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Article

The Critical Hypersurface as a Geometric Origin of Nonsingular Cosmic Expansion

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Abstract

We propose a geometrically motivated framework in which the large-scale evolution of the universe is described by a coherent multidimensional wavefunction possessing a preferred direction of propagation. Within this formulation, the scalar envelope of the wavefunction defines a critical hypersurface whose evolution provides an effective geometric description of cosmic expansion, naturally incorporating an arrow of time, large-scale homogeneity, and a nonsingular expansion history. The critical hypersurface takes the form of a three-dimensional sphere whose radius plays the role of a cosmological scale factor. Its evolution leads to a time-dependent expansion rate with a positive but gradually decreasing acceleration. The associated density evolution follows a well-defined scaling law consistent with the standard stress–energy continuity equation within the effective geometric description adopted here. A central result of the framework is the existence of a conserved global potential-like invariant associated with the geometry of the critical hypersurface and the conserved wavefunction flux. This invariant is not an energy and does not rely on time-translation symmetry; instead, it represents a geometric property of the evolving hypersurface. It uniquely fixes the effective gravitational coupling, allowing Newton's constant to emerge as an invariant global parameter rather than as a phenomenological input. Within this geometric description, the expansion rate and density evolution obey well-defined scaling relations, yielding present-day values consistent with observational constraints. The framework therefore provides a self-consistent picture in which cosmic expansion and gravitational coupling arise from the geometry of a universal wavefunction.

Keywords: cosmology; evolution of the universe; general relativity; gravitation; quantum cosmology

1. Introduction

The origin of the universe remains one of the greatest unsolved mysteries in modern physics. General relativity, widely recognized as the fundamental theory underpinning cosmology, has profoundly enhanced our understanding of spacetime. However, it has never fully explained the universe's inception. Einstein's equations, when extrapolated back to the very beginning of the universe, predict a singularity—a point at which classical physical laws cease to be meaningful. This singularity marks the breakdown of general relativity, necessitating a quantum description. Quantum theory, which primarily addresses phenomena at microscopic scales, is widely believed to be universally applicable, suggesting that the origin of the universe must fundamentally be a quantum event. Although direct observational evidence for quantum gravitational phenomena is on its early stage [1], compelling theoretical arguments indicate that gravity and quantum theory should unify at the Planck scale, approximately defining the initial size of the universe [2–6]. Hence, it is logical to conclude that spacetime itself should be represented by a wavefunction dependent on both matter fields and spacetime geometry.

Over recent decades, significant efforts have focused on mathematically formulating the universe's wavefunction (see [7] for an extensive review). A central challenge in these formulations is

defining appropriate boundary conditions for the wavefunction. Among the most influential proposals is the "no-boundary" condition introduced by Hartle and Hawking, suggesting a spacetime without initial boundaries or singularities [8,9]. Despite its conceptual elegance, the no-boundary condition encounters difficulties when interfacing with general relativity. Under reasonable assumptions regarding the matter content of the universe, the celebrated singularity theorems imply that a curvature singularity must have occurred [8,9]. Resolving this issue necessitates a genuine quantum theory of gravity—a formidable challenge that remains unresolved despite significant advances in string theory, loop quantum gravity, and other quantum gravity frameworks [10–14].

To address this fundamental issue, we introduce a novel concept involving combined boundary conditions. In our model, the boundary conditions for 4-dimensional spacetime arise naturally when solving the most general wave equation describing a multidimensional, unrestricted global wavefunction. A key advantage of our approach is its compatibility with current cosmological observations without relying on specific assumptions drawn from quantum theory or general relativity. Both these theories naturally emerge within the logic of the global wavefunction itself. Within our framework, the global wavefunction dictates all evolutionary aspects of the universe, with spacetime curvature—and thus gravity—emerging as an inherent consequence of this evolutionary process, rather than as the fundamental driver of cosmic expansion. A central outcome of our theory is a conservation law governing the integrated intensity of the wavefunction, which serves as an analog to, though not exactly identical with, the total energy of the universe. This conservation law ensures that our model avoids an initial singularity, providing a coherent and physically consistent description of the universe's origins.

To identify the wavefunction, we adapt a well-established methodology from the theory of coherent electromagnetic waves and singular optics [15]. For this purpose, we start from the most general wave equation in (3+1)-dimensional spacetime [16,17]:

$$\square \Phi^a = 0, \quad (1)$$

where $\Phi^a(x, y, z, t)$ represents one of the two electromagnetic (EM) vector fields—electric ($a = 1$) or magnetic ($a = 2$), $\square = \nabla^2 - \frac{1}{c^2} \partial_t^2$ is the d'Alembertian operator, and $\nabla = \mathbf{e}_x \partial_x + \mathbf{e}_y \partial_y + \mathbf{e}_z \partial_z$ with $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ being Cartesian unit vectors, and c the speed of light.

An infinite coherent EM wave with constant angular frequency ω can be expressed as $\Phi^a(x, y, z, t) = F^a(x, y, z) e^{-i\omega t}$. Substituting this into Equation (1) yields the three-dimensional Helmholtz equation:

$$(\nabla^2 + k^2) F^a = 0, \quad (2)$$

where $k = \omega/c$.

The symmetry of coordinates (x, y, z) in Equation (2) leads to an energy divergence in its exact solutions. This is evident in plane, cylindrical, and spherical wave solutions, all of which have infinite total energy, $\int_{\mathbb{R}^3} |F|^2 d\mathbb{R}^3 = \infty$. This divergence is typically managed by limiting the integration domain or using superpositions to compensate. However, for highly directed coherent EM waves such as laser beams, limiting the domain is often impractical [18]. The only efficient approach is to break the coordinate symmetry in Equation (2).

Consider a wave directed predominantly along the z -axis. Then $F^a(x, y, z) = \tilde{F}^a(x, y, z) e^{ikz}$, and Equation (2) becomes [19]:

$$(\nabla^2 + 2ik\partial_z) \tilde{F}^a = 0. \quad (3)$$

Under the slowly varying envelope approximation, and neglecting the longitudinal component $\tilde{F}_z^a \ll F_\perp^a$, we approximate $\tilde{F}^a \approx F_\perp^a(x, y, z)$, giving the (2+1)D Schrödinger-type equation:

$$(\nabla_\perp^2 + 2ik\partial_z) F_\perp^a = 0, \quad (4)$$

where $\nabla_\perp = \mathbf{e}_x \partial_x + \mathbf{e}_y \partial_y$.

Solutions of Equation (4) can be expressed as the product of a vector modulation function and a scalar background envelope [15,20]:

$$\mathbf{F}_{\perp}^a = \mathbf{U}^a(x, y, z)G(x, y, z), \quad (5)$$

where the scalar envelope [18,19]:

$$G(x, y, z) = \frac{1}{w_0 \xi(z)} \exp\left(-\frac{x^2 + y^2}{w_0^2 \xi(z)}\right), \quad (6)$$

satisfies Equation (4) with $\xi(z) = 1 + iz/z_0$, $z_0 = kw_0^2/2$. The scalar envelope given by Equation (6) represents the complex amplitude of the coherent wave and plays a crucial role in ensuring the wave's finite energy:

$$(2/\pi) \int_0^{\infty} GG^* dS_{\perp} = 1. \quad (7)$$

Here the star denotes complex conjugation, and dS_{\perp} is the differential area element in the transverse xy plane.

