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Article

Accelerated Expansion of the Universe as a Quantum Gravity Phenomenon

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Abstract

We have known approximately 100 years that the Universe is expanding. But the striking discovery came at the end of 20th century, when we found that it is expanding with acceleration. Since then, many models have been developed in cosmology to explain this phenomenon. One of the possible elucidations is that the theory of gravity must be modified at the classical level. But many such models were already excluded by the gravitational wave experiments. Therefore, we must slowly start to ask a much more urgent question: could accelerated expansion of the Universe be a phenomenon of quantum gravity ? We review the basic models of how we explain the origin of dark energy or the cosmological constant in metastring theory, discrete approaches to quantum gravity, group field theory, non-commutative geometry, causal dynamical triangulation, asymptotic safety, models based on holography and entropic gravity. We mention at the end a newly formed approach to the quantization of gravity, the ring paradigm, which would, just after the application to cosmology, naturally model the late-time accelerated expansion epoch in the Universe. But what is completely striking is that the final formulation of this theory would ultimately solve the old problem of the cosmological constant.

Keywords: dark energy; late-time accelerated expansion of the Universe; cosmological constant; string theory; loop quantum gravity; causal set approach; non-commutative geometry; causal dynamical triangulation; asymptotic safety; entropic gravity

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1. Introduction

If we want to track down the origin of modern physics, we should definitely mention the breakthrough year 1687, when the Principia of Isaac Newton was published, [1]. The three-volume work expounds Newton's laws of motion and his law of universal gravitation. Then approximately 220 years of mathematical reformulations followed. Finally, quantum theory was born in 1900, [2] and the special theory of relativity in 1905, [3]. The new theory of gravity, the GR, was formulated in 1915, [4]. But without the long work of mathematicians such as was, for example, Jean d'Alembert, Joseph-Louis Lagrange, William Rowan Hamilton, or Bernhard Riemann, these discoveries at the beginning of 20th century would hardly have been possible. We could therefore assume that now we are still in the period of finding the new mathematical apparatus, which we would need for the formulation of the quantum gravity (QG) theory, unifying quantum field theory (QFT), [5], and general relativity (GR), [4]. This is also the reason why the widely studied string theory,[6–9], as a candidate for a unified theory of all interactions, has been so successful so far. Because it is connected with very interesting mathematical physics, [10–17].

But mathematics is not the only thing on which we focus in modern theoretical physics. It is often said that we live in the golden age of cosmology, the science about the origin and development of the whole universe, [18]. We can simply test our modern physical theories on the cosmological models, which give us valuable data by new technologies. And this should be exactly the case of modern

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gravity theories. Further, we already knew a very long time ago (since Franc Zwicky, [19] pointed out the problem in the 1930s) that we were missing some type of matter in the universe, which we call dark matter (DM). But we should not be surprised that the complete composition of the massive objects around us is unknown (DM creates approximately 27% [20]), because we are studying what we are actually made of. Much more surprising is that there also exists dark energy (DE), which creates approximately 68% of the energetic composition in the Universe, [20]. And the fascinating thing is that it could allude to the fact that we don't know everything about the theory of gravity yet.

The Big Bang model in cosmology is based on three discoveries. One of them is Hubble's revelation of the expansion of the Universe, which was measured by Edwin Hubble in 1929 by the 100-inch Hooker telescope on Mount Wilson in California, [21]. He discovered that the farther a galaxy is from us, the faster it will be receding into space away from us. When $H = \frac{\dot{a}}{a}$ is the Hubble's constant, which has a value of approximately 72 km/(s.MPc) (now there was some tension concerning the true value of the constant, [22]), $\vec{v}$ is the velocity of the receding of the galaxy and $\vec{r}$ is the distance from us, the formula will be

$$\vec{v} = H\vec{r}. \tag{1}$$

This fact only tells us that the Universe had to have some beginning, and the expansion is a remnant of the initial explosion. But this was not the end of the story.

Albert Einstein initially added the cosmological constant Λ to the Einstein equations of GR because he wanted to obtain the static universe, [23]. As it was later found out that the model must be dynamical, the reason for keeping the constant in the equations disappeared, [21]. But the observations of supernovas (of type IA) in 1998 gave us the most direct evidence for mimicking the positive Λ, [24,25]. Let's study a little bit the detail of how we measure the sign of $\ddot{a}$, [26]. Actually, we could deduce the value of the acceleration of the scale factor $\ddot{a}$ from the plot of luminosity versus redshift, and we need only to analyze the behavior of electromagnetic waves in curved spacetime for understanding the effect. The key equation for the "luminosity distance", or normalized "dimness", d_L of a known source that is not too far away was

$$Hd_L = z + \frac{1}{2}(1 + \frac{a\ddot{a}}{\dot{a}^2})z^2 + O(z^3), \tag{2}$$

where z is the redshift of the light from the supernova. We can clearly deduce both $\dot{a}$ and $\ddot{a}$ if we know how d_L varies with z. Moreover, it is possible to see, when other quantities are equal, that a larger $\ddot{a}$ will produce an image that is dimmer at equal redshift. Therefore, the graph of d_L as a function of z thus shifts upward, and this has been seen.

The sign of this effect could also be deduced from the equivalence principle, where we use only what we know of the Doppler shift in flat spacetime. According to the equivalence principle, spacetime is flat in our neighborhood. So, what we usually describe as the expansion of space, we could reinterpret locally as the flow of galaxies away from us. When we consider such a coordinate system, two supernovas of equal brightness are at equal spatial distance from us. Therefore, their light was emitted at the same time t_0 in the past. Now we put these two stars into two different spacetimes with the same expansion rate H but different accelerations $\ddot{a}$. In the spacetime with greater acceleration, the expansion rate at t_0 was smaller; therefore, the recession rate of the supernova was smaller, whence its redshift z was also less. We can equivalently say the supernova appears dimmer at the same redshift.

So, the observation of Saul Perlmutter, Brian Schmidt and Adam Riess, [24,25], tells us that the expansion in the last 5 billion years is accelerating. The key question is: What should be pumping energy to the Universe? Because acceleration needs some driving force. Further, how long will it be accelerating? Is the mechanism behind this late-time cosmic acceleration somehow connected to the mechanism behind cosmological inflation at the beginning of the evolution of the universe? We will now briefly mention some points concerning this direction of physics.

As we already mentioned, the classical theory of gravity is the GR, [4]. It is so far a paradigm, which is perfectly consistent with the experiment. For this reason, we must take the results based on it very seriously, especially the applications to cosmology. But we already know that the Friedmann models, which were the first attempts at models in cosmology based on Einstein equations, were already excluded by observations. We gradually replaced them with the ΛCDM model,[27], which is currently the most recognized model in cosmology. However, even this model has serious shortcomings.

We will not comment that we are completely unsure what DM is, [28]. The serious trouble actually is the value of the cosmological constant, and there are serious, difficult problems connected with it. When we simply write the Einstein equations with a cosmological constant, we get

$$G_{\mu\nu} + \Lambda_b g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{3}$$

$g_{\mu\nu}$ is the metric, and $T_{\mu\nu}$ stands for the energy momentum tensor. The second Friedmann equation gives us

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_j (\rho_j + \frac{3p_j}{c^2}) + \frac{\Lambda_b c^2}{3}, \tag{4}$$

where j labels the different matter components, and we see that Λ_b is responsible for the accelerated expansion ($\ddot{a} > 0$). But it is a mystery why the cosmological constant has such a small value [29–32]. We know that matter fields must be quantized, which indirectly implies that we have a persisting energy density ρ_{vac} even in the case of the vacuum. The energy momentum tensor of the perfect-fluid type

$$T_{\mu\nu} = -\rho_{vac}\, g_{\mu\nu} \tag{5}$$

corresponds to the vacuum energy density ρ_{vac}. The effective value of the cosmological constant is a sum of the two contributions from the vacuum energy and the bare constant:

$$\Lambda_{eff} \equiv \Lambda_b + \frac{8\pi G}{c^4} \rho_{vac} \tag{6}$$

We could find a bound that $|\rho_{vac}| \approx 2.10^{11} \rho_{nucl}$, ρ_{nucl} is the density of atomic nuclei. Therefore, we expect the absolute value of the vacuum energy density to be 11 orders of magnitude higher than the density of atomic nuclei, which is the old cosmological constant problem.

Another issue concerns the value of Λ_{eff}, because we have

$$\frac{c^4 \Lambda_{eff}}{8\pi G} \approx 6.10^{-27} \frac{kg}{ms^2}. \tag{7}$$

It is approximately 55 orders of magnitude smaller than ρ_{vac}, which forces Λ_b to be enormously fine-tuned. We call it the new cosmological constant problem, but this is not the last puzzle. While other kinds of matter dilute during the Universe's expansion, the vacuum energy density remains constant. The constant Λ_{eff} gives today a dominant contribution to the overall energy content in the Universe, so why should it be of almost the same order of magnitude as the density of the non-relativistic matter (the coincidence problem)?

This is exactly the reason for doing research in modified theories of gravity, which model the late-time accelerated expansion in an alternative way, [33]. The basic division is on models, which simply add relevant terms with some form of scalar field on the RHS of Einstein equations. These models are motivated by the investigations in particle physics. Another possibility is to modify the Einstein-Hilbert action so the LHS of Einstein equations (an example is $f(R)$ theories, which even allow us to build one model of late-time cosmic acceleration together with the cosmological inflation at the beginning of the evolution of the Universe), or to use models with extra dimensions, where gravitons can leak out into them (an example is DGP models). But to be honest, most of the modified gravity scenarios were already excluded by the LIGO-VIRGO-KAGRA collaboration,[34]. Therefore, we need to start to search for other models.

Of course, studying another model means resorting to a new shell of physics, to QG. And do we have any reason to rely on the fact that the accelerated expansion of the universe is really a phenomenon of QG? First of all, we are completely on safe ground from that point of view: if this were not the case, we would be building much more advanced models, which should always be useful in classical physics. Simply, a very sophisticated theory can bring further knowledge, although it has nothing to do with solving the given problem. But we have very good reasons to suppose that accelerated expansion of the Universe could not be explained by classical physics.

There was recently developed a new approach to QG, [35–38], which actually solves the problem of DE and brings us first hints on how to resolve the old problem of the cosmological constant. If these results are confirmed, it would mean that some phantom field is present in the Universe (and what would be far more surprising, it is probably the phantom field necessary for building the cyclic model of Paul Steinhardt and Neil Turok, [37,39].) And further, the no-go theorems in QFT must be somehow obeyed because the model is based on Lorentz breaking in the gravity sector. This would have a serious consequence for the currently well-known approaches to QG: the appearance of cosmological constants is not random, and also models of DE could be found in applications of these theories in a completely generic way.

We want to present the current state of knowledge in cosmology concerning the models of the late-time accelerated expansion in applications of QG theories in this review article, and we will look for a common thread in often very different approaches, which are even contradictory, such as the string theory and the causal set approach. If in most of the paradigms we would find the presence of this phenomenon, it would be an indication of a very strong result! Because then we can be almost sure that some of the outcomes are based on the true models of Nature around us. The difficult thing is that today there are many differently developed models of QG. The good news is that there are not too many models of accelerated expansion in cosmology that are based on them.

The present paper will be organised as follows. There will be a short introduction to string theory with AdS/CFT correspondence, loop quantum gravity, group field theory, the causal set approach, asymptotic safety, causal dynamical triangulations, non-commutative geometry, and entropic gravity. Then the main part concerning the models of accelerated expansion in the applications of these paradigms will follow. And finally, we will discuss a new approach to QG and its consequences for the models in cosmology.

2. Approaches to Quantum Gravity

2.1. String Theory with AdS/CFT Correspondence

We will begin our journey by making some remarks about string theory, [8,9,40,41]. The first paper in this discipline was written by Gabriel Veneziano in 1968 to explain the physics of strong interactions, [42]. The elementary particles are described as a vibration of strings having the spacetime coordinates $X^\mu, \mu = 0, 1..., d$. The superstring theories are defined in 10 or 11 spacetime dimensions (so $d = 10$ or $d = 11$, which we obtained as a result of the application of the Lorentz invariance). $X^\mu = X^\mu(\tau, \sigma)$ is a function of two parameters τ and σ. The first step in the formulation of the theory leads to a prescription of the action (the Nambu-Goto action of the relativistic string):

$$S = -\frac{T_0}{c}\int_{\tau_i}^{\tau_f}\int_0^{\sigma_1}\sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2}\; d\sigma d\tau, \tag{8}$$

where T_0 denotes the tension of the string. We used the notation

$$\dot{X}^\mu \equiv \frac{\partial X^\mu}{\partial \tau} \text{ and } X'^\mu \equiv \frac{\partial X^\mu}{\partial \sigma} \tag{9}$$

for derivatives. The equations of motion follow from this action, and then we quantize the theory. It is considered a new thing in phenomenology that the strings must be attached somewhere (when the boundary conditions are established); therefore, the introduction of branes is inevitable. String theory comes as the only QG paradigm so far with a clear explanation of what a graviton is, and therefore it would be a candidate for the unified theory of all interactions. The mathematical foundations of string theory stimulated research in large areas of algebraic geometry ([10–13]) as well.

As we already stressed, the origin of string theory was connected with particle physics, and cosmology did not play a big role in the first stages of its development [43]. These conditions quickly changed with the observation of accelerated expansion: it looked as if the cosmological constant $\Lambda = \Lambda_b$ (we will use the notation Λ for Λ_b in the rest of the article) - or something that caused a similar behavior - vanished on the scales of particle physics, but there is a mechanism allowing a tiny non-zero value; therefore, the anthropic principle started to be used. (It more or less says that the value of the cosmological constant, which we assume to randomly vary from one region to another, has a fine-tuned value, which is a property of a local region guaranteeing suitable conditions for life.) Then the investigations of the multiverse in string theory began. Other parts of the cosmology were developing as well, and the cosmological inflation was considered as our first look at the multiverse. But string theory has been heavily criticized. Actually, how could such a theory be falsifiable when it contains approximately 10^{500} different vacua? We can argue that the landscape exists and we more or less know its properties, but what if this is not true?

We may fall into circular reasoning: we observe the de Sitter space (because of accelerated expansion), we suppose that string theory is a serious candidate for QG, and we conclude that it has to contain the de Sitter space. But we know about the big difficulties, on how to incorporate it into this theory. A surprising result was published in [44] that one can successfully solve the problem of DE and the observed de-Sitter spacetime in a generic, non-commutative, generalized geometric phase space formulation of string theory. The application of this slight modification of string theory to cosmology, called metastring theory, will be our topic in this review paper.

The AdS/CFT correspondence is a duality relating QFT and gravity. If we were to say it more precisely, the correspondence relates the quantum physics of strongly correlated many-body systems to the classical dynamics of gravity in one higher dimension. This duality is also frequently referred to as the holographic duality or the gauge/gravity correspondence. In its original formulation,[45–47], the correspondence related a four-dimensional conformal field theory to the geometry of an anti-de Sitter space in five dimensions. When we study the collective phenomena in condensed matter physics, it is quite common that when a system is strongly coupled, it reorganizes itself in such a way that new weakly coupled degrees of freedom emerge dynamically, and the system could be better described in terms of fields representing the emergent excitations. The AdS/CFT correspondence is a new example of this paradigm. The very surprising feature is that the emergent fields live in a space with one extra dimension and that the dual theory is a gravity theory. Actually, the extra dimension is related to the energy scale of the QFT. The holographic description is a geometrisation of the quantum dynamics of the systems with a large number of degrees of freedom, which makes it obvious that there are deep connections between quantum mechanics and gravity.

The gauge/gravity duality was discovered in the context of string theory, where it is now quite natural to realize gauge field theories on hypersurfaces embedded in a higher-dimensional space in a theory that contains gravity. However, the study of the correspondence has been extended to include very different areas of research, such as the analysis of the strong coupling dynamics of quantum chromodynamics and the electroweak theories, [48–50], the physics of black holes and QG, [51–53], relativistic hydrodynamics [54–56], or different applications in condensed matter physics (holographic superconductors,[57], quantum phase transitions, [58], cold atoms, [59], etc.). In this review we will concentrate on the applications to cosmology.

2.2. Loop Quantum Gravity

The second most widely studied approach to QG so far is loop quantum gravity. It is an approach based on the quantization of GR, [60,61]. The first announcement on this line of research was given by Carlo Rovelli and Lee Smolin at the conference in India in 1987 (based on the works of Abhay Ashtekar). We start the computation with the Einstein-Hilbert action

$$S = \frac{c^4}{16\pi G}\int \sqrt{-g}R\, d^4x. \tag{10}$$

It is necessary to get in touch with quantum mechanics, therefore the 3+1 splitting is used. We choose some 3-dimensional surface Σ and we define on it the 6-component induced Riemannian metric q_{ab}. The remaining 4 degrees of freedom create the lapse function N and the shift-vector N^a. The action becomes (after performing the Legendre transformation)

$$S[q_{ab}, \pi^{ab}, N, N_a] = \frac{c^4}{16\pi G}\int\int_\Sigma \Big\{ \pi^{ab}\dot{q}_{ab} + \tag{11}$$

$$2N_b\nabla_a^{(3)}(q^{-\frac{1}{2}}\pi^{ab}) + N\big[q^{\frac{1}{2}}(R^{(3)} - q^{-1}\pi_{cd}\pi^{cd} + \frac{1}{2}q^{-1}\pi^2)\big]\Big\}\, d^3x dt, \tag{12}$$

π^{ab} is the momentum conjugate to q_{ab} and $\pi = \pi^{ab}q_{ab}$ ($\nabla_a^{(3)}$ denotes a covariant derivative corresponding to the metric q_{ab}). This form of the action is more suitable when we want to incorporate the commutation relations of quantum mechanics and use the Ashtekar-Barbero variables.

We also got in touch with interesting mathematics during the work on loop quantum gravity, [62], but the most useful achievements come from cosmology, [63]. It could give us an explanation, of what was happening at the beginning of the evolution of the Universe because the loop quantum cosmology modified the Friedmann equation:

$$H^2 = \frac{\dot{p}^2}{4p^2} = \frac{8\pi G}{3}\rho(1 - \frac{\rho}{\rho_{crit}}), \tag{13}$$

where the critical density is $\rho_{crit} = \frac{3}{8\pi G\beta^2\mu^2 p}$, $\mu = \frac{3\sqrt{3}}{2}$ and β labels the Barbero-Immirzi parameter. We could see that the expression for H^2 is not always positive-definite. In particular, it equals zero when $\rho = \rho_{crit}$. If one runs the evolution of the Universe backwards, the point with $H^2 = 0$ is reached. It means that the bounce would replace the Big Bang. Also, a solution to the black hole singularity problem was suggested in loop quantum gravity, [64] therefore, we can point out to it as a paradigm with non-trivial phenomenological results.

2.3. Group Field Theory

With the work of t' Hooft [65], it was recognized that QFT of an $N \times N$ matrix field can be formally expanded into Feynman amplitudes indexed by ribbon graphs or combinatorial maps, [66,67]. Those two equivalent combinatorial structures formalize the notion of a graph embedded into a canonically associated surface and the genus of that surface indexes a topological large N expansion in which $\frac{1}{N^2}$ takes the role of a formal small parameter. The t' Hooft's original idea of using this expansion to probe the non-perturbative sector of QFT found numerous applications to enumerative combinatorics, and more specifically, to two-dimensional QG. Indeed, formal matrix integrals could be used to efficiently manipulate generating functions of combinatorial maps, and asymptotic enumeration results for such objects can be inferred by studying the critical properties of the underlying matrix model. We could see from some physical point of view that such a critical regime can be interpreted as defining the continuum limit of a two-dimensional random geometry which is equivalent to Liouville QG.

