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A No-Go Theorem for a Strict dS to SM Boundary Duality in Four Dimensions

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Abstract

I formulate a precise obstruction to identifying the four-dimensional Standard Model with the holographic dual of a four-dimensional de Sitter (dS_4) universe. Under mild and physically motivated assumptions showing no such duality exists. This result delineates a sharp sense in which a dS/SM boundary correspondence is excluded in our universe, while leaving consistent non-boundary frameworks open.

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1. Introduction

Observations we have made are consistent with a late-time de Sitter-like universe with positive cosmological constant $\Lambda > 0$ [1–3]. The SM is a local, unitary, chiral gauge theory in 3+1 dimensions. It is natural to ask whether there exists a holographic duality that identifies dS gravity with the SM in the same way AdS/CFT identifies AdS gravity with a unitary boundary CFT [4–6].

2. Set-Up and Assumptions

I aim to address the specific conjecture:

There exists a holographic duality for which the asymptotically dS_4 *bulk is dual to a local 3D Euclidean QFT B such that B is fully equivalent to the 3+1D Standard Model, a "dS/SM".*

By "B is equivalent to the 3+1D Standard Model" I mean that there exists a dictionary mapping a distinguished sector of B to SM gauge-invariant operators such that correlation functions and Ward identities match after any required analytic continuation, including the global symmetry and anomaly matching data. If one further demands an identification with the Minkowski SM, one must also reproduce SM scattering observables I treat this as an optional strengthening, not as a baseline assumption.

The idea is that there might be a deep relationship between a universe with de Sitter geometry that includes gravity, and the Standard Model without gravity living on the boundary of that universe. In this relationship, everything that happens with gravity in the curved space is exactly equivalent to what happens in the Standard Model on its boundary, so you could describe all the physics in either language. This is saying physics in curved space with gravity, the bulk can be described by the physics of the Standard Model without gravity, living on the boundary, this is the duality.

In boundary holography of the usual type, a $(d+1)$-dimensional bulk is related to a d-dimensional boundary QFT. For dS_4, the late-time screens are $I^{\pm} \simeq S^3$, so any boundary theory extracted from the bulk wavefunctional is necessarily a 3D Euclidean QFT. The phrase "dS/SM" in this paper refers only to a strict identification claim, that this 3D boundary theory could nevertheless reproduce the full observable content of the 3+1D Standard Model. The goal is to show this strict claim fails under standard locality/positivity assumptions.

I show this conjecture is false under mild assumptions spelled out below. I consider a semiclassical dS_4 bulk with future or past null screens $\mathcal{I}^{\pm} \simeq S^3$ and the associated late-time wavefunctional

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$\Psi_{\rm dS}[h_{ij}, \{\mathcal{J}\}]$ [7,8]. The boundary theory $\mathcal{B}$, if it exists is defined by $W_{\rm dS} \equiv -i \ln \Psi_{\rm dS}$, with correlators obtained by functional differentiation with respect to sources $\{\mathcal{J}\}$.

(A1) **Asymptotically dS_4 bulk.** The bulk is semiclassical and asymptotically de Sitter, with late-time screens $I^{\pm}$ that are three-manifolds topologically S^3.

(A2) **Local 3D boundary QFT.** The putative boundary dual B is a local Euclidean QFT on $I^{\pm}$ satisfying reflection positivity, Osterwalder–Schrader positivity.

(A3) **Local operator map.** Normalizable finite-energy bulk perturbations map to insertions of local operators on $I^{\pm}$ no ad hoc nonlocal/double-trace prescriptions intended to change effective dimensionality.

(A4) **Reductio hypothesis (strict SM-equivalence).** For contradiction, assume that B is "SM-equivalent" in the strong sense above, it reproduces the SM's operator/Ward-identity data, including gauge group $SU(3) \times SU(2) \times U(1)$ and anomaly matching information.

(A5) **No spacetime–internal mixing in 3+1D scattering.** When discussing 3+1D Minkowski scattering observables if demanded, assume standard S-matrix premises, the Coleman–Mandula context.

3. Main No-Go Theorem

The main theorem I want to address is:

Theorem 1 (No strict dS_4/SM boundary identification). Under the assumptions above there is no holographic duality in which an asymptotically dS_4 bulk admits a local 3D boundary QFT B on $I^{\pm} \simeq S^3$ that is strictly SM-equivalent in the sense above to the 3+1D Standard Model.

Theorem 1 is not a claim that 3D and 4D local QFTs could be equivalent; rather, it formalizes the obstruction to the proposed strict identification sometimes denoted informally as "dS/SM".

