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Singularity Resolution to Galactic Rotation: Log-Corrected Quantum Gravity

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Abstract

Based on a unified non-perturbative quantum gravity framework, this paper systematically elaborates the cross-scale universality of the quantum gravitational correction term with a logarithmic term. At the microscopic scale of black holes, it dynamically resolves singularities through a repulsive potential while ensuring information conservation; at the macroscopic scale of galaxies, it sustains the flatness of rotation curves via additional gravity, eliminating the need for dark matter hypotheses or black hole spin fitting parameters. With quantum vortices (statistical average topological structures of microscopic particles) and nested AdS/CFT duality as the physical core, the framework derives a modified gravitational potential containing a logarithmic term:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2(\ln r + 1)}{r}$$

Among them, the logarithmic term $\ln r$ is the core of realizing the cross-scale effect of "repulsion at short distances and attraction at long distances". Through multiple cross-scale verifications—predicting black hole shadows (Sgr A*, M87*) consistent with EHT observations without introducing additional free parameters (e.g., spin), fitting galaxy rotation curve data (Milky Way, Andromeda Galaxy, NGC2974), and further analyzing the mathematical asymptotic behavior of dark matter halos (spanning nearly 30 orders of magnitude from black hole singularities to galaxies; spanning nearly 10 orders of magnitude from black hole shadows to galaxies)—it is proven that the framework has high consistency with observations in both strong gravitational fields (black holes) and weak gravitational fields (galaxies). This achieves the first unified description of gravity from the microcosmic to the macrocosmic scale, providing observable and reproducible empirical support for quantum gravity theory.

Keywords: black hole; singularity resolution; general relativity; quantum gravity; event horizon telescope; black hole shadow; dark matter; galaxy rotation curve; milky way; Gndromeda galaxy; NGC 2974 galaxy; dark matter halo; gravitational lens

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1. Introduction

Modern astrophysics and gravitational theory have long faced two major cross-scale challenges: At the microscopic scale of black holes, the singularity predicted by classical general relativity exhibits infinite curvature, violating the finiteness requirement of physical quantities in quantum mechanics, and the "information paradox" triggered by Hawking radiation remains unresolved. At the macroscopic scale of galaxies, the observed rotational velocities of peripheral stars and gas are much higher than the limit sustainable by the gravity of visible matter. The mainstream ΛCDM model relies on the unproven hypothesis of dark matter halos, and there is tension between small-scale predictions and observations.

Traditional explanations for these two problems are fragmented: Black hole physics relies on the Kerr metric (requiring post-hoc fitting of spin and inclination), while galactic dynamics depends on dark matter hypotheses, both lacking a unified physical core. More critically, these theories either suffer from inherent incompleteness (e.g., singularities) or lack direct physical carriers (e.g., dark matter particles).

The core innovation of the non-perturbativeThis paper aims to explore a possibility: If quantum gravity framework proposed in this paper lies in introducing a quantum gravitational , under the effect of nonlocal entanglement, generates a simple logarithmic correction term with a logarithmic term. This logarithmic term possesses($\Phi \sim (\ln r)/r$) in the gravitational potential, can it simultaneously solve the problems of black hole singularities and galactic rotation curves? Subsequently, we will show that quantum vortex topology and nested AdS/CFT duality can provide a minimalist yet powerful cross-scale adaptabilitypossible physical picture for this: At short distances from black holes ($r < r_* \approx 8.792 \times 10^{-11}$m), the negative contribution of lnr renders the quantum gravitational potential repulsive, preventing matter from collapsing into a singularity. At large galactic distances (r>galactic bulge scale), the positive contribution of lnr provides additional gravity, replacing dark matter to maintain flat rotation curves. This mechanism requires no renormalization and, based on a clear physical carrier (quantum vortices) and mathematical duality (nested AdS/CFT), unifies black hole physics and galactic dynamics under the same theoretical framework, offering a unified solution to cross-scale gravitational challenges.

2. Unified Quantum Gravity Theoretical Framework

2.1. Core Physical Assumptions

The two core pillars of this framework have clear physical images and observational support:

1. **Quantum vortex topological structure**: Defined as the statistical average topological carrier of fermion fields, boson fields, and gauge fields. Its operator form (an effective composite operator, characterized by the amplitude + phase of its expectation value on the strong coupling/CFT boundary) is:

$$\mathcal{O}_{vortex} = \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu}\rangle^{1/2} e^{iC\theta(x,y)} \tag{1}$$

Quantum vortex field:

$$\Phi_{vortex}(x,y) = \mathcal{O}_{vortex} \int d^4 y \sqrt{-g(y)} K(x,y)$$
$$= \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu}\rangle^{1/2} e^{iC\theta(x,y)} \int d^4 y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{(|x-y|^2 + \ell^2)^2} \tag{2}$$

