Article

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Beyond a Naive Absolute Infinite

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Abstract

This paper proposes an axiomatization of the absolute infinite within a non-recursively enumerable class theory, called MK^{meta}, that maximally extends the formal MK: Morse–Kelley with global choice (GC). Class ordinals and class cardinals avoid the Burali-Forti paradox and GC is assumed to warrant comparability of class cardinals. A Hamkinsian multiverse M_h is defined as the collection of all the models v of any syntactically consistent, formal extension of MK. MK^{meta} is then rigorously defined by ranging over M_h and has V^{meta} as its unique model. At last, the absolute infinite $\Omega^{meta} = \mathrm{Ord}^{meta}$ is derived from V^{meta}. Informal, formal, and formal-based theories, having increasingly many axioms, are strictly weaker than the meta-formal theory MK^{meta}, which has absolutely infinitely many axioms. Moreover, truth relativism is countered by MK^{meta}, which accepts those axioms that maximize V^{meta}. Consequently, the definition of M_h can be used as a rebuttal of both height and width potentialism, when combined with the argument that only the meta-formal level can capture the entire mathematical reality in a single rigid theory.

Keywords: meta-formalism; Cantor's absolute infinite; Morse-Kelley class theory; set-theoretic multiverse; categoricity; actualism vs. potentialism

MSC: 03A05; 03E70, 03E55, 03E65, 03C62

1. Introduction

In a letter to Dedekind, Cantor called his absolute infinite Ω[1] an inconsistent, absolutely infinite multiplicity and he associated it with God (Cantor 1962, Thomas-Bolduc 2016). The idea of an absolute infinite can be used for various other purposes than theology: in metaphysics and modal realism to describe the size of the plenitude (Blondé 2024) or Lewis' (1986) logical space, in computer science to have the ultimate oracle (Burgin 2017) or ordinal[2] machine (Koepke and Seyfferth 2009), in epistemology to formulate omniscience (Heylen 2020), and in the philosophy of mathematics to indicate the cardinality of the collection of all abstract entities that have a formal definition in the rich mathematical landscape (Sutto 2024).

Cantor's idea continues to provoke controversy. Welch and Horsten (2016) review Cantor's conception of the set-theoretic universe as a completed infinity and prefer it above Zermelo's (1908) conception, because Cantor's universe includes the modern large cardinals via reflection principles. Livadas (2020) also discusses Cantor's absolute infinite in light of the modern large cardinals and argues that it is proof-theoretically unattainable. Gutschmidt and Carl (2024) maintain that the foundational problem with Cantor's absolute infinite calls for humility, rather than negative theology.

Set-theoretic potentialism, which has garnered significant attention in the last decade, is the view that the process of set formation is incompletable or inexhaustible and that the classical universe of sets V and the absolute infinite Ω cannot be fully captured or defined as actual, completed totalities

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[1] Not to be confused with Woodin's (2011) Ω nor any other non-maximally large infinite cardinals.

[2] While ordinals are inherently ordered, cardinals focus on the notion of 'how many' without regard to order. Assuming the axiom of choice, every set x can be well-ordered. In this case, every cardinal $|x|$ can be identified with the first ordinal that has size $|x|$.

(Zermelo 1930, Putnam 1967, Parsons 1983, Hamkins 2012, Linnebo 2013, Hellman and Shapiro 2018, Linnebo and Shapiro 2019, Brauer et al. 2022, Sutto 2024). Height potentialism can be distinguished from width potentialism. Height potentialism is the claim that certain large collections, such as the collection of all natural numbers or the collection of all sets, are not actual, but only potential collections. Width potentialism, on the other hand, asserts that there are no privileged axiomatic truths and that there is, therefore, no privileged universe of sets. Instead, there is a wide set-theoretic multiverse of potential multiverse-universes that are equally legitimate (Hamkins 2012, Scambler 2020, Meadows 2021, Gorbow 2022). For example, a multiverse-universe in which the Continuum Hypothesis (CH) is true is not better or worse than one in which CH is false.

In spite of this, the aim of this paper is to show that an absolute infinite Ω^{meta}, that can neither be proven nor defined in any formal theory, can be proven in an idealized meta-formal theory with classes.[3] The definitions of Ω^{meta} and V^{meta} can be obtained via a definition of the set-theoretic multiverse and counter both height and width potentialism.

In the next section, class ordinals and class cardinals are formalized in Morse–Kelley set theory with the axiom of Global Choice (GC), henceforth referred to with the acronym MK (Wang 1949). In Section 3, meta-formal concepts are introduced as an idealization of formal concepts. Section 4 contains the definitions of MK^{meta}, V^{meta}, and the absolute infinite Ω^{meta}. After that, in Section 5, a range of objections to these definitions is rebutted. In Section 6, four flavors of formality are discussed: informal, RE-formal, formal-based, and meta-formal theories. At last, the conclusions follow in Section 7.

2. Class Ordinals and Class Cardinals

Class ordinals and *class cardinals* will first be defined as straightforward generalizations of ordinals and cardinals, which will be called *set ordinals* and *set cardinals* for clarity. The only rationale for this generalization is to create two terms to refer to the absolute infinite in an appropriate class theory, namely the proper class ordinal and the proper class cardinal. By adding the axiom of global choice (GC) to Morse–Kelley set theory (hence creating MK), class cardinals become well-orderable.

2.1. Class Ordinals

A set α is a set ordinal if and only if (henceforth iff):

1. α is a transitive set.
2. $(\alpha, \in)$ is a well-ordering.

