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Article

A Mechanical Formulation for Resistive Network Analysis: The Spring-Conductance Matrix Method

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Abstract

We introduce a formal and systematic analogy between static mechanical spring networks and electrical resistor networks, establishing a one-to-one mathematical correspondence where spring displacements map to electric potentials, spring constants to conductances, and forces to currents. This equivalence transforms the static equilibrium equations of point-mass-spring systems into the nodal equations of resistor networks, enabling the construction of a conductance matrix that is identical in form to a mechanical stiffness matrix and coincides with the weighted graph Laplacian. We demonstrate the versatility of this framework through a series of examples from elementary series and parallel combinations to non-planar networks such as the 3D resistor cube and the Petersen graph. We show that the method provides both an intuitive mechanical interpretation of circuit concepts and a systematic, matrix-based computational algorithm for calculating equivalent resistance. The approach naturally extends to AC networks containing resistors, capacitors, and inductors, offering a unified treatment of linear networks in both DC and AC regimes. We discuss the pedagogical value of the analogy for teaching circuit theory and network analysis, as well as its connections to graph theory and computational implementation. Limitations of the method, including its restriction to linear elements and the numerical considerations of matrix inversion, are briefly discussed.

Keywords: resistor network; equivalent resistance; matrix method; graph theory

1. Introduction

Mathematical analogies between different physical systems have long been recognized as powerful tools for understanding and problem-solving in physics and engineering [1]. In electrical engineering education, analogies between mechanical and electrical systems are particularly valuable for developing intuition [2,3]. The well-known mass-spring-damper analogy to RLC circuits is a staple of systems dynamics education [4,5]. However, a formal analogy specifically between static spring networks and resistive circuits remains underdeveloped in the literature.

This paper establishes a precise mathematical correspondence between the equilibrium conditions of point masses connected by linear springs and the nodal analysis of resistor/impedance networks. The analogy provides not only an intuitive mechanical interpretation of electrical concepts but also a systematic matrix method for calculating equivalent resistance/impedance between any two nodes in a complex network.

The spring-resistor analogy builds upon the observation that Hooke's Law for springs and Ohm's Law for resistors share the same mathematical form,

$$F = k(x_j - x_i) \quad (\text{Hooke's Law}) \quad (1)$$

$$I = G(V_j - V_i) \quad (\text{Ohm's Law}) \quad (2)$$

where F is force, k is spring constant, x is displacement, I is current, G is conductance, and V is voltage. We note the directivity of both the electrical current and the mechanical force.

2. Mathematical Formalism

2.1. The Spring-Resistor Correspondence

Consider a system of N point masses connected by ideal springs obeying Hooke's Law. For a spring connecting points i and j with spring constant k_{ij} , the force exerted by the mass i on the mass j through the spring is given by,

$$\vec{F}_{i \rightarrow j} = -k_{ij}(x_j - x_i) \vec{e}_{i \rightarrow j} \quad (3)$$

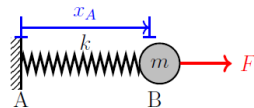
where x_i and x_j are the positions (abscissa) of the two endpoints relative to some reference and the unit vector $\vec{e}_{i \rightarrow j}$ is directed from point mass i to point mass j . We consider a spring with a vanishing unstretched length. For a resistor obeying Ohm's Law, the current through a resistor from node i toward node j is,

$$I_{i \rightarrow j} = -\frac{1}{R_{ij}}(V_j - V_i) = -G_{ij}(V_j - V_i) \quad (4)$$

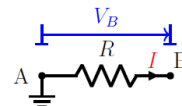
where V_i and V_j are the electric potentials at the nodes, R_{ij} is the resistance, and $G_{ij} = 1/R_{ij}$ is the conductance. The mathematical equivalence between Eqs. 3 and 4 establishes the fundamental analogy, summarized in Table 1. The pairing shown in Figure 1 allows a transfer of intuition about mechanical equilibrium, where forces balance and displacements propagate through a spring network to the electrical domain, where currents balance and potentials distribute through a resistor network. The identical mathematical structure, when paired with the visual analogy, reinforces the conceptual bridge between mechanics and circuit theory.

