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De Sitter Cores from Non-Local Quantum Field Theories

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Abstract

Non-local quantum field theories achieve perturbative ultraviolet finiteness by inserting gauge- and diffeomorphism-covariant entire-function regulators into all kinetic terms. These operators can be viewed as nonlocal smearing maps acting on sources in the Einstein equations. In this paper I show that, when the same entire-function regulator responsible for UV-finite loops is applied to the energy–momentum tensor of a point mass, the resulting static, spherically symmetric solution develops a regular de Sitter core in place of a curvature singularity. I then interpret this core in our framework as a local positive-curvature saddle in the complexified cosmological-constant plane.

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1. Nonlocal Gravitational Field Equations

Classical General Relativity, when coupled to reasonable matter sources, generically predicts curvature singularities in gravitational collapse, signaling the breakdown of the effective theory at short distances [1–4]. In the holomorphic unified framework, ultraviolet completion is achieved by combining a complexified geometric underpinning, in which gravity and gauge structure arise as complementary real slices of a single Hermitian geometry, with a covariant, entire-function form factor that renders quantum loop integrals finite while preserving locality in the infrared [5–34]. In this section we collect the nonlocal field equations on the real slice and fix notation for the regulator and its covariant action on tensor fields. This formulation will then be specialized to static, spherically symmetric sources, where the form factor acts as a nonlocal smearing of the stress–energy tensor and allows us to infer the interior structure of the corresponding regular black-hole geometries [35–39].

On a spacetime manifold $M_{\mathbb{C}}$ with coordinates [10–17]:

$$z^{\mu} = x^{\mu} + iy^{\mu}, \qquad \mu = 0, 1, 2, 3, \tag{1}$$

We formulated in terms of a Hermitian metric:

$$g_{\mu\nu}(z, \bar{z}) = h_{\mu\nu}(x, y) + iB_{\mu\nu}(x, y), \tag{2}$$

where $h_{\mu\nu} = h_{\nu\mu}$ is the Lorentzian spacetime metric on the real slice $y^{\mu} = 0$, and $B_{\mu\nu} = -B_{\nu\mu}$ packages the Yang–Mills curvature as an antisymmetric two-tensor. On the real slice M the dynamics reduce to Einstein–Yang–Mills plus matter with:

$$G_{\mu\nu}(h) = \kappa\, T_{\mu\nu}, \qquad \kappa \equiv 8\pi G_{\mathrm{N}}, \tag{3}$$

where $G_{\mu\nu}(h)$ is the Einstein tensor built from $h_{\mu\nu}$, $T_{\mu\nu}$ is the total stress–energy tensor, and G_{N} is Newton's constant. To render all loop integrals finite, we augment the real-slice action by inserting a covariant entire-function regulator constructed from the Bochner d'Alembertian:

$$\Box \equiv -h^{\mu\nu} D_{\mu} D_{\nu}, \tag{4}$$

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where D_μ is the total covariant derivative, including both spin and gauge connections. We take as regulator an entire function:

$$F\left(\frac{\Box}{M_*^2}\right) = \exp\left(-\frac{\Box}{M_*^2}\right), \tag{5}$$

with M_* a characteristic nonlocal (UV) mass scale, and F chosen such that in Euclidean momentum space it produces exponential damping, $F(\Box) \sim \exp(-p_E^2/M_*^2)$ on plane waves.

In the gravitational sector, a convenient way to encode the regulator is to let it act on the stress–energy tensor before sourcing the Einstein equations. For a given local $T_{\mu\nu}$ we define the smeared tensor:

$$S_{\mu\nu} \equiv F^2\left(\frac{\Box}{M_*^2}\right) T_{\mu\nu}, \tag{6}$$

and write the nonlocal field equations on the real slice as:

$$G_{\mu\nu}(h) = \kappa\, S_{\mu\nu} \quad \Longleftrightarrow \quad F^{-2}\left(\frac{\Box}{M_*^2}\right) G_{\mu\nu} = \kappa\, T_{\mu\nu}. \tag{7}$$

The operator $F(\Box/M_*^2)$ is defined by analytic functional calculus using its power-series expansion and acts covariantly on tensors. Because F is entire and nonvanishing in the finite complex plane, it introduces no new poles and hence no ghosts or extra propagating degrees of freedom [18,22,23,27–30,33]; at low momenta $p^2 \ll M_*^2$ we find $F \approx 1$ and classical General Relativity is recovered. For static, spherically symmetric configurations, we will treat $F^2(\Box/M_*^2)$ as a nonlocal smearing operator acting on the matter energy density. This will be sufficient to determine the interior structure of the corresponding nonlocal black-hole solution.

2. Static Limit and Nonlocal Smearing of a Point Mass

We now specialize to a static, spherically symmetric configuration sourced by a point mass M at the origin. We work on the real slice with metric signature $(-,+,+,+)$. In the static limit, all fields are independent of the time coordinate t, and the d'Alembertian decomposes as:

$$\Box = -\partial_t^2 + \nabla^2 \quad \to \quad \Box_{\text{static}} = \nabla^2, \tag{8}$$

where ∇^2 is the Laplacian on the spatial $\mathbb{R}^3$ with coordinates $\mathbf{x}$ and radial coordinate $r = |\mathbf{x}|$. We define the bare rest-mass energy density of a point source as:

$$\rho_{\text{bare}}(\mathbf{x}) = M\,\delta^{(3)}(\mathbf{x}), \tag{9}$$

so that the only nonzero component of the bare stress–energy tensor is:

$$T^0{}_0(\mathbf{x}) = -\rho_{\text{bare}}(\mathbf{x}), \tag{10}$$

$$T^i{}_j(\mathbf{x}) = 0 \quad (i,j = 1,2,3). \tag{11}$$

The minus sign comes from the convention $T^0{}_0 = -\rho$ in a static configuration.

The nonlocal regulator then defines an effective energy density:

$$\begin{aligned}\rho_{\text{eff}}(\mathbf{x}) \equiv -S^0{}_0(\mathbf{x}) &= -\left[F^2\left(\frac{\Box_{\text{static}}}{M_*^2}\right) T^0{}_0\right](\mathbf{x}) \\ &= \left[F^2\left(\frac{\nabla^2}{M_*^2}\right)\rho_{\text{bare}}\right](\mathbf{x}).\end{aligned} \tag{12}$$

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The nonlocal energy density is obtained by acting with the operator $F^2(\nabla^2/M_*^2)$ on the three-dimensional delta distribution. For the specific choice:

$$F\left(\frac{\Box}{M_*^2}\right) = \exp\left(-\frac{\Box}{M_*^2}\right), \tag{13}$$

we have:

$$F^2\left(\frac{\nabla^2}{M_*^2}\right) = \exp\left(-\frac{2\nabla^2}{M_*^2}\right). \tag{14}$$

It is convenient to parametrize the smearing operator directly by a positive length scale ℓ_*, writing:

$$\exp\left(\ell_*^2\nabla^2\right), \qquad \ell_* \equiv \frac{1}{M_*}, \tag{15}$$

since different normalizations of the argument in F can be absorbed into the definition of M_*. The essential feature is that the operator is the exponential of the Laplacian with a positive coefficient in the exponent, so that it acts as a Gaussian heat-kernel smoothing operator on $\mathbb{R}^3$ [40,41]. We now compute explicitly:

$$\rho_{\rm eff}(\mathbf{x}) = \exp\left(\ell_*^2\nabla^2\right)\left[M\delta^{(3)}(\mathbf{x})\right]. \tag{16}$$

We use the Fourier transform of the three-dimensional Dirac distribution:

$$\delta^{(3)}(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{17}$$

where $\mathbf{k}$ is the spatial momentum vector and $k \equiv |\mathbf{k}|$. The Laplacian acts on a plane wave as:

$$\nabla^2 e^{i\mathbf{k}\cdot\mathbf{x}} = -k^2\, e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{18}$$

The exponential of the Laplacian acts multiplicatively in momentum space:

$$\begin{aligned}\exp\left(\ell_*^2\nabla^2\right)e^{i\mathbf{k}\cdot\mathbf{x}} &= \sum_{n=0}^{\infty}\frac{(\ell_*^2\nabla^2)^n}{n!}e^{i\mathbf{k}\cdot\mathbf{x}} \\ &= \sum_{n=0}^{\infty}\frac{(\ell_*^2(-k^2))^n}{n!}e^{i\mathbf{k}\cdot\mathbf{x}} \\ &= \exp\left(-\ell_*^2k^2\right)e^{i\mathbf{k}\cdot\mathbf{x}}.\end{aligned} \tag{19}$$

Applying the operator to the delta distribution yields:

$$\begin{aligned}\rho_{\rm eff}(\mathbf{x}) &= M\exp\left(\ell_*^2\nabla^2\right)\delta^{(3)}(\mathbf{x}) \\ &= M\int \frac{d^3k}{(2\pi)^3}\exp\left(-\ell_*^2k^2\right)e^{i\mathbf{k}\cdot\mathbf{x}}.\end{aligned} \tag{20}$$