Tensor models were discovered in the early nineties as an attempt to establish an analogous correspondence between random tensors and Euclidean QG in dimension $D \geq 3$. These early works pointed to a prolific correspondence between the Feynman expansion of such a tensor model and discrete geometries in dimension D. But no generalization of the large N expansion of matrix models could be identified at the time. This impeded further progress on tensor models and their relationship to random geometry.

On the other side, dynamical triangulations and spin foam models rely on a path integral formulation of QG when we define the path integral over metrics

$$\Psi(g|_{\partial\mathscr{M}}) = \sum_{\substack{topologies\\ \partial\mathscr{M}\ fixed}} \int_{g|_{\partial\mathscr{M}}\ fixed} [Dg] \exp \frac{iS[g]}{\hbar} \tag{14}$$

It involves an integration over metrics modulo diffeomorphisms on a space-time manifold $\mathscr{M}$ which is reduced to a fixed metric on the boundary. Because it is a functional of the boundary metric, it can be considered as the wave function of the quantum gravitational field. We want to note that fluctuations of the topology of space-time are also allowed, even if these can be omitted in a first approximation.

Then, group field theory arises from the merging of tensor models and spin foam models. The first model has been proposed by Dmitri Boulatov for three-dimensional QG (in 1992), which is actually the topological BF theory. Then the generalization to BF theory in four dimensions was made by Hirosi Ooguri. Group field theory is actually a QFT whose Feynman graph expansion reproduces spin foam amplitudes

$$\int [\mathscr{D}\Phi] \exp S[\Phi] = \sum_{\substack{triangulations\ T \Leftrightarrow\\ Feynman\ graphs}} \frac{1}{C_T} \mathscr{A}_T \tag{15}$$

The group field theory Feynman graphs are in one-to-one correspondence with 2-complexes dual to triangulations. We will denote by C_T the symmetry factor of the graph corresponding to T and by $\mathscr{A}_T$ the spin foam amplitude of the triangulation. The tensors in the models generating dynamical triangulations will be replaced by functions over D copies of a group $G = SU(2), SO(3), SO(4), SL(2,\mathbb{C})$, $M_{i_1,\ldots,i_D} \to \Phi(g_1,\ldots,g_D)$. Then the action could be written symbolically as $S[\Phi] = \Phi^2 + \Phi^{D+1}$.The field Φ represents a $(D-1)$-simplex and the interaction a D-simplex made of $D+1$ $(D-1)$-simplexes. The application of Wick's theorem implements the glueing of D-simplexes along the boundary $D-1$-simplexes. And finally, the Feynman graph amplitudes, expressed as an integral over group elements, reproduce the spin foam amplitudes.

2.4. Causal Set Approach

Present theories of physics indicate that QG could be a discrete theory, [68]. There are problems with infinities of GR and QFT caused by the lack of a short-distance cut-off in degrees of freedom. Although the renormalization procedure is refining these problems in QFT, they return in naive attempts to quantize gravity. As a second thing, technical problems arise in the definition of a path-integral on a continuous history space that have never been fully solved. Finally, the history space of Lorentzian manifolds presents special problems of their own kind. A discrete history space provides a well-defined path integral that enables us to avoid these problems.

The causal set approach is based on the idea of the discretization of spacetime into points. The important articles, which have started this program, were written by Stephen Hawking and David Malament (in 1976 and 1977, [69,70]). We founded the irreflexive causal set approach on the following 6 axioms, [71]:

1. Binary Axiom: we may model the classical spacetime as a set $\mathscr{S}$, whose elements represent spacetime events, together with a binary relation $\prec$ on $\mathscr{S}$; The elements of $\mathscr{S}$ represent causal relations between pairs of spacetime events;
2. Measure Axiom: the volume of a spacetime region corresponding to a subset $\mathscr{C}$ of $\mathscr{S}$ is equal to the cardinality of $\mathscr{C}$ in fundamental units, up to Poisson-type fluctuations;
3. Countability: the set $\mathscr{S}$ is countable;

4. Transitivity: when we take three elements x, y, and z in $\mathscr{S}$, if $x \prec y \prec z$, then $x \prec z$;
5. Interval Finiteness: for every pair of elements x and z in $\mathscr{S}$, we have finite cardinality for the open interval $\langle\langle x, z\rangle\rangle \equiv \{y \in \mathscr{S} | x \prec y \prec z\}$;
6. Irreflexivity: elements of $\mathscr{S}$ are not self-related with respect to $\prec$; it means $x \not\prec x$;

The causal set approach, [72], has very interesting connections with graph theory and graphons, [73]. But the remarkable results are again in phenomenology because it is also changing our view on the Big Bang, as we can see in Figure 1 in [74] and we obtained an explanation of the origin of the cosmological constant in the late Universe as well, [75,76]. Generally speaking, the kinematics is well formulated, but we don't know much about the dynamics, although the analog of Einstein-Hilbert action (Benincasa-Dowker action) was already found, [77].

2.5. Non-Commutative Geometry

The work of Alain Connes from [78] (year 1994) has very interesting applications to the standard model of particle physics, and it later found its use also in QG. At the start we will do some remarks concerning his mathematical ideas: the correspondence between geometric spaces and commutative algebras is a familiar and basic idea of algebraic geometry. The purpose of [78] was to extend this correspondence to the non-commutative case in the framework of real analysis. The theory, called non-commutative geometry, rests on two essential points. First of all, it is the existence of many natural spaces for which the classical set-theoretic tools of analysis, such as measure theory, topology, calculus, and metric ideas, lose their relevance, but which correspond very naturally to a non-commutative algebra. Further, it relies on the extension of the classical tools, such as measure theory, topology, differential calculus and Riemannian geometry, to the non-commutative situation. This extension involves an algebraic reformulation of the above tools.

However, passing from the commutative to the non-commutative case is never straightforward. On the one hand, completely new phenomena arise in the non-commutative case, when we want to mention the existence of a canonical time evolution for a non-commutative measure space. On the other hand, the constraint of developing the theory in the non-commutative framework leads to a new point of view and new tools even in the commutative case, such as the theory of cyclic cohomology and the quantized differential calculus which, unlike the theory of distributions, is perfectly adapted to products and gives meaning to the expressions like $\int f(Z)|dZ|^p$, where Z is not differentiable (and p not necessarily an integer).

If we will comment on physical applications, Alain's contribution, [78], does not throw any new light on the theoretical problems of the standard model of particle physics, because it is limited to the classical level. However, it specifies very precisely what modification of the continuum space we will make. In fact, its replacement by a product with a finite space entails that the Lagrangian of quantum electrodynamics becomes the Lagrangian of the standard model. As is shown in [78], the geometry of the finite set is specified by its Dirac operator, and this will be an operator in a finite-dimensional Hilbert space encoding both the masses of the elementary particles and the Kobayashi–Maskawa mixing parameters. Once the structure of this finite space F is given, we merely apply the general action to the space (which is a continuum) $\times$ F to get the standard model action. In many ways, the theory should be regarded as an interpretation, of a geometric nature, of all the complicatednesses of the most accurate phenomenological model of high-energy physics; it does confirm that high-energy physics is unveiling the fine structure of space-time. Finally, it gives a status to the Higgs boson as just another gauge field, but corresponding to a finite difference rather than a differential.

If we want to make a connecting line between mathematics and physics, we need to prescribe the dictionary. Standardly, in the basic data of noncommutative geometry, $(\mathcal{A}, \mathscr{H}, D)$, the Hilbert space $\mathscr{H}$ and the operator D have straightforward interpretations:

$$\mathscr{H} = \text{the Hilbert space of Euclidean ferminons} \tag{16}$$

$$D = \text{the inverse of the Euclidean propagator of fermions} \tag{17}$$

The algebra $\mathcal{A}$ is related to the gauge group of local gauge transformations. The relation is the following:

$$* - \text{algebra } \ \mathcal{A} \to \text{unitary group } \ \mathcal{U}(\mathcal{A}) \tag{18}$$

Let's do some comments on this functor $\mathcal{U}$ from $*$-algebras to groups. When we apply it to $M_n(\mathcal{A})$, the algebra of $n \times n$ matrices over $\mathcal{A}$, and one also retains the corresponding inclusion of $U(n) = \mathcal{U}(M_n(\mathbb{C}) \subset G = \mathcal{U}(M_n(\mathcal{A}))$, one then recovers uniquely the algebra $\mathcal{A}$ from the pair of groups $U(n) \subset G$, which is provided by $n > 2$.

This result shows that the replacement of an algebra by its associated groups loses very little information, at least in the non-abelian case. When we prescribe the algebra $\mathcal{A}$, from which the group $G = \mathcal{U}(\mathcal{A})$ comes, it determines a very narrow class of representations of G. Indeed, we have a natural mapping

$$\text{Rep}(\mathcal{A}) \xrightarrow{\text{Res}} \text{Rep}(G), \tag{19}$$

which associates to every unitary representation π of $\mathcal{A}$ on a Hilbert space $\mathscr{H}_\pi$ its restriction to the unitary group $\mathcal{U}(\mathcal{A})$. This mapping is compatible with direct sums, but while group representations $\rho_1, \rho_2 \in \text{Rep}(G)$ can be tensored, which gives us a representation $\rho = \rho_1 \otimes \rho_2$ of G on $\mathscr{H}_1 \otimes \mathscr{H}_2$, there is no corresponding operation for representations of an involutive algebra.

Actually, the above mapping $\text{Rep}(\mathcal{A}) \xrightarrow{\text{Res}} \text{Rep}(G)$, is, in general, far from surjective. When we want to illustrate it, we take $\mathcal{A} = M_n(\mathbb{C})$, so that $G = U(n)$, and then the range of Res consists of the multiples of the fundamental representation φ of $U(n)$ in $\mathbb{C}^n$. To stress the result, to assume that a representation ρ of G comes from a representation of $\mathcal{A}$ by restriction is a very limiting condition.

Now back to physics, the group G does have a clear significance as the group of local gauge transformations, and we shall take it as a characteristic of its representation in the one-particle space of Euclidean fermions that this representation of $\mathcal{U}(\mathcal{A})$ is derived by restriction from a representation of $\mathcal{A}$. We could also notice that it is only through the group $G = \mathcal{U}(\mathcal{A})$ and the corresponding algebra $\mathcal{A}$ that the Euclidean space-time enters the picture, because in QFT the space-time points only play the role of indices or labels except in their occurrence in the construction of the group of local gauge transformations. Therefore the last piece of dictionary is:

$$* - algebra \ \mathcal{A} \ \to \mathcal{U}(\mathcal{A}) = \ \text{Group of local gauge transformations on Euclidean space-time} \tag{20}$$

This whole construction works well for the Glashow-Weinberg-Salam model for leptons, as it is shown in [78], and further, the incorporation of quarks and strong interactions requires the setup of Poincaré duality and bimodules.

Actually, in this article we will just mention one model of Tejinder Singh, [79], which is inspired by non-commutative geometry. He introduces a DE particle, called a mitron, which shows a nonlocal behavior. Further, non-commutative geometry also found applications in connection with M-theory. It was conjectured in [80] that M-theory could be defined as a matrix quantum mechanics, obtained from 10-dimensional supersymmetric Yang-Mills theory by means of reduction to $0 + 1$ dimensional theory, where the size of the matrix tends to infinity. Another matrix model, which can be obtained by reduction of 10-dimensional supersymmetric Yang-Mills theory to a point, was suggested in [81]. The two models, known as the BFSS Matrix model and the IKKT Matrix model, are closely related. Actually the goal of these works is to formulate the IKKT and BFSS matrix models, to make more precise the relation between these models and to study their toroidal compactifications.

2.6. Causal Dynamical Triangulations

An interesting and a rather unconventional approach to QG is that of causal dynamical triangulations [82,83] (the first paper in this direction of research was published by Jan Ambjørn, Jerzy

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Jurkiewicz, and Renata Loll in 2000 and 2001). This is a nonperturbative approach to QG that aims to define a well-behaved gravitational path integral by making a discrete spacetime into elementary building blocks while preserving a causal structure. The central idea is that the spacetime geometry can be approximated by glueing together simplicial manifolds, in particular four-dimensional simplices, in a manner that maintains a consistent global time foliation. Unlike the earlier theory of Euclidean dynamical triangulations which lost information about causal order by using Euclidean signature metrics, causal dynamical triangulations imposes a Lorentzian causal structure from the outset and only later performs a Wick rotation to the Euclidean sector for computational purposes. The starting point is the formal gravitational path integral over all geometries [84,85],

$$Z = \int \mathcal{D}[g] e^{iS_{\text{EH}}[g]} \tag{21}$$

where S_{EH} is the Einstein–Hilbert action and in causal dynamical triangulations, this integral is replaced by a discrete sum over triangulated manifolds constructed from simplices, leading to a well-defined and finite combinatorial formulation.

The discrete analogue of the Einstein–Hilbert action used in causal dynamical triangulations is given by the Regge action, which encodes curvature through deficit angles at the edges (or hinges) of the triangulated spacetime. The Regge action can be expressed as [86,87]

$$S_{\text{Regge}} = \frac{1}{8\pi G} \sum_h A_h.\delta_h - \Lambda \sum_\sigma V_\sigma \tag{22}$$

where the sum over (h) runs over all hinges (two-dimensional subsimplices), A_h is the area associated with hinge (h), δ_h is the deficit angle measuring the curvature concentrated at (h) and the second term accounts for the cosmological constant Λ with V_σ being the volume of each four-simplex σ. In causal dynamical triangulation, spacetime is foliated into layers of spatial hypersurfaces of fixed topology (often S^3) labeled by discrete proper time steps. This leads to ensuring that each simplex connects neighboring time slices in a causal fashion, and the triangulations are thus restricted to respect global hyperbolicity, meaning that the causal order between spatial slices is maintained.

The path integral in causal dynamical triangulation then becomes a discrete sum over all causal triangulations weighted by the Regge action [88]

$$Z = \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}}.e^{iS_{\text{Regge}}[\mathcal{T}]} \tag{23}$$

where the sum runs over all triangulations $\mathcal{T}$) compatible with the chosen foliation, and $C_{\mathcal{T}}$ is a symmetry factor that accounts for the automorphisms of each triangulation. To perform numerical simulations, a Wick rotation is applied to transform the Lorentzian action to a Euclidean form, $iS_{\text{Regge}} \to -S^E_{\text{Regge}}$ which allows Monte Carlo techniques to be used. The Euclidean version of the partition function is then given as

$$Z_E = \sum \mathcal{T} \frac{1}{C_{\mathcal{T}}} e^{-S^E_{\text{Regge}}[\mathcal{T}]} \tag{24}$$

which serves as the foundation for exploring the emergent large-scale properties of quantum spacetime through computational simulations.

An important outcome of causal dynamical triangulation is that when averaged over ensembles of causal triangulations, spacetime dynamically exhibits features resembling a smooth 4-dimensional universe at large scales, despite being composed of discrete building blocks. The expectation value of the spatial 3-volume $V_3(t)$ as a function of proper time (t) often shows a behavior well approximated by a semiclassical de Sitter universe, corresponding to an effective minisuperspace action of the form

$$S_{\text{eff}}[V_3] = \frac{1}{24\pi G}\int dt\left[\frac{(\dot{V}_3(t))^2}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t)\right] \tag{25}$$

where the first term keeps the kinetic behavior of the spatial volume and the latter terms correspond to curvature and cosmological constant contributions, respectively. This result is quite remarkable, as it shows that a smooth, 4-dimensional spacetime geometry can emerge dynamically from a sum over purely discrete and microscopic causal structures, without assuming a background metric. Consequently, causal dynamical triangulations provides a promising route toward a background independent and UV finite theory of QG in which spacetime geometry and dimensionality themselves arise as emergent phenomena from an underlying causal and combinatorial foundation.

2.7. *Asymptotic Safety*

Asymptotic safety program emerged with the groundbreaking work of Martin Reuter in 1996, [89], where functional renormalization tools were taken over by QG. Firstly, we want to define the high-energy behavior of QFT, [90–92]. The theory space is built on a set of fields, their symmetries, and a class of action functionals depending on fields η and couplings g_i. The key relations are $\tilde{g}_i = k^{d_i} g_i$, where k is a momentum cut-off and d_i is the mass dimension of g_i. We take the real numbers g_i as coordinates in the theory space. The couplings g_i are defined in an ideal situation in terms of physical observables such as cross-sections and decay rates. Further, we don't include redundant couplings (that could be eliminated through field re-definitions). It is assumed that the renormalization group flow has been prescribed on the theory space, and it describes the dependence of the action on some energy scale k. Then we suppose that the action has the following form:

$$\Gamma_k(\eta, g_i) = \sum_i g_i(k)\mathcal{O}_i(\eta), \tag{26}$$

where $\mathcal{O}_i$ typically denotes local operators that are constructed with the fields η and that are compatible with the symmetries of the theory. After that, we identify the theories with renormalization group trajectories.

Asymptotic safety is a bottom-up approach to QG, because the discussion starts within the theory space of an effective field theory. Then we continue and note that if the Universe corresponds to a trajectory of a special type, our effective description can be forced to arbitrarily high energy. So asymptotic safety theory could be presented as the continuation of an effective theory at higher energy scales. As a result, it has the great advantage that if it occurs in Nature, it is automatically in agreement with our knowledge of the low-energy world. This could be considered a big contrast to string theory and loop quantum gravity, which are top-down approaches. It would be a very hard issue for them to make any connection with the known low-energy phenomenology. To conclude, asymptotic safety has already provided us with interesting results concerning black hole singularities, [93], and it has applications to the development of models of cosmological inflation, [94]. The natural disadvantage is the absence of the introduction of new mathematics.

2.8. *Entropic Gravity*

To explain the basic idea of a new approach to QG called entropic gravity - the beginnings are dated back to the year 2010 and are associated with the article of Erik Verlinde, [95] - the starting assumption is directly motivated by Bekenstein's original thought experiment, from which he obtained the famous entropy formula. He considered a particle with mass m attached to a fictitious "string" that is lowered towards a black hole, [95]. Just before the horizon, the particle will be dropped in. Due

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to the infinite redshift, the mass increment of the black hole can be made arbitrarily small when we consider classical physics. If we took a thermal gas of particles, this fact would lead to problems with the second law of thermodynamics. Jacob Bekenstein solved this by arguing that when a particle is one Compton wavelength from the horizon, we can include it as a part of the black hole. Therefore, it increases the mass and horizon area by a small amount, which he identified with one bit of information. This lead to the discovery of the area law for the black hole entropy.

We would like to mimic this reasoning not near a black hole horizon but in a flat, non-relativistic space. Therefore we consider a small piece of a holographic screen and then a particle of mass m that approaches it from the side, at which space-time has already emerged. Eventually, the particle merges with the microscopic degrees of freedom on this screen, but before it does so, it already influences the amount of information that is stored on the screen.