The first of the problems is the dimensional obstruction as our first assumption implies the boundary is 3D, so any local $\mathcal{B}$ is a three-dimensional Euclidean QFT. A 3D local QFT cannot be isomorphic to a 3+1D local QFT with the same spectrum and operator product algebra; their engineering dimensions, renormalization group flows, and anomaly structures differ. In d Euclidean dimensions, a scalar primary $\mathcal{O}$ of scaling dimension Δ obeys:

$$\langle \mathcal{O}(x)\, \mathcal{O}(0) \rangle \;=\; \frac{C_{\mathcal{O}}}{|x|^{2\Delta}}, \tag{1}$$

while conserved currents and the stress tensor satisfy the unitarity or shortening bounds [11,12]:

$$\Delta[J] = d-1, \qquad \Delta[T] = d. \tag{2}$$

Thus for the same conserved operators one has:

$$\Delta^{(3D)}[J] = 2 \;\neq\; 3 = \Delta^{(4D)}[J], \tag{3}$$
$$\Delta^{(3D)}[T] = 3 \;\neq\; 4 = \Delta^{(4D)}[T]. \tag{4}$$

Since the OPE:

$$\mathcal{O}_i(x)\, \mathcal{O}_j(0) \;\sim\; \sum_k C_{ij}{}^{k}\, |x|^{\Delta_k - \Delta_i - \Delta_j}\, \mathcal{O}_k(0) + \cdots \tag{5}$$

is controlled by these dimensions and structure constants, the short-distance singularities in 3D and 4D are inequivalent even before dynamics is specified. No algebra isomorphism can map the 3D OPE data to the 4D SM OPE data while preserving locality. Canonical field dimensions in d spacetime dimensions are given by:

$$[\phi] = \frac{d-2}{2}, \qquad [\psi] = \frac{d-1}{2}, \qquad [A_\mu] = \frac{d-2}{2}. \tag{6}$$

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For gauge, Yukawa, and quartic interactions:

$$\mathcal{L}_{\rm YM} = \frac{1}{4g_d^2} F^a_{\mu\nu} F^{a\mu\nu}, \qquad [g_d^2] = 4-d, \tag{7}$$

$$\mathcal{L}_{\rm Yuk} = y_d\, \bar\psi\, \psi\, \phi, \qquad [y_d] = \frac{4-d}{2}, \tag{8}$$

$$\mathcal{L}_{\phi^4} = \frac{\lambda_d}{4!}\, \phi^4, \qquad [\lambda_d] = 4-d. \tag{9}$$

Hence:

$$\textbf{in 3D:} \quad [g_3^2] = 1,\ [y_3] = \tfrac{1}{2},\ [\lambda_3] = 1 \tag{10}$$

vs

$$\textbf{in 4D:} \quad [g_4^2] = 0,\ [y_4] = 0,\ [\lambda_4] = 0. \tag{11}$$

Couplings that are marginal or dimensionless in 4D are relevant in 3D. This already forbids an isomorphism of RG data. I will introduce dimensionless couplings at scale μ by:

$$\hat g^2(\mu) \equiv g_d^2\, \mu^{d-4}, \quad \hat y(\mu) \equiv y_d\, \mu^{(d-4)/2}, \quad \hat\lambda(\mu) \equiv \lambda_d\, \mu^{d-4}. \tag{12}$$

Their beta functions universally contain the engineering-dimension terms:

$$ta_{\hat g^2} \equiv \mu \frac{d\hat g^2}{d\mu} = (d-4)\,\hat g^2 - \frac{b_0}{8\pi^2}\,\hat g^4 + \mathcal{O}(\hat g^6), \tag{13}$$

$$ta_{\hat y} = \frac{d-4}{2}\,\hat y + \frac{1}{16\pi^2}\Big(a\,\hat y^3 - b\,\hat g^2 \hat y + \cdots\Big), \tag{14}$$

$$ta_{\hat\lambda} = (d-4)\,\hat\lambda + \frac{1}{16\pi^2}\Big(c\,\hat\lambda^2 + d\,\hat y^4 - e\,\hat g^2\hat\lambda + \cdots\Big), \tag{15}$$

with $b_0 = \frac{11}{3}C_2(G) - \frac{4}{3}T(R_f)n_f - \frac{1}{6}T(R_s)n_s$ in 4D and group-theory constants $a, b, c, d, e > 0$ scheme-dependently. Evaluating (13) in $d = 3$ and $d = 4$:

$$d = 3:\ \ ta_{\hat g^2} = -\,\hat g^2 - \frac{b_0}{8\pi^2}\,\hat g^4 + \cdots \tag{16}$$

vs

$$d = 4:\ \ ta_{g^2} = -\,\frac{b_0}{8\pi^2}\,g^4 + \cdots. \tag{17}$$

In 3D the linear term drives $\hat g^2$ to strong coupling in the IR, super-renormalizable, whereas in 4D the flow is logarithmic, marginal, asymptotically free for $b_0 > 0$. No change of variables can turn a linear term into a purely cubic one while preserving locality and the operator algebra and thus the RG categories are inequivalent. The 3+1D SM is chiral with anomaly cancellation encoded by the 6-form anomaly polynomial:

$$\mathcal{I}_6 = \frac{1}{6}\,\mathrm{tr}(F^3) - \frac{1}{24}\,\mathrm{tr}(F)\,\mathrm{tr}(R^2) + \cdots, \tag{18}$$

yielding the familiar ABJ relation:

$$\partial_\mu J_5^\mu = \frac{g^2}{16\pi^2}\,\mathrm{tr}\,F_{\mu\nu}\,\tilde F^{\mu\nu}, \qquad \tilde F^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{19}$$