This integral is a standard Gaussian Fourier transform. For $\ell_*^2 > 0$ we find:

$$\int d^3k\, \exp\left(-\ell_*^2k^2 + i\mathbf{k}\cdot\mathbf{x}\right) = \left(\frac{\pi}{\ell_*^2}\right)^{3/2}\exp\left(-\frac{\mathbf{x}^2}{4\ell_*^2}\right). \tag{21}$$

Dividing by $(2\pi)^3$, we obtain:

$$\begin{aligned}\rho_{\rm eff}(\mathbf{x}) &= M\frac{1}{(2\pi)^3}\left(\frac{\pi}{\ell_*^2}\right)^{3/2}\exp\left(-\frac{\mathbf{x}^2}{4\ell_*^2}\right) \\ &= M\frac{1}{(4\pi\ell_*^2)^{3/2}}\exp\left(-\frac{r^2}{4\ell_*^2}\right),\end{aligned} \tag{22}$$

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where $r = |\mathbf{x}|$ is the radial coordinate and we used:

$$(4\pi)^{3/2} = (2^2\pi)^{3/2} = 2^3\pi^{3/2} = 8\pi^{3/2}. \tag{23}$$

It is useful to express the result in terms of the nonlocal mass scale M_* via: $\ell_* = 1/M_*$. Then:

$$\rho_{\text{eff}}(r) = \frac{MM_*^3}{(4\pi)^{3/2}} \exp\left(-\frac{M_*^2 r^2}{4}\right). \tag{24}$$

This is a spherically symmetric Gaussian energy density with width of order M_*^{-1}, finite at the origin:

$$\rho_{\text{eff}}(0) = \frac{MM_*^3}{(4\pi)^{3/2}} < \infty. \tag{25}$$

By construction, the total mass is preserved:

$$\begin{aligned} \int d^3x\, \rho_{\text{eff}}(\mathbf{x}) &= 4\pi \int_0^\infty dr\, r^2 \rho_{\text{eff}}(r) \\ &= 4\pi \int_0^\infty dr\, r^2 \frac{MM_*^3}{(4\pi)^{3/2}} \exp\left(-\frac{M_*^2 r^2}{4}\right) \\ &= M, \end{aligned} \tag{26}$$

as can be verified by the standard integral:

$$\int_0^\infty dr\, r^2 e^{-\alpha r^2} = \frac{\sqrt{\pi}}{4}\alpha^{-3/2}, \qquad \alpha > 0. \tag{27}$$

The nonlocal regulator replaces the point-like mass by a smooth Gaussian ball of radius $\sim 1/M_*$ and preserves the total mass.

3. Spherically Symmetric Metric and Mass Function

We now will solve the nonlocal Einstein equations for a static, spherically symmetric metric sourced by the effective energy density (24). We work with the standard Schwarzschild-like ansatz:

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{28}$$

where $d\Omega^2$ is the line element on the unit two-sphere and $f(r)$ is an unknown function to be determined. It is convenient to introduce the Misner–Sharp mass function $m(r)$ via [42]:

$$f(r) = 1 - \frac{2G_{\text{N}} m(r)}{r}. \tag{29}$$

In ordinary General Relativity with a static, spherically symmetric stress–energy tensor $T^\mu{}_\nu = \text{diag}(-\rho(r), p_r(r), p_\perp(r), p_\perp(r))$, the Einstein equations imply:

$$m'(r) = 4\pi r^2 \rho(r), \tag{30}$$

where the prime denotes d/dr. This relation follows from the 00-component of the Einstein equations and the definition of $m(r)$ in terms of the areal radius; it can be viewed as the statement that $m(r)$ is the mass contained inside the sphere of radius r. In our nonlocal setup, the geometric side of the Einstein equations is still local, and the modification enters only through the replacement $\rho \to \rho_{\text{eff}}$. Therefore, the mass function satisfies:

$$m'(r) = 4\pi r^2 \rho_{\text{eff}}(r), \tag{31}$$

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with $\rho_{\rm eff}$ given by (24). Integrating from 0 to r with the boundary condition $m(0) = 0$ gives:

$$m(r) = 4\pi \int_0^r dr'\, r'^2 \rho_{\rm eff}(r'). \tag{32}$$

Substituting (24),

$$\begin{aligned} m(r) &= 4\pi \int_0^r dr'\, r'^2 \frac{MM_*^3}{(4\pi)^{3/2}} \exp\left(-\frac{M_*^2 r'^2}{4}\right) \\ &= \frac{4\pi MM_*^3}{(4\pi)^{3/2}} \int_0^r dr'\, r'^2 \exp\left(-\frac{M_*^2 r'^2}{4}\right). \end{aligned} \tag{33}$$

Now define:

$$a \equiv \frac{M_*^2}{4} > 0, \tag{34}$$

where a is a positive constant parameter introduced for notational convenience with dimensions of mass^2 or inverse length^2. The exponent becomes $-ar'^2$. The integral:

$$I(r) \equiv \int_0^r dr'\, r'^2 e^{-ar'^2} \tag{35}$$

can be evaluated analytically. One way is to use the identity:

$$\int r'^2 e^{-ar'^2} dr' = -\frac{r'}{2a} e^{-ar'^2} + \frac{\sqrt{\pi}}{4a^{3/2}} \operatorname{erf}(\sqrt{a}\, r'), \tag{36}$$

where erf is the error function. Evaluating between 0 and r gives:

$$I(r) = \frac{\sqrt{\pi}}{4a^{3/2}} \operatorname{erf}(\sqrt{a}\, r) - \frac{r}{2a} e^{-ar^2}. \tag{37}$$

Using $a = M_*^2/4$, we have:

$$\sqrt{a} = \frac{M_*}{2}, \tag{38}$$

$$a^{3/2} = \left(\frac{M_*^2}{4}\right)^{3/2} = \frac{M_*^3}{8}, \tag{39}$$

hence:

$$\frac{\sqrt{\pi}}{4a^{3/2}} = \frac{\sqrt{\pi}}{4} \frac{8}{M_*^3} = \frac{2\sqrt{\pi}}{M_*^3}, \tag{40}$$

and:

$$\frac{1}{2a} = \frac{1}{2 \cdot M_*^2/4} = \frac{2}{M_*^2}. \tag{41}$$

Therefore:

$$I(r) = \frac{2\sqrt{\pi}}{M_*^3} \operatorname{erf}\left(\frac{M_* r}{2}\right) - \frac{2r}{M_*^2} e^{-M_*^2 r^2/4}. \tag{42}$$

Putting this back into $m(r)$, we find:

$$\begin{aligned} m(r) &= \frac{4\pi MM_*^3}{(4\pi)^{3/2}} I(r) \\ &= \frac{4\pi MM_*^3}{(4\pi)^{3/2}} \left[\frac{2\sqrt{\pi}}{M_*^3} \operatorname{erf}\left(\frac{M_* r}{2}\right) - \frac{2r}{M_*^2} e^{-M_*^2 r^2/4}\right]. \end{aligned} \tag{43}$$

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Using $(4\pi)^{3/2} = 8\pi^{3/2}$, we have:

$$\frac{4\pi M M_*^3}{(4\pi)^{3/2}} = \frac{4\pi}{8\pi^{3/2}} M M_*^3 = \frac{M M_*^3}{2\sqrt{\pi}}. \tag{44}$$

Therefore:

$$\begin{aligned} m(r) &= \frac{M M_*^3}{2\sqrt{\pi}} \left[\frac{2\sqrt{\pi}}{M_*^3} \operatorname{erf}\!\left(\frac{M_* r}{2}\right) - \frac{2r}{M_*^2} e^{-M_*^2 r^2/4} \right] \\ &= M\left[\operatorname{erf}\!\left(\frac{M_* r}{2}\right) - \frac{M_* r}{\sqrt{\pi}} e^{-M_*^2 r^2/4} \right]. \end{aligned} \tag{45}$$

This is the exact mass function of the nonlocal black-hole solution sourced by the Gaussian effective density (24).