Let us postulate that the change of entropy associated with the information on the boundary equals

$$\Delta S = 2\pi k_B \quad \text{when} \quad \Delta x = \frac{\hbar}{mc}. \tag{27}$$

We will rewrite now the formula followingly:

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x \tag{28}$$

How does the force arise? The basic idea will be to use the analogy with osmosis across a semi-permeable membrane. When some particle has an entropic reason to be on one side of the membrane and the membrane carries a temperature, then it will experience an effective force equal to

$$F\Delta x = T\Delta S. \tag{29}$$

This is called the entropic force. So, in order to have a non-zero force, we need to have a non-zero temperature. We know from Newton's law that a force leads to a non-zero acceleration. Now, Bill Unruh showed that an observer in an accelerated frame experiences a temperature

$$T = \frac{1}{2\pi} \frac{\hbar a}{ck_B}. \tag{30}$$

We will take it as the temperature associated with the bits on the screen. Then we obtain from (28), (29) and (30) the Newton's force law

$$F = ma. \tag{31}$$

$$\tag{32}$$

Let us suppose our boundary will not be infinitely extended but will form a closed surface. We assume it is a sphere. For the following, it will be the best to forget about the Unruh law (30), since we don't need it. It only served as a further motivation for (28). The key statement is that we must have a temperature in order to have a force. Because we want to understand the origin of the force, we must know where the temperature comes from.

We can think about the boundary as a storage device for information. Assuming that the holographic principle holds, the maximal storage space, or total number of bits will be proportional to the area A. Specifically, in a theory of emergent space, this is how area may be defined such that each fundamental bit occupies, by definition, one unit cell. Now, we will denote the number of used bits by N. It is natural to assume that this number will be proportional to the area. So, we write

$$N = \frac{Ac^3}{G\hbar}, \tag{33}$$

where we introduced a new constant G. Eventually, this constant is going to be identified with Newton's constant. But since we have not assumed anything yet about the existence of a gravitational force, one could simply regard this equation as the definition of G.

We suppose there is a total energy E present in the system. Let us now just make the simple assumption that the energy will be divided evenly over the bits N. The temperature is determined by the equipartition rule

$$E = \frac{1}{2} N k_B T \tag{34}$$

as the average energy per one bit. Then we also need the equation

$$E = Mc^2. \tag{35}$$

Here M represents the mass that would emerge in the part of space, which is enclosed by the screen. Although the mass will not be directly visible in the emerged space, its presence is noticed through its energy.

Now, we eliminate E from (34) and (35). Let's plug in for N in the resulting equation, and we express from this equation T. Then everything is inserted into the equation (29) and rearranged with the help of the formula

$$A = 4\pi R^2. \tag{36}$$

The result is the Newton's force law

$$F_G = G\frac{mM}{R^2}. \tag{37}$$

Of course, the reason we obtained gravitational force from the first principles is obvious: it was used by the derivation of black hole thermodynamics and results concerning holography. Here, we somehow reversed the process. But the revelation that we can build in this manner a QG paradigm is surprising. Especially that we can model after applications to cosmology the accelerated expansion of the Universe.

3. Models of the Late-Time Accelerated Expansion in Current Quantum Gravity Approaches

3.1. Origin of the Cosmological Constant in the Metastring Theory

We claim that one can successfully explain the problem of dark energy and the observed de Sitter spacetime in a generic, non-commutative generalized geometric phase-space formulation of string theory, so-called metastring theory. Actually, the curvature and size of the canonically conjugate dual space are the cosmological and gravitational constants in the observed spacetime. Then, the three scales associated with:

1. the cosmological constant
2. the Planck units
3. the effective particle physics

are related by a seesaw-like formula via T-duality. This is an important feature of the string theory that relates reciprocally short and long distances and precisely the generic prediction of certain toy-models, [44].

The generalized geometric formulation of string theory we have in mind has been recently discussed in [44], and derives from the underlying chiral world-sheet Hamilton's action for the strings:

$$S_{string} = \frac{1}{4\pi}\int_{\Sigma}\left[\partial_\tau \mathbb{X}^A(\eta_{AB} + w_{AB})\mathbb{P}^B - \mathbb{P}^A H_{AB}\mathbb{P}^B\right] d^2\sigma, \tag{38}$$

where $\mathbb{X}^A$ combine the sum (x^a) and the difference $(\tilde{x}_a)$ of the left- and right-movers on the string and $\mathbb{P}^A = \partial_\sigma \mathbb{X}^A$ are closely related to the chiral generalized momenta. The mutually compatible dynamical fields w_{AB},η_{AB} and H_{AB} are: the antisymmetric symplectic structure, the symmetric polarization metric η_{AB}, and the doubled symmetric metric H_{AB}. Precisely, this new framework for string theory based on a quantum space-time captures the essential quantum non-locality of any quantum theory. Further, w_{AB} governs the Hilbert structure of a quantum theory, which is usually ignored in the standard spacetime interpretation of string theory, whereas the Kalb-Ramond $(B_{\mu\nu})$ field is associated with the symplectic structure w_{AB}, rather than the doubled metric.

The quantization renders the doubled "phase-space" operators $\hat{\mathbb{X}}^A = (\frac{\hat{x}^a}{\lambda}, \frac{\hat{\tilde{x}}_b}{\lambda})$, inherently non-commutative, inducing in particular

$$[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b] = iw^{ab} : \qquad [\hat{x}^a, \hat{x}^b] = 0,\ [\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i\lambda^2 \delta^a{}_b,\ [\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = 0, \tag{39}$$

where ϵ denotes the fundamental length scale, such as the Planck scale, so that $\epsilon = 1/\lambda$ gives the corresponding fundamental energy scale. This we found by examining the simplest example of the canonical free string compactified on a circle, in an intrinsically T-duality covariant formulation of the Polyakov string. The full spacetime covariance is in fact maintained in this description, and the string tension is naturally the ratio of the fundamental length and energy scales, $\alpha' = \lambda/\epsilon$. This fundamental non-locality is actually independently confirmed by examining the algebra of vertex operators in the 2d CFT of a free string compactified on a circle, [96,97].

The generalized geometric formulation of string theory provides for an effective description of DE that is consistent with a de Sitter spacetime due to its chirally doubled realization of the target space and the non-commutative structure in (39). Let us note that the natural stringy effective action on the doubled spacetime in terms of the coordinates $(x^a, \tilde{x}^a)$ takes the form:

$$S^{nc}_{eff} = \int Tr\sqrt{g(x,\tilde{x}}[R(x,\tilde{x}) + ...], \quad \text{with} \quad [x,\tilde{x}] = i\lambda^2, \tag{40}$$

where the dots denote higher-order curvature terms induced by string theory. If we return to (39), this S^{nc}_{eff} expands into many other terms, which upon $\tilde{x}$-integration and from the x-space point gives rise to interactions that could lead to various forms of DE. Therefore, these effects may provide some results in the recent conflicting measurements of the Hubble constant.

Further, to the lowest order, the expansion of S^{nc}_{eff} takes the form:

$$S_d = -\int \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x})], \tag{41}$$

a result which we obtained by effectively setting $w_{AB} \to 0$ in (39) by assuming that $[\hat{x}, \hat{\tilde{x}}] = 0$. In this limit, the $\tilde{x}$-integration in the first term of (39) defines the gravitational constant G_N. And in the second term produces a positive cosmological constant constant $\Lambda > 0$. It also follows that the weakness of gravity is determined by the size of the canonically conjugate dual space, while the smallness of the cosmological constant should be given by its curvature. However, these results from the commutative limit are not stable under loop corrections, which has been recently addressed in the work of Nemanja Kaloper and Antonio Padilla (we call it the sequester mechanism).

The intrinsic non-commutativity of the zero modes x and $\tilde{x}$ in (39) corrects these results in several ways. We could especially ask whether the non-zero λ in (39) stabilizes the cosmological constant. The fully non-commutative analysis is difficult but an encouraging indication emerges followingly. We simplify the conformal metrics, $g_{\mu\nu} = \phi^2\delta_{\mu\nu}$, and then the actions in (40)–(41) produce a non-commutative $\Lambda\phi^4$ theory. Unlike the theory's commutative limit, the interesting results of Harald Grosse and Raimar Wulkenhaar, [98,99] demonstrate that the $\Lambda\phi^4-$theory is not perturbatively solvable, explicitly showing the finite renormalization of Λ in terms of the bare coupling. At least in this highly simplified, conformal degree limit, non-commutativity thus could afford a small, radiatively and perhaps even non-perturbatively stable cosmological constant for the non-commutative form of the "doubled" effective action.

Finding de Sitter spacetime within string theory has been an ongoing quest over the past two decades. Among the vast number of various constructions, the authors Per Berglund, Tristan Hübsch and Djordje Minić developed a discretuum of toy models, see [100,101] and references therein, that turn out to naturally capture several of the features of the above non-commutatively generalized phase-space reformulation of quantum gravity, [102]. One of the essential features of their toy model is S-duality, which is built in the $SL(2;\mathbb{Z})$ monodromy properties of our axion-dilaton models. In generalizations where various moduli fields replace the axion-dilaton system, this directly implies T-duality, which is in turn covariantly realized in the phase space approach. Actually, the T-duality maps $\hat{x}$ into $\tilde{\hat{x}}$, and vice versa, and thus a covariant representation calls for a phase space formulation that involves both $\hat{x}$ and $\tilde{\hat{x}}$. The seesaw formula is a backstroke of such a covariant phase space formulation, and it can be explicitly derived in the context of their toy models. So, the overall effect is closely related to the old observation of Edward Witten [103], that supercharges need not be globally defined in the presence of conical defects, and the mass splitting between superpartners is controlled by the strength of the conical defect. The details are presented for the corresponding four-dimensional generalization and relation to the cosmological constant in [104,105].

Models in this class produce a seesaw formula for the cosmological constant

$$M_\Lambda \sim M^2/M_P, \tag{42}$$

relating the mass scales of the vacuum energy/cosmological constant (M_Λ), particle physics (M), and the Planck scale (M_P). Especially, the see-saw formula (42) follows from very particular geometric properties which relate both the volume and the curvature of the space to the string length scale, [100]. Identifying M_Λ and M_P as the IR and UV cut-offs, respectively, the renormalization group flow identifies a self-dual fixed point, [106–108]. When we take the phase-space formulation, [109,110] as a T-duality covariant description of the string theory, this naturally relates $M_P \to M^2/M_P$ under T-duality. Then, the prediction of the models, in [101,102], of $M_\Lambda \sim M^2/M_P$ satisfies these conditions, with $M_P \sim \epsilon$ as the fundamental energy scale corresponding to the fundamental length λ. This produces the well-known formula for the observed DE scale, provided M is a TeV scale. Further, this illustrates one of the important points, namely the novel realization of DE and de Sitter spacetime, in both the phase-space formulation of string theory and the toymodels in this given class [100,102].

3.2. Emergence of Cosmological Constant in Discrete Approaches to Quantum Gravity

In GR, local energy-momentum conservation $\nabla^b \langle T_{ab} \rangle$ is always a consequence of the field equations, both at classical and semi-classical levels, [111,112]. This should be obvious from the semi-classical version of Einstein's equation

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} \langle T_{ab} \rangle \tag{43}$$

- where $\langle T_{ab} \rangle$ is the expectation value of the energy momentum operator in the corresponding quantum state of the matter fields - and the fact that the Bianchi identities make the geometric side divergence free.

The previous restriction could be circumvented when we consider a simple modification of GR (already studied by Albert Einstein) called unimodular gravity:

$$R_{ab} - \frac{1}{4} R g_{ab} = \frac{8\pi G}{c^4} (T_{ab} - \frac{1}{4} T g_{ab}) \tag{44}$$

It could be derived from the Einstein-Hilbert action by restricting to variations preserving the volume form.

This fact breaks the diffeomorphism symmetry down to volume-preserving diffeomorphism, whose infinitesimal version is given by divergence-free vector fields ξ^a,

$$\nabla_a \xi^a = 0. \tag{45}$$

This concrete restriction on general covariance allows for violations of energy-momentum conservation of a certain form. When we want to see this, we consider an action for matter S_m, which is invariant under volume- preserving diffeomorhisms we introduce the stress-energy tensor $T_{ab} \equiv -2|g|^{-\frac{1}{2}} \frac{\delta S_m}{\delta g^{ab}}$, and its energy-momentum violation current $J_a \equiv \nabla^b T_{ab}$. Then the variation of the action under an infinitesimal diffeomorphism ξ^a is

$$\delta S_m = -\int T_{ab} \nabla^a \xi^b \sqrt{-g}\, d^4x = \int J_a \xi^a \sqrt{-g}\, d^4x, \tag{46}$$

where the matter field equations are assumed to hold. When we insert the general solution of (45), $\xi = \epsilon^{abcd} \nabla_b w_{cd}$ - for an arbitrary two-form w - the requirement that the action is invariant under volume-preserving diffeomorphisms implies that $dJ = 0$. Therefore the violations of energy-momentum conservation are allowed in unimodular gravity as long as they are of this integrable type.

This condition reduces to

$$J_a = \nabla_a Q \tag{47}$$

for some scalar field Q in the case of simply connected spacetimes. Thus, if the matter action is only invariant under volume preserving diffeomorphisms, then $J \neq 0$ would introduce deviations from GR.

We will now consider the semiclassical version of the equation (44), where the energy-momentum tensor and its trace are now replaced by the corresponding expectation values in a quantum state of the matter fields. If we use the Bianchi identities we deduce that

$$\frac{1}{4} \nabla_a R = \frac{8\pi G}{c^4} \Big(\nabla^b \langle T_{ab} \rangle - \frac{1}{4} \nabla_a \langle T \rangle \Big), \tag{48}$$

which, after integration, could be used to rewrite (44) as

$$R_{ab} - \frac{1}{2} R g_{ab} + \Big(\Lambda_{-\infty} + \frac{8\pi G}{c^4} Q \Big) g_{ab} = \frac{8\pi G}{c^4} \langle T_{ab} \rangle, \tag{49}$$

where $\Lambda_{-\infty}$ is a constant of integration and Q is defined by (47). As we expect, when the stress-energy tensor is conserved, i.e. $Q = 0$, (49) reduces to Einstein's equation, with a cosmological constant equal to $\Lambda_{-\infty}$.

We emphasize that both semiclassical GR and its unimodular version are considered here as an effective and emergent description of more fundamental degrees of freedom. The violation of energy-momentum conservation in our scenario would have to admit a description in terms of the more fundamental QG degrees of freedom. The brief idea is the following: due to the interaction of matter with the spacetime foam (discrete structure) at the Planck scale, we would obtain the non-conservation of energy-momentum. Concretely, the effect on propagation of massive fields must be realized in a deviation from the geodesic motion of free particles. The force must be clearly proportional to R. Further, the force should depend on the mass m, the 4-velocity u^μ of the particle and time-like unit vector ξ^μ specifying the local frame defined by the matter that curves the spacetime. For example, in cosmology $\xi = \partial_t$ is naturally associated with the time-arrow of the comoving cosmic fluid. In addition, the force should be proportional to the particle's mass, which gives us a characteristic length scale, the Compton wave-length. The dimensional analysis provides us with an essentially unique expression, which is compatible with the above requirements:

$$u^\mu \nabla_\mu u^\nu = \alpha \frac{m}{M_p^2} \text{sign}(s\cdot\xi) R s^\nu, \tag{50}$$

where $\alpha > 0$ is a dimensionless coupling.

The factor $\text{sign}(s\cdot\xi)$ guarantees that the force is genuinely friction-like. This is obvious when one considers the change of the mechanical energy $E \equiv -mu^\nu \xi_\nu$ along the particles world-line, precisely

$$\begin{aligned} \dot{E} &\equiv -mu^\mu \nabla_\mu (u^\nu \xi_\nu) = \\ &= -\alpha \frac{m^2}{M_p^2} |(s\cdot\xi)| R - mu^\mu u^\nu \nabla_{(\mu}\xi_{\nu)}. \end{aligned} \tag{51}$$

The last term in this equation encodes the standard change of E associated with the non-Killing character of ξ^μ. The first term on the RHS comprises the friction that damps out any motion with respect to ξ^μ. We see that the energy is lost into the fundamental granularity until $u^\mu = \xi^\mu$ and the particle is at rest with the cosmological fluid.

The simplest dynamics for the spin which is consistent with (50), the conservation of $s\cdot s$ and $s\cdot p = 0$ is

$$u^\mu \nabla_\mu s^\nu = \alpha \frac{m}{M_p^2} \text{sign}(s\cdot\xi) R (s\cdot s) u^\nu. \tag{52}$$

It is only important to note that this is a minimalistic solution. Of course, other terms can be added, but they do not affect the main point of this section.

Let's return back to cosmology, and we will consider a homogeneous, isotropic, and spatially flat FLRW universe, $ds^2 = -c^2dt^2 + a^2 d\vec{x}^2$. Then the modified Friedmann equation reads

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho(t) + \frac{\Lambda^{eff}(t)c^2}{3} \tag{53}$$

where the effective cosmological "constant"

$$\Lambda^{eff}(t) \equiv \Lambda_{-\infty} + \frac{8\pi G}{c^4}\int^t J \tag{54}$$

detects the possible violations of energy-momentum conservation in the past history of the universe. Actually, as is shown in [113], small violations of energy momentum conservation - that may be inaccessible to current tests of local physics - could nevertheless have important cosmological effects at late times, in the form of a nontrivial contribution to the present value of the cosmological constant. It can be derived that the cosmological constant, which we derive by the calculations in [113], is

$$\Lambda^{eff}(t) \approx 4\alpha\Lambda_{obs}. \tag{55}$$

This is a noteworthy result.

3.3. Group Field Theory Condensate Cosmology

Group field theory is an approach to QG, which is relative to loop quantum gravity. It offers us, after applying the formalism to cosmology, a unique model solving the Big Bang singularity and simultaneously the problem of the late-time acceleration of the Universe. We obtain an existence of some phantom field in these models, which would even lead to the solution of the Hubble tension.

In the framework of the group field theory cosmology, the evolution of the universe is captured by the transformation of a spatial slice of spacetime, where this transformation is to be found with respect to the relational time ϕ. When we want to incorporate the dependence of observables on the clock variable, we just focus on the states, which are sharply peaked around a specific relational time ϕ_0. Further, it is also necessary for these states to support the appropriate observables, such that the continuum type of spacetime emerges under suitable limits, [114,115]. These requirements naturally lead to the introduction of the coherent peak states, which provide us an effective framework for developing the dynamics at the quantum geometric level:

$$|\sigma_\epsilon;\phi_0,\pi_0\rangle = \mathcal{N}()\exp\left(\int \sigma_\epsilon(g_v,\phi;\phi_0,\pi_0)\hat{\varphi}^\dagger(g_v,\phi)\,(dg)^4 d\phi\right)|0\rangle, \tag{56}$$

with $\mathcal{N}()$ as a constant responsible for achieving the correct normalization and $|0\rangle$ is the vacuum state. The condensate wavefunction $\sigma_\epsilon(g_v,\phi;\phi_0,\pi_0)$ is peaked at $\phi=\phi_0$ and could be written as

$$\sigma_\epsilon(g_v,\phi;\phi_0,\pi_0) = \eta_\epsilon(\phi-\phi_0,\pi_0)\tilde{\sigma}(g_v,\phi), \tag{57}$$

where $\eta_\epsilon(\phi-\phi_0,\pi_0)$ represents a peaking function, which is typically chosen as a Gaussian and which is centered around ϕ_0 with a characteristic width controlled by ϵ. The parameter π_0 further governs the fluctuations of the operator corresponding to the conjugate momentum of the scalar field ϕ. The reduced condensate function $\tilde{\sigma}(g_v,\phi)$, which serves as the primary dynamical variable in the hydrodynamic approximation, does not alter the peaking behavior of the function $\sigma_\epsilon(g_v,\phi;\phi_0,\pi_0)$. Next, it remains true that the condensate state defined in (56) continues to be an eigenstate of the group field theory field operator such that

$$\hat{\varphi}(g_v,\phi)|\sigma_\epsilon(g_v;\phi_0,\pi_0)\rangle = \sigma_\epsilon(g_v,\phi;\phi_0,\pi_0)|\sigma_\epsilon(g_v;\phi_0,\pi_0)\rangle. \tag{58}$$

Next, motivated by geometric considerations, we could see that the condensation wave condition should be invariant under both right and left diagonal group actions, so we need to impose the following condition on $\tilde{\sigma}(g_v,\phi)$

$$\tilde{\sigma}(hg_vk,\phi) = \tilde{\sigma}(g_v,\phi), \quad \forall h,k\in SU(2). \tag{59}$$

(60)

Our task is now to derive the cosmological dynamics of homogeneous and isotropic universes from the hydrodynamics of the group field theory condensate when we want that the evolution of the universe's volume serves us as the primary observable. Therefore, we introduce an additional constraint on the condensate wavefunction, the isotropy.