In 3D there is no chirality and gauge anomalies are replaced by a parity anomaly with a Chern–Simons contact term [22–24]:

$$S_{\rm CS}[A] = \frac{k}{4\pi}\int \mathrm{tr}\Big(A\wedge dA + \tfrac{2}{3}A\wedge A\wedge A\Big), \tag{20}$$

whose quantized level k shifts under integrating out massive fermions. These topological responses have no counterpart in 4D SM anomaly cancellation data. Therefore Ward identities and 't Hooft anomalies cannot match across 3D and 4D. So, because protected operator dimensions differ, engineering dimensions force relevant 3D couplings versus marginal 4D couplings with distinct beta-function structure, and anomaly content in 3D is incompatible with 4D chiral anomaly cancellation, no local 3D Euclidean QFT on $\mathcal{I}^{\pm}$ can be isomorphic to the 3+1D SM as an operator algebra with matching RG and anomaly data. This is the dimensional obstruction claimed in the main theorem.

The second problem refers to reflection-positivity or the unitarity obstruction as the dS wavefunctional satisfies $W_{\mathrm{dS}} = i^{d-1} W_{\mathrm{AdS}} + (\mathrm{local})$ under $L \to i/H$ continuation [28]. For d=3, the overall phase generically violates reflection positivity for a subset of operators. Thus the boundary captured by Ψ_{dS} is not a unitary 3D Euclidean QFT with all the properties demanded by our second assumption and certainly not a 3+1D unitary SM as in assumption four. Under the continuation of the AdS radius $L \to i/H$ with sources held fixed, the nonlocal part of the on-shell generator obeys [8,10]:

$$W_{\mathrm{dS}}^{\mathrm{nloc}}[J] \;=\; i^{\,d-1}\, W_{\mathrm{AdS}}^{\mathrm{nloc}}[J] \;+\; (\text{local counterterms}). \tag{21}$$

I want to emphasize that AdS enters only as a calculational reference through the standard Euclidean AdS $\to$ dS analytic continuation $L \to i/H$ but the boundary object of interest is still defined by the de Sitter wavefunctional $W_{dS} \equiv -i \ln \Psi_{dS}$.

Local terms are contact as they do not affect long-distance correlators nor Osterwalder–Schrader (OS) positivity [14,15]. To show this let θ be Euclidean time reflection $(\tau, \mathbf{x}) \mapsto (-\tau, \mathbf{x})$ composed with Hermitian conjugation on operators. For any polynomial functional $F[\mathcal{O}]$ supported in the half-space $\tau > 0$, unitarity of a Euclidean d-dimensional QFT demands:

$$\langle\, (\theta F)\, F \,\rangle \;\geq 0. \tag{22}$$

For a Gaussian sector generated by:
$W^{(2)}[J] = \frac{1}{2} \int d^d x\, d^d y\, J(x)\, K(x-y)\, J(y)$,
(22) is equivalent to positivity of the quadratic form:

$$\int_{\tau_x > 0,\ \tau_y > 0} d^d x\, d^d y\; f^*(\theta x)\, K(x-y)\, f(y) \;\geq 0, \tag{23}$$

for all f. The equation (21) implies at quadratic order:

$$K_{\mathrm{dS}}(x-y) \;=\; i^{\,d-1}\, K_{\mathrm{AdS}}(x-y) \;+\; (\text{local}). \tag{24}$$

When the AdS dual is unitary, K_{AdS} is OS-positive, for $d = 3$: $i^{\,d-1} = i^2 = -1$, so the nonlocal kernel flips sign:

$$K_{\mathrm{dS}}^{\mathrm{nloc}} \;=\; -\,K_{\mathrm{AdS}}^{\mathrm{nloc}}. \tag{25}$$

Because local counterterms cannot modify the nonlocal, reflection-sensitive part of the quadratic form, there exist test functions f with support in $\tau > 0$ for which the integral in (23) becomes strictly negative. Thus OS positivity fails for a subset of operators, and the boundary extracted from Ψ_{dS} is not a unitary 3D Euclidean QFT. So because $i^{\,d-1} = -1$ in $d = 3$, the AdS$\to$dS continuation reverses the sign of the nonlocal parts of correlators. This violates OS reflection positivity for a subset of operators, including conserved sectors, so the late-time boundary extracted from Ψ_{dS} is not a unitary 3D Euclidean QFT, and a fortiori cannot realize a unitary 3+1D Standard Model as assumed in our fourth hypothesis.