The Burali-Forti paradox shows that the collection of all set ordinals Ord cannot be a set. If such a collection would be a set, it would exceed itself in size. However, a collection that is too large to be a set, such as Ord, can still be a class, namely a proper class (von Neumann 1928). Every set is a class, namely a class that is an element of another class, but not every class is a set. In particular, proper classes, which are classes that are not an element of any other class, are not sets. In this paper, MK will be used when a formal second-order theory about sets and classes is needed. With this notion of classes, class ordinals can be defined as an extension of set ordinals. A class C is a class ordinal iff:

1. C is a transitive class.
2. $(C, \in)$ is a well-ordering.

A class C is transitive iff whenever

$$x \in C \land y \in x \to y \in C \tag{1}$$

$(C, \in)$, with C some class, is a well-ordering iff every non-empty set $s \subseteq C$ has an $\in$-least element:

$$\forall s \subseteq C, s \neq \varnothing : \exists x \in s, \forall y \in s, x \neq y \to x \in y \tag{2}$$

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[3] The requirement of a meta-formal theory makes Ω^{meta} philosophical in nature.

This definition of class ordinal is a straightforward MK-class-level generalization of the von Neumann definition of (set) ordinal. By the usual ordinal analysis (Jech 2006), if C is a class ordinal and $x \in C$, then x is a set that is a set ordinal. A proper class ordinal is defined as any class ordinal that is not a set. Like any class, any class ordinal C is either a set or a proper class. If C is a set, then by definition of class ordinal it is transitive and well-ordered by $\in$, and therefore C is a set ordinal. If C is not a set, then by definition it is a proper class ordinal. Hence we have a *dichotomical ontology* for class ordinals: every class ordinal is either a set ordinal or a proper class ordinal.

Just as for set ordinals in Jech (2006), the intersection of two class ordinals is again a class ordinal. In MK, classes are closed under intersection by class comprehension, so for class ordinals C_1 and C_2, the intersection $C := C_1 \cap C_2$ is a class. It is transitive because any element of C is in both C_1 and C_2, which are transitive. At last, $\in$ well-orders C because any non-empty subclass of C is also a subclass of C_1 and C_2, which are well-ordered by $\in$.

Another result that is paralleled by set ordinals in Jech (2006) is that a class ordinal cannot be a proper subclass of another class ordinal unless it is an element of it: $C_1 \subsetneq C_2 \to C_1 \in C_2$. From this it follows that every class ordinal is an initial segment of any larger class ordinal and that we have *trichotomical comparability* with respect to $\in$ between any pair of class ordinals: exactly one of $\alpha \in \beta$, $\alpha = \beta$, or $\beta \in \alpha$, holds.

The proper class Ord is a proper class ordinal because it is transitive and well-ordered by $\in$. Given that a proper class ordinal C_p cannot be an element of any class, only the option $C_p = \mathrm{Ord}$ remains in the trichotomical comparison. Consequently, Ord is the unique proper class ordinal in its theory. Ord is also the least upper bound (LUB) of all set ordinals and it is maximal: no class ordinal exceeds it.

The notions 'least' and 'maximal' refer to the order $\in$ imposes: $\alpha \in \beta$ iff $\alpha < \beta$. The successor $\alpha + 1$ of a set ordinal α is defined as $S(\alpha) = \alpha \cup \{\alpha\}$. By the usual ordinal analysis, the successor of a set ordinal is again a set ordinal (Jech 2006). Ord cannot have a successor because Ord, being a proper class, cannot be an element of any class, and hence no class of the form $\mathrm{Ord} \cup \{\mathrm{Ord}\}$ exists.

Because of the uniqueness and the maximality of proper class ordinals in a given theory, if the absolute infinite Ω can be defined as a class ordinal in some class theory that consistently extends MK, it must be equal to a proper class ordinal.

2.2. Class Cardinals and Global Choice

In order to prove similar results for cardinals, GC is needed, because it warrants that every definable class admits a well-ordering. Without well-orderability, cardinal comparability for proper classes is not guaranteed (Halbeisen and Shelah 2001). Cardinality comparison being lost, the idea of a maximally large collection with proper class cardinality Ω-as-cardinal (henceforth Ω_{card})[4] collapses. For example, frameworks like those supporting Reinhardt (1974) cardinals explicitly reject the axiom of choice (AC). This implies the falsehood of GC, leaving cardinal comparability – and thus the very notion of 'largest size' – underdetermined. This is the reason not to consider extensions of ZF (Zermelo–Fraenkel set theory) without the axiom of choice (AC) in this paper, but extensions of the choice-consistent ZFC (ZF + AC) (Zermelo 1908). Morse–Kelley (without GC) and MK (with GC) are such extensions.

In the presence of GC, every class cardinal is identified with its initial class ordinal (the least class ordinal equinumerous with a given class), in such a way that class cardinals inherit the well-ordering – and comparability – from class ordinals. More precisely, under GC every set cardinal is identified by its initial set ordinal and the proper class cardinal by the proper class ordinal Ord. The properties of Ord then entail that any proper class cardinal is (in its theory) also unique, the least upper bound of all set cardinals, and maximal. Consequently, if Ω_{card} can be defined in a class theory that consistently extends MK, it must be a proper class cardinal.

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[4] Various other symbols are used for this cardinal: $|V|$, $||\mathrm{Ord}||$, $|\mathrm{Ord}|$, κ_{On}, and $\beth_{Ord}$.