As $r \to \infty$, the error function tends to 1 and the exponential term vanishes:

$$\operatorname{erf}(x) \to 1 \quad (x \to \infty), \qquad e^{-x^2} \to 0, \tag{46}$$

so:

$$\lim_{r\to\infty} m(r) = M, \tag{47}$$

and the exterior solution is asymptotically Schwarzschild with mass M [43]. To determine the behavior near $r = 0$, we expand $m(r)$ for small r. Using the series expansions:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \mathcal{O}(x^5) \right), \tag{48}$$

$$e^{-x^2} = 1 - x^2 + \mathcal{O}(x^4), \tag{49}$$

with $x \equiv M_* r/2$, we have:

$$\begin{aligned} \operatorname{erf}\!\left(\frac{M_* r}{2}\right) &= \frac{2}{\sqrt{\pi}}\left(\frac{M_* r}{2} - \frac{1}{3}\left(\frac{M_* r}{2}\right)^3 + \mathcal{O}(r^5) \right) \\ &= \frac{M_* r}{\sqrt{\pi}} - \frac{M_*^3 r^3}{12\sqrt{\pi}} + \mathcal{O}(r^5), \end{aligned} \tag{50}$$

and:

$$\begin{aligned} \frac{M_* r}{\sqrt{\pi}} e^{-M_*^2 r^2/4} &= \frac{M_* r}{\sqrt{\pi}}\left(1 - \frac{M_*^2 r^2}{4} + \mathcal{O}(r^4) \right) \\ &= \frac{M_* r}{\sqrt{\pi}} - \frac{M_*^3 r^3}{4\sqrt{\pi}} + \mathcal{O}(r^5). \end{aligned} \tag{51}$$

Subtracting:

$$\begin{aligned} \operatorname{erf}\!\left(\frac{M_* r}{2}\right) - \frac{M_* r}{\sqrt{\pi}} e^{-M_*^2 r^2/4} &= \left(\frac{M_* r}{\sqrt{\pi}} - \frac{M_*^3 r^3}{12\sqrt{\pi}} + \dots \right) \\ &\quad - \left(\frac{M_* r}{\sqrt{\pi}} - \frac{M_*^3 r^3}{4\sqrt{\pi}} + \dots \right) \\ &= \left(-\frac{1}{12} + \frac{1}{4} \right) \frac{M_*^3 r^3}{\sqrt{\pi}} + \mathcal{O}(r^5) \\ &= \frac{M_*^3 r^3}{6\sqrt{\pi}} + \mathcal{O}(r^5). \end{aligned} \tag{52}$$

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Near the origin:

$$m(r) = M\frac{M_*^3 r^3}{6\sqrt{\pi}} + \mathcal{O}(r^5). \tag{53}$$

Substituting into (29) gives:

$$\begin{aligned} f(r) &= 1 - \frac{2G_{\mathrm{N}}m(r)}{r} \\ &= 1 - \frac{2G_{\mathrm{N}}}{r}\left(M\frac{M_*^3 r^3}{6\sqrt{\pi}} + \mathcal{O}(r^5)\right) \\ &= 1 - \frac{G_{\mathrm{N}}MM_*^3}{3\sqrt{\pi}}r^2 + \mathcal{O}(r^4). \end{aligned} \tag{54}$$

This has precisely the form of a de Sitter static patch near the origin:

$$f_{\mathrm{dS}}(r) = 1 - \frac{\Lambda_{\mathrm{eff}}}{3}r^2, \tag{55}$$

with an effective cosmological constant:

$$\Lambda_{\mathrm{eff}} = \frac{G_{\mathrm{N}}MM_*^3}{\sqrt{\pi}}. \tag{56}$$

Equivalently, we can read off Λ_{eff} from the effective energy density at the core. In a perfect-fluid de Sitter region, the stress–energy tensor takes the form:

$$T^{\mu}{}_{\nu} = -\frac{\Lambda}{\kappa}\delta^{\mu}{}_{\nu} \quad \Rightarrow \quad \rho = \frac{\Lambda}{\kappa}, \quad p = -\rho, \tag{57}$$

with equation of state $p = -\rho$. Here $\kappa = 8\pi G_{\mathrm{N}}$. Using (24) at $r = 0$:

$$\rho_{\mathrm{eff}}(0) = \frac{MM_*^3}{(4\pi)^{3/2}}, \tag{58}$$

the Einstein equation $G^0{}_0 = -\kappa\rho$ near the origin implies:

$$\Lambda_{\mathrm{eff}} = \kappa\rho_{\mathrm{eff}}(0) = 8\pi G_{\mathrm{N}}\frac{MM_*^3}{(4\pi)^{3/2}} = \frac{G_{\mathrm{N}}MM_*^3}{\sqrt{\pi}}, \tag{59}$$

in agreement with (56). We therefore conclude that the nonlocal black-hole solution sourced by a point mass M has a regular de Sitter core of curvature scale:

$$R_{\mathrm{core}} \sim \Lambda_{\mathrm{eff}}^{-1/2} \sim \left(G_{\mathrm{N}}MM_*^3\right)^{-1/2}. \tag{60}$$

All curvature invariants are finite at $r = 0$; the central singularity is replaced by a smooth positive-curvature region with equation of state $p \approx -\rho$.

4. de Sitter Cores as Holomorphic Saddles

In our framework, the cosmological constant arises as a holomorphic parameter on the complexified manifold $M_{\mathbb{C}}$:

$$\Lambda(\theta) = |\Lambda|e^{i\theta}, \tag{61}$$

with different real phases θ selecting AdS ($\Lambda < 0$) or dS ($\Lambda > 0$) real forms. Analytic continuation in θ provides a controlled bridge between AdS and dS backgrounds without introducing new singularities in the curvature invariants, which remain polynomial and even in Λ for maximally symmetric vacua. The de Sitter core derived above is a local realization of a positive-curvature saddle in this holomorphic

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family. The same entire-function regulator $F(\Box/M_*^2)$ that renders quantum loops finite also smears the classical matter source, replacing the point mass by a Gaussian ball of radius $\sim M_*^{-1}$. The resulting Einstein equations on the real slice admit a solution whose interior approaches a static de Sitter patch with effective cosmological constant $\Lambda_{\rm eff}$ given by (56), while the exterior remains asymptotically Schwarzschild [43]. From the non-local point of view, the de Sitter core is not an ad hoc modification but a consequence of the same holomorphic structure that governs UV-finite quantum dynamics as the nonlocal regulator is an entire function of a covariant operator through the Bochner d'Alembertian, so it commutes with diffeomorphisms and gauge transformations and preserves BRST symmetry. On the real slice, $F(\Box/M_*^2)$ can be represented as a heat-kernel smearing operator [40,41]; acting on singular sources produces smooth profiles with characteristic length $\ell_* \sim M_*^{-1}$. The de Sitter core corresponds to a region where the smeared stress–energy is effectively vacuum-like, $p \approx -\rho$, and the geometry is governed by a positive $\Lambda_{\rm eff}$. In the complex Λ-plane, this is a neighborhood of the dS branch of the holomorphic family.

As a result, black-hole singularities are resolved by the same holomorphic, UV-finite mechanism that tames loop divergences. The interior of a nonlocal black hole in non-local QFT is a regular de Sitter bubble whose curvature and size are set by the interplay of the mass M and the nonlocal scale M_*, while the geometry interpolates smoothly between this core and the asymptotically Schwarzschild exterior [43].

5. Nonlocal Gravity and the Emergence of a De Sitter Core

In non-local QFT, ultraviolet finiteness is achieved by inserting entire-function regulators built from the covariant Laplace–Beltrami operator into the action. One introduces an entire function $F(z)$ of a complex variable z and promotes it to an operator:

$$F\left(\frac{\Box}{M_*^2}\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{\Box}{M_*^2}\right)^n, \tag{62}$$

where:

$$\Box \equiv -\, g^{\mu\nu} D_\mu D_\nu \tag{63}$$

is the covariant Laplace–Beltrami built from the Lorentzian metric $g_{\mu\nu}$ of signature $(-,+,+,+)$ and the gauge- and diffeomorphism-covariant derivative D_μ. The constant M_* is the nonlocality scale. A convenient explicit choice, used throughout nonlocal gravity models is:

$$F(z) = e^z, \qquad F\left(\frac{\Box}{M_*^2}\right) = \exp\left(\frac{\Box}{M_*^2}\right), \tag{64}$$

which is entire, has no zeros in the finite complex plane, and therefore does not introduce extra poles in propagators. In the gravitational sector, we consider an effective nonlocal extension of the Einstein–Hilbert action of the form:

$$S_{\rm grav} = \frac{1}{16\pi G_N}\int d^4x\, \sqrt{-g}G^{-1}(x)\,(R-2\lambda), \tag{65}$$

where G_N is Newton's constant, R is the Ricci scalar of $g_{\mu\nu}$, λ is a cosmological constant, and the position-dependent gravitational coupling is:

$$G(x) = G_N\, F\left(\frac{\Box}{\Lambda_G^2}\right), \tag{66}$$

with Λ_G a gravitational nonlocality scale. For the purposes of non-local QFT, one may simply identify $\Lambda_G \sim M_*$ in the gravitational sector.

Varying (65) with respect to $g_{\mu\nu}$ and neglecting surface terms arising from the variation of $\Box$, a standard assumption in the static, spherically symmetric context we consider here, we obtain field equations of the form:

$$F^{-2}\left(\frac{\Box}{\Lambda_G^2}\right)\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=8\pi G_N\,T_{\mu\nu}, \tag{67}$$

where $T_{\mu\nu}$ is the energy–momentum tensor of matter. It is convenient to rewrite (67) by shifting $F^{-2}(\Box/\Lambda_G^2)$ to the right-hand side. Defining a generalized or effective energy–momentum tensor:

$$S_{\mu\nu}\equiv F^2\left(\frac{\Box}{\Lambda_G^2}\right)T_{\mu\nu}, \tag{68}$$

the equations take the canonical Einstein form:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G_N\,S_{\mu\nu}. \tag{69}$$

Thus, nonlocality is encoded in the smearing of the source so the theory is equivalent to ordinary Einstein gravity coupled to a nonlocal, smeared energy–momentum tensor $S_{\mu\nu}$.