The third problem is the S-matrix obstruction as in dS_4 there are no global timelike asymptotic regions, hence no conventional LSZ S-matrix. A duality to the SM S-matrix cannot hold [13]. One may define **in-in** correlators, but these do not furnish a unitary 3+1D scattering theory. If one interprets "dS/SM" as claiming equivalence to the Minkowski Standard Model—including its LSZ scattering matrix—then an additional obstruction follows immediately, independent of the purely 3D-vs-4D

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operator-algebra mismatch discussed. In a globally hyperbolic, asymptotically Minkowski spacetime, the LSZ construction defines:

$$S_{t\alpha} = \& \langle t\alpha, \mathrm{out} \,|\, \alpha, \mathrm{in} \rangle, \tag{26}$$

$$\mathcal{A}_{n\to m} = \prod_{a=1}^{n+m}\left[\lim_{p_a^2\to m_a^2}(p_a^2 - m_a^2)\right] \times \int \prod_a d^4x_a\, e^{ip_a\cdot x_a}\, \langle 0|T\{\phi(x_1)\cdots\phi(x_{n+m})\}|0\rangle, \tag{27}$$

where fields solve $(\Box + m^2)\phi = 0$ asymptotically and approach free plane waves $\sim e^{-ip\cdot x}$ in timelike regions $t \to \pm\infty$. This requires a global timelike asymptotic regions, a unique Poincaré vacuum, well-defined one-particle mass shells the oscillatory late-time modes, and Fock spaces $\mathcal{H}_{\mathrm{in/out}}$ built from those modes. The global dS_4 has spacelike $\mathcal{I}^{\pm} \simeq S^3$ and no timelike infinity [17–19]. In flat slicing:

$$ds^2 = a(\eta)^2(-d\eta^2 + d\mathbf{x}^2), \tag{28}$$

$$a(\eta) = \frac{-1}{H\eta}, \qquad \eta \in (-\infty, 0^-), \tag{29}$$

the future boundary is at $\eta \to 0^-$, spacelike. There is no global timelike Killing vector so the notion of particle is observer or patch dependent, the cosmological horizon. Hence, $\mathcal{H}_{\mathrm{in}}$ and $\mathcal{H}_{\mathrm{out}}$ cannot be defined as global free Fock spaces, and (27) lacks meaning. There is no oscillatory late-time modes, so s failure of on-shell limits as for a scalar ϕ with mass m and curvature coupling ξ, the mode functions in dS_4 satisfy:

$$\left[\partial_\eta^2 + k^2 - \frac{2}{\eta^2}\right]u_k(\eta) = \left(m^2 + \xi R\right)a(\eta)^2 u_k(\eta), \tag{30}$$

where $R = 12H^2$. The Bunch–Davies solution is:

$$u_k(\eta) = \frac{\sqrt{\pi}}{2}\, e^{i(\nu+\frac{1}{2})\pi/2}\, (-\eta)^{3/2} H_\nu^{(1)}(k|\eta|), \tag{31}$$

$$\nu = \sqrt{\tfrac{9}{4} - \frac{m^2 + 12\xi H^2}{H^2}}. \tag{32}$$

As $\eta \to 0^-$ in late times, $u_k(\eta) \sim (-\eta)^{3/2-\nu}$ is non-oscillatory, a freezes out rather than e^{-iEt}. Thus there is no mass-shell pole isolation via the LSZ amputation as the amputated correlators do not project onto scattering amplitudes. Even when one chooses a preferred early-time vacuum, the late-time creation or annihilation operators mix:

$$a_{\mathbf{k}}^{\mathrm{out}} = \alpha_k\, a_{\mathbf{k}}^{\mathrm{in}} + t\alpha_k\, a_{-\mathbf{k}}^{\mathrm{in}\,\dagger}, \qquad |t\alpha_k|^2 \neq 0, \tag{33}$$

reflecting gravitational production and the absence of a global, time-independent notion of particle. A single global unitary S relating $|\alpha, \mathrm{in}\rangle$ to $|t\alpha, \mathrm{out}\rangle$ therefore does not exist. The **in-in** formalism gives cosmological correlators are computed by the Schwinger–Keldysh path integral:

$$Z[J_+, J_-] = \int \mathcal{D}\phi_+ \mathcal{D}\phi_-\, \exp\Big\{ iS[\phi_+] - iS[\phi_-] + i\int (J_+\phi_+ - J_-\phi_-) \Big\}, \tag{34}$$

which yields expectation values $\langle \Omega | \mathcal{O}(\eta, \mathbf{x}) | \Omega \rangle$ at finite time. These objects satisfy a largest-time or causality structure and cutting rules adapted to cosmology, but they do not define asymptotic in or out states or a unitary 3+1D scattering matrix with $S^\dagger S = \mathbf{1}$. So because dS_4 lacks global timelike asymptotic regions, has no oscillatory late-time modes, and exhibits Bogoliubov mixing tied to horizons and expansion, the LSZ recipe (27) is inapplicable. One may compute **in-in** correlators, but these describe time-evolved expectation values—not a unitary 3+1D S-matrix. Therefore any proposed duality to the SM S-matrix cannot hold.

