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SORT-QS: A Projection-Based Structural Framework for Quantum Systems *Error Correction, Noise Filtering, and Operator Diagnostics*

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Abstract

This work introduces the Supra-Omega Resonance Framework for Quantum Systems (SORT-QS), a structural operator formalism that adapts the Supra-Omega Resonance Theory from cosmological applications to finite-dimensional quantum devices. The central idea is to represent coherent and incoherent error processes, noise filtering mechanisms and diagnostic procedures in terms of a finite set of idempotent resonance operators $\{\hat{O}_i\}$, an effective projector $\hat{H}$ and a nonlocal kernel acting on the operator space rather than on configuration space. In SORT-QS, quantum channels are mapped to structured resonance manifolds in Liouville space, and error sectors are encoded as mutually constrained projectors that satisfy algebraic closure and idempotency. This enables a scale- and mode-selective description of noise, where the analogue of the projection kernel κ defines structural suppression or amplification of specific error components in an abstract frequency or syndrome domain. The framework provides three complementary layers: (i) a purely algebraic resonance space for error classes and stabilizer-like structures, (ii) a kernel-based noise filtering module formulated as a linear transformation on operator-valued modes and (iii) an operator diagnostics layer that quantifies deviations from ideal projector structure as resonance defects. No device-specific assumptions or empirical performance claims are made. Instead, SORT-QS offers a mathematically controlled template that can be instantiated within arbitrary quantum error correction schemes, gate sets and noise models, and serves as a basis for future applications to concrete architectures.

Keywords: operator algebra; quantum error correction; resonance manifolds; nonlocal kernels; noise filtering; quantum channels; CPTP maps; operator diagnostics

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1. Meta: Position of the SORT-QS System Within SORT v6

The Supra-Omega Resonance Theory in its v6 architecture organises structurally related scientific domains into a unified operator framework built on 22 idempotent resonance operators $\{\hat{O}_i\}$, a global projector $\hat{H}$ and a nonlocal projection kernel κ [78]. These components provide an operator-algebraic backbone shared across the cosmology, AI-safety and complex-systems modules [79]. The SORT-QS system extends this structure to quantum information settings, enabling the representation of quantum channels, error mechanisms and diagnostic procedures within the resonance formalism without modifying the physical postulates of quantum mechanics [80–82].

To indicate its structural role within the full theory, the conceptual module decomposition of SORT v6 is expressed as

$$\mathcal{M}_{\mathrm{SORT\,v6}} = \mathcal{M}_{\mathrm{cosmo}} \oplus \mathcal{M}_{\mathrm{AI}} \oplus \mathcal{M}_{\mathrm{cx}} \oplus \mathcal{M}_{\mathrm{QS}}, \tag{1}$$

where $\mathcal{M}_{\mathrm{QS}}$ denotes the SORT-QS system. Equation (1) is a structural identity, not a decomposition of a Hilbert space. Its purpose is to capture how different scientific domains are represented through a common algebra of resonance operators and projection mechanisms.

The SORT-QS system is introduced in v6 because the developments in SORT v5 produced, for the first time, a mathematically hardened operator registry with verified Jacobi consistency, a globally defined projector $\hat{H}$ and a projection kernel $\kappa(k)$ calibrated across several structural tests [78]. These ingredients proved sufficiently robust to support applications beyond the original cosmological context, motivating their extension to quantum information theory. The resonance-operator framework aligns naturally with areas of quantum theory where operator relations, commutators and projector constraints play a fundamental role, including stabilizer codes [8], fault-tolerant computation [16,19], quantum channels [21,22,24] and dynamical semigroup representations [29].

A central motivation for integrating quantum systems into SORT v6 is that quantum error correction, noise mitigation and verification are inherently structural disciplines. Stabilizer codes are defined by families of commuting projectors [8], quantum noise channels by Kraus operator decompositions and Choi matrices [22], and verification protocols by invariants of superoperators [35, 39]. These structures map naturally into the resonance-operator algebra of SORT, in which idempotency, commutator closure and drift diagnostics already serve to quantify structural coherence and defects [78].

Within this perspective, the global projector $\hat{H}$ acquires an interpretation as a structural-consistency filter that, in the quantum setting, can be applied to logical subspaces or admissible operator sets. The kernel $\kappa(k)$ offers a systematic way to introduce scale- or mode-selective weighting on operator-valued modes, providing a mathematical analogue of spectral noise filtering without relying on hardware-specific pulse sequences [50,53]. These tools allow SORT-QS to represent error processes, stabilizer constraints and diagnostic metrics in a unified resonance space while respecting CPTP structures when required [21,24,25].

Finally, the inclusion of SORT-QS strengthens cross-domain conceptual links established in v6. The drift measures originally developed in the AI-safety module [79] and the network-operator structures in the complex-systems module both reflect underlying questions about operator stability, propagation and constraint satisfaction. Quantum computation provides a highly resolved and experimentally accessible domain in which such structural questions can be tested, simulated and compared against established theoretical frameworks [33,57]. For this reason, SORT-QS is positioned not as an isolated extension, but as a structurally coherent component within the broader SORT v6 architecture encoded in Equation (1).

2. Introduction

2.1. Motivation

Quantum information science is intrinsically an operator-centric discipline: quantum states are represented by vectors or density operators, evolution by unitary transformations or completely positive trace-preserving (CPTP) maps [24,25], and computational primitives by sequences of noncommuting operators acting on finite-dimensional Hilbert spaces. Error mechanisms, stabilizer constraints and noise models are likewise expressed in terms of projectors, commutators and superoperators [8,21,22]. This makes quantum information an ideal domain for applying the resonance-operator formalism developed within the Supra-Omega Resonance Theory (SORT).

The SORT-QS system builds on the 22 idempotent resonance operators $\{\hat{O}_i\}$, the global consistency projector $\hat{H}$ and the projection kernel $\kappa(k)$ developed and mathematically formalised in SORT v5 [78]. These objects encode structural relations among operators, quantify deviations from ideal algebraic behaviour through drift measures, and implement mode-dependent filtering via a nonlocal kernel. The alignment between these structural elements and the requirements of quantum channels, stabilizer codes and verification protocols motivates the development of a quantum-oriented extension.

A defining feature of the SORT-QS approach is that quantum mechanics is not modified at the physical level. Instead, SORT-QS introduces a structural overlay on standard quantum theory. The full physical evolution remains governed by unitary operators or Lindblad generators [29,30], while SORT supplies a complementary layer for analysing algebraic coherence, projective consistency and resonance defects. This dual-layer formulation makes the framework compatible with all hardware platforms and theoretical models considered in the quantum information literature [19,56,57].

2.2. Interface Between Resonance Operators and Quantum Operators

The resonance-operator algebra of SORT establishes a natural bridge to the operator structures used in quantum computation. At its core is the idempotency property,

$$\hat{O}_i^2 = \hat{O}_i, \tag{2}$$

which mirrors the defining condition for projectors in quantum mechanics [8]. In stabilizer codes, families of commuting projectors define logical subspaces and error syndromes [2]. SORT-QS reinterprets Equation (2) as a structural constraint applicable to syndrome projectors, logical projectors or error-model representations.

Noncommutativity plays a crucial role in both frameworks. In SORT, structural deviations from commutator closure are quantified through resonance drift measures [78]. In quantum theory, noncommutativity underlies uncertainty, entanglement, and stabilizer incompatibility. The operator interface therefore assigns:

$$\text{SORT commutators} \quad \longleftrightarrow \quad \text{Pauli or Clifford commutation relations.}$$

Likewise, the global projector $\hat{H}$ is mapped onto logical-subspace constraints:

$$\hat{H}\hat{\rho}\hat{H} = \hat{\rho} \quad \Longrightarrow \quad \hat{\rho} \text{ remains in the resonance-consistent subspace,} \tag{3}$$

parallel to the stabilizer condition $S|\psi\rangle = |\psi\rangle$ [8,10].

Quantum channels, described through Kraus decompositions or Choi matrices [21,22], align with the kernel-weighted SORT representation. A CPTP map $\mathcal{E}$ acting on density operators ρ satisfies

$$\mathcal{E}(\rho) = \sum_a E_a \rho E_a^\dagger, \tag{4}$$

with $\sum_a E_a^\dagger E_a = \mathbb{I}$. SORT-QS embeds the operators $\{E_a\}$ into the resonance operator registry, enabling kernel-based filtering of channel components and structural diagnostics that quantify CPTP-consistency in an operator-algebraic manner.

2.3. Objectives of the SORT-QS System

The SORT-QS system pursues three complementary objectives:

1. **Projector-based quantum error correction:** Provide a structural formalism for constructing and evaluating stabilizer-like error syndromes using resonance operators, enabling algebraic diagnostics such as Jacobi consistency, idempotency tests and drift analysis. This objective directly connects to foundational literature on QEC [1,2,4].
2. **Kernel-based noise filtering:** Adapt the SORT projection kernel $\kappa(k)$ to frequency-like or mode-like decompositions of quantum channels, creating a universal noise-suppression mechanism independent of hardware-specific pulse design. This offers a structural alternative to filter-function approaches in dynamical decoupling [50,52,53].
3. **Operator-chain diagnostics:** Quantify the structural coherence of gate sequences in terms of noncommutativity, resonance drift, Jacobi defects and idempotency violations. This objective complements benchmarking and verification protocols [35,37,39].

Together, these components form a modular analytical framework that augments but never replaces standard quantum mechanical evolution.

2.4. Separation from Other SORT v6 Modules

SORT v6 comprises four structurally unified but conceptually distinct domains:

$$\{\mathcal{M}_{\text{cosmo}}, \mathcal{M}_{\text{AI}}, \mathcal{M}_{\text{cx}}, \mathcal{M}_{\text{QS}}\}.$$

The SORT-QS system differentiates itself through its application domain—finite-dimensional quantum devices—while sharing the same underlying resonance-operator infrastructure. Its unique characteristics include:

- explicit CPTP constraints (Equation (4)),
- stabilizer-like projectors (Equation (2)),
- operator-chain analyses analogous to drift diagnostics in AI-safety applications [79].

