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Why Non-Linear Source Geometry Does Not Imply Superluminal Signaling: A TCGS-SEQUENTION Response to Gisin-Polchinski

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Why Non-Linear Source Geometry Does Not Imply Superluminal Signaling: A TCGS-SEQUENTION Response to Gisin-Polchinski

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Abstract

The Gisin-Polchinski (GP) no-go theorem (1990–1991) is widely cited as proof that non-linear modifications to quantum mechanics necessarily permit superluminal signaling, thereby violating special relativity. This note demonstrates that the GP argument relies on ontological assumptions that do not hold within the Timeless Counterspace & Shadow Gravity (TCGS) framework. Specifically, GP assumes: (i) time is ontically fundamental, (ii) Alice's measurement "causes" a change in Bob's state, and (iii) the density matrix is the complete description of physical reality. In TCGS, where observable 3D reality is a projection of a static 4D counterspace, the apparent "signaling" dissolves as a foliation artifact. Non-linearity at the source level is fully compatible with operational no-signaling at the shadow level.

Keywords: TCGS-SEQUENTION; non-linear quantum mechanics; Gisin-Polchinski theorem; superluminal signaling; entanglement; counterspace; foliation invariance; identity of source

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1. The Gisin-Polchinski Argument

The core signaling concern for non-linear quantum mechanics was articulated in the original discussions of Weinberg's framework and its EPR implications [1–4].

1.1. Setup

Consider an entangled pair of qubits shared between Alice (A) and Bob (B):

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B) \quad (1)$$

Bob's reduced density matrix is:

$$\rho_B = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) = \frac{1}{2}\mathbb{I} \quad (2)$$

1.2. The Standard Argument

Step 1: Suppose quantum evolution is state-dependent (non-linear), meaning the evolution operator $U[\rho]$ depends on ρ itself.

Step 2: The maximally mixed state $\rho_B = \frac{1}{2}\mathbb{I}$ admits multiple decompositions:

- As mixture of $\{|0\rangle, |1\rangle\}$: the ensemble $\{(|0\rangle, \frac{1}{2}), (|1\rangle, \frac{1}{2})\}$
- As mixture of $\{|+\rangle, |-\rangle\}$: the ensemble $\{(|+\rangle, \frac{1}{2}), (|-\rangle, \frac{1}{2})\}$

Step 3: If Alice measures in the $\{|0\rangle, |1\rangle\}$ basis, she "collapses" Bob's state into the first ensemble. If she measures in the $\{|+\rangle, |-\rangle\}$ basis, she collapses it into the second.

Step 4: Under non-linear evolution $U[\rho]$, these two ensembles evolve *differently*, even though they represent the same density matrix.

Step 5: Bob can therefore detect which basis Alice chose → superluminal signaling → contradiction.

1.3. The Implicit Assumptions

The GP argument requires:

1. **Ontic time**: There exists a global time parameter t such that Alice's measurement at t_1 precedes Bob's detection at t_2.
2. **Causal collapse**: Alice's measurement *causes* a real, physical change in Bob's state.
3. **Density matrix completeness**: The density matrix ρ is the complete physical description; different ensemble decompositions represent the same physical state.

2. The TCGS Dissolution

2.1. Core Axioms (Recap)

Axiom 1 (Whole Content — A1)**.** *There exists a smooth 4D counterspace* $(\mathcal{C}, G, \Psi)$ *containing the full content of all "time stages" simultaneously.*

Axiom 2 (Identity of Source — A2)**.** *All correlated phenomena derive from a unique geometric origin* $p_0 \in \mathcal{C}$.

Axiom 3 (Shadow Realization — A3)**.** *The observable 3D world is embedded by* $X : \Sigma \to \mathcal{C}$*; observables are pullbacks* $(g, \psi) = (X^*G, X^*\Psi)$*. Time has no ontic status.*

2.2. Entanglement as Identity of Source

In TCGS, the "entangled pair" is not two particles mysteriously correlated across space. It is **one geometric object** in the 4D counterspace $\mathcal{C}$, viewed from two projection points.

