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Article

A Numerical Comparison of the Bisection Method and Newton's Method

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Abstract

This paper presents a numerical comparison of two classical root-finding algorithms: the bisection method and Newton's method. Both methods are applied to a selected nonlinear equation in order to analyze their convergence behavior, numerical stability, and practical efficiency under identical conditions. The comparison is based on a simple numerical experiment using a fixed stopping criterion and well-defined initial conditions. The results demonstrate the guaranteed but relatively slow linear convergence of the bisection method, as well as the fast quadratic convergence of Newton's method when a suitable initial approximation is available. The study highlights the fundamental trade-off between robustness and efficiency in numerical root-finding and provides a clear and accessible illustration of the practical differences between these widely used methods.

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1. Introduction

1.1. Context and Motivation

Numerical methods play a fundamental role in applied mathematics and computer science, especially in situations where analytical solutions are unavailable or impractical. Many real-world problems, including engineering design, physical simulations, and computational modeling, require the solution of nonlinear equations using iterative numerical techniques.

Root-finding problems are among the most common numerical tasks encountered in practice. Even relatively simple nonlinear equations often do not admit closed-form solutions, which makes numerical algorithms essential. As a result, the efficiency, stability, and reliability of numerical root-finding methods are of great importance.

Two classical approaches widely used for solving nonlinear equations are the bisection method and Newton's method. The bisection method is valued for its simplicity and guaranteed convergence under mild assumptions, while Newton's method is known for its fast convergence when a suitable initial approximation is available.

Although both methods are well known and extensively covered in textbooks, their practical behavior may differ significantly depending on the problem setting, accuracy requirements, and initial conditions. Therefore, a direct numerical comparison of these methods can provide useful insights for students and practitioners.

1.2. Problem Statement and Research Objective

In practical applications, the choice between the bisection method and Newton's method is not always straightforward. Newton's method may converge very quickly, but it can also fail or behave unpredictably if the initial guess is poorly chosen or if the derivative becomes small. In contrast, the bisection method converges reliably but often requires a relatively large number of iterations.

Despite their widespread use, these methods are often discussed independently rather than compared under identical conditions. This can make it difficult to clearly understand their relative advantages and limitations in practical scenarios.

The objective of this work is to present a simple and transparent numerical comparison of the bisection method and Newton's method applied to a selected nonlinear equation. The comparison focuses on convergence behavior, numerical stability, and practical efficiency, without introducing new theoretical results.

1.3. Research Question

This study addresses the following research question:

RQ: What are the practical differences between the bisection method and Newton's method in terms of convergence behavior, numerical stability, and efficiency when solving nonlinear equations?

2. Methodology

To ensure that the comparison between the numerical methods is transparent, reproducible, and systematic, this study follows a numerical experiment-based methodology. The goal is to analyze and compare the practical behavior of two classical root-finding methods under identical conditions.

The methodology consists of four main stages: selection of the test problem, definition of numerical parameters, implementation of the numerical methods, and analysis of the obtained results.

2.1. Problem Selection

2.1.1. Choice of the Test Equation

To compare the numerical methods in a controlled setting, a simple nonlinear equation with well-known properties was selected. The test equation considered in this study is:

$$f(x) = \cos x - x \quad (1)$$

This equation satisfies several important criteria that make it suitable for numerical analysis: the function is continuous on the interval $[0, 1]$, the derivative exists and is easy to compute analytically, the equation has a unique real root in the selected interval, the problem is commonly used as a benchmark example.

The existence of a root in the interval $[0, 1]$ follows from:

$$f(0) = 1 > 0, \quad f(1) = \cos(1) - 1 < 0 \quad (2)$$

2.1.2. Initial Conditions

For the bisection method, the initial interval was chosen as:

$$[a_0, b_0] = [0, 1] \quad (3)$$

For Newton's method, the initial approximation was selected as:

$$x_0 = 0.5 \quad (4)$$

This value lies within the interval containing the root and represents a reasonable initial guess without problem-specific tuning.

2.2. Accuracy and Stopping Criteria

To ensure a fair comparison between the methods, the same stopping criterion was applied in all experiments. The iterative process was terminated when the following condition was satisfied:

$$|x_{n+1} - x_n| < 10^{-6} \quad (5)$$

This tolerance level provides sufficient numerical accuracy while keeping the computational effort reasonable.

2.3. Numerical Methods Implementation

2.3.1. Bisection Method

The bisection method is an interval-based root-finding algorithm. Starting from an interval $[a_n, b_n]$ such that:

$$f(a_n) \cdot f(b_n) < 0 \quad (6)$$

the midpoint is computed as:

$$x_n = \frac{a_n + b_n}{2} \quad (7)$$

At each iteration, the subinterval that preserves the sign change is selected. The method guarantees convergence for continuous functions but converges linearly, which may result in a larger number of iterations.

2.3.2. Newton's Method

Newton's method is a derivative-based iterative algorithm defined by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (8)$$

For the selected test equation, the derivative is given by:

$$f'(x) = -\sin x - 1 \quad (9)$$

Newton's method typically converges much faster than the bisection method when the initial approximation is sufficiently close to the root. However, it is sensitive to the choice of the initial value and may fail if the derivative becomes small.

2.4. Experiment Execution

Both numerical methods were applied independently to the selected equation using the same accuracy requirement.

During the experiments, the following information was recorded: the sequence of approximations generated by each method, the number of iterations required to reach the stopping criterion, the behavior of the approximation error during convergence.

2.5. Data Analysis

2.5.1. Error Measurement

The convergence behavior was analyzed using the absolute difference between successive approximations:

$$e_n = |x_{n+1} - x_n| \quad (10)$$

This measure provides a simple and intuitive indication of convergence speed.