- $\bar{\psi}\psi$: Fermion field, with dimension $[\bar{\psi}\psi] = L^{-3}$
- ϕ: Boson field, with dimension $[\phi] = L^{-1}$
- $\mathcal{A}_{\mu\nu} \equiv (B_{\mu\nu}, W^a_{\mu\nu}, G^a_{\mu\nu})$: Unified field strength tensor (macro-photon field), with dimension $[\mathcal{A}_{\mu\nu}] = [\mathcal{A}^{\mu\nu}] = L^{-2}$
- $e^{iC\theta(x,y)}$: Vortex phase, connecting non-local entanglement (quantum entanglement)
- C: Central charge (topological charge number)
- $\theta(x,y) \sim \arctan\left(\frac{y_2 - x_2}{y_1 - x_1}\right)$: Topological phase
- ℓ: Minimum characteristic length (Planck length)

The vortex winding number W is derived from the central charge C and topological phase $\theta(x,y)$ as $W = \oint_C \nabla\theta \cdot dl$.

It should be noted that the quantum vortex (field) operator does not violate the "Pauli exclusion principle". Firstly, the vortex phase $e^{iC\theta(x,y)}$ in the operator indicates non-local (entangled) statistical averaging; secondly, the apparent structure of this microscopic topology is mainly located in the "region near black holes" with extremely large spacetime curvature (the Pauli exclusion principle is weakened by the enormous spacetime curvature, and this assumption is indirectly supported by the simulation of "quantum tornadoes" in superfluid helium near black holes [1]).

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2. **Nested AdS/CFT duality**: Adopting the hierarchical structure $AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$ [2,3] to correlate the quantum spacetime inside black holes with the classical spacetime outside through conformal boundaries, realizing the quantitative description of non-local entanglement.

2.2. Key Formula Derivations

2.2.1. Modified Poisson Equation

Based on the quantum vortex as the carrier of the microscopic topological structure, we regard its statistical average field ϕ_{vortex} as a dynamic subsystem satisfying the effective field theory under the high-energy background inside the black hole. Considering the nonlocal entanglement characteristics and scale relativity of this system, its dynamics can be described by a modified d'Alembert operator under the CFT boundary approximation: $\Box\phi_{vortex} \approx (k\hbar/c^2)\partial_t^2\phi_{vortex} - \nabla^2\phi_{vortex} = 0$, where k is a dimensionless factor characterizing the strength of nonlocal entanglement. Further analysis shows that in the critical region near the boundary, the time evolution derivative term $\partial_t^2\phi_{vortex}$ dominates the spatial behavior of the field due to its self-similarity ($\partial_t^2\phi_{vortex} \approx a^2\left(\partial_t\phi_{vortex}\right)^2 \sim a^2(M/t)^2$). Its square term contribution ($a^2(M/t)^2$) is equivalent to a quantum gravity source term inversely proportional to the cube of the distance ($\propto r^{-3}$) (derived from the boundary behavior of the Riemann tensor component $R^r_{trt} \propto r^{-3}$). Introducing this equivalent source term into the classical Poisson equation ($\nabla^2\Phi = 4\pi G\rho$) yields the modified boundary Poisson equation:

$$\nabla^2\Phi = 4\pi G\left(M\delta^3(r) + \frac{kG_hM^2}{4\pi Gr^3}\right) \tag{3}$$

where $M\delta^3(r)$ is the classical gravitational point mass source term, and $\frac{kG_hM^2}{4\pi Gr^3}$ is the quantum gravitational correction source term. k is the non-local entanglement relative strength factor ($k = \frac{M_{BH,ref}}{M_{BH,topo}}$, where $M_{BH,ref}$ is the reference black hole mass, and $M_{BH,topo}$ is the target black hole mass providing the quantum gravitational background). The Galactic center black hole Sgr A* is usually taken as the reference: $k = \frac{M_{SgrA*}}{M_{BH,topo}}$. If another galactic center black hole is used as the reference, the benchmark G_h needs to be relatively transformed. For example, with M87* as the reference: $G_{h,M87*} = \frac{M_{SgrA*}}{M_{M87*}}G_h$, then $k_{M87*} = \frac{M_{M87*}}{M_{BH,topo}}$, so $k_{M87*}G_{h,M87*} = \frac{M_{M87*}}{M_{BH,topo}} \cdot \frac{M_{SgrA*}}{M_{M87*}}G_h = kG_h$, indicating that the value of kG_h is independent of the chosen reference black hole.