We now specialise to a static, spherically symmetric source and metric. Consider a point mass M at the origin as the bare source, with bare energy density:

$$\rho_{\text{bare}}(\vec{x})=M\,\delta^{(3)}(\vec{x}), \tag{70}$$

$$T^0{}_0(\vec{x})=-\rho_{\text{bare}}(\vec{x}), \tag{71}$$

and all other components determined by staticity and conservation. Here $\delta^{(3)}(\vec{x})$ is the three-dimensional Dirac delta distribution, normalised so that $\int d^3x\,\delta^{(3)}(\vec{x})=1$.

The effective energy density that gravitates is given by:

$$\begin{aligned}\rho_{\text{eff}}(\vec{x})\equiv -S^0{}_0(\vec{x}) &= -F^2\left(\frac{\Box}{\Lambda_G^2}\right)T^0{}_0(\vec{x})\\ &= M\,F^2\left(\frac{\Box}{\Lambda_G^2}\right)\delta^{(3)}(\vec{x}).\end{aligned} \tag{72}$$

For a static source, the time derivatives vanish, so on the background we may replace the d'Alembertian by the spatial Laplacian acting on static configurations. Working in the weak-field, short-distance regime where the background is approximately flat, we may write:

$$\Box\longrightarrow\nabla^2, \tag{73}$$

where ∇^2 is the flat-space Laplacian on $\mathbb{R}^3$. For the exponential regulator, (64), we then have:

$$F^2\left(\frac{\Box}{\Lambda_G^2}\right)\longrightarrow\exp\left(\frac{\nabla^2}{\Lambda_G^2}\right). \tag{74}$$

Thus:

$$\rho_{\text{eff}}(\vec{x})=M\,\exp\left(\frac{\nabla^2}{\Lambda_G^2}\right)\delta^{(3)}(\vec{x}). \tag{75}$$

The operator $\exp(\sigma\nabla^2)$ is the heat-kernel evolution operator at time σ. Acting on a delta function, it produces a Gaussian. To see this explicitly, represent $\delta^{(3)}(\vec{x})$ by its Fourier transform:

$$\delta^{(3)}(\vec{x})=\int\frac{d^3k}{(2\pi)^3}\,e^{i\vec{k}\cdot\vec{x}}, \tag{76}$$

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and note that $\nabla^2 e^{i\vec{k}\cdot\vec{x}} = -k^2 e^{i\vec{k}\cdot\vec{x}}$. Then:

$$\begin{aligned}\exp\left(\frac{\nabla^2}{\Lambda_G^2}\right)\delta^{(3)}(\vec{x}) &= \int \frac{d^3k}{(2\pi)^3}\,\exp\left(\frac{\nabla^2}{\Lambda_G^2}\right)e^{i\vec{k}\cdot\vec{x}} \\ &= \int \frac{d^3k}{(2\pi)^3}\,\exp\left(-\frac{k^2}{\Lambda_G^2}\right)e^{i\vec{k}\cdot\vec{x}}, \end{aligned} \tag{77}$$

where we used the fact that ∇^2 is diagonalised by plane waves, so that $\exp(\nabla^2/\Lambda_G^2)$ acts as multiplication by $\exp(-k^2/\Lambda_G^2)$ in momentum space. The remaining integral is a standard Gaussian Fourier transform:

$$\begin{aligned}&\int \frac{d^3k}{(2\pi)^3}\,\exp\left(-\frac{k^2}{\Lambda_G^2}\right)e^{i\vec{k}\cdot\vec{x}} \\ &= \left(\frac{\Lambda_G^2}{4\pi}\right)^{3/2}\exp\left(-\frac{\Lambda_G^2 r^2}{4}\right), \qquad r \equiv |\vec{x}|. \end{aligned} \tag{78}$$

Combining (75) and (78), we obtain:

$$\rho_{\text{eff}}(r) = \frac{M\,\Lambda_G^3}{(4\pi)^{3/2}}\exp\left(-\frac{\Lambda_G^2 r^2}{4}\right), \tag{79}$$

which is a Gaussian of width $\sim \Lambda_G^{-1}$, normalised so that:

$$\int d^3x\,\rho_{\text{eff}}(\vec{x}) = M. \tag{80}$$

In the nonlocal gravity induced by entire-function regulator, a bare point mass is replaced by a smooth, Gaussian energy distribution. The nonlocality length scale $\ell_{\text{nl}} \sim \Lambda_G^{-1}$ sets the minimal radius to which matter can be localised.

We now solve the Einstein equations (69) with the effective energy density (79). We assume a static, spherically symmetric line element:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \tag{81}$$

where $f(r)$ is a function to be determined from the field equations. The nonzero components of the Einstein tensor $G^\mu_{\ \nu} \equiv R^\mu_{\ \nu} - \frac{1}{2}\delta^\mu_{\ \nu}R$ for the metric (81) are standard. The tt-component reads:

$$G^t_{\ t}(r) = \frac{f'(r)\,r + f(r) - 1}{r^2}, \tag{82}$$

where a prime denotes differentiation with respect to r. The Einstein equation (69) gives:

$$G^t_{\ t}(r) = -\,8\pi G_N\,\rho_{\text{eff}}(r). \tag{83}$$

It is convenient to define a mass function $m(r)$ by:

$$f(r) \equiv 1 - \frac{2G_N m(r)}{r}. \tag{84}$$

Substituting (84) into (82), we compute:

$$f'(r) = -\frac{2G_N}{r} m'(r) + \frac{2G_N}{r^2} m(r), \tag{85}$$

$$\begin{aligned} f'(r)\, r + f(r) - 1 &= -2G_N m'(r) + 2G_N \frac{m(r)}{r} \\ &\quad + 1 - \frac{2G_N m(r)}{r} - 1 = -2G_N m'(r). \end{aligned} \tag{86}$$

Hence:

$$G^t{}_t(r) = \frac{f'(r)\, r + f(r) - 1}{r^2} = -\frac{2G_N}{r^2} m'(r). \tag{87}$$

Equating this to (83) yields:

$$-\frac{2G_N}{r^2} m'(r) = -8\pi G_N\, \rho_{\mathrm{eff}}(r), \tag{88}$$

so that:

$$m'(r) = 4\pi r^2\, \rho_{\mathrm{eff}}(r). \tag{89}$$

This is the usual relation between the mass function and the energy density for a spherically symmetric configuration. Substituting (79) into (89), we find:

$$m'(r) = 4\pi r^2 \frac{M\,\Lambda_G^3}{(4\pi)^{3/2}} \exp\left(-\frac{\Lambda_G^2 r^2}{4}\right). \tag{90}$$

Integrating from 0 to r gives:

$$\begin{aligned} m(r) &= 4\pi \int_0^r dr'\, r'^2\, \rho_{\mathrm{eff}}(r') \\ &= 4\pi \int_0^r dr'\, r'^2 \frac{M\,\Lambda_G^3}{(4\pi)^{3/2}} \exp\left(-\frac{\Lambda_G^2 r'^2}{4}\right). \end{aligned} \tag{91}$$

The integral can be expressed in terms of an incomplete gamma function, but for our purposes it suffices to examine the small-r limit relevant for the core. For $r\Lambda_G \ll 1$, the exponential can be expanded as:

$$\exp\left(-\frac{\Lambda_G^2 r'^2}{4}\right) = 1 + \mathcal{O}(\Lambda_G^2 r'^2), \tag{92}$$

so to leading order we may approximate:

$$\rho_{\mathrm{eff}}(r') \approx \rho_0 \equiv \frac{M\,\Lambda_G^3}{(4\pi)^{3/2}}, \tag{93}$$

a constant energy density near the origin. Then (91) simplifies to:

$$\begin{aligned} m(r) &\approx 4\pi\rho_0 \int_0^r dr'\, r'^2 = 4\pi\rho_0 \frac{r^3}{3} = \frac{4\pi}{3} \frac{M\,\Lambda_G^3}{(4\pi)^{3/2}} r^3 \\ &= \frac{M\,\Lambda_G^3}{6\sqrt{\pi}} r^3. \end{aligned} \tag{94}$$

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Substituting (94) into the definition (84), we find that for $r\Lambda_G \ll 1$:

$$f(r) = 1 - \frac{2G_N}{r}\,m(r) \approx 1 - \frac{2G_N}{r}\left(\frac{M\,\Lambda_G^3}{6\sqrt{\pi}}\,r^3\right)$$
$$= 1 - \frac{MG_N\Lambda_G^3}{3\sqrt{\pi}}\,r^2. \tag{95}$$

This is precisely the static-patch form of a de Sitter metric:

$$f(r) \approx 1 - \frac{\Lambda_{\text{eff}}}{3}\,r^2, \qquad \Lambda_{\text{eff}} \equiv \frac{MG_N\Lambda_G^3}{\sqrt{\pi}}. \tag{96}$$

The line element in the core region is therefore:

$$ds^2 \approx -\left(1 - \frac{\Lambda_{\text{eff}}}{3}r^2\right)dt^2 + \frac{dr^2}{1 - \frac{\Lambda_{\text{eff}}}{3}r^2} + r^2 d\Omega^2,, \tag{97}$$

$r\Lambda_G \ll 1$, which is the static patch of four-dimensional de Sitter space with curvature radius:

$$\ell_{\text{core}} = \sqrt{\frac{3}{\Lambda_{\text{eff}}}}. \tag{98}$$

The Ricci scalar in the core is finite and given by the de Sitter relation:

$$R(0) = 4\,\Lambda_{\text{eff}} = \frac{4MG_N\Lambda_G^3}{\sqrt{\pi}}, \tag{99}$$

and similarly higher curvature invariants remain finite. Thus, the classical $r = 0$ curvature singularity of the Schwarzschild solution is replaced by a regular de Sitter core [35–39]., whose effective cosmological constant Λ_{eff} is determined by the mass M, Newton's constant G_N, and the gravitational nonlocality scale Λ_G.