Table 1. Correspondence between mechanical and electrical quantities

Mechanical System (Springs)	Electrical System (Resistors)
Displacement x_i of point i	Electric potential V_i at node i
Spring constant k_{ij}	Conductance $G_{ij} = 1/R_{ij}$
Force $\vec{F}_{i \rightarrow j}$ in spring i - j	Current $I_{i \rightarrow j}$ through resistor i - j
Point mass (node)	Circuit node
Static equilibrium: $\sum_i \vec{F}_{i \rightarrow j} = 0$	Kirchhoff's Current Law: $\sum_i I_{i \rightarrow j} = 0$
Relative displacement $(x_j - x_i)$	Potential difference $(V_j - V_i)$
External force on mass	Current injection at node
Stiffness matrix \mathbf{K}	Conductance matrix \mathbf{K}



(a) Mechanical spring with constant k . Point A is fixed (wall), point B attaches to mass m . Force F applied at B causes displacement x_B .



(b) Electrical resistor with resistance R . Node A is grounded ($V_A = 0$), current I flows from A to B , establishing potential V_B at node B .

Figure 1. Fundamental analogy between a single spring and a single resistor.

2.2. Nodal Equilibrium Equations

For a point mass m connected to several springs, static equilibrium requires,

$$\sum_{j \in \mathcal{N}(m)} F_{mj} = \sum_{j \in \mathcal{N}(m)} k_{mj}(x_j - x_m) = 0 \quad (5)$$

where $\mathcal{N}(m)$ denotes the set of points connected to m .

For an electrical node m connected to several resistors, Kirchhoff's Current Law gives,

$$\sum_{j \in \mathcal{N}(m)} I_{mj} = \sum_{j \in \mathcal{N}(m)} G_{mj}(V_j - V_m) = 0 \quad (6)$$

The mathematical identity of Equations 5 and 6 establishes the formal analogy at the nodal level.

2.3. The Stiffness/Conductance Matrix as Graph Laplacian

For a system with n nodes, we can write the equilibrium equations in matrix form. Let $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ be the displacement vector (or potential vector \mathbf{V} in the electrical analogy). The equilibrium conditions yield,

$$\mathbf{K}\mathbf{x} = \mathbf{F} \quad (7)$$

where \mathbf{K} is the *stiffness matrix* (or *conductance matrix* in the electrical context) with elements:

$$K_{ij} = \begin{cases} \sum_{k \neq i} G_{ik} & \text{if } i = j \\ -G_{ij} & \text{if } i \neq j \text{ and nodes } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and $\mathbf{F} = (F_1, F_2, \dots, F_n)^T$ is the external force vector (current injection vector in the electrical context). We note that the matrix \mathbf{K} is exactly the *Laplacian matrix* of the underlying graph weighted by conductances [3]. This connects our method to spectral graph theory.

2.4. Equivalent Resistance Calculation: Complete Algorithm

The matrix \mathbf{K} is singular (each row sums to zero), reflecting gauge invariance of potentials (adding a constant to all potentials doesn't change physics). To solve the system, we must ground one node (set its potential to zero). The complete algorithm is outlined as follows,

Step 1: Construct \mathbf{K} : For an N -node network, build the $N \times N$ conductance matrix using Eq.8.

Step 2: Ground a node: Choose a reference node r (typically one of the two nodes between which we want resistance). Set $V_r = 0$ by removing the r -th row and column from \mathbf{K} , obtaining $(N-1) \times (N-1)$ reduced matrix $\bar{\mathbf{K}}$.

Step 3: Apply current sources: To find R_{ab} between nodes a and b , we inject current I at node a and extract it at node b .

Step 4: Solve the system: Solve $\bar{\mathbf{K}}\bar{\mathbf{V}} = \bar{\mathbf{F}}$ for potentials. Since we only need $V_a - V_b$, we can use,

$$R_{ab} = \frac{V_a - V_b}{I} = (\bar{\mathbf{K}}^{-1})_{aa} + (\bar{\mathbf{K}}^{-1})_{bb} - 2(\bar{\mathbf{K}}^{-1})_{ab}$$

where indices refer to positions in the reduced system.

Step 5: Alternative formula: If a is the injection node and b is grounded ($V_b = 0$), then,

$$R_{ab} = (\bar{\mathbf{K}}^{-1})_{aa}$$

where $\bar{\mathbf{K}}$ is obtained by grounding node b .

The matrix method is particularly suited to computational solution. Algorithm 1 provides pseudocode for calculating the equivalent resistance.

