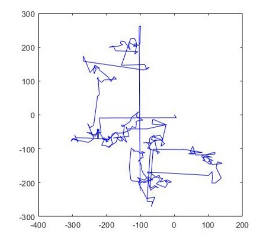
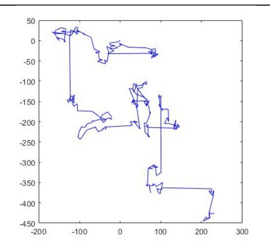
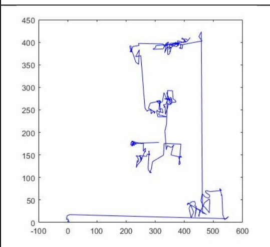
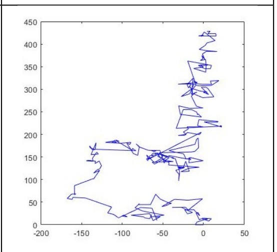
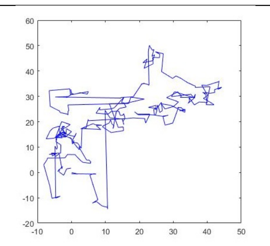
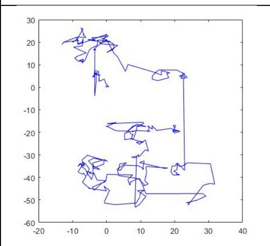
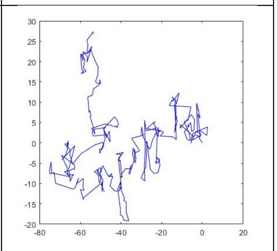
Table 6–continued

α	Movement 1	Movement 2	Movement 3
0.5			
0.75			
1.0			
1.25			
1.5			
1.75			
1.99			

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This characteristic indicates that an increase in α reduces the step length and, consequently, shrinks the search domain, thereby providing a practical reference for planning the search strategy in applications. For instance, in large regions where objectives are locally dense but globally sparse, as demonstrated in [8], a smaller α value is preferable to a larger one. This is because the shorter step length resulting from a larger α may cause the search process to fall into the large gaps between objectives, thereby increasing the time required to traverse these gaps, as pointed out in [48].

By adjusting the value of α, we conduct experiments using the algorithm and a part of the data in [8]. The results are presented in Tables 7 and 8, where the abbreviation "SRWc" denotes the SRW search within a circular region, the acronym "BSc" refers to brute-force search within a circular area, and the "Time" for each value of α is derived from the expected value of 50 independent runs. Figure 3 presents the comparison curves for different values of α. In this figure, the labels "8 digits," "9 digits," "10 digits," and "11 digits" correspond to the number of decimal digits in the integer N as listed in Tables 7 and 8, respectively. It is evident that increasing the value of α leads to an increase in the time required to find a nontrivial divisor of a bigger N, which is consistent with our analytical conclusions.

Table 7. Experiment with LF+SRWc Algorithm

Time \ α / Integer N	0.25	0.3	0.5	0.75	1.0	1.25	1.5	1.75	1.99
10909343	0.1182	0.1100	0.1006	0.1356	0.2943	0.2743	0.3051	0.3457	0.4382
392913607	0.4486	0.6461	0.5908	0.5948	0.7179	1.1923	1.4927	1.8700	1.5307
5325280633	2.9072	2.1485	2.7924	3.1753	3.7732	5.0413	9.4267	12.4941	12.7281
42336478013	2.1236	4.7007	8.1895	12.1528	15.7685	21.3863	32.2748	43.6482	54.1294

Table 8. Experiment with LF+BSc Algorithm

Time \ α / Integer N	0.25	0.3	0.5	0.75	1.0	1.25	1.5	1.75	1.99
10909343	0.2053	0.2172	0.2480	0.2513	0.2873	0.2893	0.3040	0.3464	0.3547
392913607	0.3488	0.3585	0.3750	0.3924	0.4050	0.4148	0.4643	0.4611	0.4786
5325280633	2.0949	2.7346	2.8916	3.0291	4.0847	5.4082	7.8837	10.6847	13.3391
42336478013	2.1891	2.7117	4.3389	9.3310	14.1714	17.2637	25.1580	36.2019	48.2158

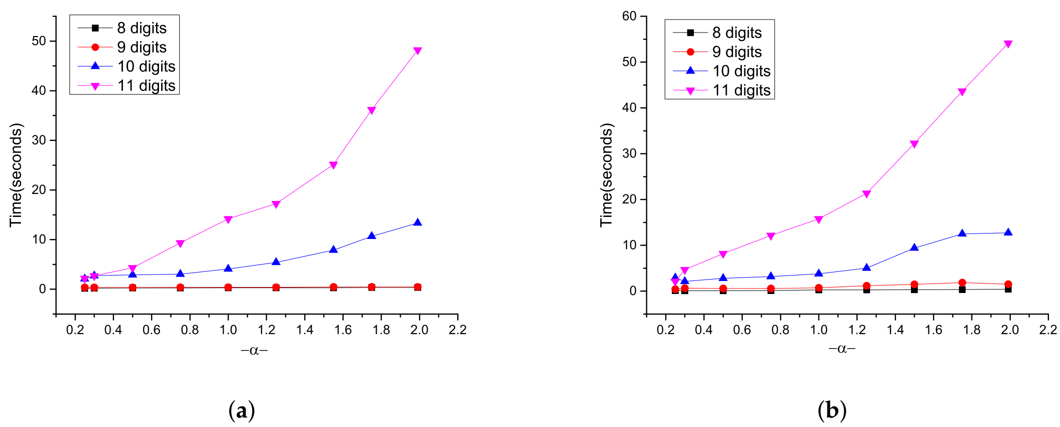

Figure 3. Comparison of varying α: (a) for LF+SRWc, (b) for LF+BSc.

5. Conclusion and Future Work

Despite its widespread application, the LF model suffers from considerable inconsistencies in its definition, parameter interpretation, and computational methodologies across the existing literature, compromising the reliability and reproducibility of empirical results. We thus propose a unified framework that standardizes the model's definition, clarifies the meaning and scope of its parameters, and establishes a consistent computational paradigm. Based on a comprehensive comparative analysis

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of relevant studies and grounded in rigorous mathematical foundations, the proposed framework can ensure logical coherence and avoid ambiguity in applying the LF model, thereby enhancing the consistency, reliability, and cross-study comparability in scientific computing.

Given the significance of the Lévy index, we conduct a preliminary investigation into its influence on Lévy flight (LF) patterns. Through experimental analysis, a governing principle is identified that characterizes how variations in the Lévy index affect both the step length distribution and the spatial extent of the explored domain in LF processes. These findings offer valuable insights into optimizing Lévy flight-based search strategies by selecting an appropriate Lévy index.

Nevertheless, several issues remain to be addressed in future research. For instance, the precise mechanism by which the Lévy index influences the LF behaviors has not yet been fully elucidated, and an accurate methodology for selecting an appropriate Lévy index tailored to specific engineering requirements is still lacking. These challenges constitute important directions for our ongoing and future work. We hope that more young scholars will engage in this field of study.

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Data Availability Statement: Matlab source codes for the numerical experiments of this paper can be downloaded at https://drive.mathworks.com/sharing/f950d972-6d9e-4026-b5a9-a7ff23255c19.

Conflicts of Interest: The authors declare no conflicts of interest.

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