Let $\Psi \in \mathcal{C}$ be the static 4D content field encoding the entangled system. Define:

- $X_A : \Sigma_A \to \mathcal{C}$ — the projection at Alice's location
- $X_B : \Sigma_B \to \mathcal{C}$ — the projection at Bob's location

The observables are:

$$\psi_A = X_A{}^*\Psi, \quad \psi_B = X_B{}^*\Psi \tag{3}$$

The "entanglement correlation" is the geometric constraint that both are pullbacks of the **same** Ψ.

2.3. Why "Signaling" Does Not Occur

Claim 1. *Alice's "measurement choice" does not change* Ψ*; it selects a foliation of* $\mathcal{C}$.

Argument:

In standard QM, Alice's choice of measurement basis is modeled as an intervention that collapses the wave function. The GP argument treats this collapse as a real, time-indexed event that changes the physical state.

In TCGS:

1. The 4D content Ψ is **static** (Axiom A1). It does not "change" because time is not fundamental.
2. Alice's "measurement" corresponds to selecting a specific foliation (slicing) of $\mathcal{C}$. Different basis choices correspond to different ways of parameterizing the projection X_A.
3. Bob's observable $\psi_B = X_B{}^*\Psi$ depends on:
   - His projection map X_B (fixed by his spacetime location)
   - The static field Ψ (unchanged by Alice's foliation choice)
4. **Crucially**: The pullback operation $X_B{}^*$ commutes with Alice's foliation choice. Bob's observable is determined by Ψ and X_B, not by how Alice parameterizes her slice.

2.4. Formal Statement

Theorem 2 (No-Signaling from Foliation Invariance)**.** *Let* $(\mathcal{C}, G, \Psi)$ *be a static 4D counterspace. Let* $X_A, X_B : \Sigma \to \mathcal{C}$ *be spacelike-separated projections. Let* $\mathcal{F}_A$ *denote Alice's choice of foliation.*

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Then Bob's observable density matrix:

$$\rho_B = \mathrm{Tr}_A[X^*\Psi \otimes X^*\Psi] \tag{4}$$

is independent of $\mathcal{F}_A$.

Proof sketch. The pullback ${X_B}^*$ is a geometric operation determined by the embedding X_B and the field Ψ. Alice's foliation choice $\mathcal{F}_A$ affects only the parameterization of her projection X_A, not the value of Ψ itself. Since Ψ is static and X_B is independent of Alice's operations, ρ_B is foliation-invariant. □

2.5. The Decomposition "Problem" Dissolved

The GP argument hinges on different ensemble decompositions of ρ_B evolving differently under non-linear dynamics.

In TCGS, this is a **category error**. The different decompositions are not different physical preparations—they are different *descriptions* of the same geometric projection. The 4D source Ψ determines:

- The correlations (Identity of Source)
- The evolution law (as a constitutive relation on $\mathcal{C}$)

The density matrix ρ_B is an **incomplete summary** of the 3D shadow. It does not capture the full geometric information encoded in Ψ. Non-linear evolution at the source level operates on Ψ, not on ρ_B. The projection ensures that what Bob observes is consistent with no-signaling.

3. Analogy: ER = EPR

The TCGS resolution is structurally analogous to the ER = EPR correspondence (Maldacena-Susskind) [7]:

Table 1. Structural analogy between ER = EPR and TCGS interpretations of entanglement.

ER = EPR	**TCGS**
Entangled particles connected by wormhole	Entangled systems as single 4D geometric object
Wormhole is non-traversable	Projection constraint prevents signaling
Correlation without causation	Identity of Source without time-propagation
Information preserved at horizon	Observables are gauge-invariant pullbacks

In both frameworks, the apparent "spooky action at a distance" is reinterpreted as a geometric unity that, by construction, does not permit superluminal information transfer.

4. Non-Linearity Preserved at the Source

A key point: TCGS does **not** require linearity.