While mechanical–electrical analogies have a long history in physics education, the present work distinguishes itself in several important respects.

Traditional nodal analysis solves for node voltages given current injections, but does not inherently provide a physical interpretation of the conductance matrix. Our formulation explicitly identifies the

Algorithm 1 Calculate Equivalent Resistance Between Two Nodes**Require:** Adjacency matrix A , resistance matrix R , number of nodes N , node indices a and b **Ensure:** Equivalent resistance R_{ab} between nodes a and b $N \leftarrow$ number of nodes

```

1: Initialize  $G$  as  $N \times N$  zero matrix
2: for  $i = 0$  to  $N - 1$  do
3:   for  $j = 0$  to  $N - 1$  do
4:     if  $A_{ij} = 1$  and  $R_{ij} \neq 0$  then
5:        $G_{ij} \leftarrow 1.0/R_{ij}$ ,  $G_{ji} \leftarrow G_{ij}$ 
6:     end if
7:   end for
8: end for
9: Build  $\mathbf{K}$ :  $K_{ii} = G_{ii}$ ,  $K_{ij} = -G_{ij}$  for  $i \neq j$ 
10:  $\bar{\mathbf{K}} \leftarrow$  remove row and column  $b$  from  $\mathbf{K}$ ,  $\bar{\mathbf{K}}^{-1} \leftarrow$  invert matrix  $\bar{\mathbf{K}}$ 
11:  $R_{ab} \leftarrow (\bar{\mathbf{K}}^{-1})_{aa}$ 
12: return  $R_{ab}$ 

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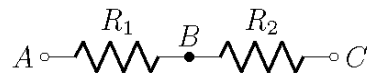
conductance matrix as a stiffness matrix arising from Hookean springs, thereby giving a concrete mechanical meaning to each term in the matrix. This allows students to visualize current flow as force transmission and potential differences as spring displacements such perspective is absent in conventional circuit pedagogy. Furthermore, nodal analysis is indeed systematic, our formulation provides a single, consistent matrix method that works for DC resistor networks, AC networks with complex impedances, Arbitrary linear networks with dependent sources and Non-planar and high-symmetry networks, all within the same intuitive mechanical framework. Thus, the novelty lies not in discovering a new circuit solution method, but in *reframing and enriching* an existing method with mechanical intuition, graph-theoretical insight, and pedagogical clarity.

3. Applications

In this section we put the matrix method to the test showing its systematic resolution of different types of simple and complex electric circuits, in DC and AC regimes.

3.1. Series Resistors

Consider two resistors R_1 and R_2 in series between nodes A and C , with intermediate node B , as shown in Figure 2.

**Figure 2.** Two resistors in series.

The conductance matrix is,

$$\mathbf{K} = \begin{pmatrix} G_1 & -G_1 & 0 \\ -G_1 & G_1 + G_2 & -G_2 \\ 0 & -G_2 & G_2 \end{pmatrix}, \quad G_i = \frac{1}{R_i} \quad (9)$$

Ground node C ($V_C = 0$),

$$\bar{\mathbf{K}} = \begin{pmatrix} G_1 & -G_1 \\ -G_1 & G_1 + G_2 \end{pmatrix}, \quad \bar{\mathbf{K}}^{-1} = \frac{1}{G_1 G_2} \begin{pmatrix} G_1 + G_2 & G_1 \\ G_1 & G_1 \end{pmatrix} \quad (10)$$

With current injection at A and extraction at C ,

$$R_{AC} = (\bar{\mathbf{K}}^{-1})_{11} = \frac{G_1 + G_2}{G_1 G_2} = R_1 + R_2$$

3.2. Parallel Resistors

Consider two resistors R_1 and R_2 in parallel between nodes A and B , as shown in Figure 3.

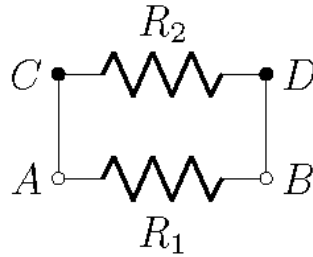


Figure 3. Two resistors in parallel (nodes $A=C$ and $B=D$ are electrically same).