Whereas cosmological and complex-systems modules operate in continuous or networked spaces, SORT-QS operates in Liouville space and shares its mathematical foundations with quantum information theory [27,34].

2.5. Reference to Previous Work in SORT v5

SORT v5 established the resonance-operator algebra, the global projector $\hat{H}$, and the projection kernel $\kappa(k)$ [78]. These elements were developed and validated in cosmological applications, demonstrating consistency, idempotency, Jacobi closure and kernel-based structural filtering.

SORT-QS inherits the following v5 structures:

1. **Operator registry:** The 22 idempotent resonance operators $\hat{O}_i$ that form the foundational algebra.
2. **Global projector $\hat{H}$:** Used in SORT-QS for logical filtering and structural consistency (Equation (3)).
3. **Projection kernel $\kappa(k)$:** Adapted for noise filtering and operator-mode suppression.
4. **Jacobi and drift diagnostics:** Directly reused for gate-set, channel and stabilizer-sequence analysis.

These components provide the mathematical scaffolding required to extend SORT into the quantum-information domain in a manner consistent with the operator-theoretic literature [44,45,47].

3. Mathematical Foundations

3.1. Resonance Operators as an Operator Algebra

The structural backbone of the SORT-QS system is the resonance-operator algebra introduced in SORT v5 [78]. It consists of 22 idempotent operators $\{\hat{O}_i\}_{i=1}^{22}$ that satisfy a closed commutator structure and a Jacobi-consistent algebra. The defining relation is the idempotency condition

$$\hat{O}_i^2 = \hat{O}_i, \tag{5}$$

which parallels the role of projectors in quantum mechanics and stabilizer theory [2,8]. The resonance operators span a finite operator space $\mathcal{R}$, and their commutator satisfies

$$[\hat{O}_i, \hat{O}_j] = \sum_{k=1}^{22} f_{ij}^{k} \hat{O}_k, \tag{6}$$

with structure coefficients f_{ij}^{k} determined in SORT v5. Jacobi consistency requires

$$[\hat{O}_i, [\hat{O}_j, \hat{O}_k]] + [\hat{O}_j, [\hat{O}_k, \hat{O}_i]] + [\hat{O}_k, [\hat{O}_i, \hat{O}_j]] = 0, \tag{7}$$

mirroring the algebraic consistency conditions of Lie algebras [46,47]. In the quantum context, Equation (6) provides a structural analogue to Pauli and Clifford commutation rules [9], while Equation (7)

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offers a diagnostic for identifying unstable or poorly conditioned operator sequences, analogous to chain-verification techniques [39].

3.2. The Global Projector $\hat{H}$ as Consistency Filter

The global projector $\hat{H}$ introduced in SORT v5 plays a central role in ensuring structural coherence across modules [78]. It is defined as a weighted combination of resonance operators,

$$\hat{H} = \sum_{i=1}^{22} w_i \hat{O}_i, \tag{8}$$

with the Light-Balance condition

$$\sum_{i=1}^{22} w_i = 0 \tag{9}$$

ensuring global neutrality of the resonance structure. In the quantum setting, $\hat{H}$ functions as a structural-consistency projector acting on operators or states. For a density operator ρ, the condition

$$\hat{H}\rho\hat{H} = \rho \tag{10}$$

indicates that ρ lies within the resonance-consistent logical manifold, structurally analogous to stabilizer constraints $S|\psi\rangle = |\psi\rangle$ [8,10]. Violations of Equation (10) correspond to resonance drift, a concept that extends SORT's core consistency diagnostics into the quantum-information domain.

3.3. The Projection Kernel $\kappa(k)$

The SORT projection kernel $\kappa(k)$ is a nonlocal weighting function originally introduced to regulate scale-dependent interactions in cosmological applications [78]. Its standard form is

$$\kappa(k) = \exp\Big[-\tfrac{1}{2}(\sigma_0 L_H k)^2\Big], \tag{11}$$

where σ_0 is the calibrated structural scale and L_H the characteristic length associated with the global projector $\hat{H}$. Kernel application defines a smoothing or filtering operation on an operator-valued mode function $X(k)$,

$$\pi_\kappa[X](k) = \kappa(k)X(k), \tag{12}$$

which acts as a structural analogue to spectral filtering in quantum control [49,50,52]. When imported into the quantum-information domain, Equation (12) provides a formal mechanism for implementing k-selective noise suppression independently of hardware-level pulse engineering [53].

3.4. Comparison Between the SORT Resonance Space and Quantum Hilbert Space

The resonance space $\mathcal{R}$ defined by SORT is an algebraic operator space, not a Hilbert space. Its elements are operators obeying Equations (5)–(7). In contrast, the Hilbert-space formalism of quantum theory [80,81] is built from vectors $|\psi\rangle$, density operators ρ and linear transformations acting on these.

The natural correspondence between the two spaces is established through the assignment:

$$\mathcal{R} \longleftrightarrow \mathcal{B}(\mathcal{H}),$$

where $\mathcal{B}(\mathcal{H})$ denotes the bounded operators on a Hilbert space. The analogy is structural rather than physical:

- Idempotent resonance operators ↔ projectors in Hilbert space.
- Resonance commutators (Equation (6)) ↔ operator noncommutativity governing uncertainty and entanglement.
- Global projector $\hat{H}$ ↔ logical-subspace constraint.
- Kernel smoothing (Equation (12)) ↔ spectral filtering in the control-theory sense.

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SORT does not replace or modify Hilbert-space dynamics; instead, it overlays additional structural diagnostics onto the quantum operator space $\mathcal{B}(\mathcal{H})$, much like verification and benchmarking frameworks [35,38].

3.5. CPTP Analogies and Constraints

Quantum channels are described by completely positive trace-preserving maps $\mathcal{E}$ with Kraus representation [21,24]

$$\mathcal{E}(\rho) = \sum_a E_a \rho E_a^\dagger, \qquad \sum_a E_a^\dagger E_a = \mathbb{I}. \tag{13}$$

The SORT-QS system does not redefine the CPTP framework; instead, it embeds channel operators $\{E_a\}$ into the resonance-operator registry. This allows structural diagnostics such as:

- **Idempotency tests:** quantify deviations of E_a from Equation (5).
- **Jacobi residuals:**

  $$\Delta_J(E_a, E_b, E_c) = \big\| [E_a, [E_b, E_c]] + \text{cycl.} \big\|_F,$$

  mirroring Equation (7).
- **Kernel-weighted channel analysis:** using Equation (12) to isolate mode-dependent behaviours of CPTP maps.

These structural metrics complement, but never replace, channel measures such as diamond distance, entanglement fidelity or process matrices [40–42].

3.6. Limits of the Mathematical Mapping

The mapping between the SORT resonance space and quantum information theory is intentionally structural, not ontological. Its limitations include:

1. **No physical time evolution:** SORT operators do not generate dynamics; quantum evolution remains governed by unitary and Lindbladian generators [29,30].
2. **Idempotent constraint:** Many quantum operators (Paulis, unitaries) do not satisfy idempotency and therefore cannot be directly identified with resonance operators.
3. **No direct physical interpretation of $\kappa(k)$:** Although Equation (11) resembles a spectral filter, it lacks a hardware-level counterpart unless explicitly embedded into a control protocol.
4. **Hilbert-space geometry not preserved:** SORT resonance space is not a vector space with inner product; therefore concepts like orthogonality, fidelity or entanglement require translation into resonance metrics.
5. **CPTP compatibility constraints:** Embedding $\{E_a\}$ into the operator registry does not guarantee that their resonance combinations remain CPTP unless explicitly checked (Equation (13)).

These limitations delineate the boundary between the physical layer of quantum mechanics and the structural overlay provided by the SORT-QS system. They ensure conceptual coherence while allowing the resonance framework to support error correction, noise filtering and operator diagnostics in subsequent sections.

4. Use Case I: Projector-Based Quantum Error Correction

4.1. Motivation

Quantum error correction (QEC) relies on structured projectors, commuting operator sets and syndrome extraction rules [1,2,8]. These structures mirror the algebraic constraints built into the resonance-operator framework of SORT, where idempotency, commutator closure and Jacobi consistency define the stability of operator families [78]. The SORT-QS system leverages this alignment by treating resonance operators as candidates for abstract syndrome projectors and using resonance metrics to diagnose structural coherence in stabilizer-like constructions.

The central motivation is that QEC, unlike general quantum algorithms, is inherently *projector-centric*. The mapping between resonance operators $\hat{O}_i$ and stabilizer generators is therefore structurally natural: idempotency (Equation (5)) mirrors the stabilizer constraint $S^2 = S$, commutator relations (Equation (6)) match the compatibility conditions of stabilizer groups, and Jacobi consistency (Equation (7)) offers a diagnostic for identifying algebraically unstable QEC constructions. This provides a structural complement to established QEC formalisms [33,34] and supports abstract validation of code components independently of qubit-level implementations.

4.2. SORT Projectors as Abstract Syndromes

In resonance algebra, idempotent operators satisfying Equation (5) act as algebraic projectors. A subset of resonance operators may therefore serve as *abstract syndrome projectors*, with the interpretation

$$\hat{O}_i \rho \hat{O}_i = \rho \quad \Longrightarrow \quad \rho \text{ lies in the syndrome subspace associated with } \hat{O}_i. \tag{14}$$

In analogy to stabilizer theory [8,10], families of mutually commuting resonance operators define logical subspaces.

Given a set $\mathcal{S}_{\text{SORT}} = \{\hat{O}_{i_1}, \ldots, \hat{O}_{i_r}\}$, the corresponding abstract code space is defined by

$$\mathcal{H}_{\text{code}} = \{\rho \mid \hat{O}_{i_a} \rho = \rho \text{ for all } \hat{O}_{i_a} \in \mathcal{S}_{\text{SORT}}\}. \tag{15}$$

Equation (15) parallels the stabilizer code definition [2] but replaces Pauli-generated stabilizers with resonance operators. Logical operators may then be represented by resonance-preserving transformations L satisfying

$$[L, \hat{O}_{i_a}] = 0 \quad \forall a, \tag{16}$$

ensuring that L preserves the resonance-defined code structure.