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3. Meta-Formal Concepts

3.1. Leaving Formality Aside

An axiomatic theory is *formal* iff all its axioms can be recursively enumerated (RE) by a classical Turing machine, which is a computer (or an ordinal Turing machine) that can only make a finite number of computational steps at each stage (Hamkins and Lewis 2000, Koepke and Seyfferth 2009). Consequently, a formal theory cannot have more than countably many axioms. Alternative expressions are that a formal theory is algorithmically enumerable or effectively axiomatizable. The most common examples of formal theories about sets are ZF and ZFC, and about classes are NBG (von Neumann–Bernays–Gödel; von Neumann 1928) and Morse–Kelley.[5]

Many technical consequences follow from the study of set-theoretic infinities via formal theories (Jech 2006, Kunen 2011): Gödel's (1931) incompleteness theorems, Tarski's (1936) undefinability of arithmetical truth, the Löwenheim-Skolem theorem (Löwenheim 1915), the great number of actually investigated theories, infinitely many non-isomorphic models of a single theory, formal theories reasoning about other formal theories, an incomplete large cardinal hierarchy, relative consistency, independence, forcing, and many more. Even though formal theories have enabled set theorists to prove an abundance of mathematical theorems (not just meta-theoretical theorems), they have one major drawback: there is no ultimate formal theory. For every formal theory, a stronger formal theory can be built (e.g. by adding a Gödel sentence). This is to say that no single formal theory can capture all mathematical truths.

The inability of a formal theory to capture the entire mathematical reality is a serious drawback for its utility in certain domains of philosophy, such as metaphysics. Moreover, because different formal theories adopt incompatible axioms (e.g., CH versus $\neg$CH) the union of all formal theories is not itself a single consistent axiomatic framework. This paper proposes that this problem – consistently defining the collection of all mathematical or set-theoretic objects in a single theory – can only be overcome by leaving the requirement of formal recursive enumeration of an axiomatic theory aside.

For this reason, an idealized non-formal theory is proposed that is more theoretical and abstract, although less technical, as compared to formal theories. The cardinality of its syntax rules, symbols, formulas, axioms, and theorems can be unrestricted in the sense that this cardinality can be more than any set cardinal that can be proven to exist in some RE-formal extension of an appropriate foundational theory. Consequently, every set x can, in a single theory, consistently be proven to exist by its own explicit axiom or theorem that states: "x exists." Let us call such an idealized theory a *meta-formal* theory. The forementioned technical consequences of formal theories, from Gödel's incompleteness theorems to forcing, are not readily applicable to a meta-formal theory.

If Gödel's incompleteness theorems cannot be generalized to meta-formal theories, the latter may be both syntactically consistent and complete (Franzén 2005). By the lack of restrictions, a meta-formal theory can achieve this completeness by including all true statements about a domain, such as arithmetic, as axioms. It will also remain syntactically consistent as long as no contradictory statements are included. Even though it is clear that a meta-formal theory is not RE-formal and not mechanically checkable by finite agents, it can aim for something formal theories cannot do, namely capturing the entire set-theoretic reality in a single consistent theory.

3.2. Meta-Formal Symbols and Definitions

In a formal theory, quantification, satisfaction, provability, and the notion of a model are all constrained by recursive enumerability of syntax on a classical Turing machine with time and memory resources ω. Meta-formality will be grounded in RE-formality by being a non-RE-formal limit case

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[5] Both NBG and Morse–Kelley extend ZFC by introducing classes alongside sets, but they differ in strength: NBG is a conservative extension of ZFC (i.e., it proves no new theorems about sets), while Morse–Kelley is strictly stronger. All these theories are formal because they have a recursively enumerable axiom set.

of it. To achieve this, the same symbols are enumerated on an ordinal Turing machine[6] (Koepke 2005, Koepke and Seyfferth 2009) with unrestricted resources: more time and memory than any set ordinal that can be defined in an RE-formal theory that extends MK. In order to indicate better where meta-formality is grounded in formality, the superscripts *meta* and *m* will indicate the limit case: meta-formality; *form* and *f* point to the familiar case: formality; and no symbol means that the level can be derived from the status of its operands (e.g. $\forall^{meta}, \vdash^{f}, or \models$). Codewords (MetaFormal versus Formal) and calligraphy ($\mathcal{T}$ versus T) will also help:

- V^{meta}: The intended universe of all sets that is a model of a meta-formal theory $\mathcal{T}$.

$$\text{MetaFormal}(\mathcal{T}) \wedge V^{meta} \models^{m} \mathcal{T} \tag{3}$$

- V^{form} (also referred to as v): A multiverse-universe that is a model of some formal theory T.

$$\text{Formal}(T) \wedge v \vDash^{f} T \tag{4}$$

The same distinctions can be made between Ω^{meta} and Ω^{form} (and hence between Ord^{meta} and Ord^{form}). Let us now define an MK-consistent (and hence GC-consistent) Hamkinsian[7] multiverse as follows (Hamkins 2012):

$$M_h = \{v \mid \exists^{m} T[\text{Formal(T)} \wedge \text{MK} \subseteq T \wedge v \vDash^{f} T]\} \tag{5}$$

$\exists^{m} T$ means that T exists in the collection of all set-theoretic theories. By convention, $\vDash^{f}$ is the usual Tarskian satisfaction relation. By ranging over $\forall^{meta}\phi : \exists v \in M_h(v \vDash^{f} \phi \vee v \vDash^{f} \neg\phi)$, we can create a rigorous definition of what it means to be a meta-formal theory in the context of this paper:

$$\begin{aligned}&\text{MetaFormal}(\mathcal{T}) \leftrightarrow \text{MK} \subseteq \mathcal{T} \wedge \\ &\forall^{meta}\phi(\exists^{m} S[\text{Formal}(S) \wedge \text{Con}(S, \mathcal{T}) \wedge \text{MK} \subseteq S \wedge S \vdash^{f} \phi] \leftrightarrow \mathcal{T} \vdash \phi)\end{aligned} \tag{6}$$