Another problem is because the dS_4 isometry group $SO(4,1)$ acts as the conformal group on the late–time screens $\mathcal{I}^{\pm} \simeq S^3$, any putative local, reflection–positive boundary theory B on $\mathcal{I}^{\pm}$ must realize conformal Ward identities on S^3 [16]. In odd boundary dimension $d = 3$ there is no local Weyl trace anomaly, so in flat space one has $T^i{}_i = 0$ up to improvement terms and contact terms on curved backgrounds. In particular, in flat space this requires:

$$\langle T^i{}_i \rangle_B \; = \; 0 \qquad (d = 3), \tag{35}$$

and correlation functions transform covariantly under $SO(4,1) = \mathrm{Conf}(S^3)$. By contrast, the 3+1 dimensional Standard Model is not conformal. Its stress–tensor trace is nonzero due to explicit masses and running couplings:

$$\begin{aligned} T^{\mu}{}_{\mu} \; &= \; \sum_a \frac{ta_{g_a}}{2g_a} F^a_{\mu\nu} F^{a\,\mu\nu} \\ &+ \; (1+\gamma_m)\, m\, \bar{\psi}\psi \; + \; ta_{\lambda}\, |H|^4, \end{aligned} \tag{36}$$

plus some curved–space trace–anomaly terms, and electroweak symmetry breaking introduces a physical scale $v \simeq 246$ GeV. Therefore no algebra isomorphism can map a conformal 3D Euclidean QFT B with $T^i{}_i = 0$ to the non-conformal 3+1D SM with $T^{\mu}{}_{\mu} \neq 0$ while preserving locality and Ward identities. One might try to evade this by relevant deformations of a 3D CFT or by introducing a dilaton to nonlinearly realize scale invariance, but then B is not conformal and the $SO(4,1)$ covariance implied by the dS isometries is lost. Hence, independently of the dimensional and S-matrix obstructions, the conformal–structure mismatch rules out a strict dS/SM boundary identification in 4D.

The last problem I will note on is the chirality or anomaly obstruction due to the fact that 3+1D SM possesses chiral fermions and anomaly cancellation conditions. In 3D, chirality is absent and gauge anomalies are replaced by Chern–Simons contact terms so there is no isomorphism preserving Ward identities and 't Hooft anomaly data across dimensions [20,21]. There is no chirality in 3D, as chirality is defined by:

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad P_{L,R} = \tfrac{1}{2}(1 \mp \gamma_5), \tag{37}$$

allowing genuinely chiral couplings $J^{\mu}_a = \bar{\psi}\gamma^{\mu} P_L T_a \psi$ as in the SM. In 3D Euclidean signature, the Clifford algebra admits a $2{\times}2$ representation with $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$, and the product $i\gamma^1\gamma^2\gamma^3$ is proportional to the identity, so there is no independent matrix that anticommutes with all γ^i. Hence no projectors $P_{L,R}$ exist, no Weyl decomposition is available, and a chiral gauge theory cannot be formulated as a local 3D QFT. In four dimensions, gauge and mixed anomalies are captured by a 6-form anomaly polynomial:

$$\mathcal{I}_6 \; = \; \frac{1}{6}\,\mathrm{tr}\big(F^3\big) - \frac{1}{24}\,\mathrm{tr}(F)\,\mathrm{tr}\big(R^2\big) + \cdots, \tag{38}$$

whose descent encodes the non-invariance of the generating functional [8,9]. Equivalently, the axial anomaly reads:

$$\partial_{\mu} J^{\mu}_5 \; = \; \frac{g^2}{16\pi^2}\,\mathrm{tr}\big(F_{\mu\nu}\widetilde{F}^{\mu\nu}\big) \; + \; \frac{1}{384\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\big(R_{\mu\nu}R_{\rho\sigma}\big), \tag{39}$$

$$\widetilde{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \tag{40}$$

For the SM, anomaly cancellation imposes the well-known constraints [25], per generation of left-handed Weyl fermions:

$$[SU(3)_c]^2U(1)_Y: \quad \sum_{\text{colored } f} Y_f\, T(R_f) = 0, \tag{41}$$

$$[SU(2)_L]^2U(1)_Y: \quad \sum_{\text{doublets } f} Y_f\, T(\mathbf{2}) = 0, \tag{42}$$

$$U(1)_Y^3: \quad \sum_f Y_f^3 = 0, \tag{43}$$

$$\text{grav}^2\, U(1)_Y: \quad \sum_f Y_f = 0,$$

$$\text{Witten anomaly:} \quad N_{SU(2)\text{ doublets}} \text{ is even} \tag{44}$$

$$(\pi_4(SU(2)) = \mathbb{Z}_2),$$

with $T(\mathbf{2}) = \frac{1}{2}$ and $T(\mathbf{3}) = \frac{1}{2}$. These relations crucially use chirality, such as left or right charges differ. In three dimensions there is no chirality and no ABJ triangle. Instead, integrating out a massive Dirac fermion in representation R produces a parity-odd contact term:

$$\Delta S_{\text{eff}}[A] = \frac{\Delta k}{4\pi} \int \text{tr}\Big(A \wedge dA + \tfrac{2}{3} A \wedge A \wedge A\Big), \tag{45}$$

$$\Delta k = \tfrac{1}{2}\, \text{sgn}(m)\, C_2(R), \tag{46}$$

shifting the Chern–Simons level k by a half-integer in the absence of a regulator that preserves both parity and gauge invariance. Gauge invariance under large transformations quantizes $k \in \mathbb{Z}$; a single massless fermion thus exhibits the parity anomaly. At the level of currents, this appears as a parity-odd contact in the two-point function:

$$\langle J_i(x) J_j(0) \rangle_{\text{odd}} = \frac{k_{\text{eff}}}{2\pi}\, \varepsilon_{ij\ell}\, \partial_\ell \delta^{(3)}(x), \tag{47}$$

$$k_{\text{eff}} = k + \sum_f \tfrac{1}{2}\, \text{sgn}(m_f)\, C_2(R_f), \tag{48}$$

while $\partial_i J_i = 0$ remains a local Ward identity, so no chiral non-conservation exists to mimic 4D. Inflow organizes 't Hooft anomalies as boundaries of higher-dimensional invertible actions:

$$d = 4: \quad dI_5 = \mathcal{I}_6 \in H^6(BG) \tag{49}$$

$$\text{(or appropriate spin-cobordism class)},$$

$$d = 3: \quad dI_3 = \mathcal{I}_4 \in H^4(BG). \tag{50}$$

Four-dimensional chiral anomalies are classified by degree-6 data or 5D Chern–Simons inflow, while three-dimensional parity anomalies are classified by degree-4 data or 4D θ-like inflow. These belong to different cohomology or cobordism groups and cannot be related by any local isomorphism that preserves anomaly matching and Ward identities. Any isomorphism mapping a 3D boundary QFT to the 3+1D SM must preserve the chiral representation content and its anomaly-cancellation equations, reproduce the ABJ and mixed-gravitational Ward identities, and match global anomalies. But 3D has no chirality at all and replaces gauge anomalies with Chern–Simons contact terms determined by k_{eff} in (46). Therefore no local map can preserve both the operator algebra and the 't Hooft anomaly data across dimensions. This is the chirality or anomaly obstruction.

These contradictions establish the no-go theorem I have presented. There is no 4D boundary without extra dimensions as if one insists that the dual of a gravitational bulk be a four-dimensional unitary QFT giving the actual SM, then the bulk must be at least 5D so that ∂(bulk) has dimension 4. In particular, any literal dS/SM boundary duality compatible with our assumptions requires extra dimensions such as branes or double-holography. Our 4D universe alone cannot realize this. There is as well no curvature–SM algebra identification as any attempt to identify spacetime curvature generators with SM internal generators violates the Coleman–Mandula no-go theorem in an interacting 3+1D QFT [26,27]. Hence even aside from our assumptions, there is no algebraic route to say curvature equals the SM in 4D.

4. 2+1-Dimensional Gauge-Theory Target

The no-go theorem established above excludes a strict identification in which an asymptotically dS_4 bulk admits a local 3D Euclidean boundary QFT $\mathcal{B}$ on $\mathcal{I}^{\pm} \simeq S^3$ that is fully equivalent, as an operator algebra with matching Ward identities and anomaly data, to the 3+1-dimensional Standard Model. This strict dS/SM claim is formulated under the hypotheses (A1)–(A4), in particular the assumption that the boundary theory is a local Euclidean QFT satisfying Osterwalder–Schrader (OS) reflection positivity. For dS_4 the late-time screens are necessarily three-manifolds, so any putative boundary theory extracted from the late-time wavefunctional is three-dimensional and Euclidean.

It is therefore natural to ask whether the conclusion depends essentially on the fact that the target theory is 3+1 dimensional, and whether one could instead posit a weaker conjecture in which the boundary is equivalent to some unitary 2+1-dimensional SM-like gauge theory, such as a Lorentzian 2+1 dimensional theory whose Euclidean continuation defines a reflection-positive local 3D Euclidean QFT on $\mathcal{I}^{\pm}$. In this dimensionally matched variant, the purely dimensional obstruction used in Theorem 1 is no longer the relevant contradiction, and one also removes the specifically four-dimensional chirality or anomaly matching requirement that is built into hypothesis (A4) as stated.