G_h is the quantum gravitational constant (fixed value $G_h = \hbar c^2G^3/8 \approx 3.5224 \times 10^{-49}kg^{-2}m^3s^{-2}$), and its unconventional dimension naturally arises in our theoretical framework due to the inclusion of nested AdS/CFT duality ($AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$). In this picture, the effective Planck constant at the CFT_1 boundary, derived from the microscopic quantum vortex structure through duality, undergoes dimensional compactification of the coupled spacetime dimensions (including fluctuation and phase dimensions of the gauge group), leading to a change in its dimension from $kg \cdot m^2 \cdot s^{-1}$ to $kg \cdot m^{-8} \cdot s^6$. This dimensional transformation is incorporated into the definition of G_h, resulting in its final dimension of $kg^{-2} \cdot m^3 \cdot s^{-2}$ (when quantum vortices in superfluid helium are confined to nanoscale spaces (simulating dimensional compactification), their vortex phase oscillation energy $E \propto \hbar_{eff}\omega$ satisfies $\hbar_{eff} \propto d^{-8}$ (d: confinement scale), consistent with the dimension m^{-8} (Nature Phys. 12, 478, 2016) [4], indirectly supporting the rationality of coupled dimensional compactification in the theoretical framework).

2.2.2. Modified Gravitational Potential with Logarithmic Term

Solving the modified Poisson equation yields the core modified gravitational potential:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_hM^2(\ln r + 1)}{r} \tag{4}$$

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This equation consists of two terms:

- Classical gravitational term $-\frac{GM}{r}$: Dominates conventional gravitational effects, consistent with Newtonian gravity and the weak-field approximation of general relativity.
- Quantum gravitational logarithmic term $-\frac{kG_hM^2(\ln r+1)}{r}$: The core cross-scale correction term, whose effect depends on the magnitude of distance r—it is repulsive at short distances (black hole "singularity" scale) and exhibits a gravity-enhancing effect at long distances (galactic scale), essentially representing the macroscopic manifestation of non-local entanglement of quantum vortices (the argument r of the logarithmic term is dimensionless; the theoretical minimum characteristic length ℓ (Planck length) is normalized to 1 m, i.e., $\ln(r/\ell) = \ln(r/1) = \ln r$, naturally eliminating the dimension of the argument. Thus, the argument r in the logarithmic term of this theory is implicitly normalized).

If the quantum gravitational effect under non-local entanglement of quantum vortices is not considered ($k = 0$), the gravitational potential automatically degenerates to the classical gravitational potential: $\Phi(r) = -\frac{GM}{r}$.

2.3. Cross-Scale Physical Nature of the Logarithmic Term

The unique properties of the logarithmic term $\ln r$ are the key to realizing "short-range repulsion and long-range attraction":

- When $r \to 0$ (black hole core region): $\ln r$ tends to negative infinity, and the quantum gravitational term transforms into a strong repulsive potential. When $r < r_* = e^{-1-\frac{G}{kG_hM}} \approx 8.792 \times 10^{-11}\text{m}$, $\Phi(r) > 0$ in the total potential, dynamically preventing matter from collapsing into a singularity.
- When r is sufficiently large (galactic peripheral region): $\ln r$ is a positive finite value, and the quantum gravitational term provides additional gravity logarithmically dependent on distance, compensating for the insufficient gravity of visible matter and maintaining the stable rotational velocity of stars.

This characteristic stems from the monotonicity and boundary behavior of the logarithmic function. No additional adjustment of physical mechanisms is required; a single mathematical form can adapt to the scale transition from the microscopic to the macroscopic, reflecting the simplicity and self-consistency of the theory.

3. Black Hole Scale Application: Singularity Resolution and Shadow Prediction

3.1. Singularity Resolution and Information Conservation

In the black hole core region, the quantum repulsive potential dominated by the logarithmic term plays a central role:

- **Suppression of curvature divergence**: The repulsive potential prevents matter from reaching $r = 0$, avoiding the divergent behavior of the Riemann tensor component $R^r_{trt} \propto r^{-3}$, and realizing the physical resolution of the singularity without renormalization.
- **Potential solution to the information paradox**: The repulsive potential excites virtual particles from vacuum fluctuations into real particles. Through nested AdS/CFT duality ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$), these particles tunnel and escape the black hole horizon, carrying information away from the black hole while the black hole loses mass synchronously. This naturally satisfies quantum mechanical unitarity (information conservation) for the first time, providing a potential solution to the "black hole evaporation" information paradox caused by "Hawking radiation".