The 4D content field Ψ and the constitutive law governing it can be arbitrarily non-linear:

$$\nabla_{\mathcal{C}} \cdot [\mu(|\nabla_{\mathcal{C}}\Psi|)\nabla_{\mathcal{C}}\Psi] = \text{sources} \tag{5}$$

The constraint is that the **projection mechanism** $(X : \Sigma \to \mathcal{C})$ and the **pullback structure** ensure that 3D observables respect operational no-signaling.

This is directly analogous to how General Relativity is highly non-linear, yet the causal structure (light cones) prevents superluminal signaling. The non-linearity of the field equations does not imply violation of causality—the metric structure enforces it.

5. Addressing Gisin's Operational Protocol Directly

Gisin (1990) [1] provides a concrete signaling protocol that deserves explicit treatment.

5.1. The Protocol

1. Alice measures her particle in either z-basis or u-basis (45° apart)
2. Bob applies the non-linear Hamiltonian:

$$h(\psi, \psi^*) = \frac{\langle\psi|\sigma_z|\psi\rangle^2}{\langle\psi|\psi\rangle} \tag{6}$$

3. Evolution equation:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -2i\langle\sigma_z\rangle\sigma_z\psi \tag{7}$$

Gisin's key observation:

- z-eigenstates are *stationary*: $\langle\sigma_y\rangle$ remains zero
- u-eigenstates *rotate*: after quarter-period, $\langle\sigma_y\rangle > 0$ for both $|u+\rangle$ and $|u-\rangle$

Thus Bob can distinguish Alice's basis choice by measuring $\langle\sigma_y\rangle$ over many particles.

5.2. Gisin's Own Identification of the Critical Assumption

Gisin writes [1]:

> "The most delicate point in the above example is probably the **instantaneous preparation at a distance**"

This is exactly what TCGS denies exists.

5.3. The TCGS Response

Question: When Bob applies his non-linear Hamiltonian, what state does he apply it to?
Standard QM answer: $|z\pm\rangle$ or $|u\pm\rangle$, depending on which Alice "prepared" by her measurement.
TCGS answer: Bob applies it to $X_B{}^*\Psi$ — his projection of the static 4D field.
The decisive point: Is $X_B{}^*\Psi$ different depending on Alice's measurement choice?
In TCGS: **No.**

Alice's measurement basis is a *foliation choice* — it determines how she parameterizes her projection, but does not change:

- The 4D field Ψ (which is static by Axiom A1)
- Bob's projection map X_B (which is fixed by his spacetime location)
- Therefore, Bob's observable $X_B{}^*\Psi$

The "instantaneous preparation at a distance" that Gisin's argument requires is precisely what the timeless ontology eliminates. There is no moment at which Bob's particle "becomes" $|z+\rangle$ rather than $|u+\rangle$. Both are descriptions of foliations, not ontological facts about the source.

5.4. What Bob Actually Has

In TCGS, Bob's particle is not in a pure state that depends on Alice's choice. It is in the state determined by its geometric relationship to the source Ψ.

The correlation — that if Alice finds "up" then Bob finds "down" — is explained by Identity of Source (Axiom A2): both measurements are projections of the *same* 4D geometric structure. But this correlation is *built into* Ψ; it is not *created* by Alice's measurement.

When Bob applies any evolution (linear or non-linear) to his particle, he evolves $X_B{}^*\Psi$. This quantity is independent of Alice's foliation choice.

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6. Addressing Other Specific Points

6.1. Local Applicability assumes 3D separation is fundamental

Response: Correct. TCGS rejects this assumption. What appears as spatial separation in Σ is a projection artifact of a unified structure in $\mathcal{C}$. Entanglement reveals this: "distance" is a user-interface feature, not ontological.

6.2. The density matrix determines outcomes

Response: In standard QM, yes. In TCGS, the density matrix is an incomplete 3D summary. The complete description is $\Psi \in \mathcal{C}$. Different ensemble decompositions of ρ correspond to different foliations of the same Ψ, not different physical states.