After merging identical nodes:

$$\mathbf{K} = \begin{pmatrix} G_1 + G_2 & -(G_1 + G_2) \\ -(G_1 + G_2) & G_1 + G_2 \end{pmatrix} \quad (11)$$

Ground node B :

$$\bar{\mathbf{K}} = [G_1 + G_2], \quad \bar{\mathbf{K}}^{-1} = \frac{1}{G_1 + G_2} \quad (12)$$

Thus:

$$R_{AB} = \frac{1}{G_1 + G_2} = \frac{R_1 R_2}{R_1 + R_2}$$

3.3. Square-Cross Bridge

Consider the square-cross bridge in Figure 4 where we look for the equivalent resistance R_{AD} between the nodes A and D . This example shows the systematic nature of the matrix method irrespective of the complexity of the network.

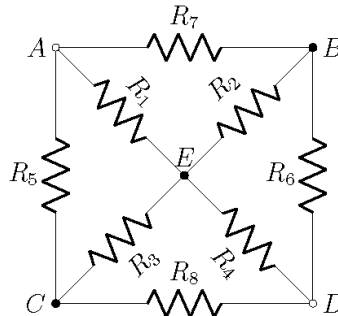


Figure 4. Square-Cross bridge circuit.

The 5×5 conductance matrix is given by,

$$\mathbf{K} = \begin{pmatrix} G_1 + G_5 + G_7 & -G_7 & -G_5 & 0 & -G_1 \\ G_7 & G_2 + G_6 + G_7 & 0 & -G_6 & -G_2 \\ -G_5 & 0 & G_3 + G_5 + G_8 & -G_8 & -G_3 \\ 0 & -G_6 & -G_8 & G_4 + G_6 + G_8 & -G_4 \\ -G_1 & -G_2 & -G_3 & -G_4 & G_1 + G_2 + G_3 + G_4 \end{pmatrix} \quad (13)$$

For R_{AD} with node D grounded, we obtain after reduction to a 4×4 conductance matrix,

$$R_{AD} = \left(\bar{\mathbf{K}}^{-1} \right)_{11} = \frac{N}{D}, \quad (14)$$

where,

$$N = (G_2 + G_6 + G_7)(G_3 + G_5 + G_8)(G_1 + G_2 + G_3 + G_4) - (G_2 + G_6 + G_7)G_3^2 - (G_3 + G_5 + G_8)G_2^2, \quad (15)$$

and,

$$\begin{aligned} D = & G_3G_4G_5G_6 + G_3G_4G_7G_6 + G_3G_5G_7G_6 + G_4G_5G_7G_6 + G_3G_4G_5G_7 \\ & + G_8(G_3 + G_4) \left[G_5G_6 + (G_5 + G_6)G_7 \right] \\ & + G_2(G_4 + G_6) \left[G_3G_5 + (G_3 + G_5)G_7 \right] \\ & + G_2G_8 \left[G_5(G_3 + G_4 + G_6) + (G_3 + G_4 + G_5 + G_6)G_7 \right] \\ & + G_1(G_3 + G_5)(G_4G_6 + G_2(G_4 + G_6) + (G_4 + G_6)G_7) \\ & + G_1G_8 \left[(G_3 + G_4 + G_5)G_6 + G_2(G_3 + G_4 + G_5 + G_6) + (G_3 + G_4 + G_5 + G_6)G_7 \right], \end{aligned} \quad (16)$$

which simplifies for equal resistors $R_i = R$ to,

$$R_{AD} = \frac{2}{3}R. \quad (17)$$

3.4. The 3D Resistor Cube

The resistor cube in Figure 5 has 8 nodes and 12 edges. Traditional series-parallel reduction fails due to 3D symmetry. Our matrix method solves it systematically,

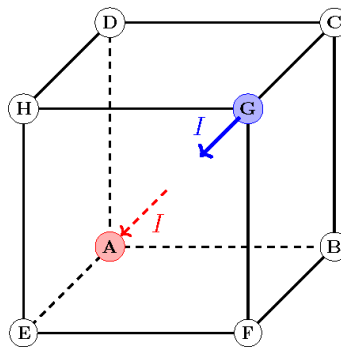


Figure 5. 3D resistor cube: 12 identical resistors R on edges of a cube. Finding R_{AG} (corner to opposite corner) is nontrivial with traditional methods.

Step 1: Label nodes

$$1=A, 2=B, \dots, 8=G.$$

Step 2: Build K

Each node connects to 3 others with conductance $G = 1/R$,

$$K_{ii} = 3G, \quad K_{ij} = \begin{cases} -G & \text{if } i, j \text{ connected} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Step 3: Ground node G (node 8)

We Remove the 8th row and column.

Step 4: Solve

With current injection at A (node 1) and extraction at G,

$$R_{AG} = (\bar{\mathbf{K}}^{-1})_{11}. \quad (19)$$

Result:

After symbolic or numerical computation and for identical resistors $R_i = R$,

$$R_{AG} = \frac{5}{6}R. \quad (20)$$

This matches known results from symmetry arguments, but obtained systematically.

3.5. A Non-Planar Network: The Petersen Graph