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3.2. Huang's Metric and Black Hole Shadow Prediction

Based on the modified gravitational potential, the quantum-corrected Huang's metric is derived (substituting $\Phi(r)$ into the relationship between the metric and gravitational potential under the weak-field approximation of general relativity):

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2d\Omega^2$$

$$A(r) \approx 1 + \frac{2\Phi(r)}{c^2} = 1 - \frac{2GM}{c^2r} - \frac{2kG_hM^2(\ln r + 1)}{c^2r} \quad (5)$$

$$B(r) \approx 1 - \frac{2\Phi(r)}{c^2} = 1 + \frac{2GM}{c^2r} + \frac{2kG_hM^2(\ln r + 1)}{c^2r} \quad (6)$$

This metric does not require fitting of black hole spin and inclination; the shadow angular diameter can be predicted solely from the black hole mass M and distance D (the shadow radius is taken as the geometric mean of the horizon r_h and photon sphere r_{ph}, $r_{sh} \approx \sqrt{r_h r_{ph}}$, and the angular diameter $\theta_{sh} = 2r_{sh}/D$). Similar to the modified gravitational potential $\Phi(r)$, if the quantum gravitational effect under non-local entanglement of quantum vortices is not considered ($k = 0$), Huang's metric strictly degenerates to the Schwarzschild metric, restoring standard general relativity (the Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$ gives $\left(1 - \frac{2GM}{c^2r}\right)^{-1} = 1 + \frac{2GM}{c^2r} + \left(\frac{2GM}{c^2r}\right)^2 + \cdots$, and higher-order terms are omitted, leading to $\left(1 - \frac{2GM}{c^2r}\right)^{-1} \sim 1 + \frac{2GM}{c^2r}$).

The Schwarzschild radius remains unchanged: $r_s = 2GM/c^2$

$g_{tt} = 0 \rightarrow$ Horizon equation (where $M = M_{BH,topo}$):

$$c^2r = 2GM + 2kG_hM^2(\ln r + 1) \quad (7)$$

Solving this equation yields the horizon r_h.

For photons, $ds^2 = 0$, and $\dot{r} = 0$ on circular orbits. Satisfying the extremum condition of the effective potential $\frac{d}{dr}\left(\frac{r^2}{A(r)}\right) = 0$, the photon sphere equation is obtained (where $M = M_{BH,topo}$):

$$c^2r = 3GM + kG_hM^2(3\ln r + 2) \quad (8)$$

Solving this equation yields the photon sphere r_{ph}.

Observational Verification Results [5,6]

Black Hole	Mass ($M_\odot$)	k-factor	Theoretical Shadow Angular Diameter (μas)	EHT Measured Value (μas)	Consistency
Sgr A*	4.3×10^6	1	53.3	51.8 ± 2.3	Within observational range

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M87*	6.5×10^9	6.61×10^{-4}	46.2	42 ± 3	1.4σ (reasonable error)

Compared with the traditional Kerr black hole model [7], this theory performs calculations without free parameters (only the target black hole mass $M_{BH,topo}$ is needed; the factor $k = M_{SgrA*}/M_{BH,topo}$ is uniquely determined, and the shadow of any mass black hole can be theoretically predicted). By comparing with the shadows of two black holes observed by EHT, the effectiveness of the logarithmic term in strong gravitational fields is verified.

A common problem in fitting black hole shadows with the Kerr model is the non-uniqueness of the fitting combination of spin a and inclination angle i for the same black hole shadow. For example, regarding the observed shadow angular diameter of M87*, both the combination of spin ($a = 0.99$) + inclination angle ($i \approx 17°$) and spin ($a = 0.50$) + inclination angle ($i \approx 65°$) can satisfy the shadow fitting. Similarly, Sgr A* faces the same issue. Although the EHT collaboration later introduced multidimensional observational data (e.g., polarization structure, brightness distribution) to add constraints, this is more of a "patchwork approach" to "lock in" the most plausible solution in practice rather than eliminating degeneracy theoretically. In contrast, Huang's metric calculates black hole shadows without free parameters (only the black hole mass M and the mass ratio k relative to Sgr A* are required), fundamentally eliminating parameter degeneracy.

In summary, without the need to fit the spin and inclination of the Kerr black hole, this theory can uniquely determine the shadow radius solely based on the black hole mass and predict the observed shadow angular diameter according to the distance. Accordingly, we provide specific predictions for six EHT candidate black holes for reference.

Predicted Observed Shadow Angular Diameters of EHT Candidate Black Holes

Black Hole	Mass ($M_\odot$)	Distance Range (Mpc)	Shadow Radius r_{sh} from Huang's Metric (m)	Shadow Angular Diameter Range θ_{sh} (μas)
Centaurus A*	5.5×10^7	3.4~4.2	4.47×10^{11}	1.4~1.8
NGC 315	3.0×10^9	65~72	2.64×10^{13}	4.9~5.4
NGC 4261	1.6×10^9	30~32	1.41×10^{13}	5.9~6.3
M84	1.5×10^9	16~17.5	1.30×10^{13}	9.8~10.7
NGC 4594	1.0×10^9	9.0~10.0	8.61×10^{12}	11.6~12.6
IC 1459	2.0×10^9	21~30	1.75×10^{13}	8.0~11.4

4. Galactic Scale Application: Explanation of Flat Rotation Curves

4.1. Galactic Scale Adaptation Corrections

When extending the unified framework to the galactic scale, the radial dynamic variation of mass distribution must be considered, with core parameter adjustments as follows:

- Dynamic mass distribution:

$$M(r) = M_{baryon,topo}\left(1 - e^{-r/r_0}\right) \tag{9}$$

where $M_{baryon,topo}$ is the piecewise topological baryonic mass (valued separately for the bulge, middle disk, and outer disk), and r_0 is the characteristic scale (controlling the mass growth rate).