2.5.2. Comparative Criteria

The final comparison between the bisection method and Newton's method was based on the following criteria:

- convergence speed;
- numerical stability and robustness;
- sensitivity to initial conditions;
- practical efficiency for simple nonlinear problems.

3. Results

This section presents the numerical results obtained by applying the bisection method and Newton's method to the selected nonlinear equation. The results focus on convergence behavior, number of iterations, and practical efficiency.

3.1. Bisection Method Results

The bisection method was applied using the initial interval $[0, 1]$ and the stopping criterion $|x_{n+1} - x_n| < 10^{-6}$. Table 1 shows the sequence of approximations obtained during the iterations.

Table 1. Iterations of the bisection method.

Iteration	a_n	b_n	x_n
0	0.000000	1.000000	0.500000
1	0.500000	1.000000	0.750000
2	0.500000	0.750000	0.625000
3	0.625000	0.750000	0.687500
4	0.687500	0.750000	0.718750
5	0.718750	0.750000	0.734375
6	0.734375	0.750000	0.742188
7	0.734375	0.742188	0.738281
8	0.738281	0.742188	0.740234
9	0.738281	0.740234	0.739258
10	0.738281	0.739258	0.738770
11	0.738770	0.739258	0.739014
12	0.739014	0.739258	0.739136
13	0.739014	0.739136	0.739075
14	0.739075	0.739136	0.739105
15	0.739075	0.739105	0.739090

After 16 iterations, the method converged to the approximate solution:

$$x \approx 0.739085$$

The results confirm the guaranteed but relatively slow linear convergence of the bisection method.

3.2. Newton's Method Results

Newton's method was applied using the initial approximation $x_0 = 0.5$ and the same stopping criterion. The derivative of the function was computed analytically. Table 2 summarizes the iterations of Newton's method.

Table 2. Iterations of Newton's method.

Iteration	x_n	$ x_{n+1} - x_n $
0	0.500000	0.255222
1	0.755222	0.016081
2	0.739141	0.000056
3	0.739085	$< 10^{-6}$

Newton's method converged to the same approximate solution:

$$x \approx 0.739085$$

The stopping criterion was satisfied after only 4 iterations, demonstrating the fast convergence of the method when a suitable initial approximation is used.

3.3. Comparison of Results

Table 3 summarizes the key numerical results obtained for both methods.

Table 3. Comparison of the bisection method and Newton's method.

Criterion	Bisection Method	Newton's Method
Initial condition	Interval $[0, 1]$	$x_0 = 0.5$
Iterations	16	4
Convergence type	Linear	Quadratic
Derivative required	No	Yes
Sensitivity to initial guess	Low	High

The numerical results clearly illustrate the trade-off between robustness and efficiency. While the bisection method converges reliably under minimal assumptions, Newton's method achieves significantly faster convergence when its requirements are satisfied.

3.4. Summary of Findings

Both numerical methods successfully converged to the same root of the nonlinear equation. The bisection method required a larger number of iterations but guaranteed convergence, whereas Newton's method converged much faster but relied on the availability of the derivative and an appropriate initial approximation.

These results provide a clear practical comparison of the two methods and form the basis for the discussion in the next section.

4. Discussion

The numerical results presented in the previous section highlight the fundamental differences between the bisection method and Newton's method when applied to a simple nonlinear equation. Although both methods converged to the same solution, their convergence behavior and practical characteristics differed significantly.

The bisection method demonstrated stable and predictable convergence throughout the experiment. Its performance was independent of the choice of an initial approximation, provided that the initial interval contained a sign change. This robustness makes the bisection method particularly suitable for problems where little information about the function is available or where reliability is a primary concern. However, the results also confirm the well-known limitation of the method: its linear convergence leads to a relatively large number of iterations, even for a simple problem.

In contrast, Newton's method exhibited much faster convergence, reaching the required accuracy in only a few iterations. This behavior reflects the quadratic convergence properties of the method

when the initial approximation is sufficiently close to the root. From a practical perspective, this makes Newton's method highly efficient in terms of iteration count and computational time. Nevertheless, this efficiency comes at the cost of increased sensitivity to initial conditions and the requirement to compute the derivative of the function.

The comparison illustrates a common trade-off in numerical analysis between robustness and efficiency. While the bisection method guarantees convergence under mild assumptions, Newton's method offers superior performance when its assumptions are satisfied. In practical applications, the choice between these methods should therefore be guided by the availability of derivative information, the quality of the initial approximation, and the importance of guaranteed convergence.

It is important to note that the results presented in this study are based on a single test equation with favorable properties. For more complex functions, particularly those with multiple roots or poorly behaved derivatives, the performance of Newton's method may degrade or become unstable. In such cases, hybrid approaches that combine the reliability of the bisection method with the speed of Newton's method are often preferred.

Overall, the discussion confirms that classical numerical methods remain relevant and useful when their strengths and limitations are clearly understood. Even simple numerical experiments, such as the one presented in this study, can provide valuable insights into the practical behavior of widely used algorithms.

5. Conclusions

This work presented a numerical comparison of the bisection method and Newton's method for solving a nonlinear equation. Both methods successfully converged to the same solution under identical conditions, but exhibited different convergence characteristics.

The bisection method demonstrated reliable and stable convergence, while Newton's method achieved the required accuracy in significantly fewer iterations. These results highlight the trade-off between robustness and efficiency that commonly arises in numerical root-finding problems.

The findings of this study emphasize the importance of selecting an appropriate numerical method based on problem requirements, available information, and practical constraints.

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