In other words, a theory $\mathcal{T}$ is meta-formal iff $\mathcal{T}$ extends MK and proves that 'a sentence is true iff there is some formal extension of MK, consistent with $\mathcal{T}$, that formally proves that this sentence is true.' Also here, $\exists^{m} S$ means that S is a set-theoretic theory. $\mathcal{T} \vdash$ means syntactic derivability in either a formal or a meta-formal theory, not derivability in MK. The relative consistency $\text{Con}(S, \mathcal{T})$ is the well-known relative consistency $\text{Con}^{form}(S, \mathcal{T})$ if $\mathcal{T}$ is formal, and means the *meta-consistency* $\text{Con}^{meta}(S, \mathcal{T})$ if $\mathcal{T}$ is meta-formal. In this way, the right-hand side of the definition does not presuppose that $\mathcal{T}$ is meta-formal. $\text{Con}(\mathcal{T}, \mathcal{T})$ and $\text{Con}(\mathcal{T})$ are equivalent: $\mathcal{T}$ is syntactically consistent. *Meta-maximal extendedness* (i.e. *meta-completeness*) is ranging over $\forall^{meta}\phi$, not just over the ϕ satisfied by a single background universe among them.

The clause $\text{MK} \subseteq \mathcal{T}$ is needed to prove the existence of the absolutely infinite level and structural truths about this level by explicitly preferring an MK^{meta} over, respectively, a ZFC^{meta} and an NBG^{meta}. Theories that are too weak to be meta-formal are more easily consistent with the formal S's, but they will not make the bi-conditional true for all ϕ in the $\forall^{meta}$-quantifier. Theories that are too strong, such as those that contain sets isomorphic with the multiverse, sets with a Reinhardt cardinal size, or set ordinals larger than Ord, will not be consistent with any MK-consistent formal S. The combination of these two filters defines meta-formality precisely.

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[6] Ordinal-length proofs are based on axioms and prove theorems with any ordinal length and they require a number of steps indexed by an ordinal, each of which follows a rule. Limit steps need to be defined over a union of infinitely many predecessor steps.

[7] Hamkins' multiverse definition is not adopted verbatim and is restricted to suit the needs of meta-formal analysis.

4. Defining the Absolute Infinite

Let us introduce the absolute/meta concepts in the previous section more rigorously. We can define MK^{meta} as the meta-maximally consistent extension of MK (Enderton 2001), with no RE-formally definable restrictions on the cardinality or enumerability of axioms,[8] in such a way that:

1. $\mathrm{MK} \subseteq \mathrm{MK}^{meta}$
2. MK^{meta} is meta-complete: $\mathrm{MK}^{meta} \vdash \forall^{meta}\phi(\mathrm{MK}^{meta} \vdash \phi \vee \mathrm{MK}^{meta} \vdash \neg\phi)$
3. MK^{meta} is meta-consistent: $\mathrm{MK}^{meta} \vdash \mathrm{Con}(\mathrm{MK}^{meta})$

Let us say that a theory $\mathcal{T} \supseteq \mathrm{MK}$ is an MK-theory. It can then be shown that MK^{meta} is meta-formal:

Theorem 1. *MetaFormal(MK^{meta})*

Proof. $\mathrm{MK} \subseteq \mathrm{MK}^{meta}$ is given by definition. By definition of a meta-maximally consistent MK-extension, there is no problem with independence. Therefore, MK^{meta} proves a sentence ϕ if ϕ can be proven to exist in an MK^{meta}-consistent, formal MK-theory S. This proves left-to-right for all ϕ in the $\forall^{meta}\phi$ bi-conditional of the equation 6. Assume right-to-left does not hold for some ϕ in that bi-conditional. Then ϕ provably holds in MK^{meta}, but no MK^{meta}-consistent formal MK-theory S proves ϕ. However, we can construct the formal theory $S = \mathrm{MK} \cup \{\phi\}$. Since ϕ is proven by MK^{meta}, S is MK^{meta}-consistent. As S is a formal MK-theory that proves ϕ, we have a contradiction. Thus, right-to-left holds for all ϕ in $\forall^{meta}\phi$. Therefore, MetaFormal(MK^{meta}). ☐

The following theorem shows that MK^{meta} is semantically closed:

Theorem 2. $\forall^{meta}\phi(MK^{meta} \vdash \phi \longleftrightarrow MK^{meta} \vdash (MK^{meta} \vdash \phi))$

Proof. Right-to-left holds trivially. Left-to-right: If $\mathrm{MK}^{meta} \vdash \phi$ holds, then $\mathrm{MK}^{meta} \vdash (\mathrm{MK}^{meta} \vdash \phi)$, the right-hand clause, can consistently be added to MK^{meta}. Because MK^{meta} is meta-maximally consistently extended, without any RE-formally definable bounds on its language, this consistent addition has already taken place. Paradoxical sentences are not in the $\forall^{meta}\phi$-quantifier, because neither they nor their negations are satisfied by any model $v \in M_h$. Therefore, left-to-right also holds. ☐

From this it follows that MK^{meta} can serve as its own semantic meta-theory and that truth and provability coincide in MK^{meta}. That MK^{meta} can be both syntactically consistent and complete follows from the next theorem:

Theorem 3. $MK^{meta} \vdash$
$(\forall^{meta}\phi(MK^{meta} \vdash \phi \vee MK^{meta} \vdash \neg\phi) \wedge Con^{meta}(MK^{meta}))$

Proof. Because there is no RE-formal restriction on the cardinality and the enumerability of axioms in a meta-formal theory, it can be meta-maximally extended: it can include every sentence ϕ or its negation in the $\forall^{meta}$-quantifier as an axiom. By consistency, the sentence 'MK^{meta} is meta-complete' is also decided as true by MK^{meta}. Using Theorem 2, this proves that MK^{meta} proves its own meta-completeness.