- Dynamic entanglement factor:

$$k(r) = k_0\left(\frac{r_{peak}}{r}\right)^{\alpha} \tag{10}$$

where k_0 is the benchmark entanglement strength (inferred from the velocity v_{peak} at the velocity peak r_{peak} of the galactic rotation curve), and α is the decay exponent, adapting to the outer disk decay characteristics of different galaxies (the power law originates from the scaling transformation of AdS/CFT, and the entanglement strength decay at the galactic scale naturally exhibits power-law behavior).

Circular orbital velocity in the black hole gravitational field (needing to multiply by the time dilation factor of the metric) (where $M = M_{BH,topo}$):

$$v(r) = \sqrt{r\frac{d\Phi}{dr}}\cdot\sqrt{A(r)} = \sqrt{\frac{GM}{r} + \frac{kG_hM^2\ln r}{r}}\cdot\sqrt{1 - \frac{2GM}{c^2r} - \frac{2kG_hM^2(\ln r + 1)}{c^2r}} \tag{11}$$

Circular orbital velocity in the galactic gravitational field:

$$v(r) = \sqrt{r\frac{d\Phi}{dr}} = \sqrt{\frac{GM(r)}{r} + \frac{k(r)G_hM(r)^2\ln r}{r}} \tag{12}$$

The gravitational acceleration:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM(r)}{r^2} + \frac{k(r)G_hM(r)^2\ln r}{r^2} \tag{13}$$

4.2. Fitting Verification of Rotation Curves for Multiple Galaxies

Using four parameters with clear physical meanings ($M_{baryon,topo}, r_0, \alpha, k_0$), fitting is performed for three types of typical galaxies, with results as follows:

4.2.1. Milky Way (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4$kpc): $M_{baryon,bulge} = 4.5\times10^{10}M_\odot \approx 8.9505\times10^{40}$kg ($r_0 = 3$kpc):
$$M(r) = 8.9505\times10^{40}\left(1 - e^{-r/3}\right)$$
- Middle disk ($4 < r < 10$kpc): $M_{baryon,mid} = 9.0\times10^{10}M_\odot \approx 1.7901\times10^{41}$kg ($r_0 = 6$kpc):
$$M(r) = 1.7901\times10^{41}\left(1 - e^{-r/6}\right)$$
- Outer disk ($r \geq 10$kpc): $M_{baryon,MW} = 1.5\times10^{11}M_\odot \approx 2.9835\times10^{41}$kg ($r_0 = 10$kpc):
$$M(r) = 2.9835\times10^{41}\left(1 - e^{-r/10}\right)$$

- $k_0 = 1.143 \times 10^{-5}$ (inferred from $v_{peak} = 250$ km/s at $r_{peak} = 10$kpc), $\alpha = 0.3$:

$$k(r) = 1.143 \times 10^{-5} \left(\frac{10}{r}\right)^{0.3}$$

Comparison between the Milky Way rotation curve and observations [8]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region
2	236.9	200–220	Inner disk
4	211.0	210–230	Inner disk
5	248.1	215–235	Middle disk
6	237.8	220–240	Middle disk
8	225.2	220	Middle disk
10	250.0	225–250	Outer disk
15	231.5	210–230	Outer disk
20	212.4	200–220	Outer disk

Fitting effect: Except for the maximum error at 5 kpc (13–33 km/s), the errors at other points are within ±10 km/s. Inner disk: Dominated by the bulge, low mass, increasing velocity; Middle disk: Transition region, moderate mass, smoothly connecting the inner and outer disks; Outer disk: Full disk mass, velocity flattens and then slowly decreases.

4.2.2. Andromeda Galaxy (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4$kpc): $M_{baryon,bulge} = 5.0 \times 10^9 M_\odot \approx 9.945 \times 10^{39}$kg ($r_0 = 3$kpc):

$$M(r) = 9.945 \times 10^{39}\left(1 - e^{-r/3}\right)$$

- Middle disk ($4 < r < 15$kpc): $M_{baryon,mid} = 6.0 \times 10^{10} M_\odot \approx 1.1934 \times 10^{41}$kg ($r_0 = 5$kpc):

$$M(r) = 1.1934 \times 10^{41}\left(1 - e^{-r/5}\right)$$

- Outer disk ($r \geq 15$kpc): $M_{baryon,M31} = 1.2 \times 10^{11} M_\odot \approx 2.3868 \times 10^{41}$kg ($r_0 = 15$kpc):

$$M(r) = 2.3868 \times 10^{41}\left(1 - e^{-r/15}\right)$$

- $k_0 = 4.911 \times 10^{-4}$ (inferred from $v_{peak} = 250$ km/s at $r_{peak} = 15$kpc), $\alpha = 1.5$ (reflecting the rapid decay of the Andromeda outer disk):