A principal strength of the formulated matrix method is its universal applicability to any connected graph structure. Formally, a graph $G = (V, E)$ is defined by a set of vertices V and a set of edges E connecting them [6,7]. The conductance matrix \mathbf{K} constructed in Eq. 8 is precisely the weighted Laplacian matrix of this underlying graph [8], where edge weights are given by conductances G_{ij} . Consequently, the algorithm for computing equivalent resistance is not limited to circuits drawn in a textbook; it provides a general solution for the *resistance distance*, a well-known metric in graph theory, between any two nodes in any weighted, undirected graph. This makes the method directly applicable to the analysis of celebrated and well-studied graph families such as the Petersen graph [9], complete graphs (K_n), cycle graphs (C_n), cubic graphs, and bipartite graphs ($K_{n,m}$) [10]. The formalism thus bridges circuit theory and structural graph theory, offering a computational tool for problems in network science, chemistry such as analyzing fullerene graphs [11], and beyond.

The Petersen graph in Figure 6, presents a significant challenge for traditional circuit analysis methods. With 10 nodes and 15 edges, it is non-planar, that is, cannot be drawn in a plane without edge crossings, and lacks the symmetry or series/parallel combinations to be simplified. Moreover, Delta-wye transformations introduces more complexity than simplifications. In general, each resistor has an arbitrary value, eliminating symmetry-based approaches. The matrix method provides the only *systematic* approach for such networks.

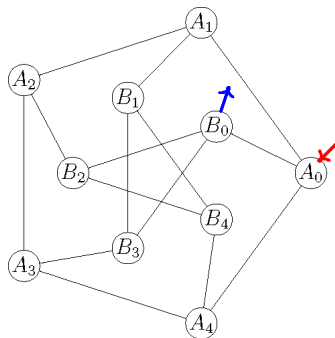


Figure 6. Petersen graph with 10 nodes and 15 edges. This non-planar graph cannot be simplified using series-parallel or delta-wye transformations. All 15 resistors have arbitrary values R_1, R_2, \dots, R_{15} .

Matrix Method Solution

1. **Node labeling:** Label nodes as A_0, A_1, \dots, A_4 (outer pentagon) and B_0, B_1, \dots, B_4 (inner pentagon).
2. **Conductance matrix construction:** The 10×10 conductance matrix \mathbf{K} has,
 - Diagonal elements: $K_{ii} = \sum_{j \in \mathcal{N}(i)} G_{ij}$ where $G_{ij} = 1/R_{ij}$
 - Off-diagonal elements: $K_{ij} = -G_{ij}$ if nodes i and j are connected, 0 otherwise
3. **Grounding:** To find $R_{A_0 B_0}$ between nodes A_0 and B_0 , ground node B_0 ($V_{B_0} = 0$).

4. **Matrix reduction:** Remove row and column corresponding to B_0 , obtaining 9×9 matrix $\bar{\mathbf{K}}$.
5. **Solution:** The equivalent resistance is,

$$R_{A_0 B_0} = (\bar{\mathbf{K}}^{-1})_{A_0 A_0}. \quad (21)$$

Symbolic Result:

For arbitrary resistors, the symbolic expression is extremely complex (over 100 terms even for this modest network). However, for equal resistors $R_{ij} = R$, symmetry yields,

$$R_{A_0 B_0} = \frac{2}{5}R. \quad (22)$$

Algorithm 2 solves for the equivalent resistance in an arbitrary Petersen graph.

Algorithm 2 Solve Arbitrary Petersen Graph Resistance

Require: 15 resistance values R_1, R_2, \dots, R_{15} for edges

Ensure: Equivalent resistance $R_{A_0 B_0}$

- 1: Define node connectivity: 10 nodes, 15 edges with specific connections
 - 2: Build 10×10 adjacency matrix A (1 if connected, 0 otherwise)
 - 3: Build resistance matrix R_{mat} with corresponding values
 - 4: Compute conductance matrix: $G_{ij} = 1/R_{mat,ij}$ for connected nodes
 - 5: Build \mathbf{K} : $K_{ii} = \sum_j G_{ij}$, $K_{ij} = -G_{ij}$ for $i \neq j$
 - 6: $\bar{\mathbf{K}} \leftarrow$ remove row/column for node B_0 from \mathbf{K}