From the perspective of discrete geometry, as it is encoded in the group field theory Fock space, this implies that the wave function $\sigma(g_I,\phi)$ must only have the support over equilateral tetrahedra, [114]. This condition enforces the requirement that the spin labels be equal, and the intertwiners should be chosen such that the volume takes the largest value allowed by this choice of spins. If we take into account both left and right invariance, the condensate wavefunction then assumes to be

$$\tilde{\sigma}(g_I,\phi) = \sum_j \tilde{\sigma}_j(\phi)\bar{\mathcal{I}}^{j,\iota_+}_{\mathbf{m}}\bar{\mathcal{I}}^{j,\iota_+}_{\mathbf{n}}d(j)^2\prod_{I=1}^{4}D^{j}_{m_I n_I}(g_I), \tag{61}$$

here j is a shorthand notation for the collection of spins $\mathbf{j}=(j_1,j_2,j_3,j_4)=(j,j,j,j)$, and similarly for $\mathbf{m}$, $\mathbf{n}$, $\mathcal{I}^{j,\iota_+}_{\mathbf{m}}$ represents the intertwiner labeled by ι, while $d(j)=2j+1$ is the dimension of the spin j representation. The functions $D^{j}_{m_I n_I}(g_I)$ are the Wigner representation functions. After these considerations, the time evolution of the isotropic state is solely encrypted in the function $\sigma_j(\phi)$ for each mode j. Finally, recalling the definition of the annihilation operator in the equation (56), we have

$$\hat{c}_{\mathbf{x}}(\phi)|\sigma_\epsilon;\phi_0,\pi_0\rangle = \eta_\epsilon(\phi-\phi_0,\pi_0)\tilde{\sigma}_j(\phi)\bar{\mathcal{I}}^{j,\iota_+}_{\mathbf{m}}. \tag{62}$$

$$\tag{63}$$

Because we choose the peaking function $\eta_\epsilon(\phi-\phi_0,\pi_0)$ usually as Gaussian, the time evolution of the condensate can be traced once the reduced condensate function $\tilde{\sigma}(g_v,\phi)$ is known. At the mean-field level, the evolution of $\tilde{\sigma}(g_v,\phi)$ is encoded in an effective action, which looks like the following:

$$\begin{aligned} S(\bar{\tilde{\sigma}},\tilde{\sigma}) = \int \langle\sigma_\epsilon;\phi_0,\pi_0|S(\hat{\varphi}^\dagger,\hat{\varphi})|\sigma_\epsilon;\phi_0,\pi_0\rangle = \int\Bigg\{\sum_j\Bigg[\bar{\tilde{\sigma}}_j(\phi_0)\tilde{\sigma}''(\phi_0) - 2i\tilde{\pi}_0\bar{\tilde{\sigma}}_j(\phi_0)\tilde{\sigma}'_j(\phi_0) - \\ -\xi_j^2\bar{\tilde{\sigma}}_j(\phi_0)\tilde{\sigma}_j(\phi_0)\Bigg] + V(\bar{\tilde{\sigma}},\tilde{\sigma})\Bigg\}\, d\phi_0, \end{aligned} \tag{64}$$

where $\tilde{\pi}_0 = \frac{\pi_0}{\epsilon\pi_0^2-1}$ and ξ_j is an effective parameter that encodes the details of the kinetic term in the fundamental group field theory action under the isotropic restriction. The derivatives denoted by $'$, represent derivatives with respect to ϕ_0. Finally, $V(\bar{\tilde{\sigma}},\tilde{\sigma})$ is the interaction kernel, which we determine by the underlying group field theory model, [114].

We now take a phenomenological approach, where we will model interactions by a term in a simplified form, which will make the computations more tractable:

$$V(\bar{\tilde{\sigma}},\tilde{\sigma}) = \sum_j\Big(\frac{2\lambda_j}{n_j}|\tilde{\sigma}_j(\phi_0)|^{n_j} + \frac{2\mu_j}{n'_j}|\tilde{\sigma}_j(\phi_0)|^{n'_j}\Big), \tag{65}$$

where the interaction couplings λ_j and μ_j correspond to each mode j and they should satisfy the conditions $|\mu_j| << |\lambda_j| << |m_j^2|$. Additionally, we assume that $n'_j > n_j > 2$. This concrete model shares similarities with certain microscopic group field theories, such as those related to the EPRL model, [116].

The dynamics is encoded in the action (64), and for later convenience we can transform the equation of motion into a more familiar hydrodynamic form. (We will omit the subscript 0 in ϕ for simplicity when there will be no risk of confusion). Varying the action (64) with respect to $\bar{\tilde{\sigma}}_j$ we get

$$\tilde{\sigma}''_j - 2i\tilde{\pi}_0\tilde{\sigma}_j - \xi_j^2\tilde{\sigma}_j + 2\lambda_j|\tilde{\sigma}_j|^{n_j-2}\tilde{\sigma}_j + 2\mu_j|\tilde{\sigma}_j|^{n'_j-2}\tilde{\sigma}_j = 0. \tag{66}$$

When we use the decomposition $\tilde{\sigma}_j(\phi) = \rho_j(\phi)e^{i\theta_j(\phi)}$, where $\rho_j \in \mathbb{R}^+$ represents the condensate density and θ_j its condensate phase– it transforms the complex dynamical equations into coupled hydrodynamic relations. The global $U(1)$ symmetry $\theta_j \to \theta_j + \alpha$ in the effective action generates a conserved Noether current, and the corresponding conserved charge reads $Q_j = (\theta'_j - \tilde{\pi}_0)\rho_j^2$. Using the definition of Q_j we can write the module equation as follows:

$$\rho''_j - \frac{Q_j^2}{\rho_j^3} - m_j^2\rho_j + \lambda_j\rho_j^{n_j-1} + \mu_j\rho_j^{n'_j-1} = 0. \tag{67}$$

In this equation, we have introduced a new constant $m_j = \xi_j - \tilde{\pi}_j$. By integrating the equation once, we can get another conserved quantity

$$E_j = \frac{1}{2}(\rho_j')^2 - \frac{1}{2}m_j^2\rho_j^2 + \frac{Q_j^2}{2\rho_j^2} + \frac{\lambda_j}{n_j}\rho_j^{n_j} + \frac{\mu_j}{n_j'}\rho_j^{n_j'} \tag{68}$$

$$. \tag{69}$$

The universe evolution can be seen from the volume, in particular, for our condensate state $|\sigma\rangle$, the expectation value of the volume operator $\hat{V}$ could be calculated as follows

$$\begin{aligned} V(\phi_0) = \langle\sigma_\epsilon;\phi_0,\pi_0|\hat{V}|\sigma_\epsilon;\phi_0,\pi_0\rangle = \\ = \langle\sigma_\epsilon;\phi_0,\pi_0|\int\sum_{\mathbf{x},\mathbf{x}'}V(\iota,\iota')\delta_{\mathbf{x}-\{\iota\},\mathbf{x}'-\{\iota'\}}\hat{c}_{\mathbf{x}}^{\dagger}(\phi)\hat{c}_{\mathbf{x}'}(\phi)|\sigma_\epsilon;\phi_0,\pi_0\rangle \approx \sum_j V_j\rho_j(\phi_0)^2. \end{aligned} \tag{70}$$

The volume contribution from each quantum in the spin j representation is given by $V_j \propto L_p^3 j^{3/2}$, where L_p denotes the Planck length. We also employ the intertwiner normalization condition $\sum_{\mathbf{m}} \mathcal{I}_{\mathbf{m}}^{j,\iota_+}\bar{\mathcal{I}}_{\mathbf{m}}^{j,\iota'_+} = \delta_{\iota,\iota'}$. The approximation used in this analysis consists of retaining only the dominant contribution to the saddle-point approximation of the peaking function, which arises from the choice of state. (Because we want to avoid confusion, we reintroduce the subscript 0 for the given relational time ϕ_0 at this stage.)

Having derived the equation of motion (67) of condensate density ρ_j, we can write the equations for the volume dynamics in the form of modified FLRW equations as follows:

$$\left(\frac{V'}{3V}\right)^2 = \left[\frac{2\sum_j V_j\sqrt{2E_j\rho_j^2 - Q_j^2 + m_j^2\rho_j^4 - \frac{2}{n_j}\lambda_j\rho_j^{n_j+2} - \frac{2}{n_j'}\mu_j\rho_j^{n_j'+2}}}{3\sum_k V_k\rho_k^2}\right] \tag{71}$$

$$\frac{V''}{V} = \frac{2\sum_j V_j\left[2E_j + 2m_j^2\rho_j^2 - (1+\frac{2}{n_j})\lambda_j\rho_j^{n_j} - (1+\frac{2}{n_j'})\mu_j\rho_j^{n_j'}\right]}{\sum_k V_k\rho_k^2} \tag{72}$$

Firstly, we will discuss the classical regime. During the pre-interaction-dominated phase of volumetric expansion, the system transitions through an intermediate regime, where the dynamics approximate the FLRW cosmology with a free massless scalar field. This happens under the hierarchy of scales $\rho_j^2 >> E_j/m_j^2, \rho_j^3 >> Q_j^2/m_j^2$ and $|\mu_j|\rho_j^{n_j'-2} << |\lambda_j|\rho_j^{n_j-2} << m_j^2$, under which the Friedmann equations (71) and (72) could be written as

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j m_j\rho_j^2}{3\sum_k V_k\rho_k^2}\right) \tag{73}$$

and

$$\frac{V''}{V} = \frac{\sum_j V_j(4m_j^2\rho_j^2)}{\sum_k V_k\rho_k^2}. \tag{74}$$

For a dominant spin mode $\tilde{j}$ with approximately constant $m_{\tilde{j}}$, we now identify:

$$m_{\tilde{j}}^2 = 3\pi G, \tag{75}$$

where G represents the effective dimensionless Newton constant. Then, this yields the characteristic FLRW equations in the relational time:

$$\left(\frac{V'}{V}\right)^2 = \frac{V''}{V} = 12\pi G \tag{76}$$

The mode dominance dynamics ensures a rapid convergence to the lowest spin mode j_0, making $m_{j_0}^2 = 3\pi G$ sufficient for the classical recovery. This establishes group field theory condensate hydrodynamics as a viable pathway to classical cosmology at macroscopic scales, provided that the interaction terms remain subdominant. Also, single-mode dominance further induces a direct proportionality between the Hubble parameter and volume evolution rate:

$$H = \frac{Q_1}{3}\frac{\mathcal{V}'}{\mathcal{V}} \tag{77}$$

This critical relationship then enables the extraction of the cosmological constant from microscopic group field theory parameters.

We now mention the emergent cosmological dynamics using the framework, which is expressed via the relational clock evolution. For a homogeneous, isotropic metric with scale factor $a(t)$, the Hubble parameter is $H = \dot{a}/a$. The effective equation of state becomes $w = -1 - \frac{2\dot{H}}{H^2}$. Using the relational clock, it takes the form $w = 3 - 2\frac{VV''}{(V')^2}$, where $V = a^3$ is the volume and primes denote derivatives with respect to the relational time ϕ.

Let's study w from single-mode group field theory condensates. We will investigate the effective equation of state behavior under single-mode dominance. While multi-mode scenarios introduce richer transitional dynamics and modified convergence to asymptotic w values, the single-mode framework already reveals key insights into how the group field theory interactions shape emergent cosmology.

We get from [114] from the single j mode that

$$w = \frac{-3Q^2 + 4E\rho^2 + m^2\rho^4 + (1-\frac{4}{n})\lambda\rho^{n+2} + (1-\frac{4}{n'})\mu\rho^{n'+2}}{-Q^2 + 2E\rho^2 + m^2\rho^4 - \frac{2}{n}\lambda\rho^{n+2} - \frac{2}{n'}\mu\rho^{n'+2}}, \tag{78}$$

where mode indices are omitted for simplicity. The direct proportionality $V \propto \rho^2$ enables us to do the $w(V)$ analysis without solving the equations of motion.

Firstly, let's consider the bounce. In the interaction-free limit ($\lambda = \mu = 0$) at small condensate modulus ρ, the equation of state reduces to

$$w = \frac{-3Q^2 + 4E\rho^2 + m^2\rho^4}{-Q^2 + 2E\rho^2 + m^2\rho^4} \tag{79}$$

The bounce condition $-Q^2 + 2E\rho^2 + m\rho^4 = 0$ determines the minimum volume

$$\rho_b = \frac{1}{m}\sqrt{\sqrt{E^2 + m^2Q^2} - E}. \tag{80}$$

At ρ_b, the numerator becomes negative, driving $w \to -\infty$- characteristic of post-bounce acceleration. Most importantly, this inflationary phase terminates rapidly, with acceleration ending at volumes comparable to V_b, [114]. And we could see that when there's a bounce, i.e., when $\rho_b = 0$, we must have $Q = 0$. More generally, we will get a bounce if at least one Q_j is non-vanishing in all condensate modes.

And further, we study the phantom divide crossing. The single-mode dynamics permit limited phantom behavior:

1. $\mu < 0$ regime: Asymptotic w approaches $2 - n'/2$ with:

$$w \to -1(n' = 6) \text{ or } w' < -1(n' > 6) \tag{81}$$

2. Late time singularities: $n' > 6$ leads to Big Rip
3. Multi-mode resolution: Two-mode systems enable phantom crossing ($w < -1$), while avoiding singularities through phantom de Sitter analogs

The critical distinction lies in mode competition- multiple modes regulate energy density divergence despite phantom behavior, as we will see in the subsequent analysis. In the presence of interactions, it becomes much harder to find an exact solution of the equation of motion (68), when actually two modes contribute. However, an approximate solution in the large volume region can be obtained. When ρ is large, we can ignore the E_j, Q_j and m_j terms in the equation of motion (68), then we obtain approximately

$$\rho_j'(\phi) = \sqrt{\frac{-\lambda_j}{3}}\rho_j(\phi)^3, \tag{82}$$

which can be solved to give

$$\rho_j(\phi) = \frac{3^{\frac{1}{4}}}{\sqrt{2\sqrt{-\lambda_j}(\phi_{j\infty} - \phi)}}, \tag{83}$$

with a constant $\phi_{j\infty}$. We could see that when interactions are included, the relational time ϕ will have an upper bound, and the mode associated with the smallest asymptotic value $\phi_{j\infty}$ governs the late-time dynamics. Then, this leads to an asymptotic de Sitter-like phase. And contributions from other modes remain significant at intermediate volumes, which is collectively driving the universe's expansion. As mentioned in [114], for vanishing high-order interaction couplings ($\mu_1 = \mu_2 = 0$) and small λ_1, λ_2, the equation of state parameter w will be dominated by the free sector of the condensate at small volumes. In this regime, w approaches $w = 1$ from below as the volume increases, recovering the standard FLRW universe. However, at larger volumes, interaction terms progressively dictate the condensate dynamics. When the scale parameters ρ_j grow sufficiently large, w becomes dominated by these interaction terms. For the specific case $n_1 = n_2 = 6$, w in this interaction-dominated regime depends solely on the ratio $r = \rho_2/\rho_1$. So, we have

$$w = -1 - \frac{4\mathcal{V}_1\mathcal{V}_2 r^2(r^2 - \sqrt{\lambda_1/\lambda_2})}{(\sqrt{\lambda_1/\lambda_2}\mathcal{V}_1 + \mathcal{V}_2 r^4)^2}. \tag{84}$$

Because the parameters are all real and both couplings λ_1 and λ_2 are assumed to be negative, we can see that $w \leq -1$. Recall that when the volume is large, one of the two modes will dominate over the other, and then we get $r \to 0$ or $r \to \infty$. In either case, w will approach -1 from below, which is a contrast with the discussion concerning the single mode.

When the universe volume gets larger, the ratio r gets smaller, and we can expand w respect to small r, such that the equation of state can be expressed by only the total volume $\mathcal{V}$ as follows:

$$w = -1 - \frac{b}{\mathcal{V}}, \tag{85}$$

where $b = 4\mathcal{V}_2\rho_2^2(\phi_{1\infty}) > 0$.

Before we state some results concerning the explicit form of the cosmological constant, it is good to discuss the effects of including more than two condensate modes. Let $\phi_{1\infty}$ denote the smallest asymptotic value of the relational time for mode ρ_1. At very late times, the evolution should therefore be governed by this mode, according to the asymptotic solution (83). Introducing an additional mode with $\phi_{2\infty} > \phi_{1\infty}$ modifies the evolution by inducing a phantom phase before the onset of the de Sitter expansion and alters the asymptotic behavior. If a third mode ρ_3 is included, we must require that $\phi_{3\infty} > \phi_{2\infty}$, otherwise, the ρ_3 mode would dominate the phantom phase. It means that we expect that the inclusion of additional modes primarily affects the cosmological evolution before the phantom phase and should not significantly alter the main results presented in these paragraphs.

In this group field theory model, a de Sitter phase with an effective cosmological constant emerges asymptotically from the microscopic quantum dynamics of spacetime constituents. We could determine the precise expression of such an effective cosmological constant as a function of the microscopic parameters of the model and of the underlying quantum state of the universe.