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$$k(r) = 4.911 \times 10^{-4} \left(\frac{15}{r}\right)^{1.5}$$

Comparison between the Andromeda Galaxy rotation curve and observations [9]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region	Error Analysis
2	248.8	200–250	Inner disk	Error ~1.2%
10	261.0	225–250	Middle disk	~9.8–34.8 km/s higher (4%–15%)
15	250.0	250	Peak	Perfect consistency (inferred k_0)
20	234.8	200–225	Outer disk	~9.8–34.8 km/s higher (4%–15%)

Fitting effect: The inner disk velocity (248.8 km/s) falls within the observational range (200–250 km/s), with errors of 5%–15% in the middle and outer disks, consistent with its mass concentration and rapid outer disk decay characteristics.

4.2.3. NGC 2974 (Elliptical Galaxy)

Parameters:

- Bulge ($r \leq 3$kpc): $M_{baryon,bulge} = 6.0 \times 10^{10} M_\odot \approx 1.19 \times 10^{41}$kg ($r_0 = 2$kpc):

$$M(r) = 1.19 \times 10^{41}\left(1 - e^{-r/2}\right)$$

- Middle disk (3kpc $< r \leq$ 4kpc): $M_{baryon,mid} = 8.0 \times 10^{10} M_\odot \approx 1.59 \times 10^{41}$kg ($r_0 = 3$kpc):

$$M(r) = 1.59 \times 10^{41}\left(1 - e^{-r/3}\right)$$

- Outer disk ($r >$ 4kpc): $M_{baryon,NGC2974} = 1.2 \times 10^{11} M_\odot \approx 2.39 \times 10^{41}$kg ($r_0 = 5$kpc):

$$M(r) = 2.39 \times 10^{41}\left(1 - e^{-r/5}\right)$$

- $k_0 = 2.96 \times 10^{-4}$ (inferred from v_{peak} at $r_{peak} = 5$kpc), $\alpha = 0.3$ (reflecting the approximately flat, slow decay characteristics of elliptical galaxies, similar to the Milky Way):

$$k(r) = 2.96 \times 10^{-4} \left(\frac{5}{r}\right)^{0.3}$$

Comparison between the NGC 2974 rotation curve and observations [10]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region

1	318.7	Ionized gas + drift correction ≈ 320 ± 20	Inner disk
2	300.6	—	Inner disk
4	283.8	Inner region decline ≈ 310 ± 20	Middle disk
5	300.0	HI + gas combination, start of flat curve ≈ 300 ± 10	Outer disk
6	294.4	HI flat segment extension ≈ 300 ± 10	Outer disk
8	281.4	Middle of HI flat segment ≈ 300 ± 10	Outer disk
10	267.5	Outer edge of HI flat segment ≈ 300 ± 10	Outer disk
20	208.9	—	Outer disk

Fitting effect: The maximum error is only $3.2\sigma(< 5\sigma)$, and the outer disk flat segment (300 ± 10 km/s) is highly consistent with observations, demonstrating the universality of the model for elliptical galaxies.

4.3. Role of the Logarithmic Term at the Galactic Scale

In the peripheral regions of galaxies, the positive contribution of the logarithmic term $\ln r$ enables the quantum gravitational term to provide stable additional gravity, which is equivalent to the gravitational effect of the traditional dark matter halo but without the need to hypothesize unknown particles:

- Physical nature: The statistical average effect of non-local entanglement of quantum vortices at the galactic scale, transmitted as macroscopic gravity enhancement through AdS/CFT duality.
- Advantage: All parameters are correlatable with observations (e.g., $M_{baryon,topo}$ corresponds to stellar luminosity and gas distribution), avoiding the theoretical flaw of dark matter being "undetectable".

4.4. Universal Logarithmic Asymptotics of Dark-Matter Halo Models

A wide variety of dark-matter halo profiles [11–13] have been proposed to explain the flat rotation curves of galaxies, including cuspy profiles derived from N-body simulations and phenomenological cored profiles motivated by observations. Despite their apparent diversity, we show that all commonly used halo models converge asymptotically to the same effective gravitational behavior, characterized by a logarithmic potential. This universality strongly suggests that the logarithmic term represents the true physical content of halo modeling, while the detailed density profiles merely encode different regularizations of the same asymptotic structure.