In our framework, Λ_G arises from the same gauge- and diffeomorphism-covariant entire-function regulator $F(\Box/M_*^2)$ that renders loop amplitudes finite and smears fields over a length scale $\ell_* \sim M_*^{-1}$. The emergence of a de Sitter core inside a black hole is therefore not an ad hoc modification: it is the direct manifestation, in the strong-gravity regime, of the nonlocal geometric structure already required by non-local QFT for UV completeness.

6. Regular Black Holes from Entire-Function Nonlocal Regulators

Recall we are working in four spacetime dimensions with signature $(-,+,+,+)$. Greek indices $\mu,\nu,\ldots$ run over $0,1,2,3$. The metric is $g_{\mu\nu}$, the Levi–Civita connection is ∇_μ, the Ricci tensor is $R_{\mu\nu}$, the Ricci scalar is $R \equiv g^{\mu\nu}R_{\mu\nu}$, and the Einstein tensor is $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. Newton's constant is G_N and we define $\kappa \equiv 8\pi G_N$. The generally covariant d'Alembertian is:

$$\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu. \tag{100}$$

An ultraviolet-improved ghost-free nonlocal modification is encoded by an entire form factor F that is analytic everywhere in the finite complex plane, with no zeros or poles built from $\Box$ and a UV mass scale Λ_G (or M_*). A representative choice used throughout is exponential damping in Euclidean momentum space:

$$F\!\left(\frac{\Box}{\Lambda_G^2}\right) \sim \exp\!\left(-\frac{\Box}{2\Lambda_G^2}\right), \tag{101}$$

in this normalization conventions vary by factors of 2. The nonlocal gravitational field equations can be written in either of two exactly equivalent forms:

$$F^{-2}\left(\frac{\Box}{\Lambda_G^2}\right) G_{\mu\nu} = \kappa\, T_{\mu\nu}, \tag{102}$$

$$G_{\mu\nu} = \kappa\, S_{\mu\nu}, \tag{103}$$

$$S_{\mu\nu} \equiv F^2\left(\frac{\Box}{\Lambda_G^2}\right) T_{\mu\nu}. \tag{104}$$

Here $T_{\mu\nu}$ is the bare local stress–energy tensor, while $S_{\mu\nu}$ is the smeared stress–energy tensor that gravitates. Because $\nabla^\mu G_{\mu\nu} = 0$, consistency requires $\nabla^\mu S_{\mu\nu} = 0$.

Equation (102) says geometry is nonlocal, equation (104) says geometry is local Einstein, but matter is nonlocally smeared. They are physically equivalent so we will use (104) because it cleanly isolates how UV nonlocality modifies black-hole interiors as it replaces point sources by extended, finite-width sources.

If we consider a static configuration then the time derivative vanishes $\partial_t(\cdots) = 0$ and on such fields the d'Alembertian reduces to the spatial Laplacian:

$$\Box \to \nabla^2, \tag{105}$$

where ∇^2 is the Laplacian on the spatial slice, locally $\mathbb{R}^3$ in the weak-field regime. Take a bare point mass M at the origin with bare energy density:

$$\rho_{\text{bare}}(\mathbf{x}) = M\,\delta^{(3)}(\mathbf{x}), \tag{106}$$

$$T^0{}_0(\mathbf{x}) = -\rho_{\text{bare}}(\mathbf{x}), \tag{107}$$

$$T^i{}_j = 0 (i,j = 1,2,3). \tag{108}$$

We can define the effective energy density by:

$$\rho_{\text{eff}}(\mathbf{x}) \equiv -\,S^0{}_0(\mathbf{x}) = F^2\left(\frac{\nabla^2}{\Lambda_G^2}\right)\rho_{\text{bare}}(\mathbf{x}). \tag{109}$$

For the exponential regulator, $F^2(\nabla^2/\Lambda_G^2) = \exp(\nabla^2/\Lambda_G^2)$ up to conventions, the operator $\exp(\sigma\nabla^2)$ is precisely the heat-kernel evolution operator at time σ, when acting on a delta distribution, it produces a Gaussian.

To show this explicitly, we use the Fourier representation:

$$\delta^{(3)}(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{110}$$

$$\nabla^2 e^{i\mathbf{k}\cdot\mathbf{x}} = -k^2 e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{111}$$

then:

$$\begin{aligned}\rho_{\text{eff}}(\mathbf{x}) &= M\exp\left(\frac{\nabla^2}{\Lambda_G^2}\right)\delta^{(3)}(\mathbf{x}) \\ &= M\int \frac{d^3k}{(2\pi)^3}\exp\left(-\frac{k^2}{\Lambda_G^2}\right)e^{i\mathbf{k}\cdot\mathbf{x}} \\ &= M\left(\frac{\Lambda_G^2}{4\pi}\right)^{3/2}\exp\left(-\frac{\Lambda_G^2 r^2}{4}\right),\end{aligned} \tag{112}$$

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$r \equiv |\mathbf{x}|$. This is a smooth Gaussian energy distribution of width $\ell_{\rm nl} \sim \Lambda_G^{-1}$, normalized so that $\int d^3x\, \rho_{\rm eff} = M$.

The regulator forces an effective minimal localization length for gravitating sources so a point mass cannot source curvature at scales below $\ell_{\rm nl}$; it is replaced by a finite ball.

If we assume a static, spherically symmetric metric in Schwarzschild gauge ($g_{tt} = -g_{rr}^{-1}$):

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{113}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ is the line element on the unit two-sphere. We introduce the Misner–Sharp mass function $m(r)$ by [42]:

$$f(r) \equiv 1 - \frac{2G_N m(r)}{r}. \tag{114}$$

For a static, spherically symmetric effective stress tensor $S^\mu{}_\nu = \mathrm{diag}(-\rho_{\rm eff}(r), p_r(r), p_\perp(r), p_\perp(r))$, the tt-component of Einstein's equations gives the standard relation:

$$m'(r) = 4\pi r^2\, \rho_{\rm eff}(r), \tag{115}$$

with boundary condition $m(0) = 0$, then integrating gives:

$$m(r) = 4\pi \int_0^r dr'\, r'^2\, \rho_{\rm eff}(r'), \tag{116}$$

substituting the Gaussian density (158) yields an exact closed form in terms of incomplete gamma functions. One convenient representation we can use is:

$$\begin{aligned} m(r) &= M\,\frac{\gamma\!\left(\frac{3}{2}; \frac{\Lambda_G^2 r^2}{4}\right)}{\Gamma\!\left(\frac{3}{2}\right)} \\ &= M\left[\mathrm{erf}\!\left(\frac{\Lambda_G r}{2}\right) - \frac{\Lambda_G r}{\sqrt{\pi}}\, e^{-\Lambda_G^2 r^2/4}\right], \end{aligned} \tag{117}$$

where $\gamma(s; x)$ is the lower incomplete gamma function and $\Gamma(s)$ is Euler's gamma function. The corresponding metric function is:

$$f(r) = 1 - \frac{2G_N M}{r}\, \frac{\gamma\!\left(\frac{3}{2}; \frac{\Lambda_G^2 r^2}{4}\right)}{\Gamma\!\left(\frac{3}{2}\right)}. \tag{118}$$

As $r \to \infty$, $\gamma(3/2; \Lambda_G^2 r^2/4) \to \Gamma(3/2)$ and $m(r) \to M$, so $f(r) \to 1 - 2G_N M/r$, the ordinary Schwarzschild at macroscopic distances.

For $r\Lambda_G \ll 1$, we are in the small-r limit and the Gaussian is approximately constant:

$$\rho_{\rm eff}(r) \approx \rho_0 \equiv M\left(\frac{\Lambda_G^2}{4\pi}\right)^{3/2}, \qquad (r\Lambda_G \ll 1). \tag{119}$$

Then (115) integrates to:

$$m(r) \approx \frac{4\pi}{3}\rho_0\, r^3, \qquad (r\Lambda_G \ll 1), \tag{120}$$

so by (114) we see:

$$f(r) \approx 1 - \frac{2G_N}{r}\left(\frac{4\pi}{3}\rho_0 r^3\right) \\ = 1 - \frac{8\pi G_N \rho_0}{3} r^2 = 1 - \frac{\Lambda_{\text{eff}}}{3} r^2, \tag{121}$$

with:

$$\Lambda_{\text{eff}} \equiv 8\pi G_N \rho_0 = \frac{G_N M \Lambda_G^3}{\sqrt{\pi}}. \tag{122}$$

Therefore the core geometry is the static patch of de Sitter space:

$$ds^2 \approx -\left(1 - \frac{\Lambda_{\text{eff}}}{3} r^2\right) dt^2 + \frac{dr^2}{1 - \frac{\Lambda_{\text{eff}}}{3} r^2} + r^2 d\Omega^2, \tag{123}$$

for $r\Lambda_G \ll 1$. The de Sitter curvature radius is $\ell_{\text{core}} = \sqrt{3/\Lambda_{\text{eff}}}$.