 - 7: $\bar{\mathbf{K}}^{-1} \leftarrow \text{invert}(\bar{\mathbf{K}})$
 - 8: $R_{A_0 B_0} \leftarrow (\bar{\mathbf{K}}^{-1})_{A_0 A_0}$ (where A_0 is index 0)
 - 9: **return** $R_{A_0 B_0}$
-

3.6. AC Network with RLC Elements

The network in Figure 7 contains both energy storage elements (inductor L_1 , capacitors C_1, C_2) and dissipative elements (resistors R_1, R_2, R_3). In the AC regime at angular frequency ω , we use complex impedances,

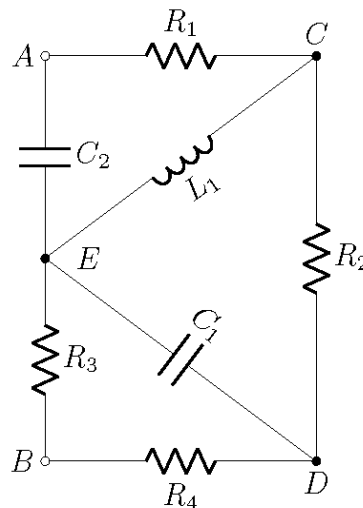


Figure 7. AC network with resistors, inductors, and capacitors. The goal is to find the equivalent impedance $\tilde{Z}_{AB}(\omega)$ between nodes A and B at angular frequency ω .

$$\tilde{Z}_R = R \quad (\text{resistor}), \quad \tilde{Z}_L = j\omega L \quad (\text{inductor}) \quad \text{and} \quad \tilde{Z}_C = \frac{1}{j\omega C} \quad (\text{capacitor}) \quad (23)$$

where $j = \sqrt{-1}$. The corresponding complex admittances are $\tilde{Y} = 1/\tilde{Z}$.

Matrix Formulation for AC:

The spring-resistor correspondence extends naturally to AC networks by replacing conductances with complex admittances,

1. **Complex admittance matrix:** For each element between nodes i and j , compute complex admittance $\tilde{Y}_{ij} = 1/\tilde{Z}_{ij}$.
2. **Complex stiffness matrix:** Construct $\tilde{\mathbf{K}}$ with elements,

$$\tilde{K}_{ij} = \begin{cases} \sum_{k \neq i} \tilde{Y}_{ik} & \text{if } i = j \\ -\tilde{Y}_{ij} & \text{if } i \neq j \text{ and connected} \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

3. **Grounding:** Ground node B , removing corresponding row/column to get $\tilde{\mathbf{K}}$.
4. **Complex solution:** Solve $\tilde{\mathbf{K}}\tilde{\mathbf{V}} = \tilde{\mathbf{I}}$ where $\tilde{I}_A = 1$ A (unit current source), $\tilde{I}_i = 0$ for $i \neq A, B$.
5. **Equivalent impedance:** $\tilde{Z}_{AB} = \tilde{V}_A$ (since $\tilde{I}_A = 1$ and $V_B = 0$).

Symbolic Solution:

For the network in Figure 7, after grounding node B,

$$\tilde{\mathbf{K}} = \begin{pmatrix} \tilde{Y}_{R_1} + \tilde{Y}_{C_2} & -\tilde{Y}_{R_1} & 0 & -\tilde{Y}_{C_2} \\ -\tilde{Y}_{R_1} & \tilde{Y}_{R_1} + \tilde{Y}_{L_1} + \tilde{Y}_{R_2} & -\tilde{Y}_{R_2} & -\tilde{Y}_{L_1} \\ 0 & -\tilde{Y}_{R_2} & \tilde{Y}_{R_2} + \tilde{Y}_{C_1} & -\tilde{Y}_{C_1} \\ -\tilde{Y}_{C_2} & -\tilde{Y}_{L_1} & -\tilde{Y}_{C_1} & \tilde{Y}_{C_2} + \tilde{Y}_{L_1} + \tilde{Y}_{C_1} + \tilde{Y}_{R_3} \end{pmatrix} \quad (25)$$

where rows/columns correspond to nodes A, C, D, E.

The equivalent impedance is,

$$\tilde{Z}_{AB} = (\tilde{\mathbf{K}}^{-1})_{11}. \quad (26)$$

The circuit has five nodes: A, B, C, D , and E . Node B is chosen as the reference (ground). The independent nodes are A, C, D , and E . The admittances of the components at angular frequency ω are,

$$Y_{R_1} = G_1 = \frac{1}{R_1}, \quad Y_{C_2} = j\omega C_2, \quad Y_{L_1} = \frac{1}{j\omega L_1} = -j\frac{1}{\omega L_1}, \quad Y_{C_1} = j\omega C_1, \quad (27)$$

$$Y_{R_2} = G_2 = \frac{1}{R_2}, \quad Y_{R_3} = G_3 = \frac{1}{R_3}, \quad Y_{R_4} = G_4 = \frac{1}{R_4}. \quad (28)$$

The admittance matrix \mathbf{Y} for nodes A, C, D, E is,

$$\mathbf{Y} = \begin{pmatrix} G_1 + j\omega C_2 & -G_1 & 0 & -j\omega C_2 \\ -G_1 & G_1 + G_2 - \frac{j}{\omega L_1} & -G_2 & \frac{j}{\omega L_1} \\ 0 & -G_2 & G_2 + G_4 + j\omega C_1 & -j\omega C_1 \\ -j\omega C_2 & \frac{j}{\omega L_1} & -j\omega C_1 & G_3 + j\omega(C_1 + C_2) - \frac{j}{\omega L_1} \end{pmatrix}.$$