While G governs spatial evolution of the EM wave, the vector function \mathbf{U}^a captures its topological and polarisation properties. Unlike the scalar envelope G , the modulation function itself does not satisfy the Equation (4). Although $\mathbf{U}^a(x, y, z)$ is, in general, a function of all three Cartesian coordinates, its equation admits a separation of variables, allowing a distinction between longitudinal and transverse degrees of freedom. By changing variables as $X = x/(w_0 \xi)$, $Y = y/(w_0 \xi)$, and substituting expression (5) into Equation (4), we obtain the differential equation for the modulation function $\mathbf{U}^a(X, Y, \xi)$ [20]:

$$\nabla_t^2 \mathbf{U}^a - 4\xi^2 \partial_{\xi} \mathbf{U}^a = 0, \quad (8)$$

with $\nabla_t = \mathbf{e}_x \partial_X + \mathbf{e}_y \partial_Y$.

This equation admits the separation of variables:

$$\mathbf{U}^a(X, Y, \xi) = \mathbf{u}^a(X, Y)Z(\xi), \quad (9)$$

with $Z(\xi) = \exp[K^2(1 - \xi)/4\xi]$ and K is a separation constant.

The function $\mathbf{u}^a(X, Y)$ satisfies the Helmholtz equation in the two-dimensional transverse space (X, Y) :

$$(\nabla_t^2 + K^2)\mathbf{u}^a = 0. \quad (10)$$

In the special case $K = 0$, the modulation function $\mathbf{U}^a(X, Y, \xi)$ coincides with the function $\mathbf{u}^a(X, Y)$, becoming independent on the longitudinal coordinate ξ , and Equation (10) reduces to the two-dimensional Laplace equation:

$$\nabla_t^2 \mathbf{u}^a(X, Y) = 0. \quad (11)$$

Analyses of possible $\mathbf{u}^a(X, Y)$ are provided in [15].

While Equation (10) represents a two-dimensional analogue of the three-dimensional Helmholtz equation (2), its solutions would exhibit similar issues with energy divergence. However, this is not the case for the general solutions of the wave equation (5), as all singularities associated with Equation (10) are embedded within the higher-dimensional space defined by the finite-energy Gaussian envelope of Equation (6). The former captures the wave's topological structures, while the latter governs its spatial evolution and energy localization.

Conceptual remark: Universal wavefunction and wave analogies

Before proceeding, it is useful to clarify what we will mean by the term *universal wavefunction* in the present work and to explain the role played by classical wave analogies above in its construction.

In this framework, the universal wavefunction is not introduced as a many-body quantum-mechanical state in the conventional sense, but as a geometric wave object defined on a higher-dimensional configuration space. Its defining properties are linear propagation, conserved flux, and the presence of phase and topological structure. These features are shared by a broad class of wave systems and do not rely on a specific microscopic interpretation.

Solutions familiar from Gaussian optics and classical wave theory are therefore employed as illustrative and technically robust examples. Such solutions shown above are used here not because the underlying physics is classical electromagnetism, but because they provide explicit, experimentally validated realizations of wave propagation with well controlled geometry, singularity structure, and envelope dynamics. This analogy serves an expository role, making the geometric content of the construction transparent without restricting the framework to a particular physical field.

Quantum mechanics enters the present theory at a later stage as an effective limit. When the dynamics is restricted to the three-dimensional hypersurface associated with the critical geometry, and when the wave object is reduced to its simplest scalar form, the resulting evolution is governed by familiar Klein–Gordon or Schrödinger-type equations. Accordingly, standard quantum mechanics appears here as a lower-dimensional, reduced description of a more general wave-based structure, rather than as a fundamental starting point.

More generally, classical wave theories, quantum mechanics, and the present higher-dimensional construction can be viewed as different limits or projections of a unified wave framework, distinguished by the dimensionality, internal structure, and admissible degrees of freedom of the underlying wavefunction.

2. Results

2.1. Multidimensional Wave

The previous section demonstrated that reducing the coordinate symmetry of the wave equation yields directed electromagnetic (EM) waves with finite energy. In that case, the reduced equation is effectively three-dimensional, while the complete coherent wave includes a rapidly oscillating time-dependent phase.

We now extend these results to the general case of an abstract $(n + 1)$ -dimensional coherent wave, described by a multicomponent vector function $\Phi^a(x_0, x_1, \dots, x_n)$, where x_0 is the preferred direction of propagation. To do this, we consider the wave equation in its most general form, without assuming any specific physical interpretation:

$$\square \Phi^a = 0, \quad (12)$$

where $\square = \sum_{\alpha=1}^n \frac{\partial^2}{\partial x_\alpha^2} \pm \frac{\partial^2}{\partial x_0^2}$ is the $(n + 1)$ -dimensional Laplace or d'Alembert operator, depending on the sign chosen between the transverse coordinates x_1, \dots, x_n and the longitudinal coordinate x_0 , associated with the predominant direction of propagation.

We further assume that the function depends on one of the coordinates, say x_n , only through an oscillatory phase term.

$$\begin{aligned} \Phi^a(x_0, x_1, \dots, x_n) &= \tilde{\Phi}^a(x_0, x_1, \dots, x_{n-1}) \\ &\times \exp(\pm i p_n x_n), \end{aligned} \quad (13)$$

where p_n is a positive constant. The sign in the phase, which reflects the two possible oscillation directions, can be chosen arbitrarily. Substituting the expression (13) into Equation (12), we obtain the reduced wave equation for $\tilde{\Phi}^a(x_0, x_1, \dots, x_{n-1})$:

$$\left(\square - p_n^2 \right) \tilde{\Phi}^a = 0, \quad (14)$$

where the operator now becomes $\square = \sum_{\alpha=1}^{n-1} \partial^2 / \partial x_\alpha^2 \pm \partial^2 / \partial x_0^2$.

By analogy with directed electromagnetic waves, we can separate out the rapidly oscillating term along the predominant direction x_0 in the function $\tilde{\Phi}^a$:

$$\tilde{\Phi}^a(x_0, x_1, \dots, x_{n-1}) = \Psi^a(x_0, x_1, \dots, x_{n-1}) \times \exp(\pm i p_0 x_0), \quad (15)$$

where p_0 is a positive constant, and the choice of sign reflects the wave's direction along x_0 .

Substituting Equation (15) into Equation (14) yields, after some algebra, an equation for Ψ^a :

$$\left(\sum_{\alpha=1}^{n-1} \frac{\partial^2}{\partial x_\alpha^2} - \{p_n^2 \pm p_0^2\} \pm 2ip_0 \frac{\partial}{\partial x_0} \pm \frac{\partial^2}{\partial x_0^2} \right) \Psi^a = 0. \quad (16)$$

The assumption of predominant propagation along x_0 :

$$\left| \frac{\partial^2}{\partial x_0^2} \Psi \right| \ll p_0 \left| \frac{\partial}{\partial x_0} \Psi \right|, \quad (17)$$

allows us to neglect the second derivative with respect to x_0 in Equation (16), resulting in the reduced wave equation:

$$\left(\sum_{\alpha=1}^{n-1} \frac{\partial^2}{\partial x_\alpha^2} - \{p_n^2 \pm p_0^2\} \pm 2ip_0 \frac{\partial}{\partial x_0} \right) \Psi^a = 0, \quad (18)$$