Because the core is de Sitter, all curvature invariants are finite at $r = 0$, in particular the Ricci scalar approaches the de Sitter value:

$$R(0) = 4\Lambda_{\text{eff}} = \frac{4G_N M \Lambda_G^3}{\sqrt{\pi}}, \tag{124}$$

and thus the classical Schwarzschild curvature singularity at $r = 0$ is replaced by a regular finite-curvature core.

In Schwarzschild gauge (113), the Einstein tensor satisfies $G^t{}_t = G^r{}_r$, so Einstein's equations imply:

$$p_r(r) = -\rho_{\text{eff}}(r). \tag{125}$$

The tangential pressure follows from conservation $\nabla_\mu S^{\mu\nu} = 0$, which gives for $\nu = r$:

$$p_r'(r) + \frac{2}{r}(p_r(r) - p_\perp(r)) = 0 \\ \Rightarrow \quad p_\perp(r) = p_r(r) + \frac{r}{2}p_r'(r) = -\rho_{\text{eff}}(r) - \frac{r}{2}\rho_{\text{eff}}'(r). \tag{126}$$

Near the origin, $\rho_{\text{eff}} \to \rho_0$ and $\rho_{\text{eff}}' \to 0$ implies:

$$p_r(0) \approx -\rho_0, \tag{127}$$

$$p_\perp(0) \approx -\rho_0, \tag{128}$$

so the core has the de Sitter equation of state $p \simeq -\rho$.

For an anisotropic fluid, the strong energy condition (SEC) involves $\rho_{\text{eff}} + p_r + 2p_\perp$. Using $p_r = -\rho_{\text{eff}}$ and $p_\perp \approx -\rho_{\text{eff}}$ near $r = 0$:

$$\rho_{\text{eff}} + p_r + 2p_\perp \approx \rho_0 - \rho_0 - 2\rho_0 = -2\rho_0 < 0, \tag{129}$$

so the SEC is violated in the core[1]. Similarly, the weak and dominant energy conditions are generically violated in a finite region near the origin. This exotic behavior is not inserted by hand as it is an effective stress tensor generated by the nonlocal smearing map $T_{\mu\nu} \mapsto S_{\mu\nu}$ [40].

[1] Violating the SEC in General Relativity means matter or energy behaves in a way that causes spacetime to accelerate its expansion or contract less, essentially exhibiting negative gravity, where tidal forces on freely falling objects are repulsive rather than attractive, allowing for exotic phenomena like wormholes or warp drives, as it breaks the classical expectation that normal matter should cause converging attractive geodesics. It means your matter or energy has properties like negative pressure like dark energy or exotic quantum effects like the Casimir effect that defy standard, well-behaved matter

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Because the regulator acts as a heat-kernel smearing operator, a bare point source is replaced by a smooth Gaussian effective density profile of width $\ell_{\rm nl} \sim \Lambda_G^{-1}$, $\rho_{\rm eff}(r) \propto \exp\left(-\Lambda_G^2 r^2/4\right)$. For radii $r \ll \Lambda_G^{-1}$ the Gaussian is approximately constant, so the effective equation of state becomes de Sitter–like, $p \simeq -\rho$, yielding a finite-curvature core that removes the would-be $r = 0$ singularity. This is also precisely the regime in which the strong energy condition must fail sd the de Sitter–like negative pressure produces a defocusing repulsive contribution that evades the classical geodesic-focusing logic behind the singularity theorems. Away from the origin the density decreases, $\rho'_{\rm eff}(r) < 0$, and the conservation relation $p_\perp = -\rho_{\rm eff} - (r/2)\rho'_{\rm eff}$ forces the tangential pressure $p_\perp$ to increase with r, consequently the SEC combination $\rho_{\rm eff} + p_r + 2p_\perp = 2p_\perp$ can become nonnegative outside a radius of order $\ell_{\rm nl}$. Thus, SEC violation is confined to the nonlocal core scale, while the core itself is unavoidably SEC-violating.

For the exponential regulator in the static limit, the effective density sourced by a bare point mass is a Gaussian:

$$\rho_{\rm eff}(r) = \frac{M\,\Lambda_G^3}{(4\pi)^{3/2}} \exp\left(-\frac{\Lambda_G^2 r^2}{4}\right), \tag{130}$$

of width $\sim \Lambda_G^{-1}$, the nonlocality length $\ell_{\rm nl} \sim \Lambda_G^{-1}$ sets the minimal localisation scale. Using $G^t{}_t = G^r{}_r$ in Schwarzschild gauge gives:

$$p_r(r) = -\rho_{\rm eff}(r), \qquad p_\perp(r) = -\rho_{\rm eff}(r) - \frac{r}{2}\rho'_{\rm eff}(r). \tag{131}$$

For the Gaussian (130) we have:

$$\rho'_{\rm eff}(r) = -(\Lambda_G^2 r/2)\rho_{\rm eff}(r) < 0, \tag{132}$$

for $r > 0$, so the tangential pressure can be written explicitly as:

$$p_\perp(r) = \rho_{\rm eff}(r)\left(\frac{\Lambda_G^2 r^2}{4} - 1\right). \tag{133}$$

The strong energy condition combination for an anisotropic fluid is $\rho_{\rm eff} + p_r + 2p_\perp$, which here reduces to:

$$\rho_{\rm eff} + p_r + 2p_\perp = 2p_\perp = 2\rho_{\rm eff}(r)\left(\frac{\Lambda_G^2 r^2}{4} - 1\right). \tag{134}$$

Since $\rho_{\rm eff}(r) > 0$, the sign of (134) is controlled by the bracket:

$$\rho_{\rm eff} + p_r + 2p_\perp < 0 \quad \text{for} \quad r < \frac{2}{\Lambda_G} \sim \mathcal{O}(\ell_{\rm nl}), \tag{135}$$

$$\rho_{\rm eff} + p_r + 2p_\perp \geq 0 \quad \text{for} \quad r \geq \frac{2}{\Lambda_G}. \tag{136}$$

Thus the SEC violation is confined to the nonlocal core of radius $\mathcal{O}(\ell_{\rm nl})$. In particular, as $r \to 0$ one has $\rho_{\rm eff}(r) \to \rho_0$, approximately constant, so $p_r(0) \simeq p_\perp(0) \simeq -\rho_0$ and the core is de Sitter–like; this is exactly the regime that replaces the Schwarzschild singularity with a regular finite-curvature core, but it is unavoidably SEC-violating. Horizons occur at radii r_h satisfying $f(r_h) = 0$:

$$r_h = 2G_N m(r_h). \tag{137}$$

Because $m(r)$ interpolates smoothly between $m(r) \sim r^3$ for small r and $m(r) \to M$ for large r, the equation (137) can admit two solutions an outer and inner horizon, one degenerate solution an extremal case, or no solution such as a horizonless compact object, depending on the dimensionless combination $G_N M \Lambda_G$. The extremal case additionally satisfies $f'(r_h) = 0$:

$$f'(r_h) = 0 \quad \Leftrightarrow \quad m'(r_h) = \frac{m(r_h)}{r_h}, \tag{138}$$

which, using $m'(r) = 4\pi r^2 \rho_{\text{eff}}(r)$ and $r_h = 2G_N m(r_h)$, yields an explicit extremality condition in terms of ρ_{eff} evaluated at the horizon radius.

For a metric of the form (113), the surface gravity is $\kappa = \frac{1}{2} f'(r_h)$, so the Hawking temperature is [51–53]:

$$T_H = \frac{\kappa}{2\pi} = \frac{f'(r_h)}{4\pi}. \tag{139}$$

In the extremal limit, for a degenerate horizon we have $f'(r_h) \to 0$, hence $T_H \to 0$. Therefore, if evaporation drives the black hole toward the extremal configuration, the terminal phase is a cold, finite-mass remnant in which semiclassical backreaction is suppressed.

7. White-hole Sector in the Maximal Analytic Extension

We let $(\mathcal{M}, g)$ be an asymptotically flat spacetime with future/past null infinity $\mathcal{I}^\pm$ and causal future/past maps $J^\pm(\cdot)$. The black-hole region $\mathcal{B}$ and white-hole region $\mathcal{W}$ are defined by:

$$\mathcal{B} \equiv \mathcal{M} \setminus J^-(\mathcal{I}^+), \tag{140}$$

$$\mathcal{W} \equiv \mathcal{M} \setminus J^+(\mathcal{I}^-). \tag{141}$$

Thus, $p \in \mathcal{B}$ means no future-directed causal curve from p reaches $\mathcal{I}^+$, while $p \in \mathcal{W}$ means no past-directed causal curve from p reaches $\mathcal{I}^-$.