The equivalent impedance between nodes A and B is given by,

$$\tilde{Z}_{AB}(\omega) = (\mathbf{Y}^{-1})_{11} = \frac{\det(\mathbf{Y}_{11})}{\det(\mathbf{Y})},$$

where \mathbf{Y}_{11} is the 3×3 submatrix obtained by deleting the first row and first column of \mathbf{Y} ,

$$\mathbf{Y}_{11} = \begin{pmatrix} G_1 + G_2 - \frac{j}{\omega L_1} & -G_2 & \frac{j}{\omega L_1} \\ -G_2 & G_2 + G_4 + j\omega C_1 & -j\omega C_1 \\ \frac{j}{\omega L_1} & -j\omega C_1 & G_3 + j\omega(C_1 + C_2) - \frac{j}{\omega L_1} \end{pmatrix}.$$

The determinants $\det(\mathbf{Y})$ and $\det(\mathbf{Y}_{11})$ are complex polynomials in ω . Their explicit expansions are,

$$\det(\mathbf{Y}_{11}) = G_2 G_3 G_4 + \frac{C_1}{L_1} (G_3 + G_4) + j \left(\omega C_1 G_2 (G_3 + G_4) - \frac{1}{\omega L_1} (G_2 G_4 + G_3 G_4 + G_2 G_3) \right).$$

and

$$\det(\mathbf{Y}) = (G_4 + G_3) \left[\frac{1}{L_1} (G_1 C_1 + G_2 C_2) - \omega^2 C_1 C_2 (G_1 + G_2) + j \left(\omega G_1 G_2 (C_1 + C_2) - \frac{G_1 G_2}{\omega L_1} + \omega \frac{C_1 C_2}{L_1} \right) \right].$$

These expressions provide a complete analytical solution for $\tilde{Z}_{AB}(\omega)$. For specific component values, the determinants can be evaluated numerically or symbolically to obtain the impedance as a function of frequency. The matrix method generalizes to any linear network containing resistors, capacitors, inductors, and linear dependent sources,

Complex Admittance Matrix:

For network with N nodes at frequency ω :

$$\tilde{\mathbf{Y}}(\omega) \tilde{\mathbf{V}}(\omega) = \tilde{\mathbf{I}}(\omega) \quad (29)$$

where $\tilde{\mathbf{Y}}$ is the $N \times N$ complex admittance matrix, $\tilde{\mathbf{V}}$ is the node voltage vector, and $\tilde{\mathbf{I}}$ is the current injection vector.

Dependent Sources:

Voltage-controlled current sources (VCCS) with transconductance g_m add terms to $\tilde{\mathbf{Y}}$,

$$\tilde{Y}_{ij} \leftarrow \tilde{Y}_{ij} + g_m \quad \text{for VCCS } I_i = g_m V_j \quad (30)$$

Matrix Solution:

After grounding one node,

$$\tilde{\mathbf{V}}_{\text{red}} = \tilde{\mathbf{Y}}_{\text{red}}^{-1} \tilde{\mathbf{I}}_{\text{red}} \quad (31)$$

Equivalent Impedance:

$$\tilde{Z}_{\text{eq}}(\omega) = \frac{V_a - V_b}{I} = (\tilde{\mathbf{Y}}_{\text{red}}^{-1})_{aa} + (\tilde{\mathbf{Y}}_{\text{red}}^{-1})_{bb} - 2(\tilde{\mathbf{Y}}_{\text{red}}^{-1})_{ab} \quad (32)$$

These examples demonstrate that while traditional methods fail for complex or non-planar networks, the matrix approach provides a systematic solution. The extension to AC networks shows the method's generality, introducing students to complex numbers and frequency-domain analysis in a unified framework.

4. Discussion and Conclusion

The spring-resistor analogy provides a powerful pedagogical bridge between mechanics and circuit theory. The matrix formulation offers a systematic computational approach for equivalent resistance that complements traditional methods [2]. Table 2 compares the matrix method to other well established circuit reduction methods, highlighting its unique position as a general-purpose, analytic and systematic technique that bridges physical intuition with mathematical formalism.