However, the unitarity obstruction arising from OS positivity remains decisive, because it is intrinsic to the AdS→dS continuation of the late-time generating functional in three boundary dimensions. Define the source-dependent generator by:

$$W_{dS}[J] \equiv -i \ln \Psi_{dS}[J], \tag{51}$$

with boundary correlators obtained by functional differentiation with respect to sources J. Under analytic continuation of the AdS radius $L \to i/H$ with sources held fixed, the nonlocal part of the on-shell generator obeys the phase relation:

$$W_{dS}^{\text{nloc}}[J] = i^{d-1} W_{AdS}^{\text{nloc}}[J] + (\text{local counterterms}), \tag{52}$$

where local denotes contact terms that do not affect long-distance correlators nor the reflection-sensitive nonlocal kernel.

OS reflection positivity in a d-dimensional Euclidean QFT can be stated as follows. Let θ denote Euclidean time reflection $(\tau, \mathbf{x}) \mapsto (-\tau, \mathbf{x})$ composed with Hermitian conjugation on operators. Then for any polynomial functional $F[\mathcal{O}]$ supported in the half-space $\tau > 0$, unitarity requires:

$$\langle (\theta F)\, F \rangle \geq 0. \tag{53}$$

In a Gaussian sector generated by a quadratic functional:

$$W^{(2)}[J] = \frac{1}{2} \int d^d x\, d^d y\; J(x)\, K(x-y)\, J(y), \tag{54}$$

this is equivalent to positivity of the quadratic form:

$$\int_{\tau_x > 0,\, \tau_y > 0} d^d x\, d^d y\; f^*(\theta x)\, K(x-y)\, f(y) \geq 0 \tag{55}$$

for all test functions f supported in $\tau > 0$.

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Equation (52) implies at quadratic order that the nonlocal kernel satisfies:

$$K_{dS}(x-y) = i^{d-1} K_{AdS}(x-y) + (\text{local}). \quad (56)$$

In $d = 3$ one has $i^{d-1} = i^2 = -1$, so the nonlocal part flips sign:

$$K_{dS}^{\text{nloc}}(x-y) = -K_{AdS}^{\text{nloc}}(x-y), \quad (57)$$

while local counterterms cannot modify the nonlocal contribution that controls reflection positivity. Therefore, even when the AdS dual kernel is OS-positive, there exist test functions f for which the reflection form becomes strictly negative after continuation, and OS positivity fails for a subset of operators. It follows that the late-time boundary object extracted from Ψ_{dS} is generically not a unitary 3D Euclidean QFT satisfying (A2).

Consequently, replacing the 3+1-dimensional Standard Model target by a unitary 2+1-dimensional SM-like gauge theory does not rescue a strict boundary duality. Under assumptions (A1)–(A3) and the reflection-positivity requirement in (A2), there is no strict holographic identification of an asymptotically dS_4 bulk with a unitary local 2+1-dimensional gauge theory on $\mathcal{I}^{\pm}$. Any proposal of this type must either abandon OS positivity or unitarity at the level of the boundary, or weaken locality/dictionary assumptions so that the boundary is not an ordinary unitary 2+1-dimensional QFT in the usual sense.

5. Kaluza–Klein 5D bulks and 3+1D Standard-Model Targets

The no-go theorem proven above targets a strict $(D \leftrightarrow D-1)$ boundary identification in a purely four-dimensional asymptotically de Sitter universe. A natural objection is that the dimensional mismatch responsible for the strict dS_4/SM failure is an artifact of working in four bulk dimensions, since a genuine boundary dual of a gravitational bulk in $(d+1)$ dimensions is expected to live in d dimensions. If one insists that the dual theory be the actual $3+1$-dimensional Standard Model, then one must at minimum enlarge the bulk so that the boundary is four-dimensional. This motivates considering a five-dimensional bulk, for example a Kaluza–Klein (KK) spacetime with one compact direction, in which the effective macroscopic geometry is de Sitter-like and the would-be holographic screen is a four-manifold.

Concretely, let $\mathcal{M}_5$ be a semiclassical bulk which is asymptotically de Sitter in five dimensions, or more generally a KK bulk whose late-time screen Σ_4 is a four-dimensional Euclidean manifold, for instance $\Sigma_4 \simeq S^4$, or $\Sigma_4 \simeq S^3 \times S^1$ when the KK circle survives on the screen. Suppose one attempts to formulate a strict holographic duality in which the late-time wavefunctional:

$$\Psi_{dS_5}[\gamma_{ij}, \{J\}] \quad (58)$$

defines a local Euclidean $d = 4$ boundary QFT B_4 on Σ_4, via the source-dependent generator:

$$W_{dS_5}[J] \equiv -i \ln \Psi_{dS_5}[J], \quad (59)$$

with correlation functions obtained by functional differentiation with respect to the boundary sources J. In this five-dimensional setting the purely dimensional obstruction that rules out a strict dS_4/SM identification is no longer automatic as it is now kinematically possible for a local boundary theory to be four-dimensional and hence to accommodate a $3+1$-dimensional Lorentzian QFT after Wick rotation, provided the Euclidean theory is reflection positive.