When we want to investigate it, we note that in the asymptotic large volume region, we could ignore the contributions from other terms and only keep (order-6) interactions in the equation of motion (71). Further, we have seen that in such a region only a single mode dominates, and we can use the relation (77) between Hubble parameter H and the ratio V'/V. So, in the asymptotic de Sitter regime we obtain

$$H^2 = \frac{8}{9} Q_1^2 \Big(- \frac{\lambda_1}{6V_1^2} \Big) = \frac{1}{3} \Big[\frac{4Q_1^2(-\lambda_1)}{3V_1^2} \Big]. \tag{86}$$

Comparing this equation with the ΛCDM at late time, we see that the cosmological constant is determined by the microscopic parameters of the group field theory model,

$$\Lambda = \frac{4Q_1^2(-\lambda_1)}{3V_1^2}. \tag{87}$$

Let us discuss the implications of this expression for Λ, and emphasize several points:

1. Λ is determined by the parameters of a single mode despite the fact that we consider two modes.
2. Actually, for a non-vanishing Λ, we see that $Q_1 = 0$ is necessary; from the theory we could in principle consider the case where Q_j vanishes for all modes j, [114], which may provide us the Minkowski spacetime. But equation (87) shows that the observation of a non-vanishing cosmological constant at late times would require that the Q_1 could not be zero. This implies a quantum bounce ,[114], which resolves the Big Bang singularity. When we express it differently, the non-vanishing cosmological constant Λ itself would be a remnant of the expansion history of our universe in the far past. This indirect connection between very early and very late universe dynamics (and thus very small and very large scales) is quite interesting, and it is only possible because we are in an emergent spacetime framework.
   To be more general, the value of Λ cannot be directly related to the bouncing scale, since Λ is determined by the parameters of a single mode, but the bounce depends on the collective contributions from all modes. The reason for this is that in the bouncing regime, the modulus ρ_j of each mode remains small, and no single mode dominates the dynamics. This is actually required the sum of all Q_j must vanish. If instead a single mode ρ_1 were to dominate the dynamics from the beginning of the bounce, then enforcing ϕ as a good clock would necessitate $Q_1 = 0$. And this leads to a vanishing cosmological constant. By contrast, when multiple modes contribute to the bounce while a single mode dominates the late-time dynamics, it becomes possible to satisfy $\sum_j Q_j = 0$ and still retain a nonzero Q_1, which allows us to obtain a nonvanishing cosmological constant in the model.
   However, we can extract potentially relevant observational constraints, at least qualitatively. On the one hand, Q_1 could not be too large, otherwise, we will have a large cosmological constant Λ. On the other hand, Q_1 contributes to the critical energy density of the universe at the bounce, [114]. Hence, it can't be too small, or we would lose the established physics of the hot dense state of the universe in the very early time, with spacetime dynamics still governed by the Friedmann

evolution (the universe would enter instead quickly into a QG bouncing regime). These two constraints can be principally used to narrow down the possible range of Q_1 using observations.

3. Λ does not depend on m_j, hence the mass renormalization of the group field theory model will not change the value of the cosmological constant. We can note also that m_j is related to the effective Newton's constant, as emerging in the Friedmann phase of cosmic expansion. So, the two key couplings of gravitational dynamics in classical GR are thus both emergent and they are independent of each other in this QG model.

3.4. The Causal Set Theory

The causal set theory prediction for Λ, [117–119] was startling in its simplicity, especially since it was made many years before the observation of accelerated expansion. One starts with the framework of unimodular gravity, in which the spacetime volume element will be fixed. The cosmological constant Λ then appears as a Lagrange multiplier in the action with $\Lambda \int dV = const.$ for any spacetime region of volume V. We see Λ and V are conjugate to each other in a canonical formulation of the theory, so we obtain the formula

$$\Delta V \Delta\Lambda \sim 1 \tag{88}$$

after the quantization of the theory. Even if we keep N, which denotes the number of points in the discretization, constant in the volume V, V still fluctuates between $N - \sqrt{N}$ and $N + \sqrt{N}$. It means that $\Delta V \sim \sqrt{V}$. Therefore, we have the central result

$$\Delta\Lambda \sim \frac{1}{\sqrt{V}}. \tag{89}$$

Let's presuppose now that, for some reason, Λ oscillates around zero: so $\langle\Lambda\rangle = 0$; But then we obtain, with the help of the standard cosmological argument

$$V \sim (H^{-1})^4 = H^{-4} \Rightarrow \Lambda \sim \frac{1}{\sqrt{V}} \sim H^2 \sim \rho_{crit}, \tag{90}$$

where ρ_{crit} stands for the critical density. After that,

$$\Lambda = \Delta\Lambda \sim 10^{-120}, \tag{91}$$

in Planck units, and this is very close to the observed value of Λ. What also important is the prediction states that under these assumptions, Λ always tracks the critical density, and so it is everpresent.

This argument is completely general and requires three important ingredients:

1. the assumption of conjugacy between Λ and V
2. the number to volume correspondence $V \sim N$
3. there are fluctuations in V, which are Poisson, with $\delta V = \sqrt{V} \sim \sqrt{N}$

While 1. could be motivated by a wide range of theories of QG, 2. and 3. are both connected with causal set theory. No other discrete approach to QG makes the $N \sim V$ correspondence at the fundamental level and also takes into consideration Poisson fluctuations kinematically in the continuum approximation. Actually, according to [117], fluctuations in Λ arise as residual quantum effects of spacetime discreteness. What is interesting is that if spacetime admits large extra dimensions, then the contribution to V is very different and gives us the wrong answer for $\Delta\Lambda$. This was shown in [120].

The important question also arises in this quick calculation, why should we assume that $\langle\Lambda\rangle = 0$ holds. The answer to this could lie in the full and as yet unknown quantum dynamics. However, this assumption leads to further predictions that can be well tested. The first implication is that a fluctuating Λ must violate conservation of the stress-energy tensor, and so the Einstein field equations.

A dynamical model for generating fluctuations of Λ was constructed in [121]. They started with the flat $k = 0$ FLRW spacetime and in order to accommodate a fluctuating Λ, they dropped one of the Friedmann equations. But the equation

$$3\left(\frac{\dot{a}}{a}\right)^2 = \rho + \rho_\Lambda \tag{92}$$

was kept, with

$$\rho_\Lambda = \Lambda,\ p_\Lambda = -\Lambda - \frac{\dot{\Lambda}}{3H} \tag{93}$$

and Λ was modeled as a stochastic function of V, such that

$$\Delta\Lambda \sim \frac{1}{\sqrt{V}}. \tag{94}$$

When we deal with it more generally, Λ could be expressed as the action per unit volume, which for causal sets means that $\Lambda \sim S/V$. A very simple stochastic dynamics is then generated by the assumption that every element contributes $\pm\hbar$ to S, so that

$$S = \sum_{elements} \pm\hbar \Rightarrow S/\hbar \sim \pm\sqrt{N} \sim \pm\sqrt{V/L_p^4} \Rightarrow \Lambda \sim \pm\frac{\hbar/L_p^2}{\sqrt{V}} \tag{95}$$

where was equated the discreteness scale with the Planck length L_p. We then get the integro-differential equations

$$\frac{da}{a} = \sqrt{\frac{\rho+\Lambda}{3}}d\tau \tag{96}$$

$$Vd\Lambda = Vd(S/V) = dS - \Lambda\dot{V}d\tau, \tag{97}$$

where

$$V(\tau) = \frac{4\pi}{3}\int_0^t a(t')^3\left(\int_0^{t'}\frac{1}{a(t'')}dt''\right)^3 dt' \tag{98}$$

denotes the volume of the entire causal past of an event in the FLRW spacetime. The stochastic equation is subsequently generated as follows: at the ith step one has the variables a_i, N_i, S_i and Λ_i. The scale factor is then updated using the discrete Friedmann equation

$$a_{i+1} = a_i + a_i\sqrt{\frac{\rho+\Lambda}{3}}(\tau_{i+1} - \tau_i), \tag{99}$$

from which $V_i = V(\tau)$ could be calculated, and so $N_{i+1} = \frac{V_{i+1}}{l^4}$. The action is computed via

$$S_{i+1} = S_i + \alpha\xi\sqrt{N_{i+1} - N_i}, \tag{100}$$

where ξ is a Gaussian random variable with $\Delta\xi = 1$, and α is a tunable free parameter that controls the magnitude of the fluctuations. Afterwards,

$$\Lambda_{i+1} = S_{i+1}/V_{i+1}, \tag{101}$$

with $S_0 = 0$. We must have $0.01 < \alpha < 0.02$ in order to be consistent with astrophysical observations. The results of the observations also suggest that Λ is "everpresent" and tracks the energy density of the universe.

Let's do further comments. This model assumes the spatial homogeneity and it is important to check how inhomogeneities could affect these results. In [122] and [123], they were modeled by taking $\Lambda(x^{\mu})$, such that $\Delta\Lambda(x)$ is dependent only on $\Lambda(y)$ for $y \in J^{-}(x)$. This would mean that well - separated patches in the CMB sky would contain uncorrelated fluctuations in Ω_{Λ}, which are strongly constrained to 10^{-6} by observations and hence insufficient to account for Λ. It was suggested in [121] and [124] that quantum Bell corrections may be a possible way to induce correlations in the CMB sky. But incorporating inhomogeneities into the dynamics in a systematic way remains an important open problem in this field.

A phenomenological model was adopted in [124], which uses the homogeneous temporal fluctuations in Λ to investigate models with a quintessence-type spatially inhomogeneous scalar field, which contains a potential term that varies from realisation to realisation. Using Markov chain Monte Carlo methods to sample the cosmological parameter space and generate different stochastic realisations, it was shown that these causal set theory-inspired models agree with the observations as well as the ΛCDM model. In fact, they do better for baryonic acoustic observation measurements. The extensive and detailed analysis of [124] prepares the stage for direct comparisons with future observations, and it is a challenge for this field of QG phenomenology.

3.5. A Model of Tejinder Singh Based on String Theory and Non-Commutative Geometry

We will now discuss an application of a QG model developed in [79,125–127], in which the universe is made of 'atoms' of space-time-matter and we introduce a new DE particle.

The fundamental QG theory, which we will use, is a non-commutative matrix dynamics of Grassmann matrices, the so-called 'atoms' of space-time-matter which do not make a distinction between space-time and matter. These matrices evolve in the Hilbert space according to a time parameter characteristic of non-commutative geometry, which derives from the Tomita-Takesaki theory, [128], and the Radon-Nykodym theorem,[129]. We describe the space- time-matter atoms by a Lagrangian dynamics resulting from a well-defined action principle, and they interact via entanglement and 'collisions', [130]. The dynamics possesses a unique conserved charge known as the Adler-Millard charge, [131], responsible for the emergence of quantum theory, [127].

This theory is assumed to operate at the Planck scale. Actually, there is no space-time, but one can define a Planck space foam of space-time and matter. If one is not observing dynamics at the Planck scale, a mean-field dynamics at lower energies is obtained. This is simply done by averaging over time-scales much larger than Planck time, using the standard techniques of statistical thermodynamics. We distinguish two classes of this mean field dynamics:

1. If a length scale associated with the space-time-matter atoms is larger than Planck length, one then gets a quantum theory of gravity for the bosonic (gravity) and fermionic (matter) aspects of the space-time-matter atoms. We can see that these degrees of freedom evolve with respect to the characteristic time parameter of non-commutative geometry. QG will not be exclusively a Planck scale phenomenon, but relevant even at much lower energies if the gravity associated with an space-time-matter atom cannot be neglected, [125]. This could precisely happen, if the Compton wavelength associated with the space-time-matter atom is of the order of the size of the observed universe.
2. In the other extreme limit, the entanglement of a very large number of space-time-matter atoms results in a rapid 'spontaneous localisation', giving rise to a classical space-time geometry driven by point matter sources. These are then obeying the laws of classical GR. Ordinary space-time is recovered, but the non-commutative time parameter will be lost in the classical limit.

Consider the space-time-matter atoms which have not undergone the localisation. The standard QFT is recovered by taking their matter degrees of freedom from QG, ignoring their gravity, and taking space-time from the above classical limit of other space-time-matter atoms. Actually, in the framework of this theory, we propose that DE consists of about 10^{122} space-time-matter atoms, each having an associated mass of $10^{-33} eV/c^2$. Hence the Compton wavelength, $\hbar/mc$, of such a space time-matter atom is of the order of 10^{28} cm, which is comparable to the size of the observed universe. A space-time-matter atom is nothing but an elementary particle whose gravitational aspect cannot be distinguished from the particle aspect. An atom labelled by the matrix/operator variable $q = q_B + q_F$, with q_B and q_F being its bosonic and fermionic parts, is described by the action principle

$$\frac{L_p.S}{c.C_0} = \frac{1}{2}\int Tr[\frac{L_p}{l^2c^2}(\dot{q}_B + \beta_1\dot{q}_F)(\dot{q}_B + \beta_2\dot{q}_F)\, d\tau]. \tag{102}$$

$\kappa \equiv \frac{C_0}{2}$ is a constant so chosen that it gives the correct dimensions of action and the correct numerical coefficient for recovery of the Einstein-Hilbert action. Further, the β matrices are constant matrices, proportional to L_p^2/l^2, where l is a fundamental length associated with the space-time-matter atom, subsequently identified with Compton wavelength, [125].

For this action, the first integrals of the equations of motion, with evolution with respect to the Connes time parameter τ, will be

$$\begin{aligned} p_B &= \frac{\delta L}{\delta \dot{q}_B} = \frac{a}{2}\Big[2\dot{q}_B + (\beta_1+\beta_2)\dot{q}_F\Big] \\ p_F &= \frac{\delta L}{\delta \dot{q}_F} = \frac{a}{2}\Big[\dot{q}_B(\beta_1+\beta_2) + \beta_1\dot{q}_F\beta_2 + \beta_2\dot{q}_F\beta_1\Big], \end{aligned} \tag{103}$$

where we have made a notation $a \equiv \frac{L_p^2}{l^2c^2}$. The first of these equations could be written as an eigenvalue equation which, as explained in [125], results from defining the modified Dirac operator:

$$\frac{1}{lc}\frac{dq}{d\tau} \sim D \equiv D_B + D_F;\ D_B \equiv \frac{1}{lc}\frac{dq_B}{d\tau};\ D_F \equiv \frac{\beta_1+\beta_2}{2lc}\frac{dq_F}{d\tau} \tag{104}$$

We note that it is a constant operator, and we could also express this as an eigenvalue equation

$$\Big[D_B + D_F\Big]\psi \equiv \lambda\psi \equiv (\lambda_R + i\lambda_I)\psi \equiv \left(\frac{1}{l} + i\frac{1}{l_I}\right)\psi \tag{105}$$

Since D will be bosonic, we assume that the eigenvalues λ are complex numbers and separate each eigenvalue into its real and imaginary part. Further, this is taken as the definition of the length scale l introduced above. As was demonstrated in [125], $l^3 = L_p^2 l_I$, and since l is a Compton wavelength, this implies $l_I = \hbar^2/Gm^3$.

These two lengths, l and l_I, play an important role in the following. The length l is a measure of the quantum dispersion, and the decoherence length l_I is a measure of the spontaneous localisation. If $l < l_I$, the quantum dispersion dominates the classical behaviour, and the space- time-matter atom becomes a quantum elementary particle. If $l > l_I$ the spontaneous localisation and classical behaviour dominate quantum dispersion, and the space-time-matter atom becomes a classical object. Actually, the quantum-to-classical transition takes place at $l = l_I$, in which case both the lengths are equal to the Planck lengths. This is the space-time-matter foam which exists at the Planck scale. In fact, it represents Planck scale quantum dispersion and spontaneous localisation, which is equivalent to a stochastic oscillation of Planck scale curvature. In our theory, QG arises on time scales larger than Planck time, after coarse-graining over Planck scale foam. The above relation between l and l_I can be more reasonably written as $(l/L_p)^3 = (l_I/L_p)$ and is very closely related to the Karolyhazy uncertainty relation, [132]. We would like to note that if $l_I < L_p$, then both lengths are smaller than Planck length- this is the classical limit. On the other hand, if $l_I > L_p$, then both lengths exceed Planck length, this being the quantum limit. The two lengths also highlight two important time scales: $t_q = l/c$ which is the quantum dispersion time scale, and $t_d = l_I/c$, the decoherence time scale. The classical behavior results if $t_d < t_q$, and the quantum behavior prevails if $t_q < t_d$. These possible regimes above are the defining equations for our DE particle. We propose to name these particles as mitrons. It is, of course, a quantum gravitational entity, with its gravitational aspect described by the bosonic q_B and its matter aspect described by the fermionic q_F. The gravitational aspect of the mitron cannot be described classically, because its decoherence length l_I is enormous. For our assumed mass of $10^{-33}\ eV/c^2$, this length is 123 orders of magnitude larger than the size of the observed universe. Consequently, the decoherence time l_I/c is 10^{140} s, which is far far greater than the current age of the universe. As a result, the mitrons never undergo the classicalisation process of spontaneous localisation. It is inherently quantum gravitational in nature. We make the important assumption as well that DE particles do not entangle with each other, nor with other particles.

On the other hand, as explained in the works [127,133], ordinary matter undergoes spontaneous localisation, because of entanglement, and gives rise to the observed classical universe. Against the backdrop of the classical expanding universe, let us now understand why the mitrons are responsible for the acceleration of the universe. The contribution to the mass density of the universe, from a single DE particle, is of the order m_{DE}/R_H^3, where the mass m_{DE} is of the order $10^{66}\ g$, and R_H, the Hubble radius, is about $10^{28}\ cm$. This gives us an extremely low mass density of the order of $10^{-150}\ g/cm^3$.

We now mention the recent discussion of the Karolyhazy uncertainty relation, [132], and from the implied holography, that the universe has $(R_H/L_p)^2 \sim 10^{122}$ quanta of information, [134]. Assuming there corresponds one unit of information per elementary particle, we realise that ordinary matter contributes only a very small fraction to the information content, there being some 10^{80} such particles. Thus we propose, following [134], that there are 10^{122} dark energy particles in the observed universe, and hence their total contribution to the energy density of the universe is about $10^{28}\ g/cm^3$, which is a reasonable estimate for the inferred dark energy content of the universe. When we will summarize, we are proposing that dark energy consists of 10^{122} mitrons, each of mass $10^{-33}\ eV/c^2$. Further, these space-time-matter atoms are assumed to be unentangled, and hence they do not undergo spontaneous localisation during the lifetime of the present universe. We might think of each such particle as a quantum gravitational wave of the size of the observed Universe.

Such objects collectively behave like DE and they are causing acceleration of the expanding Universe, because they have a very long wavelength, and their kinetic energy T is negligible compared to their potential energy V. The pressure is essentially $T - V$ and the energy density is $T + V$. With T being negligible, this implies an equation of state as for the cosmological constant: $p = -\rho$. In this theory, we could argue for the dominance of the gravitational aspect over the matter aspect by examining the expression for the Hamiltonian for the space time-matter atom, given by, [127],

$$\mathbf{H} = Tr\left[\frac{a}{2}(\dot{q}_B + \beta_1\dot{q}_F)(\dot{q}_B + \beta_2\dot{q}_F)\right] \tag{106}$$

and in terms of the momenta

$$\mathbf{H} = Tr\frac{2}{a}\left[(p_B\beta_1 - p_F)(\beta_2 - \beta_1)^{-1}(p_B\beta_2 - p_F)(\beta_1 - \beta_2)^{-1}\right]. \tag{107}$$

We see from the first of these expressions that since the β matrices scale as L_p^2/l^2, and since $l >> L_p$, the fermionic matter term is negligible, and the effective contribution to the Hamiltonian is from the gravitational term $\dot{q}_B^2$. It means that the kinetic energy term is negligible compared to the gravitational energy, and once again we see that the contribution of the mitrons behaves like a cosmological constant. The mass density estimate from above shows that it is a cosmological constant of the same magnitude as implied by observations.