A black hole is a future-trapped region where things can fall in and cannot send signals out to infinity. A white hole is the exact time reverse, a past-trapped region where signals can emerge to infinity, but nothing from $\mathcal{I}^-$ can enter it [3,60–73].

We consider static, spherically symmetric geometries of the form:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{142}$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$. A Killing horizon is a radius $r = r_h$ at which $f(r_h) = o$, and for a simple horizon $f'(r_i) \neq 0$. We define the tortoise coordinate r_* by:

$$\frac{dr_*}{dr} = \frac{1}{f(r)}. \tag{143}$$

Then the (t, r)-sector becomes conformally flat:

$$ds^2_{(2)} = -f(r)\,dt^2 + f(r)\,dr_*^2 = f(r)\big(-dt^2 + dr_*^2\big). \tag{144}$$

Near a simple horizon $r = r_h$, we expand:

$$f(r) = f'(r_h)(r - r_h) + \mathcal{O}((r - r_h)^2), \tag{145}$$

so integrating (143) gives:

$$r_* = \int^r \frac{dr'}{f(r')} = \frac{1}{f'(r_h)} \ln|r - r_h| + (\text{finite}). \tag{146}$$

Hence $r_* \to \pm\infty$ at the horizon and the original (t, r) chart is incomplete there.

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Now introduce Eddington–Finkelstein null coordinates [54]:

$$u \equiv t - r_*, \qquad v \equiv t + r_*, \tag{147}$$

so radial null curves satisfy $u =$ const or $v =$ const. We define the surface gravity at the simple horizon by:

$$\kappa_h \equiv \frac{|f'(r_h)|}{2}. \tag{148}$$

A Kruskal-type extension across $r = r_h$ is obtained with [4,55,60]:

$$U_h \equiv -e^{-\kappa_h u}, \qquad V_h \equiv e^{\kappa_h v}. \tag{149}$$

Then:

$$U_h V_h = -e^{2\kappa_h r_*}. \tag{150}$$

Using (146), one finds $U_h V_h \propto (r - r_h)$ near the horizon, so $U_h = 0$ or $V_h = 0$ is a regular null surface representing the horizon. Crucially, the mapping (149) covers four sign choices:

$$(U_h, V_h) \in \{(+,+), (+,-), (-,+), (-,-)\}, \tag{151}$$

corresponding to four Kruskal quadrants. In the maximally extended spacetime of a static, eternal solution, two distinct interior regions appear, the black-hole interior is the region bounded by the future event horizon $\mathcal{H}^+$ where $V_h = 0$ in an appropriate convention into which future-directed causal curves can enter but cannot return to $\mathcal{I}^+$. The white-hole interior is the region bounded by the past event horizon $\mathcal{H}^-$ where $U_h = 0$ from which future-directed causal curves can emerge to the exterior, but which cannot be entered from $\mathcal{I}^-$.

Thus, in any time-symmetric maximal extension of a static black-hole metric with a simple horizon, the white-hole region arises automatically as the time-reversed Kruskal quadrant.

Many UV-improved/regular metrics exhibit two simple horizons:

$$0 < r_- < r_+, \qquad f(r_\pm) = 0, \tag{152}$$

an outer event horizon r_+ and an inner Cauchy horizon r_-. Because typically $\kappa_+ \neq \kappa_-$, one cannot construct a single global Kruskal chart that regularizes both horizons simultaneously; instead one uses a Kruskal patch adapted to r_+ and another adapted to r_-.

In the two-horizon regime, the maximally extended conformal diagram contains and exterior asymptotically flat regions with $\mathcal{I}^\pm$, both a black-hole sector bounded by $\mathcal{H}^+$) and a white-hole sector bounded by $\mathcal{H}^-$, an inner Cauchy horizon associated with $r = r_-$, and an interior region that, in classical RN, terminates at a curvature singularity at $r = 0$.

In a regular black-hole geometry, the last item is modified as the $r \to 0$ region is nonsingular. In particular, if the core approaches a de Sitter static patch:

$$f(r) = 1 - \frac{\Lambda_{\rm eff}}{3} r^2 + \mathcal{O}(r^4), \qquad (r \to 0), \tag{153}$$

then curvature invariants remain finite at $r = 0$, and the conformal diagram is RN-like except that the singular boundary is replaced by a regular timelike center a de Sitter-like core.

The discussion above concerns the maximal analytic extension of the exact static metric (142). Astrophysical black holes formed by gravitational collapse are not time symmetric as they typically contain a future horizon $\mathcal{H}^+$ but not a past horizon $\mathcal{H}^-$. Consequently, the white-hole sector is best understood as a feature of the idealized eternal solution, not a generic prediction for realistic collapse.

In two-horizon geometries, the inner horizon is a Cauchy horizon. In classical GR it is known to be sensitive to perturbations, mass inflation [56,57], in UV-complete models the core may be regular,

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but the dynamical stability of the Cauchy horizon must be analyzed separately. This does not affect the purely kinematical point proven above: the presence of $\mathcal{H}^-$ in the maximal extension is a consequence of the Kruskal construction for static horizons, and the replacement of the $r = 0$ singularity by a regular core modifies the diagram by removing the singular boundary.

8. Thermodynamics and Evaporation of the Nonlocal Black Hole

Black-hole thermodynamics in regular singularity-free geometries is controlled primarily by the near-horizon structure rather than by the detailed behavior at $r = 0$. In the present nonlocal construction the Einstein equations retain their usual local form, while the ultraviolet completion enters through the regulator $F(\Box/\Lambda_G^2)$, which smears a point source into an effective, finite energy density $\rho_{\rm eff}(r)$ and thereby modifies the mass profile $m(r)$ and the metric function $f(r) = 1 - 2G_N m(r)/r$. As a consequence, the horizon equation $f(r_h) = 0$ can admit two, one (degenerate), or no horizons, and the Hawking temperature of the outer horizon is still defined by the standard surface-gravity relation $T_H = f'(r_h)/(4\pi)$ but becomes a nontrivial function of the nonlocal scale $\ell_{\rm nl} = \Lambda_G^{-1}$. In this subsection we review the horizon structure and derive the corresponding temperature and entropy relations, emphasizing the qualitative departure from the Schwarzschild case: as the horizon approaches the nonlocal core scale, T_H can decrease to zero at an extremal configuration, suggesting a cold remnant endpoint for evaporation.

We consider the static, spherically symmetric line element in Schwarzschild gauge

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \tag{154}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ is the unit S^2 metric, and $f(r)$ is the metric function. Newton's constant is G_N and we define $\kappa \equiv 8\pi G_N$ when needed.

In the nonlocal theory, the UV/short-distance completion is encoded by an entire-function regulator $F(\Box/\Lambda_G^2)$ (or equivalently $F(\Box/M_*^2)$, with $\Lambda_G \sim M_*$ in the gravitational sector), acting covariantly through the d'Alembertian $\Box$. We denote the associated nonlocal length by

$$\ell_* \equiv \frac{1}{M_*}, \qquad \ell_{\rm nl} \equiv \frac{1}{\Lambda_G}, \tag{155}$$

and in what follows we keep both M_* and Λ_G explicit.

It is convenient to introduce the Misner–Sharp mass function $m(r)$ by:

$$f(r) = 1 - \frac{2G_N m(r)}{r}. \tag{156}$$

For a static, spherically symmetric effective smeared stress tensor $S^\mu{}_\nu = \mathrm{diag}(-\rho_{\rm eff}(r), p_r(r), p_\perp(r), p_\perp(r))$, the tt-component of Einstein's equations yields the standard relation:

$$m'(r) = 4\pi r^2 \rho_{\rm eff}(r), \tag{157}$$

where a prime denotes d/dr. In the exponential-regulator model, a bare point mass M becomes a Gaussian effective density. Using the Λ_G-normalization, width $\sim \Lambda_G^{-1}$):

$$\rho_{\rm eff}(r) = M\frac{\Lambda_G^3}{(4\pi)^{3/2}}\exp\left(-\frac{\Lambda_G^2 r^2}{4}\right), \tag{158}$$

and integrating (157) gives the closed form:

$$m(r) = M\left[\mathrm{erf}\left(\frac{\Lambda_G r}{2}\right) - \frac{\Lambda_G r}{\sqrt{\pi}}\exp\left(-\frac{\Lambda_G^2 r^2}{4}\right)\right]. \tag{159}$$

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Equivalently, we can write the same expression with M_* in place of Λ_G and $\ell_* = M_*^{-1}$. As $r \to \infty$, $m(r) \to M$, so the geometry is asymptotically Schwarzschild, while as $r \to 0$ we find $f(r) = 1 - (\Lambda_{\text{eff}}/3)r^2 + \mathcal{O}(r^4)$, a de Sitter core. A Killing horizon radius r_h is any positive root of $f(r_h) = 0$:

$$f(r_h) = 0 \quad \Longleftrightarrow \quad r_h = 2G_N m(r_h). \tag{160}$$

Depending on the dimensionless combination $G_N M \Lambda_G$, (160) can admit two horizons, an outer r_+ and inner r_-, one degenerate horizon called extremal, or no horizon.