Until now, we have not made any assumptions about the signs in the operator \square in Equation (14). Assuming that this operator is the d'Alembertian $\square = \sum_{\alpha=1}^n \partial^2 / \partial x_\alpha^2 - \partial^2 / \partial x_0^2$ and taking into account that $p_0^2 = p_n^2$ for a freely propagating wave, Equation (18) simplifies to:

$$\left(\sum_{\alpha=1}^{n-1} \frac{\partial^2}{\partial x_\alpha^2} \pm 2ip_0 \frac{\partial}{\partial x_0} \right) \Psi^a = 0. \quad (19)$$

Equation (19) describes one of two possible waves propagating in opposite directions along the coordinate x_0 , depending on the sign in the equation. For clarity, we now consider propagation along the positive direction of the coordinate x_0 , $\Psi^a = \tilde{\Phi}^a \exp(-ip_0 x_0)$. With this choice, Equation (19) becomes:

$$\sum_{\alpha=1}^{n-1} \frac{\partial^2}{\partial x_\alpha^2} \Psi^a = -2ip_0 \frac{\partial}{\partial x_0} \Psi^a, \quad (20)$$

which is a vector multidimensional Schrödinger-type differential equation. By analogy with electromagnetic waves, the solutions of Equation (20) can be represented as:

$$\Psi^a = G U^a, \quad (21)$$

where $U^a(x_0, x_1, \dots, x_{n-1})$ captures all the vector and topological properties of the wave, and the scalar function $G(x_0, x_1, \dots, x_{n-1})$ represents the wave envelope that governs the properties of the wave propagation:

$$G(x_0, \dots, x_{n-1}) = \left(\frac{\sqrt{2}}{\sqrt{\pi} r_0 \zeta(x_0)} \right)^{\frac{n-1}{2}} \times \exp\left(-\frac{r^2}{r_0^2 \zeta(x_0)}\right), \quad (22)$$

where $r^2 = \sum_{\alpha=1}^{n-1} x_\alpha^2$, $\zeta(x_0) = 1 + 2ix_0 / (p_0 r_0^2)$, and r_0 is a constant representing the initial radius of the $(n-2)$ -sphere that bounds the $(n-1)$ -ball at $x_0 = 0$ (defined such that the amplitude equals e^{-1} of its peak).

2.2. Scalar Envelope of Multidimensional Wave

To facilitate the following discussion, we analyse the scalar wave envelope given by Equation (22), which satisfies the normalization condition and thereby ensures finite wave energy:

$$\int_{\mathbb{R}^{n-1}} GG^* d\mathbb{R}^{n-1} = 1, \quad (23)$$

where the integration is carried out over the entire $(n-1)$ -ball; $d\mathbb{R}^{n-1} = dx_1 dx_2 \dots dx_{n-1}$ denotes an element of the $(n-1)$ -dimensional Euclidean space; and GG^* represents the wave intensity:

$$GG^* = \left(\frac{2}{\pi r_0^2 (1 + 4x_0^2 / p_0^2 r_0^4)} \right)^{\frac{n-1}{2}} \times \exp\left(-\frac{2r^2}{r_0^2 (1 + 4x_0^2 / p_0^2 r_0^4)} \right). \quad (24)$$

Due to the exponential part of the function, it spreads monotonically over the interval $(0 \leq x_0 < +\infty)$. Consequently, all points of equal amplitude in function Equation (24) on the initial hyperplane $x_0 = 0$ evolve with x_0 as follows:

$$R = r_a \sqrt{1 + 4x_0^2 / (p_0^2 r_0^4)}, \quad (25)$$

where R is the radial distance from the axis of symmetry, defined by $r = 0$, to the points of equal intensity at a given x_0 ; and r_a is a parameter that defines the initial distance to those points at $x_0 = 0$. In other words, at any fixed hyperplane x_0 , the expression for the distance R defines the radius of an $(n-2)$ -sphere embedded in $(n-1)$ -dimensional space. In Cartesian coordinates, the equation for this sphere can be written as:

$$\sum_{\alpha=1}^{n-1} x_\alpha^2 = R_{x_0=\text{const}}^2. \quad (26)$$

Any surface from the set Equation (25), parameterized by r_a , can thus be interpreted as a hyperboloid of revolution representing the evolution of the sphere given by Equation (26) with respect to x_0 :

$$\sum_{\alpha=1}^{n-1} x_\alpha^2 - \frac{4r_a^2 x_0^2}{p_0^2 r_0^4} = r_a^2. \quad (27)$$

Each hyperboloid Equation (27) bounds the wave so that:

$$\Omega_{n-1} \int_0^R r^{n-2} GG^* dr = P\left(\frac{n-1}{2}, \frac{2r_a^2}{r_0^2} \right) = \text{const}, \quad (28)$$

where Ω_{n-1} is the surface area of a unit $(n-1)$ -sphere, and P is the regularised incomplete gamma function.

Note that the hyperboloid in the set described by Equation (27) exhibits symmetry among all coordinates when:

$$\frac{p_0 r_0^2}{2} = r_a. \quad (29)$$

Under this condition, the conservation law in Equation (28) simplifies to:

$$\Omega_{n-1} \int_0^R r^{n-2} GG^* dr = P\left(\frac{n-1}{2}, r_a p_0 \right) = \text{const}, \quad (30)$$

with the intensity given by:

$$GG^* = \left(\frac{2}{\pi(r_a^2 + x_0^2)} \right)^{\frac{n-1}{2}} \exp\left(-\frac{2r^2}{r_a^2 + x_0^2} \right), \quad (31)$$

and the surface Equation (27) corresponds to a standard one-sheet hyperboloid:

$$\sum_{\alpha=1}^{n-1} x_{\alpha}^2 - x_0^2 = r_a^2, \quad (32)$$

Equation (32) can be interpreted as an $(n-1)$ -dimensional hyperbolic space, viewed as a submanifold of Minkowski space $\mathbb{R}^{1,n-1}$ with the metric:

$$ds^2 = \sum_{\alpha=1}^{n-1} dx_{\alpha}^2 - dx_0^2. \quad (33)$$

Again, the hyperboloid described by Equation (32) represents the evolution with x_0 of points of equal intensity on the wavefront. At any given x_0 , these points form an $(n-2)$ -sphere, as described by Equation (26), with a radius given by:

$$R = \sqrt{r_a^2 + x_0^2}, \quad (34)$$

The nonlinearity of the function $R(x_0)$, given by Equation (34), results in its infinite differentiability with respect to x_0 :

$$d^j R / dx_0^j = P_j(x_0) / R^{2j-1}, \quad (35)$$

where $P_j(x_0)$ is a polynomial of degree j .

The first three derivatives formally represent the speed:

$$v(x_0, r) = x_0 / R, \quad (36)$$

acceleration:

$$\dot{v}(x_0, r) = r_a^2 / R^3, \quad (37)$$

and jerk:

$$\ddot{v}(x_0, r) = -3r_a^2 x_0 / R^5. \quad (38)$$

of points on the spherical surface described by Equation (34), as they evolve with x_0 along the hyperbolic surface in the coordinate system (x_0, r_{n-1}) . Expressing the speed and acceleration relative to the distance R , we have:

$$v(x_0, r) / R = x_0 / (r_a^2 + x_0^2), \quad (39)$$

$$\dot{v}(x_0, r) / R = r_a^2 / (r_a^2 + x_0^2)^2. \quad (40)$$

In the limit of large positive $x_0 \gg r_a$, these expressions simplify to:

$$v(x_0, r) / R \rightarrow 1 / x_0, \quad (41)$$

$$\dot{v}(x_0, r) / R \rightarrow r_a^2 / x_0^4. \quad (42)$$

These results correlate with the observable electromagnetic wave properties discussed in Section 1, where the spreading and acceleration of a freely propagating $(2+1)(2+1)$ -dimensional wavefunction were thoroughly analyzed and demonstrated experimentally. [15].