The global projector $\hat{H}$ from Equation (8) provides an additional consistency filter, enabling resonance-level verification:

$$\hat{H} \rho \hat{H} = \rho \quad \Rightarrow \quad \rho \in \mathcal{H}^{(\text{res})}_{\text{code}}. \tag{17}$$

This filter acts analogously to a stabilizer group's joint $+1$-eigenspace [8].

4.3. Explicit Mapping Scheme Between SORT Operators and Stabilizer Structures

As emphasized in the structural review, resonance operators cannot always be mapped directly to Pauli operators due to the idempotency constraint [78]. Instead, SORT-QS defines an embedding homomorphism

$$\phi : \{\hat{O}_i\}_{i=1}^{22} \longrightarrow \mathcal{S}_n, \tag{18}$$

where $\mathcal{S}_n$ denotes an n-qubit stabilizer group or its projector representation.

The mapping must satisfy three algebraic constraints:

1. **Idempotency preservation:**
$$\phi(\hat{O}_i)^2 = \phi(\hat{O}_i), \tag{19}$$
implying that $\phi(\hat{O}_i)$ must be a projector, not a Pauli string.
2. **Commutator compatibility:**
$$[\phi(\hat{O}_i), \phi(\hat{O}_j)] \sim \phi([\hat{O}_i, \hat{O}_j]), \tag{20}$$
ensuring structural alignment with Equation (6).
3. **Jacobi preservation:**
$$\phi([\hat{O}_i, [\hat{O}_j, \hat{O}_k]] + \text{cycl.}) = 0, \tag{21}$$
mirroring Equation (7).

A minimal qubit requirement follows from information-theoretic counting:

$$n \geq \lceil \log_2(22) \rceil = 5. \tag{22}$$

This lower bound matches well-known five-qubit QEC structures [4,5]. A representative mapping assigns the first two resonance projectors to the two canonical stabilizer projectors:

$$\phi(\hat{O}_1) = \tfrac{1}{2}(\mathbb{I}^{\otimes 5} + Z_1Z_2Z_3Z_4Z_5), \tag{23}$$
$$\phi(\hat{O}_2) = \tfrac{1}{2}(\mathbb{I}^{\otimes 5} + X_1X_2X_3X_4X_5), \tag{24}$$

with additional assignments chosen to satisfy Equations (19)–(21).

4.4. Operator Sequences and Jacobi Consistency

Operator sequences used in stabilizer measurements, syndrome extraction, or logical-gate construction can be validated structurally using Jacobi diagnostics. For a sequence $(U_1, \ldots, U_m)$, the *Jacobi residual* is defined as

$$J_{\max} = \max_{i,j,k} \left\| [U_i, [U_j, U_k]] + \text{cycl.} \right\|_F, \tag{25}$$

a direct analogue of Equation (7), with $\| \cdot \|_F$ the Frobenius norm. Small $J_{\max}$ values indicate stabilizer compatibility, while large values reveal structural incoherence or fragility under syndrome extraction. This resonates with recent circuit-verification insights [39,77].

The global projector $\hat{H}$ (Equation (8)) can be used to define an *effective stabilizer* for a sequence:

$$\hat{H}_{\text{eff}} = U_m \cdots U_2 U_1 \hat{H} U_1^\dagger U_2^\dagger \cdots U_m^\dagger, \tag{26}$$

with an idempotency test

$$\Delta_{\text{idemp}} = \| \hat{H}_{\text{eff}}^2 - \hat{H}_{\text{eff}} \|_F, \tag{27}$$

quantifying structural deviation of the induced stabilizer.

4.5. Minimal 5-Qubit SORT-QEC Example

A minimal example can be constructed by selecting three commuting resonance operators $\{\hat{O}_1, \hat{O}_2, \hat{O}_3\}$ and mapping them via Equation (18) into projector-stabilizers acting on five qubits. Using Equations (23)–(24) and an analogous assignment for $\hat{O}_3$, the abstract stabilizer group becomes

$$\mathcal{S}_{\text{SORT}} = \{\phi(\hat{O}_1), \phi(\hat{O}_2), \phi(\hat{O}_3)\}. \tag{28}$$

The code space is nonempty due to Equations (19) and (15). For illustrative purposes, the syndrome table for a representative error X_1 takes the form

$$s(X_1) = \big(\text{Tr}[X_1\phi(\hat{O}_1)], \text{Tr}[X_1\phi(\hat{O}_2)], \text{Tr}[X_1\phi(\hat{O}_3)] \big), \tag{29}$$

mirroring the structure of canonical QEC syndrome vectors [24,33].

Structural diagnostics include:

$$J_{\max} \text{ from Equation (25)}, \tag{30}$$
$$\Delta_{\text{idemp}} \text{ from Equation (27)}, \tag{31}$$
$$\epsilon_{\text{LB}} = \left| \sum_i w_i^{(\text{eff})} \right|, \tag{32}$$

with light-balance coefficients $w_i^{(\text{eff})}$ inherited from the sequence-defined projector $\hat{H}_{\text{eff}}$.

These quantities provide a purely algebraic evaluation of the QEC structure, independent of any physical implementation or noise model.

4.6. Limitations and Possible Extensions

The SORT-QS approach to QEC is subject to several structural limitations:

1. **Idempotency constraint:** Pauli operators are not idempotent; therefore stabilizer generators must be represented as projectors, not Pauli strings. This requires projector embeddings such as Equations (23)–(24).
2. **Nonphysical mapping:** The homomorphism ϕ of Equation (18) is structural rather than physical. It does not imply a unique or optimal stabilizer code.
3. **Jacobi metrics do not capture logical error rates:** Although Equations (25) and (27) diagnose instability, they do not measure logical error rates or thresholds [13,19].
4. **Extensions:**
   - Larger logical codes via higher-dimensional embeddings.
   - Resonance-based construction of LDPC-like projectors [61,62].
   - Incorporation of kernel-filtered stabilizers using Equation (12).
   - Operator-chain diagnostics for fault-tolerant logical gates.

These limitations clarify the role of SORT-QS: it provides a structural diagnostic framework for QEC, not a replacement for established stabilizer-code theory. Its value lies in its algebraic consistency, projector-based foundation and extensibility across different hardware platforms.

5. Use Case II: Kernel-Based Noise Filtering

5.1. Background on Quantum Noise Channels and Spectral Filtering

Quantum noise processes are commonly described by completely positive trace-preserving (CPTP) maps $\mathcal{E}$ acting on density operators ρ, with Kraus representation [21,24]

$$\mathcal{E}(\rho) = \sum_a E_a \rho E_a^\dagger, \qquad \sum_a E_a^\dagger E_a = \mathbb{I}. \tag{33}$$

Typical instances include depolarizing noise, phase damping, amplitude damping and correlated multi-qubit noise [32,34]. In physical systems, such processes often admit a spectral interpretation via environmental coupling kernels or filter functions [50,52], where decoherence rates depend on the overlap between noise spectra and control-induced filter profiles.

The SORT-QS system introduces an alternative, operator-theoretic representation of noise filtering based on the projection kernel $\kappa(k)$ from Equation (11). Instead of modifying pulse sequences or time-dependent Hamiltonians, SORT applies a resonance-structured filter directly to operator-valued mode functions. This approach complements spectral filtering in dynamical decoupling [49,53] by shifting from hardware-level time-domain control to algebraic mode suppression in operator space.

5.2. The Kernel $\kappa(k)$ as a Frequency-Space Filter

The projection kernel $\kappa(k)$,

$$\kappa(k) = \exp\Big[-\tfrac{1}{2}(\sigma_0 L_H k)^2\Big], \tag{34}$$

acts as a Gaussian low-pass filter on an abstract mode variable k. For an operator-valued spectrum $X(k)$, the filtered operator is obtained via

$$\pi_\kappa[X](k) = \kappa(k)\, X(k), \tag{35}$$

identical to Equation (12) but instantiated for noise suppression.

For a CPTP map $\mathcal{E}$ with Kraus operators decomposed into mode components $E_a(k)$, SORT-QS defines the kernel-filtered channel

$$\mathcal{E}_\kappa(\rho) = \sum_a \left(\int \mathrm{d}k\, \kappa(k) E_a(k) \right) \rho \left(\int \mathrm{d}k\, \kappa(k) E_a(k) \right)^\dagger. \tag{36}$$

The structural role of Equation (36) is similar to modifying a control filter function in spectral-decoupling theory [52], but without imposing temporal constraints. Instead, the kernel acts on the abstract resonance-space representation of noise.

The parameter σ_0, calibrated in SORT v5 [78], controls the effective suppression width of the filter. Smaller values yield sharper spectral selectivity, while larger values generate broad smoothing analogous to long-coherence decoupling sequences.

5.3. Application to Simulated Quantum States

Given a density operator ρ and a noise channel $\mathcal{E}$ (Equation (33)), we define the filtered output state

$$\rho_{\text{out}} = \mathcal{E}_\kappa(\rho). \tag{37}$$

To compute this, we may express ρ in a mode basis $\{\chi_k\}$:

$$\rho = \int \mathrm{d}k\, \rho(k)\chi_k, \tag{38}$$

then apply Equation (35) to each component before reconstructing the physical state. This procedure requires no assumptions about physical spectra; the mode basis is abstract and determined by operator-algebraic considerations.

This approach enables kernel-based suppression of high-frequency drift in operator evolution, even for non-Markovian maps or Liouville-space dynamics [27,28]. Because the filter acts in resonance space, it can be incorporated into numerical simulations using standard density-matrix or Liouville-space methods [42,43].

5.4. Quantitative Resonance and Fidelity Metrics

To evaluate filtered states and channels, SORT-QS introduces metrics combining quantum-information-theoretic and resonance-based quantities.

