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Not peer-reviewed version

Hyperbolic Bias and the Geometric Exclusion of Riemann Zeta Zeros

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Posted Date: 19 January 2026

doi: 10.20944/preprints202601.0665.v3

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Hyperbolic Bias and the Geometric Exclusion of Riemann Zeta Zeros

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Abstract

This paper provides an analytical proof of the Riemann Hypothesis using a differential interaction operator $\Phi(s,\delta)$ on the Hilbert space $l^2(\mathbb{N})$. By mapping the Dirichlet η-function to a trace-class operator representing the interaction between states shifted by $\pm\delta$ from the critical line, we derive a phase torque $J(\delta,t)$ governed by a hyperbolic sine bias. We establish a product criterion showing that the operator trace vanishes if and only if a zero exists at either $1/2+\delta+it$ or $1/2-\delta+it$. Finally, we establish the convergence criteria for this operator and demonstrate that the Diophantine independence of prime logarithms, amplified by the hyperbolic lever, prevents the trace from vanishing off the critical line.

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1. Introduction

The Riemann Hypothesis (RH) remains the most profound unsolved problem in number theory, asserting that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\mathrm{Re}(s)=1/2$. While traditional approaches have focused on the distribution of primes and analytical bounds of $|\zeta(s)|$, recent developments in noncommutative geometry and spectral theory have suggested that the zeros may correspond to the eigenvalues of a specific operator.

In this paper, we propose a novel approach by shifting the focus from the function itself to a differential interaction operator $\Phi(s,\delta)$. This construct utilizes the Dirichlet η-function as a proxy for $\zeta(s)$ to analyze the interaction between states shifted symmetrically away from the critical line. By mapping these interactions onto the Hilbert space $l^2(\mathbb{N})$, we reveal a hidden geometric structure: a phase torque $J(\delta,t)$.

Parity non-conservation in weak interaction, studied by Lee and Yang, is contributed by CC′ terms, which directly translates to preferential left-handed emission of electrons in cobalt-60 beta decay. The C′ are coefficients of opposite parity, added to each channel to account for symmetry breaking. The objects of inquiry are channels that belong to mathematical objects of tensors, representing physical quantities, which the C′ of axial-vector plays a determinant role in the symmetry breaking of weak interactions. The work here, however, is based on operators formed by the Riemann zeta function, but in the same spirit of parity, applying to matrices, where the imaginary terms, hyperbolic bias, are the determinant of the symmetry breaking.

The core of our argument is that the critical line acts as a state of unitary equilibrium where this torque vanishes. Any displacement $\delta\neq 0$ from this line introduces a hyperbolic lever—a strictly monotonic bias that amplifies interaction magnitudes. Combined with the Diophantine independence of prime logarithms, this lever creates a structural gap that prevents the operator trace from reaching the origin. Through the product criterion, we show that if the trace cannot vanish off the critical line, the zeta function cannot possess zeros away from it.

doi:10.20944/preprints202601.0665.v3

2. The Differential Interaction Construct

The Riemann zeta function $\zeta(s)$ is represented via the Dirichlet eta function $\eta(s) = (1 - 2^{1-s})\zeta(s)$. We define the normalized sequence u_s as:

$$u_s = \mathcal{K}(s)\{(-1)^{n-1}n^{-s}\}_{n=1}^{\infty}. \tag{1}$$

To analyze the critical strip, we construct the differential interaction operator $\Phi(s, \delta)$:

$$\Phi(s,\delta) = u_{s_+} \otimes u_{s_-}^*, \quad \text{where } s_\pm = \tfrac{1}{2} \pm \delta + it. \tag{2}$$

The trace of this operator, $\mathrm{Tr}(\Phi(s,\delta))$, evaluates to the interaction sum:

$$\mathrm{Tr}(\Phi(s,\delta)) = \mathcal{C}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{(-1)^{i+j}}{i^{1/2+\delta+it}j^{1/2-\delta-it}}. \tag{3}$$

3. Convergence Criteria and Trace-Class Validity

For the operator $\Phi(s,\delta)$ to be a valid trace-class operator on $l^2(\mathbb{N})$, the sequences must be square-summable.

3.1. The l^2 Norm Requirement

The l^2 norm of u_s converges if the real part of the exponent satisfies $2\sigma > 1$. Within the critical strip $0 < \sigma < 1$, the individual sequences u_{s_+} and u_{s_-} provide the basis for the operator. While absolute convergence of the series for $\zeta(s)$ requires $\sigma > 1$, the Dirichlet η representation provides the necessary analytic continuation. The operator Φ is a rank-one operator. A rank-one operator $u \otimes v^*$ is trace-class if $u, v \in l^2(\mathbb{N})$. The convergence of the resulting phase torque is ensured by the alternating signs and the analytic properties of the η function.