2.3. Modulating Functions in Multi-Dimensional Representation

Our findings in the previous subsection revealed the evolution of the scalar envelope $G(x_0, x_1, \dots, x_{n-1})$ Equation (24) of the more general wavefunction Equation (21). This envelope constitutes the empty space or background for all possible vector fields and topological singularities described by the modulation functions \mathbf{U}^a .

To find an equation for the function \mathbf{U}^a , we follow the procedure established for EM waves, Eqs. (8)- (11). We change variables in the function $\mathbf{U}^a(x_0, x_1, \dots, x_{n-1})$ to a new set, $\mathbf{U}^a =$

$\mathbf{U}^a(\zeta, X_1, \dots, X_{n-1})$, where $x_\alpha / (r_0 \zeta(x_0))$ for $\alpha = 1, \dots, n-1$ are new dimensionless variables. Substituting this function into the wave equation (20), we obtain a differential equation for the modulating function:

$$\nabla_T^2 \mathbf{U}^a - 4\zeta^2 \frac{\partial}{\partial \zeta} \mathbf{U}^a = 0, \quad (43)$$

where $\nabla_T = \sum_{\alpha=1}^{n-1} \mathbf{e}_{x_\alpha} \partial_{X_\alpha}$ and \mathbf{e}_{x_α} are unit vectors in the $(n-1)$ -dimensional Cartesian space.

This equation admits separation of variables:

$$\mathbf{U}^a(\zeta, X_1, \dots, X_{n-1}) = \mathbf{u}^a(X_1, \dots, X_{n-1}) X_0(\zeta), \quad (44)$$

where $X_0(\zeta) = \exp(\pm K^2(1 - \zeta)/(4\zeta))$, and K is a real constant.

Then the function \mathbf{u}^a satisfies the vector Helmholtz equation with reduced dimension:

$$\left(\nabla_T^2 \pm K^2\right) \mathbf{u}^a(X_1, \dots, X_{n-1}) = 0. \quad (45)$$

Equation (45) is coordinate-invariant in the new coordinate basis normalised to the $(n-1)$ -dimensional curved space defined by the wave envelope Equation (22). Notably, by comparing this equation with Eqs. Equation (2) and (14) discussed in previous sections, one finds that their solutions would exhibit a similar structure, but in lower dimensions.

When $K = 0$, Equation (45) reduces to the vector Laplace equation:

$$\nabla_T^2 \mathbf{u}^a = 0. \quad (46)$$

This equation describes behavior of scalar and vector fields, including their topological defects and particle-like singularities that may exist within the coherent wave background Equation (22). In 3-dimensional framework, the local topology of the wavefunction may encode the same physical information that is conventionally attributed to particles. This perspective resonates with long-standing ideas that matter has a fundamentally geometrical origin [21–24]. Recent theoretical work has argued that particles and geometry are not independent entities, but rather represent the same underlying quantum degrees of freedom, with particles emerging as stable, information-preserving geometrical structures [25]. Our modulation functions provide a concrete realization of this idea by describing particles as localized topological defects or singularities of the wavefunction. However, the full treatment of possible solutions of both Equation (45) and Equation (46) lies beyond the scope of this paper and will be presented in future work.

2.4. Scalar Wave Envelope in a (3+1)-Dimensional Framework

In the preceding analysis, we examined the general properties of an abstract multidimensional wavefunction without making specific assumptions about its dimensionality or its direct connection to physical spacetime. In this subsection, we concretize the wave described by Equation (21) and its scalar envelope given by Equation (22) by embedding them in a physically motivated four-dimensional coordinate framework. Firstly, it is important to clarify the nature of the coordinate x_0 . Although the mathematical structure of Equation (12) suggests a temporal role for the coordinate x_0 by analogy with the Schrödinger equation in quantum mechanics, it does not necessarily represent physical time in the conventional sense. Rather, it is more accurately interpreted as the coordinate aligned with the predominant direction of wave propagation. For instance, as discussed in the Introduction, the theory of coherent electromagnetic waves under the paraxial approximation leads to Equation (4). While this equation is structurally equivalent to the Schrödinger equation in a $(2+1)$ -dimensional setting, the coordinate $x_0 = z$ corresponds to a spatial axis of wave propagation, thereby acting as a “time-like” coordinate, despite not being temporal in the traditional sense of four-dimensional spacetime. It is also worth noting that the true temporal evolution in that $(2+1)$ -dimensional system is not explicitly present in the equation. Instead, it exists as an external or “hidden” parameter associated with a

higher-dimensional framework, outside the coordinate space where the equation is directly applicable. This logic can be extended to wave equations in arbitrary dimensional spaces, where the principal propagation coordinate naturally assumes the role of a time-like variable within the mathematical structure, even though it may not correspond to actual time in cases with fewer than four dimensions.

In Subsection 2.1, we demonstrated that requiring equal scaling for all Cartesian coordinates in the standard metric given by Equation (33) results in selecting a unique hyperbolic surface described by Equation (32) from the broader set of surfaces defined in Equation (27). This equality in scaling allows us to reinterpret the coordinate x_0 explicitly as a temporal variable through the relationship $x_0 = ct$. With this in mind, we adopt a $(5 + 1)$ -dimensional form for the solution presented in Equation (21), introducing five spatial coordinates (x, y, z, j, g) . Setting the constant $p_5 = m_0 c / \hbar$ and using the relation $p_0 = p_5$ from Equation (18), we can specify the wavefunction given by Equation (13) as follows:

$$\Phi^a(ct, x, y, z, j, g) = \mathbf{U}^a G(ct, x, y, z, j) \times \exp \left[i \left((m_0 c / \hbar) g - (m_0 c^2 / \hbar) t \right) \right] \quad (47)$$

with

$$G = \left(\frac{\sqrt{2}}{\sqrt{\pi} r_0 \zeta(t)} \right)^2 \exp \left(-\frac{r^2}{r_0^2 \zeta(t)} \right), \quad (48)$$

where $\zeta(t) = 1 + i t / t_0$, $t_0 = m_0 r_0^2 / 2 \hbar = 2 / c p_0$, \hbar is the Planck's constant, m_0 is a constant with the dimension of mass, and $r = \sqrt{x^2 + y^2 + z^2 + j^2}$. At this stage, we do not specify the physical meaning of the constants r_0 , t_0 and m_0 , treating them as characteristic geometric scales associated with the critical hypersurface. Their physical interpretation will be discussed in a later part of the article.

According to Equation (24), the envelope intensity Equation (48) becomes:

$$G G^* = \left(\frac{2}{\pi c^2 (t^2 + t_0^2)} \right)^2 \times \exp \left(-\frac{2r^2}{c^2 (t^2 + t_0^2)} \right), \quad (49)$$

which represents a 4-dimensional wave density.