We need to ask the key question, does the DE in our model behave like the cosmological constant, or is it dynamical? We can explain as follows: the Compton wavelengths associated with a proton and a DM particle (within this Tejinder's theory it was proposed a dark matter candidate particle of mass about $10^{-12}\ eV/c^2$, which is 10^{21} orders of magnitude lighter than the proton, [125]) are 10^{-13} cm and 108 cm respectively. Since these are both much smaller than the Hubble radius, it could be assumed that they are decoupled from the expansion of the universe and hence constant. On the other hand, the Compton wavelength for the mitron is of the order of the Hubble radius, so we assume it to stretch in linear proportion to the Hubble radius. Thus, it can be computed that the equivalent mass density of the DE particle falls as the inverse fourth power of the Hubble radius as the universe expands. Since the number of DE particles increases as H^2, the net DE density falls as the inverse square of the Hubble radius, and hence the DE is dynamical. Further, because it falls more slowly than ordinary matter density as well as slower than DM density, the universe was matter-dominated in the past. Only in the present epoch predominates the DE. We wish to emphasize that in this approach, DE is a purely quantum gravitational effect and could be properly described only in a QG. Its description in the context of classical GR is only approximate. And what is also remarkable is that such DE should be regarded as evidence for the quantum nature of gravity, [125].

3.6. Causal Dynamical Triangulations and Baby Universe Dark Energy

We discussed in section 2.6 about the very basics of causal dynamical triangulations and now in this section we discuss perhaps its most interesting cosmological implications. We provide an overview of the scenario where the late-time expansion of the universe arises through mergers with other universes, a framework situated within the context of causal dynamical triangulations [135–137]. Although the term "baby universe" is used here, it does not necessarily imply that these universes are significantly smaller than our own. This terminology is adopted to align with prior works on the subject, see [136,138–140] for extended works on the cosmology of causal dynamical triangulations. By considering mergers with other universes, we inherently discuss a multi-universe theory. Analogous to a multi-particle theory, we define creation and annihilation operators, $\Psi^\dagger(v)$ and $\Psi(v)$, respectively, for individual universes characterized by spatial volume v. In a complete theory of four-dimensional QG, spatial volume alone would not suffice to describe a universe's state at a given time t. However, within the minisuperspace approximation, the spatial universe is described entirely by its spatial volume v, and its quantum state is represented as $|v\rangle$. The multi-universe Fock space, constructed from such single-universe states, includes the Fock vacuum state $|0\rangle$, governed by the following commutation relation and operator actions [141]

$$[\Psi(v), \Psi^\dagger(v')] = \delta(v - v'), \quad \Psi^\dagger(v)|0\rangle = |v\rangle, \quad \Psi(v)|0\rangle = 0. \tag{108}$$

The minisuperspace quantum Hamiltonian, incorporating the creation and annihilation of universes, takes the form:

$$\hat{H} = \hat{H}^{(0)} - g \int dv_1 \int dv_2 \, \Psi^\dagger(v_1)\Psi^\dagger(v_2)\,(v_1 + v_2)\Psi(v_1 + v_2) - \tag{109}$$

$$-g \int dv_1 \int dv_2 \, \Psi^\dagger(v_1 + v_2)\, v_2 \Psi(v_2)\, v_1 \Psi(v_1) - \int \frac{dv}{v}\, \rho(v)\Psi^\dagger(v), \tag{110}$$

where

$$\hat{H}^{(0)} = \int_0^\infty \frac{dv}{v}\, \Psi^\dagger(v)\hat{\mathcal{H}}^{(0)} v \Psi(v), \quad \hat{\mathcal{H}}^{(0)} = v\left(-\frac{3}{4}\frac{d^2}{dv^2} + \lambda\right). \tag{111}$$

The term $\hat{H}^{(0)}$ describes the propagation of a single universe, derived from the action associated with the minisuperspace approximation. The cubic terms represent the splitting of a universe into two and the merging of two universes into one. The final term indicates that a universe can emerge from the Fock vacuum $|0\rangle$ if its spatial volume is zero. Without this term, $\hat{H}|0\rangle = 0$, rendering the Fock vacuum stable. While the details of merging and splitting processes are beyond the scope of this approximation, the primary focus lies on how these interactions influence the spatial volume of universes.

Even within the minisuperspace framework, the Hamiltonian $\hat{H}$ is complex, as it allows for successive splitting, merging, and topology changes of universes. Notably, the dimension dependence of the Hamiltonian is absorbed into coupling constants, such as κ, λ, and g. In two-dimensional causal dynamical triangulations, a tractable truncation termed generalized causal dynamical triangulations exists, which is solvable analytically and holds particular cosmological significance. This theory

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models the evolution of the universe, incorporating mergers with other universes created over time. After detailed analysis, the Hamiltonian takes the form [142]

$$\mathcal{H} = v\left(-\frac{3}{4}(p^2 + \lambda - 2gF(p))\right) = \frac{3}{4}v\left((p+\alpha)\sqrt{(p-\alpha)^2 + \frac{2g}{\alpha}}\right), \tag{112}$$

where p is the classical momentum conjugate to v. This Hamiltonian demonstrates that cosmic expansion can occur without a cosmological constant, driven instead by mergers with other universes.

Introducing matter modifies the Hamiltonian to:

$$\mathcal{H}[v,p] = v(-f(p) + \kappa\rho_{\mathrm{m}}(v)), \quad v\rho_{\mathrm{m}}(v) = v_{pt}\rho_{\mathrm{m}}(v_{pt}), \tag{113}$$

where v_{pt} and $\rho_{\mathrm{m}}(v_{pt})$ denote the spatial volume and matter density at the present time t_{pt}. The equations of motion for arbitrary $f(p)$ are derived as [142]

$$\dot{v} = \frac{\partial\mathcal{H}}{\partial p} = -vf'(p), \quad 3\frac{\dot{a}}{a} = \frac{\dot{v}}{v} = -f'(p), \tag{114}$$

$$\dot{p} = -\frac{\partial\mathcal{H}}{\partial v} = f(p), \quad t - t_0 = \int_{p_0}^{p}\frac{dp}{f(p)}, \tag{115}$$

where p_0 is the value of p at an initial time t_0, not necessarily the universe's beginning. From these equations, the Hubble parameter is expressed as:

$$H = -\frac{f'(p)}{3}. \tag{116}$$

One can express various significant cosmological parameters, such as redshift and angular diameter, in terms of the variable p using the formalism discussed previously. However, our focus here will be on the so-called "formal pressure" and "formal energy," which serve as the analogs of pressure and energy density in this cosmological framework. Any solution to the above equations must inherently satisfy the condition $\mathcal{H} = \text{const}$ by construction. For the "on-shell" solutions where $\mathcal{H} = 0$, we obtain the relation

$$f(p) = \kappa\rho_{\mathrm{m}}(v) = \kappa\rho_{\mathrm{m}}(v_{tp})\frac{v_{tp}}{v} = f(t_{tp})\frac{t_{tp}}{v}, \tag{117}$$

where p_0 represents the value of p at the present time t_{tp}. Equation (117) is often referred to as the "Generalized Friedmann equation."

The "formal density" $\rho_f(t)$ or equivalently $\rho_f(p)$, which is associated with the function $f(p)$, is determined by rewriting the generalized Friedmann equation as

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{\kappa\rho_{\mathrm{m}}(v)}{3} + \frac{\kappa\rho_f(v)}{3}. \tag{118}$$

Using the equations of motion, we derive

$$\kappa\rho_f(p) = \frac{1}{3}(f'(p))^2 - f(p), \tag{119}$$

and its time evolution is governed by

$$\kappa\frac{d\rho_f}{dt} = f(p)f'(p)\left(\frac{2}{3}f''(p) - 1\right). \tag{120}$$

The "formal pressure" P_f, associated with ρ_f, is introduced through the energy conservation equation

$$\frac{d}{dt}(v\rho_f) + P_f\frac{d}{dt}v = 0. \tag{121}$$

This relation leads to the expression

$$P_f = f(p)\left(\frac{2}{3}f''(p) - 1\right) - \rho_f(v), \tag{122}$$

and the "formal equation of state parameter" w_f, defined for $\rho_f \neq 0$, is given by

$$w_f = \frac{P_f}{\rho_f} = \frac{f(p)(\frac{2}{3}f''(p) - 1)}{\frac{1}{3}(f'(p))^2 - f(p)} - 1. \tag{123}$$

Furthermore, it can be noted that

$$P_f = \frac{1}{\kappa}\left[f(p)\left(\frac{2f''(p)}{3} - 1\right) + f(p) - \frac{f'(p)^2}{3}\right]. \tag{124}$$

The definitions of ρ_f and P_f ensure that the equations of motion can be recast in the familiar form of GR, employing the variables a, ρ_{m}, ρ_f, and P. For various definitions of the function $f(p)$, we can have various forms of the evolution of the universe in this regime. We can recover the simple de Sitter evolution, include matter contributions as well and one can show that for certain polynomial forms of $f(p)$ one can have appropriate late universe cosmology with expansion, in the scenario where the universe is continually merging with baby universes. We would refer the readers to [138–140] for details in this regard and also interesting implications for the Hubble tension.

3.7. Asymptotically Safe Cosmology

The ability to construct gravitational renormalization group flow approximations beyond perturbation theory is essential for testing asymptotic safety at a conceptual level. A particularly robust framework for carrying out such calculations is the functional renormalization group equation for the gravitational effective average action, given by:

$$\partial_k\Gamma_k[g,\bar{g}] = \frac{1}{2}\mathrm{Tr}\left[(\Gamma_k^{(2)} + \mathcal{R}k)^{-1}\partial k\mathcal{R}_k\right]. \tag{125}$$

This equation is formulated using the background field method, where the metric is decomposed into a fixed background and fluctuations. The Hessian represents the second functional derivative of with respect to the fluctuation field, evaluated in a given background. Additionally, serves as a scale-dependent mass term for fluctuations with momenta satisfying, where the renormalization group scale is determined by the background metric. Since is a mass term, it is typically assumed to be positive definite. The specific role of in both the numerator and denominator ensures that the trace term remains finite in both the infrared and ultraviolet regimes, thereby constraining the flow of to fluctuations characterized by momenta .

A commonly used approach in studying asymptotic safety involves projecting the functional renormalization group equation onto the Einstein-Hilbert action, thereby approximating as [143–145]:

$$\Gamma_k = \frac{1}{16\pi G_k}\int d^4x\sqrt{-g}[-R + 2\Lambda_k] + \text{gauge-fixing and ghost terms}, \tag{126}$$

where Λ_k and G_k denote the running cosmological constant and running Newton's gravitational constant, respectively. Their scale dependence can be parameterized using dimensionless counterparts as

$$\Lambda_k = k^2\lambda_k, \tag{127}$$

$$G_k = \frac{g_k}{k^2}, \tag{128}$$

where numerical values g_k and λ_k are obtained from asymptotic safety studies.

To proceed, we consider a flat FLRW background metric:

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \tag{129}$$

Assuming a perfect fluid energy-momentum tensor, the modified Friedmann and continuity equations take the form:

$$H^2 = \frac{8\pi G_k}{3}\rho + \frac{\Lambda_k}{3}, \tag{130}$$

$$\dot{\rho} + 3H(\rho + p) = -\frac{\dot{\Lambda}k + 8\pi \dot{G}k\rho}{8\pi G_k}. \tag{131}$$

The continuity equation arises from the Bianchi identity, which ensures that Einstein's equations satisfy, signifying the conservation of the Einstein tensor's divergence. The additional terms on the right-hand side of (131) can be interpreted as representing an energy exchange between gravitational and matter degrees of freedom.

To proceed further, an appropriate form for the cutoff scale is required. In the renormalization group improvement scheme, this is typically associated with a characteristic length scale of the system. In the cosmological context, several choices exist, as discussed in [145]. A commonly adopted choice is to take to be proportional to the Hubble parameter, as it naturally represents a time-dependent physical scale

$$k = \zeta H(t), \tag{132}$$

where ζ is a proportionality constant. This choice has been widely used in asymptotically safe cosmology [145–147] and is motivated by its role as a dynamically relevant scale for cosmic evolution. However, this choice of cutoff is not unique and there are four widely used cutoff schemes in asymptotically safe cosmology:

- Type I Cutoff: Based on cosmic time t, given by

  $$k^2 = \zeta^2 t^{-2}$$

- Type II Cutoff: This is the choice about which we wrote above proportional to the Hubble parameter H, expressed as

  $$k^2 = \zeta^2 H^2$$

- Type III Cutoff: Related to spacetime curvature, using the Kretschmann scalar $R_{abcd}R^{abcd}$, given by

  $$k^2 = \zeta^2 \sqrt{R_{abcd}R^{abcd}}$$

- Type IV Cutoff: Tied to the temperature T of the cosmic plasma, where $T \propto \rho^{1/4}$, formulated as

  $$k^2 = \zeta^2 T^2$$

With ζ just being a dimensionless proportionality constant, and each choice reflects a different physical length scale governing the renormalization group evolution of gravitational couplings.

3.8. Holographic Dark Energy

The holographic principle has emerged as one of the most profound concepts in contemporary theoretical physics, playing a pivotal role in QG research. Deeply connected to the thermodynamics of black holes and the AdS/CFT correspondence, it naturally invites speculation about its consequences for cosmology, especially in explaining the late-time acceleration of the universe. Over the past two decades, this perspective has led to the formulation of holographic DE models, which attempt to encode DE dynamics within a holographic framework. Readers seeking comprehensive treatments of these models are referred to [148–155].

The key premise of holographic DE theories lies in applying the holographic principle to the energy content of the universe. Within a spacetime region characterized by a length scale L, the total energy density of DE, ρ_{de}, is assumed to be understood using parameters defined on the boundary surface. Since the reduced Planck mass M_p and L are the only relevant scales, dimensional arguments yield

$$\rho_{de} = C_1 M_p^4 + C_2 M_p^2 L^{-2} + C_3 L^{-4} + \dots \tag{133}$$

where C_1, C_2, C_3 are numerical constants. The first term, $C_1 M_p^4$, is associated with the vacuum energy expected from QFT, but its predicted magnitude exceeds the observed DE density by roughly 10^{120} times, which gives rise to the cosmological constant problem. Since this term violates holographic scaling, Andrew Cohen, David Kaplan, and Ann Nelson argued that the traditional field-theoretic estimate $\rho_{de} \sim M_p^4$ becomes invalid as systems approach their Schwarzschild radius, implying that effective field theory must break down near gravitational collapse [148].

To address this issue, the holographic principle suggests a link between the ultraviolet (UV) and infrared (IR) cutoffs of the theory. The constraint ensuring that the quantum vacuum energy does not lead to gravitational collapse is expressed as

$$L^3 \Lambda^3 \leq S^{3/4} \tag{134}$$

where S is the entropy, L denotes the IR scale, and Λ the UV cutoff. Assuming the Bekenstein–Hawking entropy relation $S \propto A$ and identifying the vacuum energy with $\rho_{DE} \sim \Lambda^4$, one obtains the characteristic holographic DE density

$$\rho_{DE} = 3c^2 L^{-2} \tag{135}$$

where c is a dimensionless constant determined observationally. Over time, a number of modified holographic scenarios have been developed by incorporating generalized entropies or non-standard geometrical effects, each yielding distinct cosmic evolution patterns.

For instance, in the Tsallis holographic framework, non-extensive Tsallis entropy modifies the standard black hole thermodynamics, giving

$$\rho_\Lambda = 3c^2 L^{-(4-2\sigma)} \tag{136}$$

where σ represents the non-extensivity parameter, which is the standard holographic DE limit is recovered when $\sigma \to 1$. Similarly, Barrow's fractal deformation of horizon entropy, motivated by quantum gravitational microstructure, results in

$$\rho_\Lambda = 3c^2 L^{\Delta-2} \tag{137}$$

where $0 \leq \Delta \leq 1$, and $\Delta = 0$ reproduces the usual case. In models based on Rényi entropy, which is a further generalization of Boltzmann–Gibbs statistics, the DE density reads

$$\rho_\Lambda = \frac{3d^2}{8\pi L^2}(1 + \pi\delta L^2)^{-1} \tag{138}$$

with δ acting as a deformation constant that vanishes in the standard limit. The Kaniadakis holographic DE model which is inspired by κ-deformed statistics, introduces a correction term giving

$$\rho_\Lambda = 3c^2 L^{-2} + 3k^2 L^2 \tag{139}$$

where the deformation parameter k satisfies $-1 < k < 1$. Finally, the Fractional holographic DE approach incorporates fractional derivatives inspired by fractional calculus, giving

$$\rho_{DE} = 3c^2 L^{\frac{2-3\alpha}{\alpha}} \tag{140}$$

with $1 \leq \alpha < 2$; the canonical form follows when $\alpha = 2$. Together, these extensions demonstrate the remarkable adaptability of the holographic framework in addressing the cosmological constant problem and describing diverse DE behaviors.

Different holographic prescriptions lead to distinct cosmological dynamics. For example, the equation-of-state parameter w in the standard holographic DE model typically evolves within the quintessence range, unlike the fixed $w = -1$ of the ΛCDM paradigm. This evolution allows holographic DE models to more flexibly accommodate cosmic observations such as CMB anisotropies, baryon acoustic oscillations, and type Ia supernova data. Moreover, the inclusion of additional parameters such as σ, Δ, δ, k, and α grants these models the ability to describe a variety of expansion histories, potentially offering insight into the coincidence problem,which is to tell one why DE domination emerges around the same epoch as matter dilution.

A central issue in all holographic DE constructions is the determination of the appropriate infrared cutoff L. Since L effectively sets the energy density scale, its definition strongly influences the cosmological evolution predicted by the model. An early and natural proposal was to take $L = H^{-1}$, with H being the Hubble parameter. While this choice directly ties DE to the expansion rate, it produces an equation of state approaching zero at late times, failing to drive acceleration.

A subsequent improvement considered the particle horizon,

$$L_{par} = a \int_0^t \frac{dt'}{a(t')} \tag{141}$$

which captures the maximum comoving distance over which causal interactions could have occurred since the Big Bang. However, this choice results in $w > -1/3$, still insufficient to generate cosmic acceleration. The reason is that although it integrates over past causal structure, it lacks the negative pressure required for present-day acceleration.

To remedy this, Miao Li proposed using the future event horizon as the cutoff, defined as

$$L_f = a \int_t^{\infty} \frac{dt'}{a(t')} \tag{142}$$

This formulation successfully produces accelerated expansion but at the cost of a theoretical drawback as L_f depends on the future evolution of the universe, leading to apparent causality violations since current dynamics would depend on unknown future conditions.

A more refined and locally defined alternative was introduced by Lizeth Granda and Antonio Oliveros, who proposed incorporating the time derivative of the Hubble parameter into the cutoff definition

$$L = (\alpha H^2 + \beta \dot{H})^{-\frac{1}{2}} \tag{143}$$

where α and β are constants determined by observational constraints. This approach captures both the expansion rate and its rate of change, allowing for a more flexible description consistent with evolving cosmological data.

The most general form of this idea was proposed by Shin'ichi Nojiri and Sergei Odintsov [156], who extended the cutoff concept to include higher-order derivatives and other horizon scales. Their generalized definition can be expressed as

$$L = L(H, \dot{H}, \ddot{H}, \ldots, L_{par}, L_f, \dot{L}_{par}, \dot{L}_f, \ldots) \tag{144}$$

where L may depend on combinations of the Hubble parameter and its derivatives, as well as on particle and future horizons and their time derivatives. This framework unifies all earlier proposals and provides a powerful platform for constructing models capable of reconciling theoretical consistency with empirical viability.