MK is syntactically consistent. Given that a meta-maximally consistent extension does not create syntactic inconsistencies, also MK^{meta} is syntactically consistent. Using Theorem 2, MK^{meta} proves this. ☐

Then it is proven that all meta-formal theories are logically equivalent:

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[8] Because of this non-formality, the properties of MK^{meta} technically do not violate Gödel's incompleteness theorems. However, it will be shown in Section 6 that the requirement to avoid Gödel-style limitations is the idea that no stronger meta-theory can be built for a given theory. The crucial distinction that will be made is formal-based versus meta-formal, not formal versus non-formal.

Theorem 4. $MetaFormal(\mathcal{T}_1) \wedge MetaFormal(\mathcal{T}_2) \to \mathcal{T}_1 \equiv \mathcal{T}_2$

Proof. Assume that $\mathcal{T}_1$ and $\mathcal{T}_2$ are meta-formal. To show $\mathcal{T}_1 \equiv \mathcal{T}_2$, it is shown that they prove the same sentences $\forall^{meta}\phi$. According to left-to-right direction of Equation 6, the knowledge that a theory $\mathcal{T}$ is meta-formal fixes, for all ϕ and by following the bi-conditional on the right in both directions, both (1) the question whether $\mathcal{T}$ proves ϕ as (2) the question whether a theory S can be found for which Con(S, $\mathcal{T}$). Given that both $\mathcal{T}_1$ and $\mathcal{T}_2$ are meta-formal, (1) implies that $\forall^{meta}\phi(\mathcal{T}_1 \vdash \phi \longleftrightarrow \mathcal{T}_2 \vdash \phi)$. Therefore, $\mathcal{T}_1 \equiv \mathcal{T}_2$. ☐

The following theorem shows that MK^{meta} proves that MK^{meta} has a model:

Theorem 5. $MK^{meta} \vdash \exists^m \mathcal{M}(\mathcal{M} \models MK^{meta})$

Proof. MK^{meta} proves its own meta-completeness and syntactic consistency. The Henkin (1949) construction (which generalizes Gödel's [1930] completeness theorem), is used here: a theory is Henkin-saturated if, for every existential sentence $\phi = \exists x\psi(x)$, there is a witness constant c in the language for which the theory proves the implication $\exists x\psi(x) \to \psi(c)$. Because MK^{meta} is meta-complete and its language lacks any RE-formal bounds, it achieves Henkin-saturation: for every ϕ in the domain of the $\forall^{meta}$-quantifier, the language already contains a witness c such that MK$^{meta} \vdash (\exists x\psi(x) \to \psi(c))$. Because every consistent Henkin-saturated theory has a model, MK^{meta} has a model. Using Theorem 2, MK^{meta} proves MK^{meta} has a model. ☐

The following theorem asserts that any model satisfying a meta-formal theory is the same unique model:[9]

Theorem 6. $MetaFormal(\mathcal{T}) \wedge \mathcal{M}_1 \models \mathcal{T} \wedge \mathcal{M}_2 \models \mathcal{T} \to \mathcal{M}_1 = \mathcal{M}_2$

Proof. Let $\mathcal{T}$ be a meta-formal theory and suppose $\mathcal{M}_1 \models \mathcal{T}$ and $\mathcal{M}_2 \models \mathcal{T}$. Assume toward a contradiction that $\mathcal{M}_1 \neq \mathcal{M}_2$. Then there exists a set $x \in v \in M_h$ for which $x \in \mathcal{M}_1$ but $x \notin \mathcal{M}_2$. Let ϕ be the sentence $\exists y(y = x)$. By Equation 6, since ϕ is satisfied in some $v \in M_h$, there exists a formal, meta-consistent MK-theory S for which $S \vdash^f \phi$. Meta-formality of $\mathcal{T}$ then implies $\mathcal{T} \vdash \phi$ (since ϕ is in the domain of the $\forall^{meta}$-quantifier). Hence every model of $\mathcal{T}$ must satisfy ϕ. But $\mathcal{M}_2 \not\models \phi$, because $x \notin \mathcal{M}_2$ contradicting $\mathcal{M}_2 \models \mathcal{T}$. Therefore $\mathcal{M}_1 = \mathcal{M}_2$. ☐

Note that $\mathcal{M}_1$ and $\mathcal{M}_2$ have exactly the same members x. By the axiom of extensionality in MK and its meta-formal extension, that makes them identical, not just isomorphic or structurally similar. In other words, MK^{meta} is not just categorical, but even rigid.