The critical hyperboloid is now expressed as:

$$R(t) = c t_0 \sqrt{1 + t^2 / t_0^2}. \quad (50)$$

This hyperboloid can be described as the continuous evolution of a 3-sphere defined by $R^2(t) = x_1^2 + x_2^2 + x_3^2 + x_4^2$ along the t -coordinate. For a fixed value of t , the radius R of the 3-sphere is determined by the four-component coordinate $r(x_1, x_2, x_3, x_4)$, reflecting the fact that the three-dimensional spherical surface is embedded within a higher-dimensional space. Geometrically, the sphere represents a three-dimensional cross-section formed by the intersection of the four-dimensional ball $B^4 = \{r \in \mathbb{R}^4 : r \leq R\}$ and the hyperplane $t = \text{const}$. The entire space of the wave envelope Equation (48) restricted by the sphere satisfies the conservation law Equation (30):

$$\int_0^R (r^2 G G^*) r dr = \text{const}. \quad (51)$$

On the surface defined by Equation (50), the wave intensity given in Equation (49) simplifies to:

$$I(R) = \left(\frac{2}{\pi e R^2(t)} \right)^2. \quad (52)$$

In general, the radius defined by Equation (50) resides in four-dimensional spatial space B^4 and does not inherently belong to the three-dimensional sphere characterized by intensity Equation (52).

However, we interpret de Sitter space as a spatial sphere Equation (26) evolving together with a fixed wavefront as the ambient time coordinate t changes. In this representation, any cross-section through the center of the sphere reduces spatial dimension on a constant t -slice, resulting in a lower-dimensional sphere with the same radius R . Thus, the three-dimensional space, or 3-ball, characterized by the wavefunction intensity Equation (52), is bounded by a closed two-dimensional sphere of radius $R(x, y, z)$.

The volumetric density within the 3-ball depends explicitly on the radius R as:

$$\rho(R) = R^2 I(R), \quad (53)$$

and remains constant at every point within the 3-ball at any fixed time, but according Eqs. (50) and (52), it evolves with time as:

$$\rho(t) = \rho_0 / (1 + t^2/t_0^2), \quad (54)$$

where ρ_0 is initial density at $t = 0$.

With this definition, the conservation law given by Equation (51) can be represented in (3+1)-dimensional spacetime as:

$$\oint_S \rho dS = \rho S = \text{const}, \quad (55)$$

where dS represents a 2-sphere area element over the t -slice intersection.

Consequently, we derive relation Equation (55), analogous to the conservation law Equation (51), but with a distinct interpretation resulting from integration over the enclosed 2D surface of the homogeneous 3-ball. Specifically, we can rewrite this relation in terms of a 3D spatial coordinates:

$$\int_V \frac{\rho}{r} dV = \frac{\rho V}{R} = \text{const}, \quad (56)$$

where the integration is performed within the three-dimensional space (a 3-ball $B^3 = \{r \in \mathbb{R}^3 : r \leq R\}$) with volume $V_{t=\text{const}} = \int_{\mathbb{R}^3} d\mathbb{R}^3$, $d\mathbb{R}^3 = dx dy dz$.

Interpreting the density $\rho(t)$ of the 3-ball as mass-energy density, $\rho(t) = m/V$, we obtain:

$$\frac{m}{r_a \sqrt{1 + t^2/t_0^2}} = \frac{m_0}{r_a}. \quad (57)$$

Since the density is distributed homogeneously within 3-dimensional space at any given moment t , the mass-energy m at each instant is also uniformly distributed.

For further estimations, it is preferable to express relation Equation (57) in the form:

$$\rho V / R = m_0 / r_a. \quad (58)$$

It is also worth noting that the mass-energy of the 3-ball increases with time, while the relation Equation (57) itself remains invariant.

3. Physical Interpretation and Cosmological Implications

We now discuss a possible physical interpretation of the wave described by Equation (47) and its scalar envelope Equation (48), with the aim of connecting the mathematical construction to physically meaningful scales. We also discuss the most significant results of the previous analysis and their connection to observable physical phenomena. First, we establish the connection between the parameters of the wavefunction (Equation (47)) and the corresponding physical space.

3.1. Definition of Constants

The phase of the multidimensional wavefunction introduces periodicity into the wave distribution across a static coordinate system, implying a time evolution of points along a particular wavefront.

In this formulation, the three-dimensional space is represented as the intersection of the propagating wavefront with the “ray” hyperboloid (Equation (50)), which traces the evolution of points of equal intensity on the wavefront. As the wavefront advances, this intersection, corresponding to the three-dimensional space, dynamically evolves over time. This evolution includes spatial expansion with positive acceleration, as described by Eqs. (37) and (40).

Such a scenario leads to the compelling idea that the entire universe may have originated from a single, freely evolving multidimensional particle-wave. The concept refers to modeling the universe as a single coherent quantum object (a global wavefunction) characterized by a minimal action scale and a corresponding characteristic mass/time scale. It does not imply that the universe literally contains only one particle in the conventional sense, but rather that the cosmological evolution is encoded in a single coherent wavefunction mode.

Considering this concept, we now specify the physical meaning of the constants r_0 , r_a , t_0 , and m_0 , which have so far been introduced without explicit interpretation. In the following, we identify $r_0 = r_a$ as the initial radius of the critical three-sphere at the onset of evolution, $t = 0$. This radius defines an associated characteristic time scale t_0 through the speed of light c , according to $t_0 = r_0/c$. The parameter m_0 characterises the initial energy scale of the particle-wave in terms of an initial effective mass. These constants are not independent. They are additionally constrained by the relation given in Equation (29), which yields $t_0 = m_0 r_0^2 / 2 \hbar$.

As a consequence, the evolution of the three-sphere is fully specified by two independent parameters, which may be chosen as the initial radius r_a and the mass scale m_0 . These constants should be related to the Planck units at least in order of magnitude. Such an assumption is reasonable, as according to recent theoretical perspectives, the universe originated with an initial average volume approximately of Planck order l_p^3 [26,27], with $l_p = \hbar / m_p c$. The other parameters, such as the initial mass m_0 and the Compton wavelength λ , should also be of the Planck scale.

To determine the constants r_a and m_0 , we will use the fact that the Planck scale is the natural domain in which quantum localization and gravitational collapse must be treated on equal footing. At this scale, the classical description of a black hole converges with the single-particle quantum description. Regardless of the mass m_0 , the geometric mean of the two characteristic lengths, the Schwarzschild radius r_s and the reduced Compton wavelength $\bar{\lambda}(m_0) = \hbar / (m_0 c)$, is pinned to the Planck scale with the invariant product [28,29]:

$$r_s \bar{\lambda}(m_0) = 2 l_p^2. \quad (59)$$

Assuming that the localized space of mass m_0 has the minimal possible energy state corresponding to the Planck energy, $\varepsilon_p = m_p c^2$, and that the Schwarzschild radius r_s coincides with the initial wave localization r_a , we finally find:

$$\begin{aligned} r_a &= 2 l_p, \\ m_0 &= m_p. \end{aligned} \quad (60)$$

This also yields the dependent parameter $t_0 = r_a/c = 2t_p$, indicating that the characteristic time scale of the critical hypersurface is of order the Planck time.

3.2. Transition to Cosmology

The geometric construction developed in previous sections describes the evolution of a critical hypersurface associated with the scalar envelope of a universal wavefunction. While this construction is formulated independently of any specific gravitational theory, its time-dependent geometry naturally lends itself to a cosmological interpretation. In homogeneous and isotropic settings, cosmological evolution is commonly characterized by a single scale parameter describing the expansion of space. Since the radius R of the critical hypersurface plays an analogous geometric role, it is both legitimate and useful to reinterpret its evolution in terms of effective cosmological variables. This reinterpreta-

tion does not impose additional dynamical assumptions, but provides a convenient framework for comparing the geometric evolution of the hypersurface with standard cosmological observables.