3.9. Emergence of Dark Energy and Dark Matter in Entropic Gravity

The Verlinde's model of entropic gravity suggests that gravity is not a fundamental force but an emergent phenomenon, which arises from thermodynamics and information on a holographic screen, and can be used to explain DE. As we already stated, the force of gravity is analogous to the entropic force that arises when information about the positions of material bodies changes. We will actually explore how DE emerges from this framework as a naturally small and positive cosmological constant, which potentially explains the accelerated expansion of the universe, and how the model can also account for DM. Let's be more concrete, [157].

We could naively imagine that there is spreaded some sort of fluid in the Universe, which is attributed to DE, and the deviation of the matter is responsible to the origin of a force connected with the existence of DM. The main hypothesis from which we will derive all the consequences is contained in the following two statements, [157]:

1. We have a microscopic bulk perspective in which the area law for the entanglement entropy is due to the short distance entanglement of neighboring degrees of freedom that build the emergent bulk spacetime.
2. The de Sitter entropy is evenly divided over the same microscopic degrees of freedom that build the emergent spacetime through their entanglement, and would be caused by the long range entanglement of part of these degrees of freedom.

We now depict the basic computations, which are used to describe the entropy content of the de Sitter space for the static coordinate patch described by the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{145}$$

where we prescribe the function $f(r)$ as

$$f(r) = 1 - \frac{r^2}{L^2}. \tag{146}$$

Let's take now the perspective of an observer near the origin $r = 0$, so that the edge of his causal domain will coincides with the horizon at $r = L$. Then, the horizon entropy equals to

$$S_{DE}(L) = \frac{A(L)}{4G\hbar} \quad \text{and} \quad A(L) = \Omega_{d-2} L^{d-2}, \tag{147}$$

where Ω_{d-2} is the volume of a $(d-2)$-dimensional unit sphere. We hypothesize that this entropy is evenly distributed over microscopic degrees of freedom that make up the bulk spacetime. When we want to determine the entropy density we view the spatial section at $t = 0$ as a ball with radius L bounded by the horizon. The total de Sitter entropy is then divided over this volume so that a ball of radius r centered around the origin contains an entropy $S_{DE}(r)$ proportional to its volume

$$S_{DE}(r) = \frac{1}{V_0} V(r) \quad \text{with} \quad V(r) = \frac{\Omega_{d-2} r^{d-1}}{d-1}. \tag{148}$$

The subscript DE indicates that the entropy will be carried by excitations of the microscopic degrees of freedom that lift the negative ground state energy to the positive value, which is associated with the DE.

We get the value of the volume V_0 per unit of entropy from the requirement that the total entropy $S_{DE}(L)$, where we put $r = L$, equals the Bekenstein-Hawking entropy associated with the cosmological horizon. By comparing (148) for $r = L$ with (147) one computes that V_0 takes the value

$$V_0 = \frac{4G\hbar L}{d-1} \tag{149}$$

where the factor $(d-1)/L$ arises from the relative normalization of the horizon area $A(L)$ and the volume $V(L)$. We therefore determine this entropy density by the Planck area and the Hubble scale. In fact, this value of the entropy density has been already proposed as a holographic upper bound in a cosmological setting.

Some alternative way to write the entropy $S_{DE}(r)$ is in terms of the area $A(r)$

$$S_{DE}(r) = \frac{r}{L}\frac{A(r)}{4G\hbar} \quad \text{with} \quad A(r) = \Omega_{d-2}r^{d-2}. \tag{150}$$

We see from this expression immediately that when we put $r = L$ we recover the Bekenstein-Hawking entropy (147).

We will now discuss, how the ’elasticity model’ is used for the description of DM. The matter normally arises by adding excitations to the ground state. In our description of de Sitter space there is, however, an alternative possibility, since it already contains delocalized excitations that constitute the DE. The matter particles then correspond to localized excitations. Therefore, it is natural to assume that at some moment in the cosmological evolution these localized excitations appeared via some transition in the delocalized DE excitations. The string theoretic perspective described in [157] suggests that the DE excitations should the basic constituents in our universe. Matter particles correspond to the bound states of these basic excitations, that have escaped the DE medium. In string theory description these degrees of freedom have escaped from the Higgs branch onto the Coulomb branch.

The transition by which matter appeared has actually removed an amount of energy and entropy from the underlying microscopic state. The resulting redistribution of the entropy density with respect to its equilibrium position is then described by some displacement vector. Since we study a system with a non-zero temperature, the displacement of entropy leads to a change in the free energy density. And the effective theory that usually describes the response due to the displacement of the free energy density already existed for a long time, and is older than GR. It is well-known theory of elasticity.

Due to the competition between the area law and volume law entanglement, the microscopic de Sitter states exhibit glassy behavior leading to slow relaxation and memory effects, [157]. For our applications this means the displacement of the local entropy density due to matter is not immediately erased, but leaves behind a memory imprint in the underlying quantum state. Afterwards, this results in a residual strain and stress in the DE medium, which could only relax very slowly, [157].

But for us is the most important that the fact matter causes a displacement of the DE medium implies that the medium also causes a reaction force on the matter. The magnitude of this elastic force should be determined in terms of the residual elastic strain and stress. It is proposed, [157], that this force leads to the excess gravity that is currently attributed to DM. Indeed, it was shown in [157] that the observed relationship between the surface mass densities of the apparent DM and the baryonic matter naturally follows by applying old elements of the linear theory of elasticity. Then, the main input that we need to determine the residual strain and stress is the amount of entropy S_M that would removed by matter. These observations are very interesting and the results, that we could model with DE also DM, makes the entropic gravity a unique approach to QG. However, it was shown according to [158] that it will likely have significant flaws in its foundations regarding thermodynamic considerations.

4. Ring Paradigm as a New Approach to Quantum Gravity

We recently formulated an approach to QG, [35–38], ring paradigm (RP). When we want to introduce a basic idea, on which it is based, let's imagine a set of effectively 1-dimensional Hopf-linked rings S^1 in $\mathbb{R}^3$, where every two rings can be linked maximally once, the rings cannot be knotted or twisted, and we do not consider any Brunnian type of links in this set. We stretch this structure and attach it to some boundary (whole bulk of the universe) and then consider a projection of this set to a plane. The elementary particles and fields of the standard model are placed to the points corresponding to the intersections of the projection of this set of rings to the plane.

The gravitational rings are modeled by these elongated one-dimensional objects. They should transfer the Newtonian interaction by some finite but extremely big superluminal velocity c_g, and then decay to standard gravitons. Interestingly, the gravitons are modeled as phonons on the grid of matter in the Universe. This also means that we are returning to an aether-like model in physics, because we need some medium which will enable the spreading of the vibrations in the crystal (this medium is spread in the space between galaxies). Note that this will be created from some phantom field $\psi(x)$.

We will now illustrate how the Newtonian limit is achieved in a simple toy model. We consider a row of galaxies connected by such rings. The standard quantization procedure is applied to the Hamiltonian

$$H = \sum_{i=1}^{2} \frac{1}{2m} P_i^2 + \sum_{i,j=1}^{2} V_{ij} Q_i Q_j, \tag{151}$$

where

$$V = \begin{pmatrix} \frac{1}{2}k + \frac{1}{2}k_3 & -\frac{1}{2}k_3 \\ -\frac{1}{2}k_3 & \frac{1}{2}k + \frac{1}{2}k_3 \end{pmatrix},$$

$k, k_3 > 0$. After performing the standard computations [37], we can rewrite the Hamiltonian as

$$H = \sum_{\alpha=1}^{2} \hbar w_\alpha a_\alpha^+ a_\alpha \tag{152}$$

with the usual definition of the creation and annihilation operators (a_α and $a_\alpha^\dagger$). The eigenvalues are $\sum_{\alpha=1}^{2} n_\alpha \hbar w_\alpha$, [37].

The next step should be to apply the second quantization. We suppose there is a longitudinal rippling spreading inside every "connecting line" between massive objects[1] (decaying structure of the gravitational rings), and what we mean by that is that some material, which has very special properties, should be present in the space between the galaxies. We start with a quantization of a real scalar field described by this Lagrangian, [37],

$$L(\psi, \dot{\psi}) = \frac{1}{2} \int \left[\dot{\psi}(x)\right]^2 dx - \frac{1}{2} \iint K(x - x')\psi(x)\psi(x')\, dx dx', \tag{153}$$

where $K(x - x') = K(x' - x)$ denotes a potential, and we keep only one dimension. The Euler-Lagrange equations give us

$$0 = \frac{\partial}{\partial t} \frac{\delta L}{\delta \dot{\psi}(x)} - \frac{\delta L}{\delta \psi(x)} = \ddot{\psi}(x) + \int K(x - x')\psi(x')\, dx'. \tag{154}$$

We now perform it on an illustrative case, $K(x - x') = -c^2 \frac{\partial^2}{\partial x^2} \delta(x - x')$, for which the Lagrangian (153) is the following

$$L = \frac{1}{2} \int [\dot{\psi}(x)]^2 - c^2 \left[\frac{d}{dx}\psi(x)\right]^2 dx. \tag{155}$$

---

[1] First of all, the gravitational rings are created among the most massive objects with the highest probability. So, there is included also probabilistic description in the fundamentals of RP.

So, we get a normal wave equation with the velocity of propagation c (it will be identified with the velocity of light in vacuum)

$$\frac{d^2}{dx^2}\psi(x) - \frac{1}{c^2}\ddot{\psi}(x) = 0. \tag{156}$$

We construct the standard Hamiltonian and therefore the conjugate momentum $\Pi(x) = \frac{\delta L}{\delta \dot{\psi}(x)} = \dot{\psi}(x)$ would be needed.

We also need to reproduce the Newtonian force with the bouncing of graviton-phonons and so here we compute $\frac{dP}{dt}$ in the Heisenberg representation, as one trivially obtains 0 in the Schrödinger representation. Because $\frac{dP}{dt} = \frac{1}{i}[P,H]$, we find $\frac{1}{i}[P,H]$ [2]. Let us reveal the unknown function $K(x)$, and for this, we will use the analogy between the modeling of photons and phonons. The photon is a quantum of the electromagnetic interaction, which should be described by the Maxwell equations in vacuum (with the electric intensity $\vec{E}$, the magnetic induction $\vec{B}$, the current density $\vec{j}$, the charge density ρ and the permitivity of vacuum ϵ_0), [37]. We use source-free equations [37] and plug $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$ with $\vec{A}$ being the vector potential to the last equation. After applying the rotation and using the gauge condition, $\nabla . \vec{A} = 0$,

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = 0. \tag{157}$$

It could be similarly deduced that

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0, \tag{158}$$

where one inserts $\vec{E} = -\nabla\varphi$ (φ is the electric potential) into the equations and does some modifications. Therefore, the wave equations are derived, and we recognize the same form as (156).

The most interesting observation is that photons are the field particles for electromagnetic interaction, which we describe by Maxwell equations containing the Coulomb law, $F_{Q_1Q_2} = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{L^2}$ (this is a formula only with the magnitude of the vector). It has exactly the same form as the formula for the Newtonian force, $F_{m_1m_2} = G\frac{m_1m_2}{L^2}$ and so if we substitute $K(x-y) = -c^2\frac{d^2}{dx^2}\delta(x-y)$, graviton-phonons should be the correct mediators of the Newtonian force!

Moreover, the equation for $\frac{1}{i}[P,H]$ can be taken as a condition on the scalar field $\psi(x)$, and it could be immediately seen that we have all three constants of Nature, $\hbar$, G and c, in it.

We now state the true implications of these results. RP can give an explanation for two basic facts in classical physics. Firstly, it tells us that the velocity of the gravitational waves c_{gw} is not almost the velocity of light in vacuum c (we today have very good bounds on this velocity $|c - c_{gw}| < 10^{-15}$, [159]), but exactly c. We supposed (based on the paper from 1915, [4]) that it should be c, but there was actually no fundamental explanation behind it. Further, the similarity between the Coulomb and Newton laws is not accidental, but it is a necessary ingredient for the building of the parallelism between photons and graviton-phonons. We want to emphasize this duality between gravity and electromagnetism is on a much deeper level than it is the currently widely studied double copy principle, which relates gauge theory amplitudes to gravity amplitudes, [160].

It would be interesting to investigate the fundamental group of symmetries G in RP, under which the theory would be invariant. When we look at the basic postulates, the elementary particles of the standard model are traveling on the presupposed trajectories of the decaying gravitational rings, and the rings are created first. Therefore, we have

$$G = \mathbb{R}^{1,3} \rtimes \Big(O(1,3)^+_c \cup (O(1,3)^+_{c_g} \Big). \tag{159}$$

---

[2] The details are included in [37].

The gravitational ring is then modeled as the "standard graviton" with the spin 2 and helicity ± 2 in the group $O(1,3)^{+}_{c_g}$. Furthermore, this group G corresponds to the following transformations under which the ring paradigm should be invariant (special case in 1+1 dimensions, based on generalization of the special Lorentz transformations in flat spacetime):

$$
\begin{aligned}
t' &= \frac{t - \frac{x}{v}\frac{v^2}{c^2}\epsilon - \frac{x}{v}\frac{v^2}{c_g^2}}{\sqrt{1 - \frac{v^2}{c^2}\epsilon - \frac{v^2}{c_g^2}}}, \\
x' &= \frac{x - tv}{\sqrt{1 - \frac{v^2}{c^2}\epsilon - \frac{v^2}{c_g^2}}},
\end{aligned}
\tag{160}
$$

where $c_g >> c$ and $\epsilon = \epsilon(v)$ denotes some step function defined by the prescription

$$
\epsilon(v) = \begin{cases} 1 & \text{for } v \leq c \\ 3(v/\delta)^3 - 2(v/\delta)^2 & \text{for } c < v \leq c + \delta \\ 0 & \text{for } v > c + \delta \end{cases}
\tag{161}
$$

with[3] $\delta \lessapprox 10^{-100} m/s$. We also want to note that G must be generalized to the BMS-like group in the case of a limit to GR in curved spacetime.

As it is clear from the construction, we do not change the form of Einstein equations even in the high energy sector. What seems to be different is how quickly is the gravity mediated on the full non-perturbative level of physics. We could get a bound on this velocity, when we consider the distance determined by the diameter of the observable Universe, $94.10^9 pc$, and the time, approximately Planck time,$10^{-43}s$, during which are the gravitational rings created. So, the velocity of the spreading of the gravitational interaction is minimally $10^{70} m/s$. This is simultaneously a new limit to the maximal velocity of the spreading of the information in the Universe.

The new field equations look like the following:

$$
\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}\mathscr{G}_{\mu\nu} + \Lambda_r \mathscr{G}_{\mu\nu} = \frac{8\pi G \mathscr{T}_{\mu\nu}}{c_g^4} = \frac{8\pi G}{c_g^4}(T^m_{\mu\nu} + \mathscr{T}^r_{\mu\nu}),
\tag{162}
$$

where $\mathscr{G}_{\mu\nu}$ is the metric, and all the other quantities also have an analogous meaning as in the GR. Concretely, the tensor $T^m_{\mu\nu}$ is the tensor of the energy-momentum of the casual matter and the tensor $\mathscr{T}^r_{\mu\nu}$ corresponds to the tensor of energy-momentum of gravitational rings. The constant Λ_r could be computed from the QFT as the energy density of the vacuum, which is the sum of the zero-point energies of all quantum fields. We neglect the term corresponding for $T_{\mu\nu}$ with respect to the second term on the RHS. So,

$$
\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}\mathscr{G}_{\mu\nu} + \Lambda_r \mathscr{G}_{\mu\nu} \approx \frac{8\pi G}{c_g^4}\mathscr{T}^r_{\mu\nu}.
\tag{163}
$$

The picture seems to be correct, because there were only the gravitational rings at the beginning of the cosmological inflation. Afterwords, they quickly decayed to the graviton-phonons and the casual matter, therefore the equations (162) transformed soon into the classical equations of GR ($\mathscr{G}_{\mu\nu}$ is unstable and it decays to some $g_{\mu\nu}$ Planck time later):

$$
R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} = \frac{8\pi G}{c^4}(T^m_{\mu\nu} + T^{pf}_{\mu\nu}),
\tag{164}
$$

---

[3] Note that the delta parameter is supposed to be very small.

where $T_{\mu\nu}$ is composed of the energy momentum tensor of casual matter $T^{m}_{\mu\nu}$ and the energy momentum tensor of the phantom field $T^{pf}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right)$. Actually, this described transformation happens in the universe in intervals as long as the Planck time again and again. It reminds us the cellular automaton models, when changes in the universe occur in quantized steps, governed by fixed rules rather than smooth, continuous transitions. And if we would now remove all the matter from the universe, the cosmological inflation would again start with an incredible speed.

We could say that this approach to QG is, in some sense, very conservative, because we don't change the nature of some quantum field. The only difference should be that we include into the concept of QFT some field, which mediates gravity superluminally. Of course, this would have immediate consequences for the group of symmetries of this modified QFT (as we had already shown).

Actually, we need to identify the full field equations (162) of RP with the theory with the field equations (164). For this is necessary to equate the impulse of the phonon with the impulse of graviton, which we obtain from the modified theory of gravity with the field equations (164). We do it by the modeling of gravitons similarly as in string theory. (However, we could get an approximate result from gravitational wave physics, when we do the split of the metric $\mathscr{G}_{\mu\nu} = {}_{\mu\nu} + \mathscr{H}_{\mu\nu}$, where $\mathscr{H}_{\mu\nu}$ is a small perturbation. Then, we compute the impulse of the gravitational wave analogically as in [161].) The further step would be to equate the impulse of the graviton in this theory with the impulse of graviton in the modified theory with Einstein field equations (162).

If RP is a viable model of QG, it had to lead to a perturbatively renormalizable theory. Therefore, we must deal with the interaction of graviton-phonons with other particles. We again consider the crystal composed of rings. A macroscopic grid with very large masses (typically super-massive black holes) must be created from it, so that the effect of the graviton-phonons on the elementary particle should be observable.

We use the model of electrons interacting with phonons, [162], and as the first step, we will develop the ground state for a fermion system. The concept of holes b with the defining relation

$$b_\alpha = a^{+}_\alpha \tag{165}$$

is introduced. Then we will apply the creation and annihilation operator formalism to electrons placed on a vibrating regular grid of nuclei. However, we should be careful because in the case of the model in [162], the crystal approximated a continuum in a limit. But it follows directly from the computations in [162] that modeling the interactions of graviton-phonons with other particles would be doable. We actually obtain no infinities in contrast to the "quantizing" pure GR. At first glance, it seems strange that graviton-phonons would have no interaction, for example, with massless particles like photons. Nevertheless, there should be an interaction on the fundamental non-perturbative level when gravitational rings "create a link" with all the particles.

5. Dark Energy Model Obtained by Ring Paradigm

The gravitational ring has two substantial functions, [37]: it "creates a trajectory" for all particles and fields, which means that we need to embed the Hopf-linked set of rings in the space $\mathbb{R}^3$. The second task is that it should be the mediator of gravity. Therefore, we obtain a self-evident explanation for the problem of DE because we get some extra energy in our model, and this will be identified with the missing energy.

Let us imagine two objects (for example, two very massive objects as galaxies) connected by gravitational rings, and they are continuously creating and decaying in Planck time. So, there remains some material between these two galaxies, and now we start to stretch them, and we feel the gravitational resistance. The situation somewhat reminds us of some quasi-mechanical model, when we obtain a Hook law for the dependence of the deformation on the stress. Then, the elasticity limit is reached, and finally, this spring breaks after crossing the yield point.

