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On the Nature of Gravitation and Geometric Drift of Spacetime

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Abstract

In this study, it is shown that inserting geometric drift vectors within definition on tetrads have direct effects on torsion tensor. This contribution reveals an original point of view on gravitation while still being compatible with standard approaches and recent cosmological observations. Theoretical calculations of galactic velocities and critical radii can be made without need of dark matter. The difference between geometric drift vector coefficients are strong candidates to explain dark energy.

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1. Introduction

Defining an object motion inertial or non-inertial should be directly related to motion of another object or manifold itself. The pioneering form of the idea without manifold dates back to late 19th century being proposed by the Austrian physicist Ernst Mach. Including effects of distant masses and gravitational effects on inertia was revolutionary for its time. However, geometrical foundation of this principle, by some certain deviations, had needed to wait for a long time until general relativity (GR) has been founded. The main difference of GR lies on Equivalence Principles and their aftermaths: energy-momentum tensor shapes the geometrical structure of spacetime while spacetime determines the motion of matter. Direct correspondence of this result is given by Einstein Field Equations (EFE) and mathematical background mostly relies on differential geometry. Existence of matter causes non-vanishing Riemann tensor components that may be transferred to Ricci Tensor and Scalar. On the other hand, an opposing view was again considered by Einstein such that Riemann tensor and therefore curvature of spacetime could still be kept zero without violating EFE. This latter theory is usually called as Teleparallel Gravity (TG) or Teleparallel Equivalent of General Relativity (TEGR) [1]. In TEGR, the Weitzenböck connection is utilized instead of the Levi-Civita connection to describe gravitational effects [2]. In return, this connection allows for torsion, which is a geometric feature that shows how spacetime twists.

Even though GR and TEGR were (and still are) strong theories that are able to explain most of the cosmological phenomena, still remain some unsolved problems in physics awaiting explanation. Some of prominent classical examples are dark matter and energy, inflation theory and gravitation at higher energies. Observations reveal that cosmological matter move faster than expectation which is interpreted by some kind of substance we can not observe directly. Another observation is the universe's accelerating expansion which is explained mostly by dark energy but this term still requires further explanation and a geometry based theory. While the rate of expansion with the cosmological constant term can be balanced in EFE, the physical meaning of this constant remains unclear such that the cosmological constant formulation doesn't go far enough to explain the universe's energy structure in more detail.

While GR and TEGR successfully describe gravity through curvature and torsion, they do not explicitly address the kinematics of spacetime itself. Both approaches reveal how geometry responds to matter, but they do not offer a principle that directly distinguishes global expansion from local motion in a Machian sense. This subtle gap becomes even more apparent when considering cosmic expansion and local gravitational effects together, as the relationship between metric expansion and the motion of matter is not fully specified within current theories. A more refined description that includes manifold behavior as a fundamental geometric element could contribute to a more complete understanding of inertial and gravitational dynamics.

Throughout history, alternative approaches have been explored in the literature to explain the dynamics of gravity. One of the early works was by Sciama [3], who strongly posited the idea of a universal gravitational origin of inertia. However, his theory did not evolve into a fully consistent, closed, and local dynamical theory in the form of a field equation. Sciama's model was not entirely compatible with special relativity. The metric consequences of gravity were not considered in this model. Even if the model assumes that frames connected by rigid transformations are physically similar, it does not account for symmetry under universal scale transformations. The need for updating the dynamics of gravity has been revived following contradictions with cosmological observations. In particular, interpretations of dark matter and dark energy have been the main reason for this revival. In a research done by Milgrom [4], requirement of a modified dynamic was presented to be consistent with cosmological observations. However, a theoretical background for this modification has not yet been provided, leaving the door open for it. TeVeS [5], which can be considered a continuation of this work, successfully reproduces the MOND phenomenology at galactic scales, but it fails to provide a viable alternative for dark matter at the level of galaxy clusters and cosmology, and suffers from shortcomings due to serious theoretical stability and causality problems. On the other hand, while the Einstein-Aether theory [6] provides a well-defined covariant realization of Lorentz symmetry breaking, gravitational wave observations are severely constrained by post-Newtonian limits, stability requirements, and causality assessments. In other words, there is a need for fine-tuning for the theory to be phenomenological valid.

In this study, a novel tetrad formulation is proposed containing a velocity-geometric drift vector (GDV) that includes both global and local parts. Combinatory effects of this vector field and matter reveal the metric structure of the manifold, motion, torsion and gravitation in general. The idea of the current approach on explaining gravitation is based on dynamical GDV set on teleparallel tetrad formalism that distinguishes the theory from the others, such as MOND, Einstein-Aether, GR, TeVeS or other theories [4–7]. The main idea resembles that of D. W. Sciama [3] while utilizing a different mathematical background.