More explicitly, the horizon structure is controlled by the dimensionless combination $\mu \equiv G_N M \Lambda_G$, equivalently $G_N M/\ell_{\text{nl}}$. There exists a critical minimal mass M_0 such that:

$$M > M_0 \;\Rightarrow\; \text{two horizons } (r_-, r_+), \tag{161}$$

$$M = M_0 \;\Rightarrow\; \text{one degenerate horizon } r_- = r_+ = r_0, \tag{162}$$

$$M < M_0 \;\Rightarrow\; \text{no horizon.} \tag{163}$$

The critical values r_0 and M_0 are determined by the double-root, extremality conditions $f(r_0) = 0 = f'(r_0)$ derived below. Let $\chi = \partial_t$ be the static Killing vector. For a metric of the form (154), the surface gravity at a simple horizon is:

$$\kappa_h = \frac{|f'(r_h)|}{2}, \tag{164}$$

and the associated Hawking temperature is:

$$T_H(r_h) = \frac{\kappa_h}{2\pi} = \frac{|f'(r_h)|}{4\pi}. \tag{165}$$

For the outer horizon r_+ one typically has $f'(r_+) > 0$ and we may drop the absolute value.

Using (156), we compute:

$$f'(r) = 2G_N\left(\frac{m(r)}{r^2} - \frac{m'(r)}{r}\right), \tag{166}$$

so at a horizon (160) implies $m(r_h) = r_h/(2G_N)$ and therefore:

$$f'(r_h) = \frac{1}{r_h} - \frac{2G_N}{r_h} m'(r_h). \tag{167}$$

Substituting into (165) gives a compact general expression:

$$T_H(r_h) = \frac{1}{4\pi r_h}\left[1 - 2G_N m'(r_h)\right]. \tag{168}$$

Using (157), this can be written directly in terms of the effective density at the horizon:

$$T_H(r_h) = \frac{1}{4\pi r_h}\left[1 - 8\pi G_N r_h^2 \rho_{\text{eff}}(r_h)\right]. \tag{169}$$

For large black holes $r_h \gg \ell_{\text{nl}}$, the Gaussian tail makes $\rho_{\text{eff}}(r_h)$ exponentially small, so (169) reduces to the Schwarzschild result $T_H \simeq 1/(4\pi r_h) = 1/(8\pi G_N M)$.

For the explicit Gaussian profile (158), we can also write (169) as:

$$T_H(r_h) = \frac{1}{4\pi r_h}\left[1 - \frac{G_N M\, r_h^2}{\sqrt{\pi}\, \ell_{\text{nl}}^3} \exp\left(-\frac{r_h^2}{4\ell_{\text{nl}}^2}\right)\right], \tag{170}$$

$\ell_{\text{nl}} = \Lambda_G^{-1}$. Eliminating M in favor of r_h using (160) and (159) yields a purely dimensionless representation. We define:

$$x_h \equiv \frac{\Lambda_G r_h}{2} = \frac{r_h}{2\ell_{\text{nl}}}, \qquad \mathcal{F}(x) \equiv \text{erf}(x) - \frac{2x}{\sqrt{\pi}} e^{-x^2}, \tag{171}$$

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so that $m(r) = M\,\mathcal{F}(x)$ and (160) gives:

$$M(r_h) = \frac{r_h}{2G_N\,\mathcal{F}(x_h)}. \tag{172}$$

Then (170) becomes:

$$T_H(r_h) = \frac{1}{4\pi r_h}\left[1 - \frac{4x_h^3 e^{-x_h^2}}{\sqrt{\pi}\,\mathcal{F}(x_h)}\right]. \tag{173}$$

When $M > M_0$, the equation $f(r) = 0$ admits an outer event horizon r_+ and an inner Cauchy horizon r_-. Each horizon has an associated surface gravity:

$$\kappa_\pm = \frac{|f'(r_\pm)|}{2}, \qquad T_\pm = \frac{\kappa_\pm}{2\pi} = \frac{|f'(r_\pm)|}{4\pi}. \tag{174}$$

In general $T_+ \neq T_-$, so the spacetime does not admit a single global equilibrium temperature. For an asymptotic observer, the semiclassical Hawking flux is governed by the outer-horizon temperature T_+, while the inner horizon controls the causal structure of the interior and may be relevant to stability analyses, but does not define the temperature measured at infinity.

A degenerate extremal horizon occurs when $f(r)$ has a double root:

$$f(r_0) = 0, \qquad f'(r_0) = 0. \tag{175}$$

Using (166), $f'(r_0) = 0$ is equivalent to:

$$m'(r_0) = \frac{m(r_0)}{r_0}, \tag{176}$$

and using (160) and (157) this becomes the particularly transparent condition:

$$8\pi G_N r_0^2\,\rho_{\text{eff}}(r_0) = 1. \tag{177}$$

From (169) we immediately see that extremality implies:

$$T_H(r_0) = 0. \tag{178}$$

Hence, if semiclassical evaporation drives the black hole toward the degenerate configuration, the endpoint is a cold object of finite mass: a remnant.

For the Gaussian profile (158), the extremality condition can be written purely in terms of $x_0 = \Lambda_G r_0/2$:

$$1 = \frac{4x_0^3 e^{-x_0^2}}{\sqrt{\pi}\,\mathcal{F}(x_0)}. \tag{179}$$

This transcendental equation has a unique positive solution:

$$x_0 \simeq 1.5112, \tag{180}$$

so the extremal remnant horizon radius is:

$$r_0 = \frac{2x_0}{\Lambda_G} \simeq \frac{3.0224}{\Lambda_G} \simeq 3.0224\,\ell_{\text{nl}}, \tag{181}$$

and similarly $r_0 \simeq 3.0224\,\ell_*$ if $\Lambda_G = M_*$. Using (172) at $r_h = r_0$ and $\mathcal{F}(x_0) \simeq 0.79366$, the remnant mass is:

$$M_0 = \frac{r_0}{2G_N\,\mathcal{F}(x_0)} = \frac{x_0}{G_N\,\mathcal{F}(x_0)}\,\ell_{\text{nl}} \simeq \frac{1.904}{G_N\,\Lambda_G} \simeq \frac{1.904}{G_N\,M_*}. \tag{182}$$

Therefore the UV scale controlling nonlocal smearing sets the parametric size and mass of the terminal object.

Because the field equations are in Einstein form with constant G_N and all UV structure is packaged into the effective source, the natural semiclassical entropy assignment is the Bekenstein–Hawking area law for the outer horizon:

$$S_{\rm BH}(r_+) = \frac{A_+}{4G_N} = \frac{\pi r_+^2}{G_N}. \tag{183}$$

Together with $T_H(r_+)$ from (169)–(173), this implies the standard qualitative evaporation behavior as for $r_+ \gg \ell_{\rm nl}$ one has Schwarzschild-like heating as the mass decreases, but once the horizon approaches the nonlocal core scale, the second term in (169) becomes important, T_H reaches a maximum and then decreases, tending to zero as $r_+ \to r_0$. In that regime, Hawking emission is suppressed and the evolution asymptotes to the cold remnant (181)–(182).

For $M < M_0$ the metric function remains positive, $f(r) > 0$ for all $r > 0$, and no event horizon forms. In this case there is no Hawking temperature and no Bekenstein–Hawking entropy, since these notions are intrinsically tied to the presence of a horizon. The solution should instead be interpreted as a regular, horizonless compact object supported by the smeared effective stress tensor, with a de Sitter–like core at small r.

In the subcritical regime $M < M_0$ one has $f(r) > 0$ for all $r > 0$, so the spacetime contains no (Killing) horizon [44]. Consequently there is no horizon surface gravity κ_h and no associated Hawking temperature, which in the horizon-present cases is defined by $T_H = \kappa_h/(2\pi) = f'(r_h)/(4\pi)$ [44,53]. Likewise, there is no Bekenstein–Hawking entropy term $S_{\rm BH} = A_h/(4G_N)$ [44,51] nor its covariant generalization as a Wald–Iyer Noether-charge integral [45,46], because these notions are intrinsically horizon quantities evaluated on a bifurcation two-surface. In this sense no entropy means the absence of a horizon-entropy contribution in the thermodynamic first law, not the absence of possible model-dependent statistical entropy carried by the effective horizonless compact object supported by the smeared stress tensor [47–50].

9. Conclusions

We have given an explicit derivation of the de Sitter core that appears in the static, spherically symmetric solutions of the nonlocal gravitational sector. Starting from the covariant entire-function regulator $F(\Box/M_*^2)$, we showed that a point-like mass source is mapped on the real slice to a Gaussian effective energy density with width $\ell_* = M_*^{-1}$. Solving the Einstein equations with this source yields a mass function $m(r)$ that approaches M at large radii and behaves as $m(r) \propto r^3$ near the origin. The corresponding metric function is and its small-r expansion matches a de Sitter static patch with effective cosmological constant $\Lambda_{\rm eff} = G_{\rm N} M M_*^3/\sqrt{\pi}$.

In our language, the de Sitter core is a local manifestation of a positive-curvature saddle in the complexified cosmological-constant plane [58,59]. The same holomorphic, gauge- and diffeomorphism-covariant regulator that achieves UV finiteness also enforces regularity of black-hole interiors, unifying singularity resolution with quantum consistency.

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