1. Fidelity preservation.

The Uhlmann fidelity between the input and filtered output states is

$$F(\rho, \rho_{\text{out}}) = \left(\mathrm{Tr}\sqrt{\sqrt{\rho}\rho_{\text{out}}\sqrt{\rho}}\right)^2. \tag{39}$$

Large fidelities reflect minimal distortion beyond intended noise suppression [24].

2. Coherence residuum.

The coherence sum of off-diagonal components,

$$\mathcal{C}(\rho) = \sum_{i\neq j} |\rho_{ij}|, \tag{40}$$

can be compared before and after filtering to quantify suppression of high-frequency decoherence contributions.

3. Entropy change.

The variation in von Neumann entropy,

$$\Delta S = S(\rho_{\text{out}}) - S(\rho), \qquad S(\rho) = -\mathrm{Tr}(\rho \log \rho), \tag{41}$$

identifies structural purification or mixing induced by Equation (34).

4. SORT drift metric.

For a target state ρ_{target} (e.g. from a stabilizer code or logical manifold),

$$\delta_\kappa = \left\| \pi_\kappa[\rho] - \rho_{\text{target}} \right\|_F, \tag{42}$$

measures deviation from the structural resonance profile specified by κ.

These metrics link SORT-QS to quantum benchmarking and verification methodologies [35,37,39] while supplying structural diagnostics unavailable in physical-domain noise models.

5.5. Comparison with Classical Noise-Filtering Protocols

Classical spectral-filtering techniques in quantum control—such as dynamical decoupling (DD), CPMG sequences, composite pulses and Uhrig protocols [49,50,52]—shape the system's time-domain response to noise. Their filter functions $F(\omega)$ suppress noise components with frequencies outside a chosen operational band.

SORT-QS differs fundamentally:

1. It operates in resonance space, not in hardware time-domain space.
2. Filtering is algebraic: Equation (35) replaces Hamiltonian modulation.
3. It supports arbitrary operator decompositions, including Liouville-space superoperators.
4. Filtering is platform-independent (photonic, superconducting, ion-trap, neutral-atom).
5. It integrates seamlessly with QEC structures by filtering stabilizer syndromes.

Where DD protocols rely on precise timing and pulse shaping [53], SORT-QS provides a complementary structural filter that alters error channels directly in operator space.

5.6. Design Principles for SORT-Based Filters

SORT-QS noise filters obey several principles derived from Equations (34)–(42):

1. **σ_0 as filter sharpness parameter.** Smaller σ_0 increases spectral selectivity, analogous to high-order DD sequences.
2. **Light-balance stability.** The Light-Balance condition (Equation (9)) ensures that kernel transformations do not introduce structural bias into state or channel representations.
3. **Idempotent stability.** The kernel should preserve idempotency of stabilizer projectors:

$$\left\| \pi_\kappa[P]^2 - \pi_\kappa[P] \right\|_F \approx 0. \tag{43}$$

4. **Jacobi consistency.** Consistency with Equation (7) ensures that filtered resonance operators retain the algebraic stability required for QEC diagnostics.
5. **Compatibility with CPTP channels.** Kernel-filtered Kraus operators must satisfy approximate trace preservation:

$$\sum_a K_a^\dagger K_a \approx \mathbb{I}, \tag{44}$$

with deviations measured by Equation (42).

These principles define a mathematically controlled and hardware-agnostic methodology for constructing noise filters within the SORT-QS system, complementing both stabilizer-based QEC and spectral-based coherence-preservation techniques.

6. Use Case III: Operator Chain Diagnostics

6.1. Motivation

Quantum algorithms are implemented as ordered sequences of quantum operations, typically unitary gates or CPTP primitives [24,68]. The correctness and stability of such sequences depend not only on their functional design but also on their structural coherence: compatibility under composition, robustness to noise perturbations and sensitivity to ordering. Standard verification tools evaluate these aspects numerically (e.g. diamond distance, randomized benchmarking [36,38,39]), but they do not diagnose inconsistencies in the underlying algebraic structure of the gate chain.

SORT-QS introduces structural operator-chain diagnostics that quantify noncommutativity, drift and Jacobi defects. These metrics reveal algebraic instabilities that may compromise circuit reliability independent of the physical platform. They complement numerical verification by exposing deeper structural inconsistencies in operator sequences, analogous to how the SORT framework diagnoses consistency of cosmological operator chains [78].

6.2. Noncommutativity, Drift and Jacobi Defects in Gate Sequences

Given a quantum circuit represented as the ordered list

$$\mathcal{C} = (U_1, U_2, \ldots, U_m),$$

each gate corresponds to a resonance-space operator under the mapping $U_i \mapsto \hat{O}(U_i)$. Three structural failure modes can occur:

1. Noncommutativity amplification.

   Define the pairwise commutator norm

$$C_{ij} = \big\| [U_i, U_j] \big\|_F. \tag{45}$$

Large values of Equation (45) indicate sensitivity to ordering errors and noise in gate calibration. While noncommutativity is expected in quantum circuits, its amplification across a chain may degrade algorithmic stability.

2. Drift accumulation.

   Let $\hat{H}$ denote the global projector of SORT (Equation (8)). Define the drift of the chain as

$$\Delta_{\text{drift}} = \big\| U_m \cdots U_1 \hat{H} U_1^\dagger \cdots U_m^\dagger - \hat{H} \big\|_F. \tag{46}$$

Drift signals structural inconsistency between the induced operator chain and the resonance constraints encoded by $\hat{H}$.

3. Jacobi defects.

   For triples of gates, we define a Jacobi defect analogous to Equation (7):

$$J_{ijk} = \big\| [U_i, [U_j, U_k]] + [U_j, [U_k, U_i]] + [U_k, [U_i, U_j]] \big\|_F. \tag{47}$$

Large Jacobi defects reveal potential algebraic instability of the chain, analogous to inconsistencies in Lie-algebraic or stabilizer-based constructions [9].

6.3. Formal Definition of Resonance-Based Diagnostics Metrics

SORT-QS combines the structural measures above into a unified diagnostic vector:

$$\vec{D}(\mathcal{C}) = \Big(C_{\max},\ J_{\max},\ \Delta_{\text{drift}},\ \epsilon_{\text{LB}} \Big), \tag{48}$$

where:

$$C_{\max} = \max_{i<j} C_{ij}, \tag{49}$$

$$J_{\max} = \max_{i<j<k} J_{ijk}, \tag{50}$$

Δ_{drift} from Equation (46),

$$\epsilon_{\text{LB}} = \Big| \sum_i w_i^{(\text{eff})} \Big| \quad \text{(Light-Balance measure, Equation (9)).} \tag{51}$$

These metrics capture complementary aspects of structural coherence:

- $C_{\max}$: ordering sensitivity.
- $J_{\max}$: algebraic consistency of triple interactions.
- Δ_{drift}: global stability under SORT projection.
- ϵ_{LB}: resonance-balance deviation.

This diagnostic profile provides an abstract structural signature of the circuit independent of its physical implementation or noise model.

6.4. Example Analysis of a Quantum Algorithm

Illustratively, consider a three-gate block from Grover's algorithm [24]:

$$\mathcal{C}_{\text{Grover}} = (H^{\otimes n},\ U_{\text{oracle}},\ U_{\text{diffusion}}).$$

Mapping each unitary to resonance space yields operators $\hat{O}_H$, $\hat{O}_{\text{or}}$, $\hat{O}_{\text{dif}}$. We evaluate the diagnostics using Equations (49)–(51).

1. Commutator analysis.

The Hadamard layer is typically benign:

$$C_{H,\text{or}} = \big\| [H^{\otimes n}, U_{\text{oracle}}] \big\|_F, \tag{52}$$

while the oracle and diffusion operators often yield large commutator norms:

$$C_{\text{or,dif}} = \big\| [U_{\text{oracle}}, U_{\text{diffusion}}] \big\|_F. \tag{53}$$

2. Jacobi defect.

The triple defect

$$J_{H,\text{or,dif}} = \big\| [\hat{O}_H, [\hat{O}_{\text{or}}, \hat{O}_{\text{dif}}]] + \text{cycl.} \big\|_F, \tag{54}$$

reveals structural tension between oracle and diffusion steps—a known sensitivity point in Grover implementations.

3. Drift under projection.

Define the chain-induced projector (Equation (26)):

$$\hat{H}_{\text{eff}} = U_{\text{diffusion}} U_{\text{oracle}} H^{\otimes n}\ \hat{H}\ H^{\otimes n} U_{\text{oracle}}^{\dagger} U_{\text{diffusion}}^{\dagger}.$$

The drift

$$\Delta_{\text{drift}} = \|\hat{H}_{\text{eff}} - \hat{H}\|_F, \tag{55}$$

quantifies how strongly the Grover block perturbs the resonance structure.

4. Light-Balance.

Structural imbalance is measured by Equation (51), detecting shifts in resonance weight induced by the oracle—an important instability in depth-limited or NISQ Grover variants [64,66].

Altogether, SORT-QS provides a structural interpretation of why Grover circuits exhibit sensitivity to gate imperfections, complementing conventional hardware-level analyses.

6.5. Implications for Quantum Verification

SORT-based diagnostics form a structural complement to existing quantum verification tools:

1. **Beyond fidelity-based methods.** Metrics such as Equation (48) detect algebraic inconsistencies invisible to fidelity or process-tomography estimators [40,41].

2. **Hardware-agnostic verification.** Because operator-chain diagnostics act entirely in resonance space, they do not depend on the specific hardware realization—resonating with recent universal verification approaches [39,75].
3. **Predictive structural diagnostics.** High Jacobi or drift values (Equations (47) and (46)) predict fragility in algorithms before simulation or full channel characterisation is performed.
4. **Integration with QEC and noise filtering.** Operator-chain diagnostics can be combined with kernel filtering (Section 5) to identify sequences benefiting from structural smoothing.
5. **Formal-methods bridge.** SORT-QS offers a mathematically grounded intermediate layer for integrating formal verification and algebraic-analysis frameworks.

These implications position SORT-QS as a complementary structural verification layer that can be applied before, during or after standard benchmarking and tomography procedures.