Let Ord^{meta} be the class of all set ordinals proven to exist by MK^{meta}. Then we can define the maximal meta-consistent height (MMH) of an axiom ϕ as follows:

> **Definition:** The *maximal meta-consistent height* of a sentence ϕ is the LUB (or supremum)[10] of the class of all set ordinals α in Ord^{meta} for which there exists a multiverse-universe $v \in M_h$ in which ϕ holds and in which $\alpha \in$ Ord^v. (Ord^v denotes the class of set ordinals of v):
>
> $$\mathrm{MMH}(\phi) := \sup\{\alpha \in \mathrm{Ord}^{meta} | \exists^m v \in M_h(v \models^f \phi \wedge \alpha \in \mathrm{Ord}^v)\} \tag{7}$$

Intuitively, the MMH is the maximal value that Ord^{meta} can possibly still reach when ϕ is made true. Indeed, the MMH ranges over the part of the multiverse where ϕ is true and then, just like Ord^{meta}, derives the LUB of all set ordinals it found. GC warrants that the class of all these Ω^{meta}_{card}-many

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[9] The Löwenheim-Skolem theorem proves that if a countable first-order theory has an infinite model, it has models of every infinite cardinality (Löwenheim 1915). This does not apply here, because meta-formal theories are non-countable and, more crucially, have no stronger meta-theory.

[10] Note that the supremum of any class of set ordinals is a class ordinal.

set ordinals is well-orderable. The following theorem provides a generally intractable but ontologically fixed criterion that determines which axioms are true and which are false:[11]

Theorem 7. $\forall^{meta}\phi(MMH(\phi) > MMH(\neg\phi) \to MK^{meta} \vdash \phi)$

Proof. Let ϕ be a sentence in the $\forall^{meta}$-quantifier for which MMH(ϕ) > MMH($\neg\phi$). By the definition of MMH, there exists a set ordinal $\alpha \in \mathrm{Ord}^{meta}$ for which $\alpha > \mathrm{MMH}(\neg\phi)$. Because $\alpha \in \mathrm{Ord}^{meta}$, the existence of α can be proven by MK^{meta}. However, if $\mathrm{MK}^{meta} \vdash \neg\phi$, then, again by the definition of MMH, the existence of α cannot be proven by MK^{meta}. Therefore, $\mathrm{MK}^{meta} \vdash \neg\phi$ cannot be true. Because of the meta-completeness of MK^{meta}, it follows that $\mathrm{MK}^{meta} \vdash \phi$. ☐

This result implies that even if CH is independent of MK, it possesses an MMH in a GC-consistent multiverse. If MMH(CH) > MMH($\neg$CH), we have an ontologically grounded reason to prefer CH as an absolute truth of the meta-formal level, regardless of its independence from a formal theory like MK (Gödel 1947). This is Maddy's (1997) MAXIMIZE principle.

By Theorem 4, MK^{meta} is the unique meta-formal theory. By Theorem 5, MK^{meta} has a model. By these two theorems and Theorem 6, every model of a meta-formal theory is equal to the same unique model of MK^{meta}. Moreover, by Theorem 7, MK^{meta} decides every sentence ϕ in such a way that the model of MK^{meta} is maximized. Now, we can define V^{meta} as that unique model of MK^{meta}:

$$\exists! V^{meta}(V^{meta} \models \mathrm{MK}^{meta}) \tag{8}$$

This enables us to axiomatize a unique absolute infinite Ω^{meta}_{card} as the proper class cardinality of V^{meta}:

$$\text{Axiom of absolute infinity (AI)} : \exists\Omega^{meta}_{card}(\Omega^{meta}_{card} = ||V^{meta}||) \tag{9}$$

Ω^{meta} is then the proper class ordinal with size Ω^{meta}_{card}.

5. Objections

In this section, it is argued that the provided definition of Ω^{meta} is robust by answering a range of objections to it. Any competing definition has to deal with most of these objections.

5.1. Ω^{meta} *Succumbs to the Burali-Forti Paradox*

The Burali-Forti paradox demonstrates that naively constructing the set of all set ordinals leads to a contradiction, namely that the constructed set is both an element of itself and not an element of itself (Burali-Forti 1897). By introducing class ordinals in Section 2.1, Ω^{meta} can be constructed as a proper class ordinal that is equal to the class of all set ordinals Ord^{meta}, rather than the set of all set ordinals (Jech 2006). This is the application of the axiom of limitation of size to ordinals. Moreover, no class is ever an element of itself, because MK^{meta}, which extends MK, maintains the axiom of foundation. Given that MK^{meta} is meta-complete and thus settles every membership relation, there cannot be any loops or non-well-founded structures hiding in its model V^{meta}. This avoids the most common paradoxes.

5.2. Ω^{meta} *Is Smaller Than* $\Omega^{meta} + 1$

An ordinal $\Omega^{meta} + 1$ is not defined in any of the formal or meta-formal axiomatic theories that are meta-consistent. (Without this requirement, it becomes possible to redefine Ω^{meta} as, for example, ω.) Since any successor construction requires $\{\Omega^{meta}\}$, which is not defined, $\Omega^{meta} + 1$ cannot be formed. In MK, it is a category mistake to apply a successor operator to Ω^{meta}, because Ω^{meta} is a proper class and not a set. If both a theory-specific Ω^{form} and $\Omega^{form} + 1$ are well defined in the

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[11] The inverse direction of the theorem is beyond the scope of this paper. Nevertheless, it can informally be argued that MMH(ϕ) is never equal to MMH($\neg\phi$) if both ϕ and $\neg\phi$ are meaningful. In this case, the infinite richness of a meta-formal model will always break the symmetry.

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same meta-consistent, formal MK-theory, then they are both MK^{meta} sets and therefore smaller than Ω^{meta}. Any theory that formally constructs $\{\Omega^{meta}\}$ effectively reduces Ω^{meta} to an Ω^{form} and is thus meta-inconsistent.