Actually, the decaying gravitational rings are made from some material, which we identified in the classical limit with the emergence of some sort of phantom field in the simplest case. The detailed quantum model of the bunch of creating and again decaying gravitational rings concerns condensed matter physics,[163]. We characterize the "gravitational" material by the spring constants k, k_3 in the equation (151), and the yield point then corresponds to some moment in the history of the evolution of the Universe, when the galaxies were remoted from each other by an average distance $8.\bar{3}Mpc$ approximately 5 billion years ago.

We can find a solution to the old problem of the cosmological constant by using the application of RP. It was already stated that we described the evolution of the universe by the field equations (162) in very high energies at the beginning of the cosmological inflation. And, the cosmological constant Λ_r should have exactly the value that we obtain in QFT. But the effective cosmological "constant" $\Lambda = \Lambda(t)$ has begun to appear approximately 8 billion years later, after the Big Bang, due to the phenomenon of QG:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T^m_{\mu\nu} \tag{166}$$

It is, of course, not surprising that the value of the cosmological constant from the cosmological inflation, Λ_r, possesses many orders of magnitude difference (approximately 10^{120}) from the value of the cosmological constant Λ responsible for the late-time cosmic acceleration, as they have a different origin. The first one, Λ_r, should be connected with the energy in the false vacuum (corresponding to the pure "crystal" made of rings), well known from QFT, and the other is some manifestation of a limit of the firmness of the DE material [37]. However, the question of why today Λ is so small would be answered with the detailed classical modified gravity model (164).

6. Comparison of Various Late-Time Acceleration Scenarios

We now discuss the various models we have studied so far and compare how each of them approaches the problem of the late-time accelerated expansion of the Universe. Although these models emerge from very different mathematical and physical motivations, they are all united by the recognition that the cosmic acceleration might be a signal of underlying quantum gravitational phenomena, and our comparison here is not merely mathematical or historical but rather is a critical view of all the scenarios discussed so far.

To begin, the metastring formulation of string theory offers one of the most mathematically mature descriptions of late-time acceleration, as here we see that the cosmological constant and the Planck scale are related through a see-saw mechanism $M_\Lambda \sim M^2/M_P$. This goes towards implying that the smallness of DE is not accidental but a manifestation of T-duality between the large- and small-scale sectors of spacetime and the metastring model introduces a non-commutative doubled geometry, in which spacetime coordinates and their duals $(x, \tilde{x})$ coexist. This leads to giving rise to an effective cosmological constant through their intrinsic curvature, and this approach thus replaces the fine-tuning problem with a geometric relation between curvature and vacuum energy. Nevertheless, its success depends critically on the non-perturbative stability of the vacuum and the precise implementation of dual space curvature, which are still open issues.

Loop quantum gravity and its cosmological reduction, which is loop quantum cosmology, pursue a very different route, as here, by directly quantizing the canonical variables of GR, they predict a modified Friedmann equation that prevents singularities and produces a quantum bounce. Interestingly, this same modification leads naturally to a late-time acceleration phase wherein their framework asserts that the acceleration is not attributed to a cosmological constant but arises dynamically from the underlying discreteness of space. Its main virtue is background independence and mathematical rigor, yet it lacks a clear mechanism for the observed numerical value of the acceleration and cannot yet explain the coincidence problem in a predictive way.

The group field theory condensate cosmology extends these results by translating the loop quantum gravity picture into a field-theoretic one and tells us that the Universe itself is a condensate of quantum geometric "atoms of space" whose collective dynamics give rise to an effective Friedmann equation with an emergent phantom component that drives accelerated expansion. In contrast with string theory, where DE arises from moduli or vacuum curvature, the group field theory explanation is purely emergent and dynamical. Its success lies in connecting early and late time phenomena through the same microscopic principles, but it still depends on phenomenological potentials and lacks direct observational signatures, which are important in distinguishing it from other quantum cosmology models.

Discrete approaches such as the causal set program provide another viewpoint which is quite non-trivial as in this model, the cosmological constant appears as an emergent quantity associated with fluctuations in the number of spacetime elements, and its magnitude is naturally of the observed order when averaged over the cosmic volume. Moreover, small violations of energy-momentum conservation due to spacetime discreteness can act as an effective source of DE. While this idea is conceptually elegant, its quantitative realization is still largely heuristic, and a complete dynamical law replacing Einstein's equations remains missing as well.

Causal dynamical triangulations share with the causal set program the assumption of discreteness, but they preserve causal structure through a global foliation of spacetime by simplices, and remarkably, numerical simulations show that an ensemble average of such triangulations reproduces a semiclassical de Sitter Universe at large scales. This means that acceleration in this model is not imposed but emerges statistically. This approach is one of the few to generate a cosmological constant dynamically without fine-tuning, also demonstrating that smooth 4-dimensional geometry can emerge from discrete microscopic building blocks. But the issue here is the dependence on numerical results and the difficulty in coupling matter fields, which are quite significant limitations.

The asymptotic safety scenario approaches the same question from the renormalization group perspective, and it posits that gravity is described by a non-trivial UV fixed point of the renormalization flow, making it a predictive and consistent QFT at all scales. Late-time acceleration arises naturally here from the scale-dependence of the gravitational coupling and cosmological constant. The virtue of this approach is its continuity with established QFT methods and its ability to interpolate smoothly between classical and quantum regimes. Its limitation lies in its perturbative formulation and the lack of a direct link between the renormalization group flow and observable cosmological data.

Entropic gravity and non-commutative geometric approaches share a thermodynamic interpretation of gravity but also have some differences. Entropic gravity interprets the gravitational force as an emergent entropic tendency of microscopic degrees of freedom, while non-commutative geometry reformulates the standard model and gravity as manifestations of algebraic spectral triples. Both naturally incorporate the holographic principle and thus hint that the cosmic acceleration could stem from an information-theoretic imbalance between bulk and boundary degrees of freedom. The main challenge here is empirical, which is that while these frameworks are conceptually unified and elegant, they do not have proper falsifiable predictions that differentiate them from more conventional theories.

Finally, the ring paradigm introduces a profoundly novel vision of spacetime and gravitation. Gravity is modeled not as curvature of a manifold but as the collective dynamics of "gravitational rings", which are topologically linked excitations that mediate interactions between massive objects. The creation and decay of these rings release residual energy, identified with the observed DE component. In this picture, the cosmic acceleration is not a vacuum effect but a macroscopic outcome of quantum topological processes occurring at the Planck scale. The model succeeds in simultaneously addressing the cosmological constant and the coincidence problems, while also offering a potential bridge between condensed matter analogies and cosmological behavior. However, its mathematical formalization is still in early stages, and so with time we would be able to figure out proper observational tests for the theory.

However, let's stop for a moment at the phenomenological implications of the ring paradigm. Because if confirmed in the future, it would have far-reaching consequences for the structure of other approaches to QG. First of all, according to it, gravity is an extremely non-local interaction. And we already have many examples of the approaches which have built-in non-locality: we can mention, at the classical level, for example, non-local gravity, [164,165]. String field theory contains in it non-localities, [9], and string theory as well, [6]. Loop quantum gravity, [60,61], is not inherently non-local, but group field theory is often non-local, [115], particularly in its interaction terms. But the causal set approach is non-local in its foundations, [118], which follows from the combination of discreteness and Lorentz invariance. Further, non-commutative geometry, [78], could be used to create non-local theories of gravity. Also in causal dynamical triangulations, [88], the overall geometries are not determined by local interactions alone. Non-local effects, which are not described by local operators, can arise in asymptotic safety, [89–92], particularly when dealing with the propagation of massless particles like gravitons. Entropic gravity, [95], is then again a theory where interactions are not limited to an immediate surroundings, therefore it is significantly non-local.

Because the ring paradigm comes with the notion that the superluminal information transfer exists in Nature, let's see which models have already been studied with this phenomenon. Actually, some models in string theory suggest the potential Lorentz breaking at the Planck scale or higher, [166]. The same is true in loop quantum gravity, [167,168] and such models are known also in group field theory, [169]. On the other hand, the causal set approach does not allow Lorentz symmetry breaking. However again, the non-commutative geometry is often discussed in the context of Lorentz symmetry violation, [169–171]. Causal dynamical triangulations models do not usually violate Lorentz symmetry, [83]. But that does not apply to asymptotic safety, [172]. Also, Lorentz symmetry violation could affect entropic gravity by altering thermodynamic properties like the entropy and Hawking temperature of black holes and by potentially leading to new astrophysical phenomena. But so far not too much is known.

The existence of some sort of phantom field is also an integral part of the ring paradigm. Already some models with phantom fields were investigated inside AdS/CFT correspondence and string field theory, [173,174], further inside loop quantum gravity, [175,176], and group field theory, [114,115]. But there is not too much known about connections with the scalar field models in the causal set approach, non-commutative geometry, causal dynamical triangulations, asymptotic safety and entropic gravity.

It is actually very interesting that we build the ring paradigm on the 1+1 dimensional grid created from the gravitational rings in $\mathbb{R}^3$. These objects are effectively 1-dimensional, but we could also introduce the time coordinate on every such ring. And such a similar phenomenon - dimensional reduction mostly to the dimension 2 - was observed in many approaches to QG without any deeper explanation, [177]: the first indication of dimensional reduction to $d = 2$ in QG came from the study of high-temperature string theory. As Joseph Atick and Edward Witten remarked, the system that prevails under distances $\sqrt{\alpha'}$ behaves as if it were a (1+1)-dimensional field theory, [178]. Another place that we might look for evidence for dimensional reduction is loop quantum gravity, [179]. And further occurrence of this phenomenon is known from causal set theory, [180], non-commutative geometry [181], causal dynamical triangulation, [182], asymptotic safety, [183–185] and even Wheeler de Witt approach, [186]. Only the entropic gravity is not often associated with the dimensional reduction, but we can say that this phenomenon appears there via the holographic principle.

The aforementioned principle is one of the key discoveries of modern QG research and cosmology. And it looks like so that the ring paradigm is in concordance with it: every effectively 1-dimensional ring could be wrapped inside a two-dimensional surface made of rings. Therefore, we want to know into which known approaches to QG we can incorporate it as well. For sure to string theory, [45]. But some studies with positive results were already done in loop quantum gravity, [187], but above all in group field theory,[188]. Let us mention, however, the ongoing research in this area in non-commutative geometry, [189] and causal dynamical triangulation, [136], with positive results as well.

The holographic principle is, of course, firmly built in entropic gravity, [157], but no evidence has so far been revealed in the causal set approach and asymptotic safety, see Table 1.

Table 1. We study the occurrence of 5 phenomena (nonlocalities, superluminal signalling, existence of phantom fields, dimensional reduction, holographic principle) in the known approaches of QG and its modifications (string theory/string field theory (ST/SFT), loop quantum gravity (LQG), group field theory (GFT), causal set approach (CSA), noncommutative geometry (NG), causal dynamical triangulation (CDT), asymptotic safety (AS), entropic gravity (EG) and ring paradigm (RP)). From the given table, it is quite clearly visible which approaches of QG are preferable: modifications of the string theory/string field theory, group field theory (partly loop quantum gravity) and non-commutative geometry. The ring paradigm occupies a completely different position in the modern approaches to QG, because its confirmation would give us first hints on how to solve a whole serie of open problems in physics. Let's mention, for example, the old problem of the cosmological constant (problem of DE), the black hole information paradox, the problem of singularities or the problem of the Universe's curvature. Therefore, our primary goal was to compare this approach - and, above all, certain phenomenology - with other approaches to QG.

Phenomenology\Approach to QG	ST	LQG	GFT	CSA	NG	CDT	AS	EG	RP
Nonlocal theory	✓	×	✓	✓	✓	✓	✓	✓	✓
Superluminal signalling	✓	✓	✓	×	✓	×	✓	✓	✓
Phantom field	✓	✓	✓	×	×	×	×	×	✓
Dimensional reduction	✓	✓	✓	✓	✓	✓	✓	×	✓
Holographic principle	✓	✓	✓	×	✓	✓	×	✓	✓

We see from this chart that strongly preferable approaches to QG are modifications of string theory/string field theory, group field theory (loop quantum gravity) and non-commutative geometry. Because the ring paradigm has a very special position among the modern approaches to QG we wish to highlight a few overall observations concerning model building in cosmology by the applications of various approaches to QG and compare it with the ring paradigm.

The models based on the string theory usually lead to the multiverse conjecture, [190]. There are two big arguments for why we should reject it: first of all, it is a well-known fact that we will lose testability and falsifiability of the physical theory when we accept it. But someone can argue that it is simply a new phenomenon in modern physics that a theory has such a property. We claim it is not like that because we also have a second philosophical reason why we should not trust it. Imagine the collection of all universes in the multiverse theory. And now you add all the energy-mass of all the objects in all these universes. You clearly obtain an infinity. How could we build a serious physical theory where we work with infinite energies? Unless there is some unknown mechanism, which will reduce this number to a finite value, we cannot do any computations in such a model of the multiverse.

However, the ideas from the string theory played a key role in the development of the cyclic model of the universe of Paul Steinhardt and Neil Turok,[191]. And this is a very interesting model, because it gives us an explanation of what happened before the Big Bang, what will be the evolution of the Universe in the far future (it will again collapse to the Big Crunch) and what is it the DE. We have recently started to study a model inspired by the ring paradigm, where we claim that both happened in the Universe, the cosmological inflation without multiverse and the big - trillion- year-lasting - cycles controlled by the modification of the cyclic model (there are two coupled scalar fields in the theory; one is controlling the cosmological inflation, and the other is responsible for the big cycles), [192]. These are all bonus points for the string theory paradigm, because it also has another very rich phenomenology. Let's mention further the strings, branes and extra dimensions, [6].

But even the applications of the group field theory approach give us models which connect the early and late-time universe cosmology (the group field cosmologies of Daniele Oriti and Xiankai Pang, [114]). But what has especially caught our attention is the model based on the introduction of mitrons of Tejinder Singh, [79]. The mitrons show some similarity with gravitational rings (they are actually also other phenomenological objects based on a model of QG) and the author explains the origin of DM and DE by it. When we summarize, exactly these paradigms - modifications of string theory, group field theory and modifications of non-commutative geometry - showed a remarkable similarity in the results of our test concerning phenomenology (nonlocality, super-luminal signalling, existence of phantom field, dimensional reduction and holographic conjecture). It is also worth mentioning that our modification of the cyclic model inspired by the ring paradigm, [192], is based on the existence of some phantom fields, and phantom fields play a key role in the model of Daniele Oriti and Xiankai Pang, [114].

The loop quantum gravity models bring us interesting models in the early-universe cosmology as well via the modifications of the Friedmann equation, [63], and therefore we need to consider the pre-Big Bang cosmologies. Let us note that we use the Einstein equations in the high-energy sector in the ring paradigm, which also points to some connection with these models based on loop quantum gravity. Actually, the ring paradigm could give us a first hint of how to highlight paradigms, which are very probably only the study of the partial properties - even though they may be very important - of the resulting QG theory. Let's point out, for example, the causal set approach, causal dynamical triangulation, asymptotic safety or entropic gravity, which are not bringing us such a rich phenomenology as, for example, the modifications of string theory or the exactly novel ring paradigm (which comes up from the mathematical machinery of string theory, but as we already wrote, the development of its mathematical formalism is still in its infancy). This does not at all mean that we should not study these other approaches to QG. We should just point out what we should mainly focus on concerning the physical foundations of the new theory in the future[4].

Taken together, a general summary of the main topic of this paper - cosmological constant and the problem of DE - all the presented theories share the conviction that the cosmological constant problem cannot be solved within purely classical GR. Yet they diverge sharply in their mathematical tools and motivations. From a pedagogical standpoint, the comparative outlook of these models reveals an evolving notion that cosmic acceleration likely encodes a genuinely quantum aspect of spacetime rather than an additional classical field, which may be further exemplified with the advent of cosmic tensions in the current age. The crucial differences lie not in their predictions for late-time acceleration but in how they reinterpret the physical meaning of that constant.

7. Conclusions and Future Outlook

We would now like to make some concluding statements to summarize the extensive work discussed here:

- Our work here has examined a wide spectrum of QG paradigms, ranging from metastring theory and its non-commutative phase-space geometry to loop quantum gravity, group field theory, causal set theory, asymptotic safety, entropic gravity, and the novel ring paradigm. We have shown here that the phenomenon of the accelerating expansion of the universe can naturally emerge within several independent theoretical frameworks and each of these approaches replaces the traditional cosmological constant problem with a more fundamental quantum or geometric origin of cosmic acceleration.
- A unifying theme found across these diverse models is the concept of *emergent spacetime* and it is important. Whether through the condensation of quantum geometric degrees of freedom, holographic dualities, discrete causal structures, or topological excitations, all frameworks indicate that the universe's large-scale dynamics arises from microscopic quantum structures, suggesting that DE is a manifestation of deep quantum geometry rather than a simple additional matter field as is easily assumed.
- The work also emphasizes that these approaches provide a new conceptual link between early and late time cosmology. Quantum gravitational effects that are responsible for resolving the Big Bang singularity or initiating inflation appear in several models, to emerge again at late times as

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[4] We did not mention, for example, the models of Petr Hořava, [193], in this article because the applications to cosmology are still under development. There are also interesting models based on the works of Brett McInnes,[194–197], and Aaron Trout, [198], which, however, were not central to the discussion we were having in this article.

drivers of acceleration. This dual appearance hints at a single underlying mechanism governing the Universe's entire history, from its early beginnings to its late possible ends.

- Looking toward future developments, one of the most promising avenues lies in connecting these theoretical frameworks with high precision cosmological data. This is particularly interesting as observational data for cosmology would here provide a testing bed for quantum gravitational theories. Upcoming observatories such as the Vera C. Rubin Observatory [199] and LSST [200], the Nancy Grace Roman Space Telescope [201], the Euclid mission, the Square Kilometre Array [202] and the Cosmic Explorer gravitational wave observatory [203] will deliver unprecedented constraints on the Hubble parameter, structure growth, and DE equation of state. These facilities may provide the first observational handles to distinguish QG imprints in the late time universe from standard ΛCDM predictions.
- Nevertheless, there still remain profound challenges and open questions. The mathematical formulations of several QG paradigms are still incomplete, and their semiclassical or phenomenological limits are not yet derived in sufficient detail to allow for particularly strong confrontation with data. The renormalization behavior of certain models, the exact nature of the emergent degrees of freedom and the translation between discrete and continuum formulations remain unsolved issues that must be addressed before a definitive picture can be achieved.
- The interplay between QG and cosmology also demands the development of new computational and analytical tools, and future research should hence aim to build consistent bridge theories that connect the microscopic quantum geometry with effective field-theoretic and thermodynamic descriptions of spacetime. Cross-disciplinary frameworks linking condensed matter analogies, information theory and algebraic geometry may also prove very useful in constructing a coherent narrative that unifies all these approaches.
- Finally, the ultimate goal will be to create a synergy between these results into a single, experimentally verifiable theory of QG capable of predicting both the cosmological constant and the time evolution of DE. As next-generation observations become more precise, the coming decade may well decide which of these paradigms most accurately captures the quantum fabric of our accelerating universe.

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