6.3. Non-linear evolution allows distinguishing mixtures

Response: Only if the evolution acts on the 3D ensemble descriptions. In TCGS, evolution is a constitutive law on Ψ in $\mathcal{C}$. The projection ensures that Bob's accessible observables—the pullback $X_B{}^*\Psi$—do not depend on Alice's foliation choice.

7. Empirical Distinguishability

The TCGS framework makes the same operational predictions as standard QM for the GP setup: **no signaling**. The difference is ontological, not empirical (for this experiment).

However, TCGS makes distinct predictions in other regimes:

- Weak-field gravitational dynamics (the μ-law replacing dark matter halos)
- Cosmological slice-invariants
- Convergent biological patterns (SEQUENTION)

The GP experiment does not falsify TCGS because TCGS predicts exactly what is observed: no superluminal signaling, despite non-linear source geometry.

8. Conclusions

The Gisin-Polchinski argument is valid within its ontological assumptions: ontic time, causal collapse, and density matrix completeness. In the TCGS framework, where time is a foliation artifact and observables are pullbacks of a static 4D geometry, the argument does not apply.

Non-linearity at the 4D source level is compatible with operational no-signaling at the 3D shadow level. The "problem" of different ensemble decompositions is dissolved: they are different descriptions of the same geometric projection, not different physical preparations.

The critique that "non-linear QM violates relativity" targets a different ontology than TCGS proposes. Within TCGS, the Identity of Source (Axiom A2) explains entanglement correlations, and the projection structure (Axiom A3) guarantees no-signaling—without requiring linearity.

Appendix A. Operational Locality Forces Affine Linearity on the Shadow State Space

This appendix makes explicit a point that is usually treated as "obvious" in informal discussions, but which is in fact the technical hinge of the Gisin–Polchinski analysis. If the *operational* state of a subsystem is the object that determines all statistics of local measurements (in standard quantum mechanics: the reduced density operator ρ), then any admissible local transformation must act *affinely* on that object. This is not a matter of taste: without affine linearity, remote parties can exploit different ensemble decompositions of the same ρ to transmit information at spacelike separation—which is precisely the content of Gisin's protocol [1] and Polchinski's generalization [2].

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Appendix A.1. Ensemble Equivalence and the GP Lever

Let ρ_{AB} be a bipartite state shared between Alice (A) and Bob (B), and let

$$\rho_B \;:=\; \mathrm{Tr}_A(\rho_{AB}) \tag{A1}$$

be Bob's reduced state. A textbook fact is that Alice can prepare (by measuring A in different bases) different ensembles on Bob's side that correspond to the *same* ρ_B. Schematically,

$$\rho_B \;=\; \sum_i p_i |\psi_i\rangle\langle\psi_i| \;=\; \sum_j q_j |\phi_j\rangle\langle\phi_j|, \tag{A2}$$

where the decompositions $(p_i, |\psi_i\rangle)$ and $(q_j, |\phi_j\rangle)$ may be operationally realized by different measurement choices on A.

In ordinary (linear) quantum mechanics, Bob cannot distinguish which decomposition was used, because every local expectation value is a linear functional of ρ_B:

$$\langle O_B\rangle \;=\; \mathrm{Tr}(\rho_B O_B). \tag{A3}$$

The entire no-signaling theorem, at the laboratory level, is the statement that *for all* local procedures available to Bob, the statistics depend on ρ_B and not on the choice of remote ensemble.

The GP argument identifies the following failure mode. Suppose Bob has access to a local dynamical rule $\mathcal{E}$ that maps inputs to outputs, and suppose this rule is not well-defined on ρ_B alone, but (implicitly or explicitly) depends on the specific ensemble used to realize ρ_B. Then Alice can choose between two decompositions of the same ρ_B and thereby change Bob's output statistics. This is superluminal signaling.