5.3. Ω^{meta}'s *Definition Is Inconsistent or Circular*

By defining a class ordinal Ω^{meta} that is not an MK^{meta} set, there is no obvious self-reference in AI. Furthermore, the definition of Ω^{meta} is meta-consistent as proven by MK^{meta}. This preserves syntactic consistency. The definition of the meta-formal level begins with defining what it means to be unrestricted or unbounded: being larger than any set ordinal that has a definition in some RE-formal extension of MK. This grounds meta-formality as a limit case of formality and avoids circularity.

The meta-formal theory $\mathcal{T}$ admittedly occurs at both sides of the bi-conditional at the right-hand side in Equation 6. However, this only reflects the challenge to compute MK^{meta}: even finite sentences can only be established as true or proven after having taken every sentence in the $\forall^{meta}$-quantifier into account. An ordinal Turing machine with resources Ω^{meta} can fulfill this task.

5.4. Ω^{meta} *Is Indefinable and Inexhaustible*

Gödel has described the universe of sets V^{meta} as structurally indefinable (Wang 2016, p. 280). A related objection is that V^{meta} is inexhaustible (Maddy 1988, pp. 501-3). Such remarks can also be made about Ω^{meta}. However, what Gödel means is that Ω^{meta} and V^{meta} cannot be *formally* defined. The definitions in this paper are meta-formal.

5.5. *Non-Formal Theories Should Be Avoided*

MK^{meta} is a non-RE-formal axiomatic theory that is rigorously and consistently defined as meta-formal. This theory is needed to define and prove the existence of every meta-consistently definable set in a single theory and to acquire several desirable and internally provable meta-theoretical properties, such as completeness, syntactic consistency, uniqueness, and maximality. Set theorists use formal theories all the time, as they make proofs achievable by humans. However, being manageable or directly constructible by humans cannot be invoked as a principle in defining abstract entities (Quine 1948, Putnam 1971, Feferman 1991). A meta-formal theory like MK^{meta} is indispensable, because the set-theoretic reality is too large to be captured by a single formal theory, while MK^{meta} uniquely satisfies the often sought maximality criterion in set theory (Maddy 1997). Therefore, theoretical definitions must be accepted for abstract entities, regardless of how theoretical they are. After all, any rigorous definition of a set-theoretic multiverse is not formal either, even though it is equally indispensable in the philosophy of mathematics. This paper only completes the meta-formal level with entities like MK^{meta}, V^{meta}, Ω^{meta}, and Ω^{meta}_{card}.

6. Four Flavors of Formality

Four flavors of formality that a theory can have are discussed in this section, ordered by the increasing cardinality of the axiom set they use: informal, RE-formal, formal-based, and meta-formal theories. Mathematicians inevitably rely on an *informal* semantic meta-theory to give meaning to the syntax of the theories they work with in the form of an intended model. Because informal theories are meant for practical reasoning, they are provided with a small finite number of axioms or principles.

If a theory is *RE-formal* then it has a countable axiom set.[12] They have been well investigated and some important limitative results have been derived by Gödel (1931) and Tarski (1936) for them. One way to approach these results is to investigate the relation between the axiom set cardinality of a theory and the cardinality of the models of the theory. For RE-formal systems that are sufficiently expressive (containing arithmetic), most models have strictly more elements than the axiom set cardinality of their theory.

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[12] The inverse direction does not hold: True Arithmetic has countably many axioms, but is not RE-formal (Tarski 1936).

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The following definition is introduced here: a theory is *formal based* iff it is a syntactically consistent theory for which the existence of the cardinality of its set of axioms can be proven in *some* RE-formal MK-theory. Theories that have an intermediate status between RE-formal and meta-formal theories are infinitary theories (Karp 1964, Barwise 1967), like $L_{\kappa,\lambda}$, with κ and λ some RE-formally defined ordinals. If κ and/or λ are too large (beyond countable), such infinitary theories are not RE-formal themselves. Nevertheless, they are formal-based theories because the cardinality of its axiom set can be proven to exist according to some RE-formal MK-theory. Feferman (1960) generalized Gödel's incompleteness theorems by showing that certain formal-based theories are incomplete. However, for the purposes of this paper, only the *meta*-incompleteness of formal-based theories will be shown: incompleteness given a non-RE-bounded domain. While standard incompleteness is a failure of a theory to talk about its own code, meta-incompleteness is a failure to talk about the meta-formal level. An incomplete theory is always meta-incomplete, however, proving the latter directly is less technical.

To begin with, MK has models that have strictly more elements than the countable axiom set of MK. Furthermore, for any sufficiently expressive formal-based MK-theory B it is always possible to construct at least one new set ordinal by adding a new meta-consistent[13] axiom ϕ to B, for example, a ϕ that asserts the existence of the successor x of the least upper bound of all the set ordinals that B can prove to exist. Because ϕ is proven by some RE-formal MK-theory, it can always be added to B while keeping formal-basedness of the new theory B_2. This means that the imbalance in the axiom set cardinality of B versus the cardinalities of B's models, diagnosed for MK, can only become worse for an increasing RE-formal axiom set cardinality. Because of this, a formal-based theory is inherently meta-incomplete and cannot be meta-formal.

A meta-formal theory has Ω^{meta}_{card}-many axioms. This follows from the facts that every theory with an RE-formal cardinality κ of axioms is by definition formal based, that no formal-based theory is meta-formal, that every RE-formally defined cardinality κ is a set cardinality in MK^{meta}, and that Ω^{meta}_{card} is the LUB of all set cardinals. This marks a foundational pivot: while models of sufficiently expressive formal-based theories have strictly more sets as element than these formal-based theories have axioms, this is not the case for V^{meta} versus MK^{meta}, which have Ω^{meta}_{card}-many sets and Ω^{meta}_{card}-many axioms, respectively.[14] Because of this, the class of all the sets in V^{meta} can be injected in the class of all the axioms of MK^{meta}. This in turn can explain why the Gödelian and Tarskian limitative results for RE-formal theories can often be generalized to formal-based theories that are not RE-formal, without necessarily generalizing to meta-formal theories.