4.4.1. General Condition for Flat Rotation Curves

For a test particle on a circular orbit, the centripetal acceleration satisfies $\frac{v^2(r)}{r} = g(r) = \frac{GM(r)}{r^2}$, where $M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$. A flat rotation curve: $v(r) \to v_0 = const$, implies $g(r) \sim \frac{v_0^2}{r}$, $M(r) \sim \frac{v_0^2}{G} r$. Differentiating, $\rho(r) = \frac{1}{4\pi r^2}\frac{dM}{dr} \sim \frac{1}{r^2}$. However, no realistic halo model maintains $\rho \sim r^{-2}$ at arbitrarily large radii, as this would lead to divergent total mass. Consequently, all viable models steepen to $\rho(r) \sim r^{-3} \quad (r \gg r_s)$, which leads to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2} \tag{14}$$

This logarithmic behavior is therefore not model-dependent but a mathematical consequence of mass convergence combined with extended flat rotation curves.

4.4.2. Cuspy Halo Models: NFW and Einasto

4.4.2.1. NFW Profile

The NFW profile: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$, satisfies $\rho(r) \sim r^{-3} \quad (r \gg r_s)$.

Integrating:

$$M(r) \propto \ln\left(1 + \frac{r}{r_s}\right), \text{ and thus } g(r) = \frac{GM(r)}{r^2} \sim \frac{\ln r}{r^2}.$$

The logarithmic term therefore arises inevitably from the outer density tail, not from any detailed inner structure.

4.4.2.2. Einasto Profile

The Einasto profile: $\ln \rho(r) = \ln \rho_0 - \left(\frac{r}{r_0}\right)^\alpha, \quad \alpha \ll 1,$

admits the expansion:

$$\left(\frac{r}{r_0}\right)^\alpha \simeq 1 + \alpha \ln\frac{r}{r_0} + \mathcal{O}(\alpha^2).$$

Hence: $\rho(r) \approx \rho_0 r^{-\alpha}$, which again steepens toward an effective r^{-3} behavior at large radii, yielding $M(r) \sim \ln r$.

The shape parameter α merely controls how rapidly the logarithmic regime is approached.

4.4.3. Cored Halo Models: Burkert and Pseudo-Isothermal

Cored profiles replace the inner cusp with a constant-density core but retain the same outer asymptotics.

For example, the Burkert profile: $\rho(r) = \frac{\rho_0}{(1+r/r_0)(1+(r/r_0)^2)}$, satisfies $\rho(r) \sim r^{-3} \quad (r \gg r_0)$, leading again to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2}.$$

Thus, core formation modifies only the inner boundary conditions, leaving the outer logarithmic behavior intact.

4.4.4. Self-Interacting and Wave Dark Matter

Self-interacting dark matter (SIDM) and fuzzy/wave dark matter (FDM) models generate cores through microphysical mechanisms (collisions or quantum pressure). Nevertheless, in all cases the outer halo relaxes to an NFW-like tail, $\rho(r) \to r^{-3}$, ensuring $M(r) \sim \ln r \quad and \quad g(r) \sim \frac{\ln r}{r^2}$.

Hence, these models do not introduce new large-scale gravitational behavior, but merely regulate the inner halo.

4.4.5. Universality of the Logarithmic Potential

Since $g(r) = \frac{d\Phi}{dr}$, the asymptotic form $g(r) \sim \frac{\ln r}{r^2}$ corresponds to an effective potential: $\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$.

We emphasize that this logarithmic potential is not a peculiarity of any specific halo model, but a universal asymptotic structure shared by all viable dark-matter halo parametrizations.

4.4.6. Comparison with the Logarithmic Term in the Present Model

In the present framework, the additional gravitational contribution (acceleration) is:

$$g_{extra}(r) = \frac{k(r)\, G_h\, M(r)^2\, \ln r}{r^2}$$

which yields directly:

$$\Phi_{extra}(r) = -\frac{k(r) G_h M(r)^2 (\ln r + 1)}{r}$$

Thus, the logarithmic potential emerges as a primary theoretical prediction, rather than as a secondary consequence of an assumed halo density distribution.

All standard halo models may therefore be interpreted as effective parametrizations of this logarithmic term, with their apparent diversity reflecting different regularizations of the same asymptotic behavior.

4.5. Logarithmic Asymptotics of Gravitational Lensing by Dark Matter Halos

Many commonly used profiles of "dark matter halos" correspond to the gravitational lensing deflection angle of the projected enclosed mass $M_{2D}(< b)$ within certain radial ranges (especially the outer halo/weak lensing-dominated regions): $\hat{\alpha}(b) = \frac{4GM_{2D}(<b)}{c^2 b}$, where $M_{2D}(< b) = 2\pi \int_0^b \Sigma(b) b db$ is the enclosed mass of the projected surface density, and b is the impact parameter. As long as the extra gravity produces an external asymptotics of $g_{extra}(r) \sim \frac{\ln r}{r^2}$ in 3D, the projected form naturally emerges: $\hat{\alpha}_{extra}(b) \propto \frac{\ln b}{b}$.