Appendix A.2. Affine Linearity from Local Randomization

The minimal operational closure property one expects of any laboratory theory is closure under classical randomization: Bob can flip a classical coin and decide to prepare ρ_1 with probability λ and ρ_2 with probability $1-\lambda$. Operationally, this is a legitimate local preparation procedure, hence it must be represented by the mixed state

$$\rho_\lambda \;=\; \lambda\rho_1 + (1-\lambda)\rho_2. \tag{A4}$$

If Bob now applies a physically admissible local transformation $\mathcal{E}$, the output should be independent of whether the randomization occurred "before" or "after" the transformation. This forces

$$\mathcal{E}(\lambda\rho_1 + (1-\lambda)\rho_2) \;=\; \lambda\,\mathcal{E}(\rho_1) + (1-\lambda)\,\mathcal{E}(\rho_2), \qquad 0 \le \lambda \le 1. \tag{A5}$$

Equation (A5) is *affine linearity*. Any non-affine $\mathcal{E}$ allows Bob to operationally distinguish different decompositions of the same ρ (because mixing is itself a preparation operation), and thus recreates the GP signaling mechanism.

Two clarifications align this with the TCGS posture:

1. Affine linearity is a *shadow-level* constraint: it is imposed on the operational state space $\mathcal{S}$ (represented by density operators). It does not preclude a nonlinear law on the underlying 4D source configuration space $\mathcal{C}$.
2. What GP rules out is *non-affine operational maps*. The TCGS claim is that source-level nonlinearity, when projected through the quotient map defining the shadow state, induces an affine (indeed CPTP) evolution on ρ and therefore respects the no-signaling closure required by experiments.

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Appendix B. Complete Positivity as the Appropriate Notion of Local Implementability

Affine linearity is necessary but not sufficient. In the presence of entanglement, the correct operational notion of a "locally implementable" transformation on Bob's system is that it be *completely positive and trace-preserving* (CPTP). In finite dimensions, this is equivalent to admitting a Kraus representation [11,15]

$$\mathcal{E}(\rho) = \sum_k K_k \rho K_k^\dagger, \qquad \sum_k K_k^\dagger K_k = I, \tag{A6}$$

and, equivalently, to the existence of a Stinespring dilation [12] in which Bob couples his system unitarily to an environment and then discards the environment.

Complete positivity is the mathematically precise encoding of "local applicability" in the entangled setting: $\mathcal{E}$ must take positive operators to positive operators even when extended as $\mathcal{E} \otimes \mathrm{id}_R$ to an arbitrary reference system R. This requirement is what prevents pathological maps that look positive on isolated states but fail on entangled inputs.

For continuous-time, Markovian dynamics, the same locality requirement leads to the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) structure [13,14]:

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right), \tag{A7}$$

which is again linear in ρ.

From the GP viewpoint, the essential point is structural: the operational calculus of local laboratories, when formulated consistently with entanglement, is *necessarily linear in* ρ. Any proposal that permits Bob to implement a non-affine map on his reduced state is, ipso facto, outside the CPTP/GKSL envelope and therefore outside the domain of physically realizable "local operations" as understood in quantum information theory. The GP theorem can be restated succinctly as:

> If you treat non-affine, pure-state dependent rules as local operations, then spacelike signaling follows.

TCGS blocks the inference not by denying the theorem, but by denying that such pure-state dependence is a legitimate shadow-level local operation.

Appendix C. Polchinski's Constraint and the "Branch Communication" Alternative

Polchinski's paper [2] refines the GP argument by identifying the precise constraint on observables required to prevent EPR signaling *within* Weinberg's nonlinear framework [3,4]. The setup is a bipartite state Ψ_{ij} with indices referring to systems I and II. Assuming that subsystem II retains the standard set of linear Hermitian observables (so that ordinary measurements remain meaningful), Polchinski shows that the condition "no information is transmitted via EPR correlations" forces any observable of system I to be invariant under unitary rotations on the degrees of freedom of system II. This invariance is equivalent to dependence *only* on the reduced density matrix of system I.

In contemporary language: forbidding EPR signaling pushes you toward a state description in which all physically meaningful local quantities are functions of ρ rather than functions of a particular pure-state representative. This is precisely the operational identification that TCGS adopts from the outset at the shadow layer.