However, the reflection-positivity, unitarity is the obstruction that arises from the analytic continuation structure of de Sitter holography persists, and in fact becomes sharper when the boundary dimension is even. The relevant point is that the late-time generating functional in de Sitter is related,

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at the level of nonlocal data, to the corresponding AdS functional by a universal phase. Abstractly, for boundary dimension d one has a relation of the form:

$$W_{dS}^{\text{nloc}}[J] = i^{d-1}\, W_{AdS}^{\text{nloc}}[J] + (\text{local counterterms}), \tag{60}$$

where local denotes contact terms that do not affect the nonlocal kernel controlling reflection positivity. To test unitarity in the Euclidean boundary theory, one imposes the OS reflection-positivity condition. Let θ denote Euclidean time reflection $(\tau, \mathbf{x}) \mapsto (-\tau, \mathbf{x})$ composed with Hermitian conjugation. Then for any polynomial functional $F[O]$ supported in $\tau > 0$, reflection positivity requires:

$$\langle (\theta F)\, F \rangle \geq 0. \tag{61}$$

In a Gaussian sector generated by a quadratic functional:

$$W^{(2)}[J] = \frac{1}{2} \int d^d x\, d^d y\; J(x)\, K(x-y)\, J(y), \tag{62}$$

this is equivalent to positivity of the reflection form:

$$\int_{\tau_x > 0, \tau_y > 0} d^d x\, d^d y\; f^*(\theta x)\, K(x-y)\, f(y) \geq 0 \qquad \forall \quad f. \tag{63}$$

The phase relation (60) implies at quadratic order that the de Sitter kernel is related to the AdS kernel by:

$$K_{dS}(x-y) = i^{d-1} K_{AdS}(x-y) + (\text{local}). \tag{64}$$

Now take $d = 4$, which is the boundary dimension relevant to a five-dimensional bulk. Then:

$$i^{d-1} = i^3 = -\,i, \tag{65}$$

so the nonlocal kernel undergoes a purely imaginary phase rotation:

$$K_{dS}^{\text{nloc}}(x-y) = -\,i\, K_{AdS}^{\text{nloc}}(x-y). \tag{66}$$

If the AdS dual kernel is OS-positive, then K_{AdS} defines a positive reflection form in (63). Multiplication by $-i$ cannot preserve positivity: the reflected quadratic form becomes generically purely imaginary for nonzero test functions supported in $\tau > 0$, and hence cannot satisfy the real inequality (61). Local counterterms cannot remove this obstruction because they only contribute contact terms and do not modify the nonlocal kernel that governs reflection positivity. Therefore, under the same locality and OS-positivity hypotheses used in the four-dimensional argument, the late-time boundary object extracted from Ψ_{dS_5} is generically not a unitary Euclidean QFT in $d = 4$, and so it cannot furnish a strict, unitary SM_{3+1} boundary dual.

The conclusion is that passing to a KK or otherwise five-dimensional de Sitter-like bulk does remove the simple dimensional mismatch that forbids a $3+1$-dimensional target in a dS_4 boundary construction, but it does not by itself rescue a strict holographic identification with the Standard Model. A strict dS_5/SM boundary duality would require either a nonstandard relaxation of OS reflection positivity, or a weakening of the locality/dictionary assumptions so that the boundary object is not an ordinary unitary local QFT in four Euclidean dimensions.

This is distinct from higher-dimensional brane-world or double-holographic constructions, in which the Standard Model is realized as a genuine $3+1$-dimensional theory living on a dynamical four-dimensional brane inside a five-dimensional bulk. Such scenarios can be consistent because they do not assert that the Standard Model is the literal boundary dual living on the asymptotic late-time screen of a pure de Sitter universe, rather, they place the Standard Model on an internal four-dimensional locus while bulk curvature data may be encoded holographically in an auxiliary way.

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This type of mechanism lies outside the strict boundary identification targeted by the present no-go theorem.

6. Relation to Viable Alternatives

The theorem targets only a strict $D \leftrightarrow D-1$ boundary identification in 4D. It leaves open two consistent directions like double holography or brane-world this is because a 5D bulk with a 4D brane can accommodate a genuine 4D, unitary SM on the brane while bulk curvature is encoded holographically. This is not a dS/SM boundary duality of a pure 4D universe. A 4D$\rightarrow$ 4D holomorphic projection such as presented in HUFT says one may work on a complexified ambient 4D manifold with a direct-product holomorphic symmetry, selecting a dS real slice for gravity and the compact real form for the SM [29–41]. This produces GR+Λ and the SM on the same 4D slice without any spacetime–internal generator mixing. It is, however, not a boundary duality and thus consistent with the theorem.

7. Discussion and Outlook

The result formalizes an often implicit point that a literal dS/SM boundary correspondence in 4D is excluded by basic features of de Sitter holography and of the SM itself. The only robust paths to combine dS gravity with the SM are either higher-dimensional brane constructions or 4D internal projections such as holomorphic or ambient frameworks. Practically, this clarifies claims that when speaking of dS/SM, one must specify a non-boundary mechanism rather than a boundary dual of our 4D universe.

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