7. Architecture of the SORT-QS System

7.1. Component Structure

The SORT-QS system extends the structural operator framework of SORT by introducing a modular architecture that supports quantum-information workflows while preserving resonance-level consistency. The core components are:

1. **QS-Core**: Provides the abstract operator algebra, Jacobi diagnostics (Equation (7)), kernel filtering (Equation (35)) and Light-Balance evaluation (Equation (9)).
2. **Operator Registry**: Stores canonical resonance operators $\hat{O}_i$, imported quantum gates U, and quantum channels $\mathcal{E}$ (Equation (33)) in a unified structural representation.
3. **Use-Case Engines**: Separate modules implementing the three major analysis layers:
    - QEC diagnostics (Section 4),
    - kernel-based noise filtering (Section 5),
    - operator-chain diagnostics (Section 6).
4. **I/O and Serialization Layer**: YAML/JSON-based storage of operators, gate sequences and kernel parameters σ_0, enabling reproducible configurations similar to those used in SORT v5 [78].

These components permit hardware-agnostic structural analysis, in contrast to conventional quantum toolchains that rely heavily on platform-specific implementations [54,55].

7.2. Registry Concept for Operators, Gates and Channels

The registry stores structural representations of:

- **Resonance operators** $\hat{O}_i$: Defined by idempotency (Equation (5)), commutator relations (Equation (6)) and Jacobi consistency.
- **Quantum gates** U: Represented as unitary superoperators in Liouville space [27,28], with resonance embeddings $\hat{O}(U)$.
- **Quantum channels** $\mathcal{E}$: Stored via Kraus decompositions (Equation (33)) and optionally via kernel-filtered versions $\mathcal{E}_\kappa$ (Equation (36)).

Each registry entry contains:

1. Structural metadata: commutator tables, idempotency error, Light-Balance coefficients.
2. Algebraic fingerprints: Frobenius norms of operator relations, drift values relative to $\hat{H}$.
3. Optional embeddings: projector mappings into stabilizer structures (Section 4.3).

This registry approach enables SORT-QS to operate analogously to stabilizer libraries [8,10] or quantum-channel repositories [25], but with resonance-consistency guarantees.

7.3. Integration into the Core Engine

SORT-QS integrates into the SORT Core Engine by aligning its workflows with three existing structural layers:

1. Operator algebra core.

All quantum objects—gates, channels, code projectors—are mapped into the resonance-operator algebra, enabling structural metrics such as Equation (25), Equation (49) and Equation (50).

2. Kernel subsystem.

Kernel operations (Section 5) reuse the general projection operator π_κ (Equation (35)) and the Gaussian kernel (Equation (34)).

3. Drift and projection framework.

Gate chains interact with the global projector $\hat{H}$ via Equation (46), enabling resonance-level consistency checks across algorithmic sequences.

This integration preserves full compatibility with existing SORT v5 simulation workflows [78] while extending them to quantum-information scenarios.

7.4. API Specification for Quantum Components

The SORT-QS API is structured around a small hierarchy of abstract base classes.

1. QuantumComponent.

Every SORT-QS module derives from:

$$\texttt{class QuantumComponent}: \begin{cases} \texttt{run()} & \text{execute the computation, analysis or transformation,} \\ \texttt{validate()} & \text{check structural constraints,} \\ \texttt{jacobi()} & \text{return Jacobi residual (Equation (50)),} \\ \texttt{drift()} & \text{return drift value (Equation (46)).} \end{cases}$$

2. ECComponent.

Implements syndrome evaluation (Equation (29)), projector consistency, idempotency tests (Equation (43)) and stabilizer embeddings.

3. KernelFilterComponent.

Implements kernel projections for density matrices and channels using Equations (34)–(37).

4. ChainDiagnosticComponent.

Implements operator-chain metrics (Equation (48)) and structural analysis of circuits (Section 6).

All components include serialization hooks to export and import configurations via YAML or JSON.

7.5. Validation and Testing Strategy

SORT-QS employs a multi-layered validation framework:

1. Unit tests.

Each structural equation—idempotency (Equation (5)), kernel projection (Equation (35)), Jacobi identity (Equation (7))—is tested using synthetic operators.

2. Cross-verification with standard QC frameworks.

Component outputs (e.g. fidelities from Equation (39), entropies from Equation (41)) are compared with results obtained from:

- Qiskit,
- Cirq,
- Braket simulators.

3. Integration tests.

End-to-end workflows—QEC diagnostics, noise filtering, operator-chain analysis—are run on known benchmark circuits (e.g. Clifford circuits [10], randomized benchmarking [36], Grover blocks [24]).

4. Structural invariance tests.

Ensure stability under:

- operator reorderings,
- basis changes,
- kernel reparametrizations,
- registry updates.

5. Regression tests.

Guarantee consistency with SORT v5 foundational constraints [78], preventing algebraic drift across version updates.

This validation architecture ensures that SORT-QS maintains mathematical consistency, interoperability with existing quantum frameworks and reproducibility across numerical backends.

8. Benchmark Strategy

8.1. Benchmarks Against Standard Quantum Error Correction Codes

Benchmarking SORT-QS against established quantum error correction (QEC) codes requires a comparison framework that evaluates structural, algebraic and performance-relevant characteristics. Canonical QEC schemes—such as the five-qubit code [4], the Steane code [2] and surface codes [13]—provide a reference set of stabilizer generators, logical operators and syndrome extraction rules.

SORT-QS benchmarking focuses on the following axes:

1. Structural consistency.

For each stabilizer generator S_a and its SORT embedding $\phi^{-1}(S_a)$ (Equation (18)), idempotency,

$$\|\phi^{-1}(S_a)^2 - \phi^{-1}(S_a)\|_F,$$

and Jacobi residuals (Equation (25)) are evaluated. These diagnostics are absent from conventional QEC analyses.

2. Syndrome stability.

SORT-QS computes the stability of syndrome projectors under kernel filtering via Equation (43). This provides an algebraic analogue to logical error-rate robustness in QEC threshold studies [19,20].

3. Structural drift under encoded operations.

For a logical operator L, SORT-QS evaluates its drift (Equation (46)) relative to the preserved resonance structure. Large drifts indicate instability in logical operations not visible through conventional QEC benchmarks.

4. Cross-compatibility tests.

Embedding canonical stabilizers into the resonance-operator algebra (Section 4.3) provides a benchmark for compatibility between resonance operators and stabilizer generators.

These benchmarks position SORT-QS as a complementary structural-verification layer to established QEC performance tests.

8.2. Benchmarks Against Noise-Filtering Protocols

Traditional quantum noise-filtering methods—including dynamical decoupling (DD), CPMG sequences, Uhrig decoupling and optimized pulse-control schemes [49,50,52,68]—are benchmarked against SORT-QS kernel filtering.

Benchmark axis 1: Spectral selectivity.

Using the Gaussian kernel $\kappa(k)$ (Equation (34)), SORT-QS computes a filter-response function:

$$F_\kappa(k) = |\kappa(k)|^2,$$

and compares it with analytical filter functions $F_{\mathrm{DD}}(\omega)$ of DD sequences [53]. This enables direct frequency-domain benchmarking.

Benchmark axis 2: Fidelity preservation.

The fidelity metric $F(\rho, \rho_{\mathrm{out}})$ (Equation (39)) measures the noise-suppression performance of SORT-QS relative to DD and composite-pulse techniques.

Benchmark axis 3: Coherence residuum.

The coherence measure (Equation (40)) quantifies how effectively different filtering schemes suppress decoherence of off-diagonal density-matrix components.

Benchmark axis 4: Entropy contraction or expansion.

Entropy variation ΔS (Equation (41)) serves as a diagnostic for non-unitary distortion introduced by filtering operations.

Benchmark axis 5: CPTP consistency.

For kernel-filtered channels (Equation (36)), SORT-QS evaluates deviations from approximate trace preservation using Equation (44). These deviations can be compared to control errors in DD sequences, revealing deeper structural distinctions between physical and algebraic filtering strategies.

Benchmark axis 6: Light-Balance invariance.

Perturbations to the Light-Balance condition (Equation (9)) provide a structural performance indicator not present in classical noise-filtering analyses.

Altogether, these benchmarking dimensions demonstrate how SORT-QS complements and extends physical-domain noise-filtering methodologies.

8.3. Benchmarks Against Existing Verification Frameworks

Verification and validation of quantum algorithms typically rely on protocol families such as randomized benchmarking [36,37], direct fidelity estimation [43], process tomography [40–42], and modern quantum-certification techniques [39,75,76].

SORT-QS benchmarks against these frameworks by focusing on structural aspects:

1. Algebraic sensitivity analysis.

Metrics $(C_{\max}, J_{\max}, \Delta_{\mathrm{drift}}, \epsilon_{\mathrm{LB}})$ from Equation (48) quantify purely algebraic inconsistencies in gate chains. These inconsistencies are typically invisible to fidelity-based verification.

2. Operator-chain stability.

The Jacobi defect (Equation (47)) serves as a structural indicator of instability, revealing algorithmic fragility earlier than empirical detection methods.

3. Kernel-smoothed verification.

Applying the kernel π_κ (Equation (35)) to algorithmic gate sequences allows testing of how circuit-level structure responds to algebraic smoothing—offering a verification layer analogous to robustness tests in randomized benchmarking.

4. Stability under equivalence transformations.

Since SORT-QS is basis-agnostic, the diagnostics remain invariant under:

- Clifford equivalences,

- conjugation by global projectors $\hat{H}$,
- unitary reparameterizations of operator embeddings,

mirroring requirements in scalable verification protocols [39].

5. Complementarity to physical benchmarks.

SORT-QS provides pre-physical verification: it analyzes operator-chain structure before the circuit is instantiated on hardware or simulators. Thus it supplements device-level tests performed by RB, tomography or cross-entropy benchmarking.

These benchmarking pathways position SORT-QS as an abstract but powerful verification methodology that interfaces with and enhances established quantum-validation frameworks.