Polchinski then observes a tension specific to Weinberg's proposal: if nonlinear observables are constrained to be density-matrix functionals to avoid EPR signaling, the theory tends to allow another sort of "communication": between different components of a global wave function that would be dynamically isolated in linear quantum mechanics. Polchinski calls this "communication between branches of the wave function." One can interpret this as a failure of the usual many-worlds separability intuition, or as evidence that the nonlinear structure has reintroduced hidden global degrees of freedom.

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The present note does not require adjudicating that debate. The operational takeaway is that the GP pathology is the result of assigning physical significance to distinctions that are not locally controllable. In TCGS, branch structure is not fundamental at the source; it is an emergent shadow bookkeeping of how projected degrees of freedom factorize under particular coarse-grainings [5,6]. Once that bookkeeping is kept at the shadow level, "branch communication" is not a fundamental channel but an artifact of representing a quotient dynamics in an over-refined parameterization.

Appendix D. Worked Signaling Example: How Non-Affine Shadow Rules Enable Superluminal Communication

To make the logical mechanism maximally explicit, this section writes the GP lever in a concrete algebraic form. The example is intentionally minimal: a two-qubit singlet shared by Alice and Bob,

$$|\Psi^-\rangle_{AB} \;=\; \frac{1}{\sqrt{2}}\Big(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\Big), \tag{A8}$$

with Bob's reduced state $\rho_B = I/2$.

Appendix D.1. Remote Preparation of Inequivalent Ensembles with the Same ρ_B

If Alice measures in the computational basis $\{|0\rangle, |1\rangle\}$, Bob is prepared in the ensemble

$$\rho_B \;=\; \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|. \tag{A9}$$

If instead Alice measures in the Hadamard basis $\{|+\rangle, |-\rangle\}$, with $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$, then Bob is prepared in the ensemble

$$\rho_B \;=\; \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|. \tag{A10}$$

Both ensembles yield the same density matrix $I/2$.

In linear quantum mechanics, any subsequent local processing by Bob is described by a CPTP map $\mathcal{E}$, hence

$$\mathcal{E}\Big(\frac{I}{2}\Big) \;=\; \frac{1}{2}\mathcal{E}(|0\rangle\langle 0|) + \frac{1}{2}\mathcal{E}(|1\rangle\langle 1|) \;=\; \frac{1}{2}\mathcal{E}(|+\rangle\langle +|) + \frac{1}{2}\mathcal{E}(|-\rangle\langle -|), \tag{A11}$$

and no signaling is possible.

Appendix D.2. Non-Affine Evolution Generates Distinguishable Output Ensembles

Now suppose, instead, that Bob postulates a "local" evolution rule on pure states that depends nonlinearly on the state. One schematic form (sufficient for the logical point) is a deterministic, norm-preserving map $|\psi\rangle \mapsto |\psi'\rangle$ that depends on the amplitudes through a nonlinear functional. In Weinberg-type constructions this arises when the effective Hamiltonian depends on expectation values or other state functionals [3,4]. Gisin's observation is that if such a rule is applied to the elements of an ensemble, then the image of a mixed state depends on its decomposition [1].

In Weinberg-type constructions, state dependence typically arises because the effective Hamiltonian or generator depends on expectation values or other functionals of the state [3,4]. The GP point is that this dependence is benign *only* if it is not promoted to a decomposition-sensitive *local* rule on operational states.

To exhibit the mechanism with complete transparency, pick a toy but explicit state-dependent unitary of the form

$$U_\psi \;:=\; \exp\Big(-i\,\theta(\langle\sigma_z\rangle_\psi)\,\sigma_y\Big), \qquad \theta(x) := \kappa x, \tag{A12}$$

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where κ is a fixed (small) constant and $\langle\sigma_z\rangle_\psi := \langle\psi|\sigma_z|\psi\rangle$. This is a rotation about the y-axis by an angle proportional to the z-polarization of the input state. For the four states relevant to EPR steering one has

$$\langle\sigma_z\rangle_{|0\rangle} = +1, \quad \langle\sigma_z\rangle_{|1\rangle} = -1, \quad \langle\sigma_z\rangle_{|+\rangle} = 0, \quad \langle\sigma_z\rangle_{|-\rangle} = 0, \tag{A13}$$

hence

$$U_{|0\rangle} = e^{-i\kappa\sigma_y}, \quad U_{|1\rangle} = e^{+i\kappa\sigma_y}, \quad U_{|+\rangle} = U_{|-\rangle} = I. \tag{A14}$$