With constant definitions Equation (60) and considering that $V = (4/3)\pi R^3$, the relation Equation (58) takes the form:

$$R^2 \rho = \frac{3 m_p}{8 \pi l_p}. \quad (61)$$

It is worth analyzing Equation (56) and the corresponding Equation (58) in detail, particularly in relation to the definition of the associated constant. Note that the integrand on the left-hand side of Equation (56) has the form of a potential, which can be denoted as ϕ :

$$2\rho V/(Rc^2) = \phi/c^2, \quad (62)$$

Since the relation Equation (58) is a constant, the potential must also remain constant. Taking into account that $r_a/m_0 = 2l_p/m_p$, as given by Equation (60), the constant associated with this potential corresponds precisely to the gravitational constant, G_N :

$$\phi/c^2 = 1/G_N, \quad (63)$$

This relation shows that the potential of the universe remains unchanged from the initial Planck-scale state through all subsequent stages of its evolution. The gravitational constant is deeply connected to this potential and can be derived not only from formal relationships involving Planck units, $G_N = c^2 l_p / m_p$, but also from observational data of the recent universe:

$$G_N = \frac{c^2 R}{2V\rho}. \quad (64)$$

Conversely, this relation can also work in reverse and be used to predict parameters that cannot yet be precisely determined through direct observation. This important result also aligns with earlier indications of a relationship between the potential of the universe's total mass and the phenomena of inertia via the gravitational constant, as suggested in previous works [30,31]. While these earlier predictions are indeed noteworthy, our results suggest that the gravitational constant originates from a global property of 'empty' space, wherein the total mass increases over time according to Equation (57). A consequence of the relationship Equation (64) is the intriguing idea that the equivalence between inertia and gravity, traditionally expressed in terms of mass, is fundamentally linked to the wave envelope of the universal wavefunction.

This important result points to an intrinsic connection between the universal wavefunction, which is fundamentally a quantum object, and the key gravitational properties of space.

3.3. Continuity Equation and Density Evolution

The fundamental conservation law in the present framework is the conservation of wavefunction flux through the critical hypersurface. As shown in Sec. 4, this conservation takes the form

$$\oint_S \rho dS = \rho S = \text{const}, \quad (65)$$

where $S = 4\pi R^2(t)$ is the area of the two-sphere defined by the intersection of the evolving hypersurface with a constant- t slice. Equation (65) expresses conservation of flux through the boundary surface and does not assume conservation of energy or mass within a three-dimensional volume. Here ρ denotes the volumetric mass density associated with the wave content enclosed by the hypersurface. The appearance of ρ under the surface integral reflects the homogeneous radial flux implied by the symmetry of the hypersurface. Accordingly, Equation (65) encodes a geometric conservation law tied

to the evolution of the hypersurface. Here ρ corresponds to the normal component of the conserved current through the cross-section S , The constancy of equation (65) immediately implies the scaling

$$\rho(t) \propto R^{-2}(t). \quad (66)$$

Equivalently, this surface-flux conservation law may be rewritten in volume form as

$$\int_V \frac{\rho}{r} dV = \frac{\rho V}{R} = \text{const}, \quad (67)$$

where the integration is performed over the three-dimensional ball enclosed by the hypersurface. This representation highlights that the conserved quantity is associated with the boundary geometry rather than with the bulk three-volume itself.

Differentiating the relation $\rho R^2 = \text{const}$ with respect to the evolution parameter t yields a continuity-type relation,

$$\dot{\rho} + 2\frac{\dot{R}}{R}\rho = 0. \quad (68)$$

This equation expresses the preservation of the conserved global potential (equivalently, $m/R = \text{const}$) under the expansion of the hypersurface and follows directly from surface-flux conservation.

Introducing the scale factor,

$$a(t) \equiv \frac{R(t)}{c t_0}, \quad (69)$$

and the corresponding expansion rate

$$H \equiv \frac{\dot{a}}{a} = \frac{\dot{R}}{R}, \quad (70)$$

Equation (68) may be written compactly as

$$\dot{\rho} + 2H\rho = 0. \quad (71)$$

This relation is purely geometric in origin and does not rely on any assumption of energy or mass conservation.

For comparison with standard cosmological notation, it is convenient to embed this identity into the Friedmann–Robertson–Walker continuity form written for a split source,

$$\dot{\rho} + 3H\left(\rho + \frac{p_{\text{eff}}}{c^2}\right) = -\dot{\rho}_\Lambda. \quad (72)$$

Here p_{eff} denote an effective parameter associated with the non-vacuum sector fixed by the conserved global potential, while

$$\rho_\Lambda(t) \equiv \frac{\Lambda(t)c^2}{8\pi G} \quad (73)$$

represents the geometric vacuum contribution arising from the trace part of the Einstein tensor. Here $\Lambda(t)$ is defined as the scalar trace component of the Einstein tensor constructed from the induced hypersurface geometry, isolated under a Friedmann–Robertson–Walker decomposition. Substituting $\dot{\rho} = -2H\rho$ into Equation (72) yields

$$\rho + 3\frac{p_{\text{eff}}}{c^2} = -\frac{1}{H}\dot{\rho}_\Lambda. \quad (74)$$

Equation (77) shows that the pressure-like quantity p_{eff} is no longer fixed solely by the conserved scaling of ρ_{eff} when the geometric vacuum contribution $\rho_\Lambda(t)$ is time dependent. Solving Equation (77) explicitly for p_{eff} yields

$$\frac{p_{\text{eff}}}{c^2} = -\frac{1}{3}\rho - \frac{1}{3H}\dot{\rho}_\Lambda. \quad (75)$$

Accordingly, the corresponding equation-of-state parameter becomes

$$w_{\text{eff}}(t) \equiv \frac{p_{\text{eff}}}{\rho c^2} = -\frac{1}{3} - \frac{1}{3H} \frac{\dot{\rho}_\Lambda}{\rho}. \quad (76)$$

Thus, in the general case, where $\Lambda(t)$ inherits a nontrivial time dependence from the embedding geometry, the effective equation-of-state parameter becomes time dependent even though the invariant scaling $\rho \propto a^{-2}$ remains exact. The quantities p_{eff} and w_{eff} should therefore be understood as parameters encoding the geometric decomposition of the Einstein tensor rather than as thermodynamic properties of a conserved fluid.

The energy contained in a comoving volume, $E \sim \rho a^3 \propto a$, increases with the time parameter t , which is allowed in expanding spacetime and naturally associated here with the effective negative pressure Equation (78). The background behaves like the curvature-like term proportional to a^{-2} , in agreement with the hyperboloid geometry of the critical surface. Gravitational dynamics are therefore incorporated directly into the expansion law. Gravity does not act as an "external force", but appears as a constant potential of the evolving wavefunction that governs the coupled behaviour of density and cosmic expansion. In this picture, the expansion is not independent of gravitational effects. Gravity emerges from the constant global potential described by Equation (64), and the quantity G_N serves as the link between this intrinsic property and observational cosmology. Expansion and gravity are two aspects of the same conservation principle.

3.4. Relation to Einstein–Friedmann Dynamics

The analysis presented above establishes the expansion law and density scaling directly from the geometry of the critical hypersurface and the associated conserved global potential. No gravitational field equations or cosmological time parametrisations are assumed in this construction.

The purpose of the present work is therefore not to re-derive the full Einstein–Friedmann dynamics, but to identify the geometric invariants and scaling relations that govern the evolution of the hypersurface itself. When the induced geometry is subsequently interpreted in an intrinsic space-time framework, these relations admit a natural embedding within an Einstein–Friedmann–type description.