Gödelian diagonalization operates by exploiting the cardinality mismatch between a theory's language – limited to a countable set of strings – and the uncountability of its intended model. This allows for the construction of a diagonal set (or Gödel sentence) that is provably outside the reach of the theory's naming conventions. For a meta-formal theory, any attempt to diagonalize out of the theory fails: the diagonal set is not a new, unprovable entity, but is already uniquely identified by an existing injection into the axiom set.

Tarski's theorem on the undefinability of truth states that in any sufficiently expressive RE-formal language, the property of "truth" within that language cannot be defined by a formula in the language itself. This necessitates an infinite hierarchy of always stronger meta-languages. However, for a meta-formal theory, there is no stronger meta-language. The truth of any sentence ϕ is coextensive with its meta-formal provability ($\vdash^m$).

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[13] It is always possible to find an RE-formal MK-theory S that can prove $\phi = \exists y(y = x)$. Otherwise, there would be a paradoxical least set ordinal α that cannot be proven to exist by any RE-formal MK-theory. But then MK + α is an RE-formal MK-theory that proves the existence of α. Because of Theorem 7 about maximal meta-consistent height, ϕ is then meta-consistent.

[14] This mirrors Galileo's Paradox: while the density of square numbers within the natural numbers approaches zero as the finite threshold increases, the two sets are strictly equinumerous at the transfinite limit.

7. Conclusions

This paper defines several meta-formal concepts: 1) M_h, the Hamkinsian multiverse, 2) MK^{meta}, a theory that is unique in being meta-formal, up to logical equivalence, 3) V^{meta}, the unique model of MK^{meta}, 4) $\Omega^{meta}_{card} = \kappa^{meta}_{On}$, the proper class cardinality of V^{meta}, and 5) $\Omega^{meta} = \mathrm{Ord}^{meta}$, the unique meta-formal proper class ordinal, representing the absolute height of the universe V^{meta}. The maximality and the uniqueness of V^{meta} counter height and width potentialism, respectively, when combined with the claim that the meta-formal level is a superior level because it can capture the entire set-theoretic reality in a single theory. MK^{meta}'s humanly intractable but ontologically fixed preference for maximal meta-consistent height further counters width potentialism and relativism about set-theoretic truth. This preference systematically asserts an axiom is true if MK^{meta} can prove that its maximal meta-consistent height exceeds that of its negation, which amounts to the axiom being meta-consistent. Since these rebuttals can start from a rigorous definition of a set-theoretic multiverse, we can conclude that multiversism is no safe refuge for potentialists.

One of the reasons why the set-theoretic multiverse and width potentialism appear viable is that syntactic (or relative) consistency is sometimes abbreviated as consistency. Meta-consistency, however, is a superior notion of consistency, because there exists an internal proof of meta-consistency in a meta-formal theory. Meta-*in*consistent theories in M_h lack such a proof.

While Gödel's incompleteness theorems are important and sound results, their applicability is restricted to a notion of formality that requires recursive enumeration on Turing machines that can perform only finitely many computational steps at each stage. Formal theories are inevitable tools for mathematicians, but they cannot provide a complete description of the whole mathematical reality in a single theory. Completeness, consistency, uniqueness, and maximality can be proven internally by MK^{meta}, as it is not formal in the restricted Gödelian sense and lacks a meta-theory that is strictly stronger. By having an absolutely infinitely large class of axioms, MK^{meta} is not a formal-based theory with RE-formally many axioms either. Even though a meta-formal theory can be perceived as an extreme ideal, it is philosophically relevant.

Table 1. Guide to Symbols and Notation

Symbol	**Name**	**Description**
$\vdash$	Proof	Provability within a formal or meta-formal theory
$\vDash$	Satisfaction	Satisfaction within a formal or meta-formal model
$\vdash^f$	Formal Proof	Provability within a formal theory S
$\vDash^f$	Formal Satisfaction	Satisfaction within a formal model v
MK^{meta}	Meta-formal Theory	Meta-maximally consistent extension of MK
M_h	Hamkinsian Multiverse	All models v of any formal MK-theory
$v \in M_h$	Multiverse-Universe	A model within the multiverse M_h
$\forall^{meta}\phi$	Any Sentence	Any sentence $\phi : \exists v \in M_h(v \vDash \phi \vee v \vDash \neg\phi)$
V^{meta}	Meta-Universe	The unique model of MK^{meta}
Ord^v	Model Ordinals	The class of set ordinals internal to a model v
Ord^{meta}	Absolutely All Ordinals	The proper class of all set ordinals in V^{meta}
Ω^{meta}	Absolute Infinity	The proper class ordinal Ord^{meta}
Ω^{meta}_{card}	Absolute Infinity	The proper class cardinality of V^{meta}

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Abbreviations

The following abbreviations are used in this manuscript:

AC	The Axiom of Choice
card	cardinal
CH	The Continuum Hypothesis
Con	Consistent
GC	The axiom of Global Choice
LUB	Least Upper Bound
MK	Morse–Kelley set theory with GC and class ordinals
MMH	Maximal Meta-consistent Height
Ord	The class of all set ordinals
RE	Recursively Enumerable
ZF	Zermelo–Fraenkel set theory
ZFC	ZF with AC

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