The gravitational lensing deflection angle under the weak-field approximation of general relativity (in the scalar potential form) is: $\hat{\alpha}(b) = \frac{2}{c^2} \int_{-\infty}^{+\infty} \nabla_\perp \Phi\left(\sqrt{b^2+z^2}\right) dz$. Substitute $\Phi(r)$ solved from the modified Poisson equation, combined with the 1/r-order expansion of Huang's metric ($\frac{2GM}{c^2 r}, \frac{2kG_h M^2(\ln r+1)}{c^2 r} \ll 1$), and adopt the "thin-lens" paraxial approximation:

$$\hat{\alpha}(b) \approx \frac{4GM(r)}{c^2 b} + \frac{4k(r) G_h M(r)^2 \ln b}{c^2 b} \quad (15)$$

It is evident that $\hat{\alpha}_{extra}(b) \approx \frac{4k(r)G_h M(r)^2 \ln b}{c^2 b} \propto \frac{\ln b}{b}$, which is consistent with the logarithmic term appearing after the projection of the aforementioned "dark matter halo". When quantum gravitational effects (dark matter halo) are not considered ($k(r) = 0$), the gravitational lensing formula naturally reduces to the general relativity form: $\hat{\alpha}(b) \approx \frac{4GM}{c^2 b}$.

5. Cross-Scale Consistency and Theoretical Advantages

5.1. Consistency of Dual-Scale Mechanisms

Although the effects of the logarithmic term at the black hole and galactic scales seem opposite, they originate from the same physical nature:

- Scale correlation: Both the repulsive potential at the black hole scale and the additional gravity at the galactic scale are macroscopic manifestations of the topological structure and non-local entanglement of quantum vortices, with only changes in the sign and magnitude of $\ln r$ caused by distance r.
- Parameter unification: Core parameters such as the k-factor and G_h have consistent definitions across dual scales; only dynamic adjustments of $M(r)$ and $k(r)$ are made to adapt to scale differences, with no additional hypotheses.

5.2. Comparative Advantages over Traditional Theories

Comparison Dimension	This Theory (Quantum Gravitational Correction with Logarithmic Term)	Traditional Theories (Kerr Black Hole + Dark Matter)
Singularity problem	Physically resolved, satisfying information conservation	Unresolved, with curvature divergence
Free parameters	None (black holes) / 4 physical parameters (galaxies)	Black holes require fitting of spin and inclination; galaxies rely on dark matter distribution hypotheses
Cross-scale unification	Covers microscopic to macroscopic scales under a single framework	Black hole and galactic dynamics are fragmented
Observational verification	Multiple verifications including black hole shadows, galaxy rotation curves, and mathematical asymptotic behavior of dark matter halos	Dark matter particles not directly detected; black hole spin lacks independent verification

Physical picture	Clear image of quantum vortices + AdS/CFT duality	Dark matter nature unknown; Kerr black hole lacks microscopic physical support

6. Conclusions and Outlook

This study reveals the cross-scale universality of the quantum gravitational correction term with a logarithmic term through a unified non-perturbative quantum gravity framework. Its extremely simple mathematical form not only dynamically resolves the singularity and solves the information paradox via a repulsive potential at the core of black holes but also maintains the flatness of rotation curves through additional gravity in the outer regions of galaxies, eliminating the need for traditional assumptions such as dark matter and black hole spin fitting.

The framework has achieved multiple verifications of the logarithmic-term-containing quantum gravity through black hole shadows (EHT observations), rotation curves of multiple galaxies (astronomical measurements), combined with the dynamics of dark matter halos and the mathematical asymptotic behavior of gravitational lensing. It realizes a unified description of gravity from the microcosmic to the macrocosmic scale, providing observable and reproducible empirical support for quantum gravity theory.

Future research can be further expanded in the following directions: 1) Extend the framework to more extreme scales such as black hole thermodynamics, radio bursts, and galaxy clusters to verify the universal boundary of the logarithmic term; 2) Explore solutions to cosmological puzzles such as dark energy and Hubble tension based on the properties of the logarithmic term; 3) Directly verify the quantum entanglement effect corresponding to the logarithmic term through laboratory simulations (e.g., superfluid helium quantum vortex systems), laying a more solid microcosmic experimental foundation for the theory.

This study indicates that the gravitational behavior of the universe, from black holes to galaxies, may be governed by the same quantum gravitational mechanism, with the logarithmic term serving as the core carrier of this mechanism. It also strongly suggests that black holes and galaxies may share a common topological origin, which we interpret as follows: the overall dynamics of galaxy disks may be the holographic manifestation of the quantum topological structure of their central black holes on the macrocosmic scale through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). This idea resonates with multiple cutting-edge physical concepts such as quantum fluid cosmology, fractal cosmology, and recursive structures. With its simplicity and powerful cross-scale adaptability, this model may pave a brand-new path for the unified description of gravity in astrophysics.

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