Table 2. Comparison of resistance calculation methods

Method	Advantages	Disadvantages
Series-Parallel Reduction	Intuitive, quick for simple circuits	Fails for complex networks (e.g., cube, bridges)
Delta-Wye Transformation	Powerful for certain symmetric circuits	Limited applicability, requires pattern recognition
Nodal-Mesh Analysis	Systematic, taught in standard curriculum	Requires solving linear equations manually
Spring-Conductance Matrix Method	Systematic for any network , connects to mechanics, computationally friendly	Requires matrix operations, singular matrix handling

From a pedagogical perspective, the spring-resistor analogy offers several distinct advantages that make it particularly valuable for introductory circuit theory education [1]. Students who have already developed intuition about mechanical systems can leverage their understanding of forces and displacements to grasp the more abstract concepts of current flow and potential differences. This physical grounding makes electrical theory more concrete and accessible. The matrix formulation, while introducing computational complexity, provides a systematic framework that works uniformly for any resistor/impedance network, eliminating the need for the pattern recognition and cleverness often required by traditional methods. This consistency helps students develop confidence in tackling complex problems. Moreover, the analogy naturally extends to visualizing current as *force flow* and potential as *displacement*, fostering physical intuition about circuit behavior that complements the mathematical analysis. The method's generality demonstrates the unity of mathematical approaches across disciplines, connecting circuit analysis to structural mechanics, heat transfer, and even social network theory, thereby broadening students' appreciation for mathematical modeling [3].

However, while the matrix method introduces students to computational thinking and provides a unified analytical framework, it is important to acknowledge its inherent limitations [12]. The approach fundamentally assumes linear relationships in both domains, Ohm's law for resistors and Hooke's law for springs, which restricts its direct application to nonlinear circuit elements. Practical considerations include potential numerical instability in large networks, where the conductance matrix may become ill-conditioned, and the computational cost of matrix inversion, which scales as $O(N^3)$ and becomes inefficient for very large networks compared to sparse solvers. Despite these constraints, the method retains substantial value for pedagogical applications and moderate-sized networks, where its systematic nature and intuitive mechanical connection provide significant educational benefits.

The method presented in this work is particularly useful for analyzing complex networks that resist solution by traditional series-parallel reduction or symmetry arguments. Future work could explore applications to random resistor networks [13], percolation theory, and the development of interactive educational tools based on this analogy, further enhancing its pedagogical impact and extending its reach to more advanced topics in network analysis and computational modeling.

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