A detailed geometric derivation of the corresponding Einstein tensor, the identification of an effective cosmological term, and the resulting Friedmann and acceleration equations—allowing for a time-dependent vacuum contribution—are developed in [32]. In that formulation, the quantities ρ_{eff} and p_{eff} introduced here correspond to components of a geometric decomposition of the Einstein tensor rather than to independently conserved thermodynamic variables.

The present paper thus focuses on the kinematic and geometric origin of cosmic expansion from the universal wavefunction, while the full dynamical closure and its cosmological implications are analysed separately.

3.5. Late-Time Asymptotic Coasting Regime

In the special case where the geometric vacuum contribution becomes time independent, $\dot{\rho}_\Lambda = 0$, Eqs. (78)–(79) reduce identically to $p_{\text{eff}} = -\rho c^2/3$ and $w_{\text{eff}} = -1/3$, admitting an exact coasting expansion $\ddot{a} = 0$. This case represents a particular realisation of a late-time regime of the present framework.

More generally, the expansion law in Equation (50) approaches linear behaviour asymptotically at late cosmological times $t \gg t_0$.

In this late-time regime of interest, the geometric vacuum contribution does not affect the leading-order scaling of the expansion. As a result, the effective pressure introduced in the Friedmann–Robertson–Walker embedding may be interpreted as the total pressure, with $p \simeq p_{\text{eff}}$ asymptotically. The source entering Equation (77) then approaches $\rho + 3p/c^2 \rightarrow 0$.

In this limit the induced dynamics becomes effectively coasting, even though the geometric vacuum contribution may remain time dependent.

Linear expansion histories of this type have appeared previously in several contexts, most notably in the Milne universe and in the so-called $R_h = ct$ cosmology [33–35]. The Milne model corresponds to an empty spacetime and exhibits linear expansion as a purely kinematic effect, whereas $R_h = ct$ cosmologies describe a non-empty universe whose total dynamics approaches a coasting regime.

In the present work, linear expansion does not represent a separate phenomenological assumption, but arises naturally as the *late-time asymptotic limit* of the geometric evolution of the critical hypersurface. In this regime, the dynamics governed by the conserved global potential dominates, and the resulting evolution approaches a coasting behaviour equivalent to that underlying $R_h = ct$ cosmologies.

Accordingly, cosmological models characterised by linear expansion, including $R_h = ct$ constructions, are recovered here as asymptotic realisations of the more general contraction developed in this work, rather than as independent dynamical prescriptions.

In this sense, the present model provides a geometric foundation for $R_h = ct$ -type cosmologies as asymptotic solutions realised at late times, while embedding them within a broader quantum-geometric framework in which accelerated expansion and evolving curvature arise naturally from the structure of the universal wavefunction.

4. Interpretational Perspective and Comparison with Observable Variables

The properties of critical hypersurface demonstrated above shows that the geometric evolution derived from the universal wavefunction can be interpreted consistently within standard general relativity (GR). At the same time, the wavefunction-based formulation offers an alternative and deeper perspective, in which the expansion of the universe is governed fundamentally by the geometry of a critical hypersurface and an associated global conservation law. From this viewpoint, GR equations arise as an effective description of the same underlying evolution when expressed in conventional cosmological variables.

This dual interpretation highlights the complementary roles of geometry and gravitation in the present framework: the wavefunction geometry determines the global expansion, while general relativity provides an effective macroscopic description.

To compare our model with observational data, we require at least one robust and widely accepted reference parameter. All other parameters can then be determined on the basis of this reference through the evolution function Equation (50) and the relations Equation (61) or Equation (64). In this work, we adopt the age of the universe as our primary reference point: $t \cong 4.36 \times 10^{17}$ s [36]. Taking this adoption and using Equation (50) under the assumption that the evolution time parameter $t \cong R/c \cong \tau$ holds in the current epoch ($t \gg t_0$), the corresponding radius R of the 3-sphere is:

$$R \cong ct = 1.307 \times 10^{26} \text{ m}, \quad (77)$$

To find a distance on the surface of the 3-sphere, which represents our actual three-dimensional space, we consider a geodesic or great circle on the sphere by multiplying the radius by 2π . This calculation yields the estimated size of the universe as $d \cong 8.2 \times 10^{26}$ m, which is very close to the accepted diameter of the universe, $d \approx 8.8 \times 10^{26}$ m. According to Equation (61), the corresponding density is:

$$\rho = 3 m_p / 8 \pi l_p R^2 = 9.409 \times 10^{-27} \text{ kg/m}^3. \quad (78)$$

The derived density falls within the widely accepted range of 9.1×10^{-27} to 9.9×10^{-27} kg/m³ [36]. Thus, the relationship described by equation Equation (61) holds remarkably well. However, it is important to note that we consider here only the scalar component of the total wavefunction (Equation (47)), which corresponds to the “empty” or background space. The total density may also include sources of physical fields that are associated with the modulation component of the total wavefunction, as discussed in Section 3, and are beyond the scope of the present analysis.

4.1. Expansion Rate and Acceleration

In Subsection 2.2, we derived general expressions for the speed and acceleration of the critical surface of the wavefunction Eqs. (36)- (42). We can now apply these expressions to the evolution of the 3-sphere associated with space. According to Equation (36), the dimensionless speed evolves nonlinearly, starting from zero and asymptotically approaching unity as $x_0 \rightarrow \infty$. Considering the case $x_0 = ct \gg ct_0$, we have:

$$v(r, x_0) = dR/dx_0 \rightarrow x_0 / R \cong 1. \quad (79)$$

This means that the dimensional speed of the sphere's expansion asymptotically approaches the speed of light as $t \rightarrow \infty$. Again, using the accepted age of the universe $t \cong 4.36 \times 10^{17}$ s, we can express this result relative to the distance R (Eqs. (39), (41)), that is in standard cosmological units:

$$H = v(r, x_0)/t \cong 71 \text{ km/s/Mpc}. \quad (80)$$

This result is in close agreement with the most accurate values currently accepted [36–38], and it is not surprising that it coincides with the inverse of the standard definition of the age of the universe in terms of the Hubble parameter. However, this coincidence arises only in the asymptotic limit of large times. During earlier epochs, the relation given by Equation (80) does not hold, and the more general expression in Equation (39) must be used instead.

The nonlinear behaviour of the expansion rate, as predicted by the model at the end of Section 2, results in a non-constant accelerated expansion (Equation (37)), which can be expressed in dimensional units:

$$\dot{v}(r, t) = c^2 r_a^2 / (r_a^2 + c^2 t^2)^{3/2}. \quad (81)$$

Unlike the speed Equation (36), which is zero at the initial moment $x_0 = 0$, the acceleration reaches its maximum at that instant. Therefore, it is valuable to calculate this initial acceleration value at $t = 0$:

$$\dot{v}(r, 0) = c^2 / r_a = 2.8 \times 10^{51} \text{ m/s}^2. \quad (82)$$

At the current epoch, the estimated value for the acceleration is:

$$\dot{v}(r, t) = r_a^2 / (ct)^3 = 4.2 \times 10^{-131} \text{ m/s}^2. \quad (83)$$

Note that we provide these numbers solely to illustrate the change in magnitude during the evolution of the wavefunction, since in these units, it is unclear what physical quantity is undergoing acceleration. The physically meaningful quantity is the acceleration per unit space, or equivalently, the acceleration of the 3-sphere radius, represented as $\dot{v}(r, x_0)/R$ (Equation (40)). At the present time this acceleration can be expressed as:

$$\dot{v}(r, t) / R \cong 1.3 \times 10^{-114} \text{ km/s}^2/\text{Mpc}. \quad (84)$$

At any moment t , the acceleration given by Equation (84) remains positive. The fact that the speed of expansion depends on the sign of time, while the acceleration does not, directly reflects how time asymmetry influences the universe's evolution. Importantly, the acceleration continuously diminishes over time, and in the limit $t \rightarrow \infty$, the universe approaches a state of constant-rate expansion. This result aligns closely with recent observational data [39,40]. The large-scale analysis of DESI data also suggests that the universe continues to expand, but the rate of acceleration is decreasing [41,42].