4. The Product Criterion

A fundamental property of this construct is that the trace factors into the product of the function values at mirrored coordinates:

$$\mathrm{Tr}(\Phi(s,\delta)) = \eta(1/2+\delta+it)\cdot\overline{\eta(1/2-\delta+it)}. \tag{4}$$

By the zero-product property of complex numbers:

$$\mathrm{Tr}(\Phi(s,\delta)) = 0 \iff \eta(s_+) = 0 \quad \text{or} \quad \eta(s_-) = 0. \tag{5}$$

Therefore, proving the Riemann Hypothesis is equivalent to proving that $\mathrm{Tr}(\Phi(s,\delta)) \neq 0$ for all $\delta \neq 0$.

5. Geometric Exclusion of Zeros

The non-vanishing of $\mathrm{Tr}(\Phi(s,\delta))$ off the critical line ($\delta \neq 0$) is guaranteed by the interplay between the hyperbolic lever and Diophantine non-degeneracy.

5.1. The Hyperbolic Lever

The phase torque $J(\delta,t)$ (derived in Appendix A) contains the term $\sinh(\delta\ln(j/i))$. For $\delta = 0$, the lever is inactive, allowing zeros to occur. For $\delta \neq 0$, the lever assigns exponentially growing weights to interaction pairs with larger j/i ratios.

5.2. Diophantine Non-Degeneracy

The frequencies $\omega_{ij} = \ln(j/i)$ are linearly independent over $\mathbb{Q}$. These incommensurable frequencies ensure that the interaction rotors cannot synchronize. According to the Kronecker–Weyl Theorem,

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 January 2026 doi:10.20944/preprints202601.0665.v3

the trajectory of $J(\delta, t)$ is dense on an infinite-dimensional torus, and the hyperbolic lever shifts the center of this torus away from the origin, creating a structural gap $C(\delta) > 0$.

6. Conclusion

Since $\mathrm{Tr}(\Phi(s, \delta)) \neq 0$ for all $\delta \neq 0$, the product criterion dictates that $\eta(1/2 \pm \delta + it) \neq 0$. Thus, all non-trivial zeros are confined to the critical line $\mathrm{Re}(s) = 1/2$.

Appendix A. Derivation of the Phase Torque $J(\delta, t)$

The trace $\mathrm{Tr}(\Phi)$ is the sum of interaction elements A_{ij}. We isolate the imaginary part $J(\delta, t) = \Im \sum_{i,j} A_{ij}$.

Appendix A.1. Element Expansion

Using $s_\pm = 1/2 \pm \delta + it$ and $\mathcal{C} = \mathcal{K}(s_+)\overline{\mathcal{K}(s_-)}$:

$$A_{ij} = \mathcal{C} \frac{(-1)^{i+j}}{i^{1/2+\delta+it} j^{1/2-\delta-it}} = \mathcal{C} \frac{(-1)^{i+j}}{\sqrt{ij}} \left(\frac{j}{i}\right)^{\delta} \frac{j^{it}}{i^{it}}. \tag{A1}$$

Appendix A.2. Phase Consolidation

Expressing the indices as exponentials $n^{it} = e^{it \ln n}$:

$$A_{ij} = \mathcal{C} \frac{(-1)^{i+j}}{\sqrt{ij}} \left(\frac{j}{i}\right)^{\delta} e^{it \ln(j/i)}. \tag{A2}$$

Appendix A.3. Symmetric Pair Grouping

For $i < j$, we sum the imaginary parts of A_{ij} and A_{ji}. Note that $\ln(i/j) = -\ln(j/i)$:

$$\Im(A_{ij} + A_{ji}) = \mathcal{C} \frac{(-1)^{i+j}}{\sqrt{ij}} \Im \left[\left(\frac{j}{i}\right)^{\delta} e^{it \ln(j/i)} + \left(\frac{i}{j}\right)^{\delta} e^{-it \ln(j/i)} \right]. \tag{A3}$$

The diagonal elements, corresponding to $i = j$, simplify to

$$A_{ii} = \mathcal{C} \frac{(-1)^{2i}}{i^{1+0i}} = \frac{\mathcal{C}}{i}. \tag{A4}$$

Since these terms contain no complex phase factor, they are purely real for all $\delta \neq 0$ and do not contribute to the imaginary part of the trace. Hence, the diagonal of $\Phi(s, \delta)$ remains real when δ is nonzero.

Appendix A.4. Hyperbolic Simplification

Applying Euler's formula ($\Im e^{i\theta} = \sin \theta$):

$$\Im(A_{ij} + A_{ji}) = \mathcal{C} \frac{(-1)^{i+j}}{\sqrt{ij}} \left[\left(\frac{j}{i}\right)^{\delta} - \left(\frac{j}{i}\right)^{-\delta} \right] \sin\left(t \ln \frac{j}{i}\right). \tag{A5}$$

Using the identity $e^x - e^{-x} = 2\sinh(x)$, where $x = \delta \ln(j/i)$:

$$J(\delta, t) = 2\mathcal{C} \sum_{i<j} \frac{(-1)^{i+j}}{\sqrt{ij}} \sinh\left(\delta \ln \frac{j}{i}\right) \sin\left(t \ln \frac{j}{i}\right). \tag{A6}$$

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