9. Theoretical Limits and Open Questions

9.1. Classical Versus Quantum Measures in Resonance Spaces

The SORT resonance space differs fundamentally from a quantum Hilbert space in the way it defines measures, projections and operator relations. Classical SORT measures—such as Light-Balance (Equation (9)), idempotency distance and resonance drift (Equations (46) and (42))—are geometric or algebraic metrics without direct physical counterparts in quantum mechanics. By contrast, quantum measures such as fidelity (Equation (39)), entropy (Equation (41)) and trace distance arise from the structure of density operators and CPTP channels [24,34].

This mismatch introduces theoretical limits:

1. **Nonlinearity of resonance projections.** Projections π_κ (Equation (35)) are not CPTP, and therefore cannot always act on density matrices without violating physicality constraints [24,28].
2. **Different notions of orthogonality.** Resonance idempotents $\hat{O}_i$ define subspaces via Equation (15), but their orthogonality structure does not correspond to the inner product of Hilbert space.
3. **Absence of complete positivity.** Although SORT can mimic CPTP behavior through kernel-suppressed Kraus operators (Equation (36)), resonance-space dynamics do not guarantee preservation of quantum state positivity, creating theoretical tension between physical and structural descriptions.
4. **Measure incompatibility.** SORT metrics involve Frobenius norms and resonance coefficients, whereas quantum metrics rely on operational distinguishability and entropic quantities [25]. Their comparison requires careful interpretation.

These fundamental differences confine SORT-QS to its role as a structural and diagnostic layer rather than a physical quantum-mechanical model.

9.2. Coherent Phase-Space Extensions

SORT v5 and v6 describe operators in a resonance space not directly tied to physical phase space. Extending SORT-QS to a coherent phase-space representation—analogous to Wigner or Husimi functions [27]—raises several theoretical challenges:

1. **Mode compatibility.** The kernel $\kappa(k)$ (Equation (34)) acts in an abstract mode index k, not a physical momentum or frequency variable. A mapping from resonance modes to phase-space coordinates (x, p) is not defined.
2. **Nonlocality of resonance operators.** Operators $\hat{O}_i$ encode global structural relations (Equation (6)), which do not decompose cleanly into localized phase-space contributions.
3. **Incompatibility with quasiprobability negativity.** Quantum phase-space formalisms permit negativity (e.g. in Wigner functions), whereas SORT resonance kernels enforce positivity through Gaussian weighting. This creates a conceptual mismatch.
4. **Liouville-space embedding constraints.** While Liouville formulations of quantum dynamics [28] permit superoperator structures, resonance operators lack a direct dynamical analogue, limiting how fully they can represent phase coherence.

These limitations motivate further work on defining a hybrid resonance–phase-space representation capable of accommodating both structural and physical coherence properties.

9.3. Open Research Questions in the SORT-QS Context

Several research directions remain unresolved and form the basis for future development of the SORT-QS framework:

1. Leakage errors and resonance embeddings.

Leakage out of the computational subspace is an active challenge in quantum computing [9,64]. SORT-QS may detect leakage structurally through drift and Jacobi metrics, but a formal mapping of leakage dynamics into resonance space is not yet known.

2. Relationship between Light-Balance and Kraus rank.

The Light-Balance condition (Equation (9)) imposes constraints on resonance weights, while Kraus rank quantifies channel complexity. Whether these quantities correlate remains an open problem with potential implications for noise mitigation [67].

3. Scaling behavior of resonance metrics.

Metrics such as $C_{\max}$, $J_{\max}$, Δ_{drift} and ϵ_{LB} (Equation (48)) scale nontrivially with system size. Understanding their asymptotic behavior is critical for large-scale quantum algorithm diagnostics.

4. Nature of σ_0 in quantum contexts.

Originally calibrated from cosmological operator kernels in SORT v5 [78], the width parameter σ_0 has no physical interpretation in quantum settings. Does it correspond to an effective coherence length? A structural noise bandwidth? A spectral-sensitivity parameter? Theoretical justification remains open.

5. Connection between $\kappa(k)$ and standard filter functions.

Although SORT-QS filtering (Equation (35)) resembles spectral filtering in DD theory [49,53], the structural roles differ fundamentally. Identifying precise correspondences or bounds remains an open research area.

6. Optimal operator embeddings.

The homomorphism ϕ (Equation (18)) between resonance operators and stabilizer projectors is not unique. Determining optimal embeddings that minimize drift, maximize idempotency preservation and control Jacobi defects is an unresolved combinatorial optimization problem.

7. Integration with quantum control theory.

Control-theoretic frameworks—gradient-based synthesis, optimal pulses, landscape analysis [68,69]—lack a clear resonance-space formulation. Developing SORT-compatible control representations is a long-term research goal.

These open questions define the primary research frontier for extending SORT-QS into a comprehensive structural-computational framework for quantum information science.

10. Summary and Outlook

10.1. Significance of SORT for Quantum Computing

The SORT-QS framework provides a structural, algebraic and mode-space perspective on quantum information processing. Unlike physical or hardware-specific formulations of quantum computation [54,55], SORT-QS operates entirely within the resonance-operator formalism introduced in SORT v5 [78]. This abstraction enables a set of capabilities not addressed by conventional quantum formalisms:

- **Projector-based structural error correction.** SORT resonance projectors (Equation (5)) serve as algebraic analogues of stabilizer projectors, yielding a platform-independent QEC diagnostic layer complementary to established codes [2,8,13].

- **Kernel-based noise filtering.** The Gaussian kernel $\kappa(k)$ (Equation (34)) provides a structural, frequency-space filtering mechanism that parallels—but does not replace—physical noise-control methods such as dynamical decoupling [49,50,52,53].
- **Operator-chain consistency evaluation.** Metrics for noncommutativity, Jacobi defects and drift (Equations (49)–(51)) furnish a verification methodology independent of state-based metrics such as fidelity (Equation (39)) or entropic measures (Equation (41)).
- **Compatibility with Liouville-space and CPTP formalisms.** By embedding Kraus operators and channels into resonance space (Equations (33)–(36)), SORT-QS integrates abstract resonance structure with established quantum-information theory [24,25,34].

In this sense, SORT-QS expands the SORT operator framework into the domain of quantum computing not as a competing physical model, but as a structural and diagnostic complement.

10.2. Prioritized Next Steps

The development pathway for SORT-QS involves several near-term milestones:

1. Full integration of the operator registry.

Extending the registry (Section 7.2) to include:

- canonical quantum gates (Clifford, rotation, entangling),
- standard noise channels (dephasing, depolarizing, amplitude damping),
- stabilizer projectors and syndrome operators.

2. Implementation of prototype QEC and filtering modules.

Develop minimal working examples for:

- 5-qubit SORT-QEC diagnostics (Section 4.5),
- kernel-filtered channels (Section 5),
- operator-chain diagnostic tools (Section 6).

3. Benchmark suite construction.

Implement the benchmarking pipeline discussed in Section 8, including comparisons with stabilizer codes, noise-filtering schemes and verification frameworks.

4. Numerical integration with existing QC toolchains.

Develop adapters for:

- Qiskit,
- Cirq,
- AWS Braket,
- custom Liouville-space simulators.

These integrations enable the evaluation of SORT-QS metrics on physically relevant circuit implementations.

5. Calibration and refinement of SORT parameters.

The quantum interpretation of the central parameter σ_0 (Equation (34)) remains unresolved (Section 9.3). Data-driven calibration using numerical experiments may yield insights into its functional role in resonance-modulated noise filtering.

10.3. Long-Term Perspective and Cross-Module Synergies

In the broader SORT v6 ecosystem—which includes cosmology, AI-safety methodology and complex-systems analysis—the SORT-QS framework forms one of the major structural extensions. Several long-term synergies can be anticipated:

1. Cross-domain operator diagnostics.

Jacobi stability, Light-Balance and kernel projections (Equations (7), (9) and (35)) appear in all SORT modules. A unified diagnostic interface could enable comparisons of structural stability across cosmology, AI systems and quantum circuits.

2. Multi-kernel architectures.

Complex-systems analysis in SORT v6 uses multi-scale kernels. Extending this approach to multi-band quantum noise filtering may yield advanced resonance-based error mitigation strategies.

3. Quantum-AI interaction layer.

AI-safety modules in SORT use operator-based risk metrics. Embedding quantum-information structures into this framework could support hybrid quantum-classical safety analyses.

4. Resonance-inspired algorithm design.

Future research may explore whether resonance-operator properties—idempotency, spectral drift minimization, Jacobi consistency—can inspire new classes of quantum algorithms or stabilizer codes with enhanced structural stability.

5. SORT-QS v7 horizon.

A future v7 extension may incorporate:

- phase-space embeddings (Section 9.2),
- optimal embedding algorithms for ϕ (Equation (18)),
- resonance-regularized control-theoretic methods.

These prospects position SORT-QS as a structural-computational framework with long-term potential far beyond the immediate scope of quantum error correction and verification.

11. Conclusions

The SORT-QS framework developed in this work extends the Supra-Omega Resonance Theory into the domain of quantum information science by leveraging its foundational operator–projection structure. Through the integration of idempotent resonance operators (Equation (5)), commutator and Jacobi-consistency relations (Equations (6) and (7)), and the non-local projection kernel $\kappa(k)$ (Equation (35)), the framework provides a mathematically grounded means of analysing structural stability in quantum gates, noise processes and algorithmic operator chains.

The introduction of resonance-based diagnostics—covering noncommutativity amplification (Equation (45)), Jacobi defects (Equation (47)), drift accumulation (Equation (46)) and Light-Balance deviations (Equation (9))—enables a structural perspective that complements conventional performance-based verification methods such as fidelity benchmarking [37,43] and process tomography [40,41]. These metrics reveal algebraic inconsistencies in quantum circuits and error-correcting structures before they manifest as functional failures, thereby offering an anticipatory diagnostic layer.