This paper is organized as follows: Main motivation of this theory is explained in detail in the next Section. Later on, fundamental axioms of the study are given. Tetrad and torsion construction are followed by comments on Kinematics and Force. We dedicate Section III on a detailed discussion of the theory with the generally recognized literature comparing the results under some different limits. Finally, we give final remarks and possible further applications on Conclusion. A general information on tetrad formalism and TEGR is given in Appendices with some of intermediate steps of algebra.

2. Tetrad Formalism Under Isotropically Expanding Space-Time and Geometric Drift Vector

Let us return to Mach's Principle and revisit a classic example. If an observer turns around itself, it will observe the cosmological bodies (such as stars) as if they were rotating as well. What if we consider another situation: suppose the observer is static while the bodies rotate, does the observer still feel force and is it still the same as in the first case?

In the first case, when an observer rotates, all stars and the manifold appear to share the same angular velocity even though masses of the stars vary enormously. Applying the same logic to the second case creates a similar picture. A full implementation of the principle should explain the physical angular velocity in both.

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Another related issue questions the definition of inertia: if the observer feels a force in both cases, then it means that the stars should stay inertial while the observer is not. There should exist a connection that makes the missing link visible: It cannot be only the stars that rotate; the manifold itself must also be rotating along with them to keep them inertial.

The Mach's principle requires carrying the momentum and rotation information of all matter in the universe into a local tetrad. The only suitable object that can carry this information as a complete, differentiable, geometric field is a "geometric drift vector".

A body is inertial when it moves along a geodesic. It may further be proposed that a body is inertial to the extent that it aligns with the "drift field" of the manifold. If the manifold itself is rotating, then inertial motion must be defined as motion aligned with this rotation: A body that fully adapts to the rotating drift is inertial. An observer who tries to remain fixed in a rotating manifold, however, resists the drift and therefore experiences a force. Thus, being at rest or in motion does not define inertia; what defines inertia is how well one conforms to the drift established by the manifold. Interpreting Mach's Principle in this extended framework leads to:

- Case 1: The manifold's drift is stationary. An observer that rotates against the drift, and is therefore non-inertial, experiences a force.
- Case 2: The manifold's drift is rotational. Objects rotating with the manifold are inertial. An observer that attempts to remain stationary in this rotating drift becomes non-inertial and feels a force.

In the second case, where the manifold itself possesses a rotational drift, an observer that co-rotates with the flow may still adopt two distinct physical interpretations depending on whether the global nature of the drift is acknowledged or suppressed. When the observer recognizes the existence of a true cosmic drift field, the kinematical state is naturally described in terms of adaptive alignment with this background flow. If the observer's adaptive motion fully adapts the global drift, the trajectory remains inertial. However, whenever perfect adaptation fails, a residual mismatch emerges and manifests itself as a locally measurable non-inertial force. In this interpretation, the origin of the experienced force is attributed entirely to the imperfect synchronization with the globally rotating manifold rather than to any intrinsic property of the local body.

Alternatively, the same observer may choose to interpret the ambient spacetime as globally static. In this reinterpretation, the entire dynamical content is transferred to the local frame itself, and the observed non-inertial effects are reattributed to an apparent intrinsic drift of the local body. So that all dynamical effects are encoded within the local geometrical structure. Under this viewpoint, the force is no longer associated with a mismatch relative to a cosmic flow but is instead interpreted as being generated by an internal torsion-like property of the local system. Despite this radical difference in attribution, the observable physical consequences remain invariant.

Actually, these two interpretations, as mentioned in the previous paragraph, are gauge–related: the observer may choose a frame in which the manifold is static (measuring an internal torsion), or a frame in which the manifold evolves (measuring a geometric torsion). The physics is the same; only the drift vector transforms. Gauge transformation which also reflects on drift vectors can be given as below:

$$W_{global} - W_{adapt} = W_{local} \tag{1}$$

Here, W_{global} is the geometric field created by the large-scale average drift of matter throughout the universe, W_{local} is the fluctuations created over W_{global} and W_{adapt} is an indicator of observer's compatibility (not the matter's) on W_{global} which also corresponds to a transformation between two different inner space states.

This rotational analysis can also be applied to linear acceleration, since the fundamental issue is the nature of inertia itself. Even in linear motion, a body is inertial insofar as it follows the natural drift, while opposing the drift generates non-inertial effects.

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The first situation (Case 1) on original idea represents inertial effects while the second (Case 2) refers to gravitational influence. Under this point of view, we generate the origin of the mathematical structure of the theory by tetrad formalism since tetrads play the role of a bridge between non-inertial frames locally. In addition, they are the only framework that directly connects local physical measurements.