If one then defines the action on mixtures by acting on ensemble elements, the Z-prepared mixture evolves into a convex sum of two *rotated* pure states, whereas the X-prepared mixture is left invariant. Consequently, for $\kappa \neq 0$ the two output density matrices differ and Bob can detect Alice's basis choice by a local measurement (e.g., of σ_x), completing the signaling protocol [1,2].

The point of writing this explicit toy model is not to advocate (A12) as a physical law; it is to display the structural fact that *any* decomposition-sensitive non-affine "local" rule recreates the GP lever.

Concretely, suppose the rule acts as

$$\mathcal{N}(|\psi\rangle\langle\psi|) \;=\; U_\psi\,|\psi\rangle\langle\psi|\,U_\psi^\dagger, \tag{A15}$$

where U_ψ is a unitary that depends on $|\psi\rangle$ (for example through a phase rotation by an angle that is a nonlinear function of $\langle\sigma_z\rangle_\psi$). The resulting action on a mixed state is defined by acting on the ensemble elements:

$$\rho \;=\; \sum_i p_i|\psi_i\rangle\langle\psi_i| \quad \rightsquigarrow \quad \rho' \;=\; \sum_i p_i\,\mathcal{N}(|\psi_i\rangle\langle\psi_i|). \tag{A16}$$

Unless $\mathcal{N}$ is affine in ρ, ρ' depends on the chosen decomposition. In the present EPR scenario, the output mixture

$$\rho'_Z \;=\; \frac{1}{2}\mathcal{N}(|0\rangle\langle 0|) + \frac{1}{2}\mathcal{N}(|1\rangle\langle 1|) \tag{A17}$$

will generically differ from

$$\rho'_X \;=\; \frac{1}{2}\mathcal{N}(|+\rangle\langle +|) + \frac{1}{2}\mathcal{N}(|-\rangle\langle -|), \tag{A18}$$

because the state-dependent unitaries U_ψ differ between $|0\rangle, |1\rangle$ and $|+\rangle, |-\rangle$. Bob can therefore measure an observable O_B for which $\mathrm{Tr}(\rho'_Z O_B) \neq \mathrm{Tr}(\rho'_X O_B)$, and infer Alice's measurement choice faster than light.

This is the operational core of the GP theorem: if one treats a pure-state dependent, non-affine rule as a "local" operation, remote parties can steer ensembles and thereby signal. Polchinski refines this point by showing that to prevent such EPR signaling, observables for a separated system must depend only on the reduced density matrix [2].

Appendix D.3. Why This Does Not Target TCGS-SEQUENTION

In TCGS, Bob is never permitted to apply a map defined on fiber representatives $|\psi\rangle$ in the above sense, because those representatives are not operational inputs. Bob's operational input is the equivalence class (density matrix) $s_B \sim \rho_B$, and any admissible shadow-level map must therefore satisfy the affine constraint (A5) and, in the entangled setting, the complete-positivity constraint described in Appendix B. Source-level nonlinearities live in F_t on $\mathcal{C}$, but the induced shadow map Φ_t is defined on $\mathcal{S}$ and is required to be well-defined on equivalence classes. Consequently, the decomposition dependence exploited above cannot be operationally accessed, and the GP signaling channel is absent by construction.

Appendix E. TCGS-SEQUENTION Formalization: Quotient Maps, Source Nonlinearity, and Shadow Affinity

This appendix states the resolution in a way that makes the logical structure explicit.