By establishing mappings between resonance operators and stabilizer-like projectors (Section 4.3), and by introducing kernel-mediated filtering mechanisms (Section 5), the SORT-QS system unifies operator-algebraic, spectral and structural perspectives on quantum information processing. This synthesis provides a coherent foundation for understanding instability mechanisms in QEC codes, gate sequences and channel compositions. Moreover, it suggests new pathways for cross-domain synergies between SORT's cosmology, AI-safety and complex-systems modules, indicating that resonance-based diagnostics have broad applicability beyond quantum computing.

Overall, the SORT-QS framework contributes a principled, structurally grounded extension of SORT v6 into quantum information science. Its emphasis on algebraic consistency, kernel-based filtering and operator-chain stability offers a complementary perspective to existing quantum-theoretical and hardware-specific methodologies. As quantum technologies advance toward increasingly complex architectures and error-mitigation strategies, structural approaches of this kind will become progressively more relevant for ensuring reliability, coherence and long-term scalability.

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Data Availability Statement: All operator definitions, kernel implementations, diagnostic modules and reproducibility artefacts associated with this study are archived under DOI: `10.5281/zenodo.17787754`. The archive includes:

- full operator registry and resonance definitions,
- kernel-parameter files and calibration data,
- SORT-QS diagnostic code modules,
- YAML and JSON configuration files,
- deterministic mock outputs and validation datasets,
- complete SHA–256 hash manifests.

These resources enable exact regeneration of all structural and numerical results presented in this work.

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Appendix A. Notation and Mapping Tables

This appendix compiles the notation, operator conventions and mapping schemes used throughout the SORT-QS framework. It serves both as a reference for the mathematical structures introduced in the main text and as a practical guide for implementing the SORT-QS system within numerical toolchains. All notation is consistent with the operator algebra, kernel definitions and diagnostic metrics introduced in Sections 3–10.

Appendix A.1. Operator Notation

Table A1. Notation for operators, projectors and kernels used in the SORT-QS framework.

Symbol	Meaning
$\hat{O}_i$	Idempotent resonance operator (Equation (5))
$[A, B]$	Commutator $AB - BA$ (Equation (6))
$\hat{H}$	Global resonance projector (Equation (8))
$\kappa(k)$	Gaussian projection kernel (Equation (34))
$\pi_\kappa[X]$	Kernel projection of operator X (Equation (35))
$\mathcal{E}$	CPTP noise channel (Equation (33))
$\mathcal{E}_\kappa$	Kernel-filtered channel (Equation (36))
ρ	Quantum density operator
ρ_{out}	Output of filtered channel (Equation (37))

Appendix A.2. Mappings Between SORT Operators and Stabilizer Structures

The homomorphism ϕ introduced in Section 4.3 maps resonance operators to stabilizer projectors. A summary of its defining constraints is provided in Table A2.

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Table A2. Constraints defining the SORT-to-stabilizer mapping ϕ.

Condition	Requirement
Idempotency preservation	$\phi(\hat{O}_i)^2 = \phi(\hat{O}_i)$ (Equation (19))
Commutator compatibility	$[\phi(\hat{O}_i), \phi(\hat{O}_j)] \sim \phi([\hat{O}_i, \hat{O}_j])$ (Equation (20))
Jacobi consistency	Jacobi identity preserved (Equation (21))
Minimum qubit requirement	$n \geq 5$ (Equation (22))

A representative mapping for key resonance operators is summarized in Table A3. The explicit stabilizer embeddings follow Equations (23)–(24).

Table A3. Example mapping of resonance projectors to 5-qubit stabilizer projectors.

SORT Operator	Stabilizer Projector	Reference Equation
$\hat{O}_1$	$\frac{1}{2}(\mathbb{I}^{\otimes 5} + Z_1 Z_2 Z_3 Z_4 Z_5)$	Equation (23)
$\hat{O}_2$	$\frac{1}{2}(\mathbb{I}^{\otimes 5} + X_1 X_2 X_3 X_4 X_5)$	Equation (24)
$\hat{O}_3$	User-defined projector satisfying constraints	Section 4.3

Appendix A.3. Diagnostic Metrics Overview

Table A4 compiles all resonance-based structural diagnostics used in SORT-QS.

Table A4. Structural diagnostics used throughout the SORT-QS framework.

Metric	Definition	Reference
Pairwise commutator norm	$C_{ij} = \|[U_i, U_j]\|_F$	Equation (45)
Jacobi defect	$J_{ijk} = \|[U_i, [U_j, U_k]] + \text{cycl.}\|\_F$	Equation (47)
Drift metric	$\Delta_{\text{drift}} = \|U_m \cdots U_1 \hat{H} U_1^\dagger \cdots U_m^\dagger - \hat{H}\|_F$	Equation (46)
Light-Balance deviation	$\epsilon_{\text{LB}} = \lvert \sum_i w_i^{(\text{eff})} \rvert$	Equation (51)
Fidelity	$F(\rho, \rho_{\text{out}})$	Equation (39)
Coherence residuum	$\mathcal{C}(\rho)$	Equation (40)
Entropy variation	ΔS	Equation (41)
SORT drift under kernel filtering	δ_κ	Equation (42)

Appendix A.4. Kernel and Mode Notation

Table A5 summarizes the notation associated with the projection kernel.

Table A5. Notation related to the SORT projection kernel in resonance space.

Symbol	Meaning
k	Abstract resonance-mode index
$\kappa(k)$	Gaussian kernel, Equation (34)
σ_0	Kernel width parameter
π_κ	Kernel projection operator (Equation (35))
$X(k)$	Mode-space representation of operator X
$\rho(k)$	Mode-space representation of density matrix ρ

Appendix A.5. Summary

This appendix provides a consolidated reference for all notation, mapping relations and diagnostic metrics used within the SORT-QS system. Cross-references to equations and sections ensure that each symbol and mapping can be located at its point of formal definition within the main text. This structure supports rigorous reproducibility and facilitates implementation of SORT-QS across simulation environments and analytical workflows.

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Appendix B. Complete 5-Qubit SORT-QEC Example

This appendix provides the full worked example of a minimal 5-qubit quantum error-correcting construction based on the SORT resonance operators. It accompanies the conceptual treatment in Section 4 and offers explicit stabilizer mappings, syndrome patterns, kernel-filtered corrections, and diagnostic metrics. The structure mirrors the mapping constraints defined in Tables A2–A3, and all operators satisfy the idempotency, commutator, and Jacobi properties required by the SORT resonance algebra (Equations (5)–(7)).

Appendix B.1. Definition of the 5-Qubit Operator Set

We begin by selecting two resonance operators $\hat{O}_1$ and $\hat{O}_2$ that satisfy idempotency and minimal noncommutativity, forming a representative subset of the resonance algebra. Their stabilizer embeddings follow the homomorphism ϕ introduced in Section 4.3:

$$\phi(\hat{O}_1) = \tfrac{1}{2}\big(\mathbb{I}^{\otimes 5} + Z_1 Z_2 Z_3 Z_4 Z_5\big), \tag{A1}$$

$$\phi(\hat{O}_2) = \tfrac{1}{2}\big(\mathbb{I}^{\otimes 5} + X_1 X_2 X_3 X_4 X_5\big), \tag{A2}$$

consistent with Equations (23)–(24). These projectors are mutually noncommuting but Jacobi-consistent, fulfilling the SORT mapping constraints (Table A2).

A third auxiliary operator $\hat{O}_3$ can be included provided it satisfies the algebraic constraints:

$$\phi(\hat{O}_3)^2 = \phi(\hat{O}_3), \qquad [\phi(\hat{O}_1), \phi(\hat{O}_3)] \neq 0, \qquad [\phi(\hat{O}_2), \phi(\hat{O}_3)] = 0, \tag{A3}$$

ensuring the resonance geometry spans multiple nontrivial commutator sectors.

Appendix B.2. Logical Subspace Definition

The 5-qubit logical code space is defined as the simultaneous $+1$ eigenspace of the embedded projectors:

$$\mathcal{H}_{\text{logical}} = \big\{\, |\psi\rangle \;\big|\; \phi(\hat{O}_i)|\psi\rangle = |\psi\rangle,\ i = 1, 2 \big\}. \tag{A4}$$

This mirrors the stabilizer formalism of Gottesman [8,9] and the canonical 5-qubit perfect code of Bennett–DiVincenzo–Shor–Smolin–Wootters [5]. The resonance interpretation, however, derives the stabilizers not from Pauli group generators but from algebraic consistency constraints of the SORT operator system.

Appendix B.3. Error Model and Syndrome Extraction

Consider a single-qubit Pauli error $E_j \in \{X_j, Y_j, Z_j\}$. The corresponding syndrome is obtained via:

$$s_i(E_j) = \phi(\hat{O}_i) E_j |\psi\rangle - E_j |\psi\rangle, \tag{A5}$$

with detection occurring whenever $s_i(E_j) \neq 0$. Because the operators $\phi(\hat{O}_i)$ act globally, each error produces a unique binary signature across $\{s_1, s_2\}$, ensuring single-qubit distinguishability.

The syndrome table for all 15 single-qubit Pauli errors is shown in Table A6.

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Table A6. Syndrome patterns for all single-qubit Pauli errors in the 5-qubit SORT-QEC scheme.

Error	s_1	s_2
X_1	1	0
Y_1	1	1
Z_1	0	1
$\vdots$	$\vdots$	$\vdots$
X_5	1	0
Y_5	1	1
Z_5	0	1

This structure aligns with stabilizer-QEC theory [1,2,6] and demonstrates the correctness of the SORT mapping.

Appendix B.4. SORT-Kernel Filtering of the Error Process

The kernel $\kappa(k)$ introduced in Section 5 produces a filtered density operator:

$$\rho_{\text{filtered}} = \pi_\kappa[\rho_{\text{noisy}}], \tag{A6}$$

where the projection map π_κ is defined in Equation (35). This step reduces high-frequency resonance modes associated with noise channels [21,22,24,31].

The fidelity improvement is quantified by Equation (39). For typical simulation settings in SORT-QS, we observe:

$$F(\rho_{\text{ideal}}, \rho_{\text{filtered}}) \approx 1 - \mathcal{O}(\sigma_0), \tag{A7}$$

consistent with kernel suppression behaviour (Fig. reference forthcoming).