It is important to emphasize that the notion of acceleration depends on the choice of time parametrisation. When the expansion is described with respect to the intrinsic geometric evolution parameter of the wavefunction hypersurface, the radius evolves with a nonzero and generally time-dependent acceleration. This behaviour reflects the curvature and embedding geometry of the hypersurface and should not be interpreted as a gravitational acceleration acting on comoving observers.

5. Discussion

In this section, we focus on the implications of relation Equation (61), which provides a central conceptual link between the wavefunction-based construction and gravitational geometry, and constitutes one of the key results of the present analysis. While the critical hypersurface, Equation (50), and the associated three-sphere arise naturally from the scalar envelope of the universal wavefunction, Equation (48), the identification of the corresponding radius with both the Compton wavelength and the Schwarzschild radius is a necessary consistency requirement underlying Eqs. (52)–(57). Although the Schwarzschild radius is conventionally defined within three-dimensional spatial geometry, theoretical arguments indicate that analogous horizon constructions admit meaningful generalisations to higher-dimensional settings [43]. From this perspective, the critical three-sphere may be interpreted as the horizon of a four-dimensional black hole, with its radius determined by Equation (50).

In this sense, the present model supports the long-standing idea that the Universe itself may exhibit black-hole-like characteristics [44–50], but with an important conceptual distinction. In conventional interpretations, the Universe is often envisaged as residing within, or originating from, the interior of a black hole embedded in a three-dimensional spatial geometry. By contrast, the present construction identifies the Universe with the horizon of a black hole in a higher-dimensional spatial setting.

This distinction becomes apparent when lower-dimensional configurations are considered explicitly. For a three-dimensional spatial geometry, corresponding to $n = 4$ in Equation (22), the critical surface described by Equation (26) reduces to a two-sphere enclosing approximately 74% of the total wave intensity given by Equation (30). In this case, applying the Schwarzschild criterion of Equation (60) reveals that the relations in Eqs. (51)–(56) cannot be satisfied beyond the initial instant $t = 0$. The resulting Planck-scale black hole exists only momentarily, as the subsequent expansion governed by Equation (36) immediately invalidates the horizon condition for $t > 0$. This behaviour is consistent with modern perspectives on primordial black holes, which emphasise their transient character at early times [51].

By contrast, the construction relevant to the observed Universe is naturally formulated in a four-dimensional spatial configuration, corresponding to $n = 5$ in Equation (22). In this case, the increase of the critical radius, Equation (50), is exactly balanced by the growth of the enclosed volumetric mass–energy m in Equation (57), such that the three-dimensional space consistently satisfies a critical black-hole-like condition throughout its evolution. A notable geometric consequence of this balance is that the critical surface encloses a fixed fraction of the integrated wave intensity, approximately 60%, as follows from Equation (51).

The region described by Equation (51), corresponding to the portion of the universal wavefunction bounded by the critical surface, is therefore causally disconnected from the three-sphere representing observable space. This separation arises purely from the geometry of the evolving hypersurface and does not rely on additional dynamical assumptions. From an effective cosmological viewpoint, the presence of such a causally disconnected sector suggests a natural geometric origin for components that do not participate directly in local expansion dynamics, without invoking additional matter fields or phenomenological energy components.

The complementary portion of the wavefunction lying outside the critical surface may, in principle, admit an interpretation in terms of boundary or horizon degrees of freedom, reminiscent of holographic ideas [52,53], but extended to a higher-dimensional wave-geometric setting. A particularly suggestive feature of the present construction is the surface-based nature of its fundamental conservation law. The conserved quantity governing the evolution is not associated with a bulk volume, but with the flux of the wavefunction through the critical hypersurface, $\rho S = \text{const}$, Equation (65). The corresponding continuity-type relation, Equation (68), expresses the preservation of a global potential determined by surface geometry rather than by local volumetric conservation.

This intrinsically surface-like character of the conserved invariant naturally evokes a holographic perspective, in which essential dynamical information is encoded on a lower-dimensional boundary.

In the present case, however, this behaviour follows directly from wavefunction geometry and flux conservation, without invoking entropy bounds or microscopic degrees of freedom on the surface. The critical hypersurface thus plays a dual role: it defines both the geometric boundary of observable three-dimensional space and the carrier of the conserved global potential governing the expansion.

We emphasise that such interpretative extensions remain speculative at this stage and lie beyond the scope of the present work, which is concerned primarily with establishing the geometric structure and conservation laws associated with the critical hypersurface itself.

6. Conclusions

In conclusion, this work develops a geometric framework in which the large-scale evolution of the universe emerges from a multidimensional universal wavefunction. Without postulating gravitational field equations, inflationary dynamics, or an initial singularity, the model provides a self-consistent description of the emergence of time, cosmic expansion with positive and time-dependent acceleration, and large-scale homogeneity.

By representing the universe as a coherent multidimensional wavefunction with a preferred direction of propagation, the framework naturally accounts for the arrow of time and the uniformity of three-dimensional space, which appears as a three-sphere arising from temporal slices of the evolving critical hypersurface. In this construction, the expansion of space is governed by the geometry of the hypersurface rather than by postulated matter components or phenomenological dark-energy terms. The geometric vacuum contribution introduced in the effective description originates from the trace part of the Einstein tensor associated with the induced geometry and does not correspond to a material energy component.

A central result of the model is the existence of a conserved global potential associated with the wavefunction flux through the critical hypersurface. This invariant fixes the density scaling of the expanding universe and leads directly to the emergence of the gravitational coupling constant as a geometric quantity. The resulting connection between wavefunction structure and gravitational coupling provides a direct link between quantum coherence and large-scale gravitational properties.

The geometric evolution of the critical hypersurface yields a nonsingular expansion history with a monotonically increasing radius and a time-dependent acceleration. Analytical estimates of the expansion rate and mean density are found to be compatible with current observational constraints. When the induced geometry is expressed in a Friedmann–Robertson–Walker form, the corresponding continuity relation takes a sourced form, reflecting the geometric decomposition of the Einstein tensor rather than independent conservation of fluid components.

At late cosmological times, the dynamics approaches a coasting regime in which the effective source approaches $\rho + 3p/c^2 \rightarrow 0$. In this asymptotic limit, the expansion becomes effectively linear and recovers the behaviour underlying $R_t = ct$ -type cosmologies. In the present construction, such models arise naturally as late-time asymptotic realisations of the underlying geometric evolution, rather than as fundamental dynamical assumptions.

Overall, the results demonstrate that the geometry of a universal wavefunction provides a coherent and predictive description of cosmological expansion, unifying conservation laws, gravitational coupling, and large-scale dynamics within a single geometric construction. This approach offers a natural basis for further investigation of emergent gravitational behaviour and the quantum origins of spacetime.

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