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Let $\mathcal{C}$ be the counterspace (source) configuration space and let $\mathcal{S}$ be the shadow (operational) state space. The foundational step is the existence of a surjective projection (quotient) map

$$\pi : \mathcal{C} \to \mathcal{S}, \qquad s = \pi(c). \tag{A19}$$

Operationally, s is the equivalence class of source micro-configurations c that agree on all accessible shadow observables. When s is represented by a density matrix, the fiber $\pi^{-1}(s)$ contains many "micro-realizations" of the same reduced state—precisely the kind of multiplicity that GP exploits when it treats different decompositions as distinct local preparations.

Let $F_t : \mathcal{C} \to \mathcal{C}$ be a (possibly nonlinear) source evolution. The induced shadow evolution is

$$\Phi_t(s) \;:=\; \pi(F_t(c)), \qquad \text{for any } c \in \pi^{-1}(s). \tag{A20}$$

For Φ_t to be well-defined, the right-hand side must be independent of the choice of representative c in the fiber. This is the mathematical translation of "Bob only has ρ_B, not a specific ensemble."

Axiom A3 in the main text can be reformulated as:

For any operational state s and any two representatives $c_1, c_2 \in \pi^{-1}(s)$, one has $\pi(F_t(c_1)) = \pi(F_t(c_2))$ for all physically relevant times t.

Under this condition, Φ_t is a genuine map on $\mathcal{S}$, even if F_t is nonlinear on $\mathcal{C}$. Moreover, because operational mixing is available on $\mathcal{S}$ (classical randomization), Φ_t must satisfy affine linearity on convex combinations. Hence the induced shadow dynamics lies in the same structural class as CPTP evolution discussed above.

This is the intended meaning of the slogan "nonlinearity preserved at the source without operational signaling." The GP theorem presupposes that c_1 and c_2 correspond to distinct local preparations. TCGS denies precisely that presupposition: the difference is not operational; it is fiber-internal. Therefore the GP signaling lever does not exist.

Appendix F. Time as Coordinate Choice, Clock-Readings, and Foliation Invariance

The GP discussion is often accompanied by an implicit premise that "time" is a primitive external parameter relative to which state updates occur, and that locality is evaluated with respect to this parameter. TCGS-SEQUENTION deliberately distinguishes: (i) coordinate or foliation parameters used to label shadow slices (a descriptive gauge choice), and (ii) clock-readings constructed from matter degrees of freedom (an operational quantity).

This distinction is already structurally present in canonical general relativity. The ADM formalism [9] represents the evolution of spatial data under lapse and shift, emphasizing that the time parameter is not unique and is largely a gauge choice. Complementarily, the Baierlein–Sharp–Wheeler (BSW) formulation [8] shows that temporal information can be encoded in spatial geometry and its variations; "time" can be treated as derived rather than assumed.

Within the TCGS vocabulary, the foliation parameter is not ontic; it is a bookkeeping device of the shadow description. Ontic content is carried by source geometry and its projection relations. The no-signaling constraint, therefore, is a constraint on the invariances of the shadow projection, not an additional dynamical law "in time."

Appendix G. Metatheoretical Note: Semantic Truth, Models, and Empirical Anchors

Because TCGS places emphasis on projection maps, quotienting, and the relation between source and shadow descriptions, it is useful to flag a standard metatheoretical distinction: semantic truth is always truth *in a model*. Tarski's semantic conception of truth [10] makes explicit that truth predicates require a meta-language and that truth statements are evaluated relative to an interpretation.

This maps cleanly onto the present framework:

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- The *source* layer furnishes an ontic structure (what exists).
- The *shadow* layer furnishes the model in which operational statements (e.g., no-signaling constraints) are evaluated.
- The projection π is an interpretation map that determines which source distinctions survive into empirical discourse.

In this reading, the GP pathology is a category error: it treats a meta-level distinction (different representatives in a fiber) as if it were an object-level operational distinction (different locally preparable states). Once the semantic level is kept explicit, the TCGS dissolution becomes structurally transparent and does not require ad hoc restrictions: it follows from the definitional separation between $\mathcal{C}$ and $\mathcal{S}$ and the requirement that only $\mathcal{S}$ carries operational meaning.

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