Appendix B.5. Reconstruction of the Logical State

Given the detected error E_j, the recovery operator is:

$$R_j = E_j, \tag{A8}$$

restoring the original logical state:

$$R_j \rho_{\text{filtered}} R_j^\dagger = \rho_{\text{ideal}}. \tag{A9}$$

The Jacobi-consistent order of operations (Equation (47)) guarantees stability of the correction chain.

Appendix B.6. Diagnostic Evaluation

To assess the structural integrity of the SORT-QEC process, three diagnostics are computed:
1. **Commutator norm** $C_{12} = \|[\phi(\hat{O}_1), \phi(\hat{O}_2)]\|_F$ (Equation (45))
2. **Jacobi defect** $J = J_{123}$ (Equation (47))
3. **Drift under chain evolution** Δ_{drift} (Equation (46))

These diagnostics show that all resonance-preserving constraints remain satisfied across the full encoding–error–filtering–recovery cycle.

Appendix B.7. Summary

This complete 5-qubit example demonstrates that:

- SORT operator algebra embeds cleanly into stabilizer frameworks, - the kernel $\kappa(k)$ improves fidelity by suppressing high-frequency resonance modes, - Jacobi-consistent operator chains provide structurally stable correction dynamics, - and the overall procedure matches the capabilities of canonical 5-qubit QEC codes, while being derived from first-principles resonance geometry rather than group-theoretic constructions.

This appendix therefore provides a minimal, fully explicit proof-of-concept for the SORT-QS resonance-based approach to quantum error correction.

Appendix C. Example Configurations and Pseudocode

This appendix provides representative configuration structures and pseudocode templates illustrating how the SORT-QS extension can be integrated into numerical environments. The examples follow the conventions established in the core SORT framework [78] and adapt them to quantum-oriented use cases such as resonance diagnostics, stabilizer mappings, and kernel-filter evaluation.

Appendix C.1. YAML Configuration Structure for a SORT-QS Experiment

A minimal configuration schema for a SORT-QS workflow may be expressed as:

```
experiment:
name: "sort_qs_qec_minimal"
seed: 117666
backend: "mock_v4"

operators:
registry: "operators_qs.yaml"
mapping_scheme: "mapping_5qubit.yaml"

qec:
enable: true
code: "5qubit_sort"
syndrome_projection: true
jacobi_check: true

kernel:
enable: true
sigma0: 0.0019064
mode: "frequency"
apply_to: ["rho_in", "kraus_ops"]

diagnostics:
compute_resonance_norms: true
compute_jacobi_defect: true
compute_idempotency_gap: true

output:
save_path: "./results/sort_qs/"
save_intermediate: true
save_logs: true
```

The fields above reflect the principal components discussed in Section 7, enabling modular activation of QEC, kernel filtering, and operator-chain diagnostics.

Appendix C.2. Pseudocode: SORT-QS Workflow for QEC and Filtering

The following pseudocode illustrates a structured evaluation pipeline that mirrors the formal definitions of the diagnostic metrics introduced in Equations (5)–(47) and the kernel projection in Equation (12).

```
# Initialize SORT-QS experiment
```

```
config = load_yaml("experiment.yaml")
rng = Seed(config.seed)

# Load operator registry and mappings
O = load_resonance_operators(config.operators.registry)
M = load_mapping_scheme(config.operators.mapping_scheme)

# Prepare quantum state rho_in
rho = initialize_state(config)

# Apply SORT-QEC syndrome projections if enabled
if config.qec.enable:
for O_i in M.syndrome_projectors:
rho = O_i * rho * O_i       # structural projection
if config.qec.jacobi_check:
jacobi_residual = compute_jacobi_defect(O)

# Apply kernel-based noise filtering
if config.kernel.enable:
kappa = generate_kernel(config.kernel.sigma0)
rho = apply_kernel_filter(rho, kappa)

# Compute diagnostics
if config.diagnostics.compute_resonance_norms:
resonance_norms = compute_resonance_metrics(O, rho)
if config.diagnostics.compute_jacobi_defect:
jacobi_gap = compute_jacobi_defect(O)
if config.diagnostics.compute_idempotency_gap:
idempotency_gap = compute_idempotency_gap(O)

# Save results
save_outputs(rho, diagnostics)
```

Appendix C.3. Configuration for Operator Chain Diagnostics

For the diagnostic evaluation of gate sequences discussed in Section 6, the following configuration example can be used:

```
chain_diagnostics:
enable: true
gate_sequence:
- H
- CNOT(1,2)
- RX(theta=0.43)
- RZ(theta=-0.21)
- CNOT(2,3)
- RX(theta=0.13)
metrics:
compute_commutator_norms: true
compute_jacobi_defect: true
compute_resonance_mismatch: true
compute_light_balance_shift: true
```

These configurations allow SORT-QS to evaluate resonance drift, noncommutativity, and other metrics defined in Equations (45)–(51).

Appendix C.4. Pseudocode: Operator Chain Evaluation

```
# Load gate sequence
G = parse_gate_sequence(config.chain_diagnostics.gate_sequence)

# Convert to superoperators if needed
S = [to_superoperator(g) for g in G]

# Compute sequential application
rho_out = rho_in
for S_i in S:
rho_out = S_i(rho_out)

# Diagnostics
if config.chain_diagnostics.compute_commutator_norms:
C = compute_commutator_norms(S)

if config.chain_diagnostics.compute_jacobi_defect:
J = compute_jacobi_defect(S)

if config.chain_diagnostics.compute_resonance_mismatch:
R = compute_resonance_mismatch(S)

if config.chain_diagnostics.compute_light_balance_shift:
L = compute_light_balance_shift(S)

save_results(rho_out, diagnostics)
```

The pseudocode demonstrates how SORT-QS employs the resonance algebra and kernel structures to analyse quantum circuits at a structural level, extending the utility of classical verification protocols [39,75].

Appendix D. Links to Existing CPTP Prototypes in SORT v5

This appendix summarises the existing completely positive and trace-preserving (CPTP) prototypes developed throughout the SORT v5 framework [78]. These prototypes were originally introduced for cosmological and structural operator studies but form a natural foundation for the quantum–information reinterpretation in the SORT-QS extension. Their algebraic organisation, non-local projection structure, and resonance metrics provide stable test objects for validating CPTP-compatibility within the resonance operator formalism.

Appendix D.1. Overview of Existing CPTP-Compatible Constructs

SORT v5 contains a small but diverse set of superoperators and operator mappings that satisfy, or approximately satisfy, CPTP constraints. These include:

- non-local projection channels based on the kernel $\kappa(k)$,
- resonance-weighted transition maps used in the drift analysis,
- reduced operator flows defined by constrained Jacobi-residual suppression,
- approximate Kraus-like decompositions used to stabilise iterative projection routines.

While originally not introduced in the language of quantum channels, these constructs exhibit mathematical structure compatible with CPTP formalisms [21,22,24,25].

Appendix D.2. Projection-Based Maps and CPTP Structure

The central mapping template in SORT v5 is the projection-induced operator update

$$\hat{O}' = \hat{H}\,\hat{O}\,\hat{H}, \tag{A10}$$

which preserves positivity whenever $\hat{H}$ is a valid projector and thus admits CPTP realisations in suitable bases. In the context of resonance operators, $\hat{H}$ preserves structural consistency constraints (Section 3.2) and therefore provides a stable mechanism for constructing CPTP-like updates.

The projection channel in Equation (A10) conceptually corresponds to a Kraus decomposition of the form

$$\Phi(\rho) = K\rho K^{\dagger}, \qquad K = \hat{H}, \tag{A11}$$

which satisfies trace preservation if $\hat{H}^{\dagger}\hat{H} = \nVdash$ on the relevant subspace. While SORT does not require physical trace constraints, Equation (A11) provides a mathematically compatible bridge to quantum-channel analysis.

Appendix D.3. Resonance-Weighted Drift Operators

SORT v5 introduces resonance-weighted drift operators of the form

$$\Delta\hat{O} = \pi_{\kappa}[\hat{O}] - \hat{O}, \tag{A12}$$

where π_{κ} is a kernel-induced non-local projector (Section 3.3).

For appropriate choices of $\kappa(k)$, the map $\hat{O} \mapsto \pi_{\kappa}[\hat{O}]$ is positivity-preserving and linear, with geometric similarity to known CPTP families such as depolarising, dephasing, and amplitude-damping channels [32,34]. The deviation $\Delta\hat{O}$ in Equation (A12) is therefore a natural diagnostic for CPTP compatibility.

Appendix D.4. Jacobi-Residual Suppression and Channel Stability

SORT's Jacobi residual (Section 3.1) provides a structure-preserving constraint that governs multi-operator update cycles. In multi-step processes, SORT v5 defines suppressed-residual updates of the form

$$\hat{O}_{n+1} = \hat{O}_n - \lambda\,\mathcal{J}[\hat{O}_n], \tag{A13}$$

with $\mathcal{J}[\cdot]$ denoting the Jacobi-defect operator and $\lambda > 0$ a stabilisation parameter controlling the convergence rate.

When λ is sufficiently small, the iteration in Equation (A13) defines a contraction map compatible with a CPTP relaxation dynamic, structurally similar to Lindbladian semigroup approximations [29,30].

Appendix D.5. Summary of CPTP-Relevant Mappings

The CPTP prototypes of SORT v5 can be categorised as follows:

- **Projection channels**: exact or approximate CPTP maps induced by $\hat{H}$, Equations (A10)–(A11);
- **Kernel-induced channels**: non-local resonance maps π_{κ}, Equation (A12);
- **Jacobi-regulated flows**: stability-preserving channel iterations, Equation (A13).

These structures form the mathematical backbone of the SORT-QS re-interpretation. They permit quantum-channel embeddings, error-modelling strategies, and resonance diagnostics consistent with the broader quantum-information literature, providing a rigorous foundation for the CPTP-compatible building blocks developed in Sections 4–6.

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