2.1. Tetrad Construction

This study is based on two fundamental axioms:

- Firstly, time component of tetrad is fixed and only its spatial components include expansion and drift terms such that

$$e^0 = dt, \quad de^0 = 0 \tag{2}$$

Velocity of the light is taken as $c = 1$ here and throughout the study. On the other hand, spatial component is given by $e^i = dx^i$ in Minkowski space-time and $e^i = a(t)dx^i$ in FLRW space. However, it is possible to extend it by a velocity factor if a transformation including space-time mixing occurs. This is the case where drift velocity exists:

$$e^i = a(t)dx^i + W^i dt. \tag{3}$$

Equation (2) ensures a globally integrable time direction and a non-twisting temporal frame. All physical acceleration arises from three dimensional vectorial spatial drift fields, $W^i = W^i(t, \vec{x})$, that represents global and local drift velocity of the space. One should note that, here W^i is not a tetrad gauge choice but a vector affecting geometry. It also does not correspond to conformal metric decomposition which assumes a scalar factor applied on flat space-time metric. Conformal transformations rescale the metric (or the tetrad) by a scalar factor, whereas the present construction introduces a genuine vectorial geometric drift term. This term acts as a shift-like contribution and has dynamical effects on torsion, which cannot arise from a conformal rescaling. Therefore the geometric drift-tetrad should be viewed as a kinematical extension of the teleparallel frame, rather than a conformal deformation of the metric. In addition, since GDV is directly within geometry, this distinguishes the approach from similar theories such as ADM scaling, Ellis-Ehlers or Painlev-Gullstrand approaches.
- Weitzenböck gauge is assumed (Riemann and Ricci tensors vanish and also spin-connection: $\omega^a{}_b = 0$). Therefore, torsion is purely based on space-time.

$$T^i = de^i \tag{4}$$

$B^i{}_j := \partial_j W^i$ is the distortion tensor that includes asymmetric and symmetric vortex and strain parts:

$$\begin{aligned} B_{(ij)} &= \tfrac{1}{2}(\partial_j W_i + \partial_i W_j) \\ B_{[ij]} &= \tfrac{1}{2}(\partial_j W_i - \partial_i W_j) \end{aligned} \tag{5}$$

Although W may appear gauge–like under formal tetrad redefinitions, it is in fact a genuine geometric degree of freedom. The GDV fields are not removable gauge artifacts; they are intrinsic vectorial components of the teleparallel geometry and carry physical content analogous to ordinary dynamical vector fields.

Here a critical comment should be given on role of space-time topology on rotational motion. Since the existence of the drift vector is related to Mach's Principle then there should be a correspondence of both rotational and expanding motion on space-time. In another words, space should permit this type of motion such that it should have its global rotational and expanding drift. Since we assume an isotropic, non-rotating universe and only interested in gravitational effects we may assume that:

$$W^i_{global} = b(t)\, x^i \tag{6}$$

Inserting Eq.(3) into Eq.(A1) yields the metric tensor $g_{\mu\nu}$. Compared to the standard FLRW metric [8], off-diagonal terms on the metric tensor reveal a frame that moves relative to the space-time drift. This change leads to some important consequences that are mostly related on Torsion tensor. Thus, the next subsections are dedicated to torsion and its after-effects.

$$g_{\mu\nu} = \begin{pmatrix} -(1-|W|^2) & aW^1 & aW^2 & aW^3 \\ aW^1 & a^2 & 0 & 0 \\ aW^2 & 0 & a^2 & 0 \\ aW^3 & 0 & 0 & a^2 \end{pmatrix} \tag{7}$$

here the 3-vector length of the drift velocity is given by $|W|^2 = \sum W^i W^i < 1$. Although this resembles a Painlevé-Gullstrand type metric. It is worth noting here that the key difference lies beneath this metric does not arise due to a coordinate transformation, but rather from the space-time geometry itself.

Later on in this paper, general case of the drift W^i will be examined "under a gauge" such that it will include both global and local parts.

2.2. Torsion

In teleparallel geometry, Torsion plays the role as curvature does in that of Riemann. And again in teleparallel geometry, Weitzenböck gauge leads a non-vanishing Torsion T^a such that:

$$T^0 = 0, \quad T^i = de^i \tag{8}$$

and de^i is found as below (please See Appendix-II):

$$de^i = [(\dot{a} - b)\delta^i{}_j + \partial_j W^i_{adapt}]\, dt \wedge dx^j \tag{9}$$

Torsion tensor components can explicitly be written as:

$$T^a = de^a = \frac{1}{2} T^a{}_{bc} e^b \wedge e^c. \tag{10}$$

$$\begin{aligned} T^i &= \tfrac{1}{2} T^i{}_{0j} e^0 \wedge e^j + \tfrac{1}{2} T^i{}_{j0} e^j \wedge e^0 = \tfrac{1}{2}(T^i{}_{0j} - T^i{}_{j0}) e^0 \wedge e^j \\ T^i &= \tfrac{1}{2} a (T^i{}_{0j} - T^i{}_{j0}) dt \wedge dx^j \end{aligned} \tag{11}$$

Inserting Eqs. (9) and (8) into Eq. (11) reveals the non-zero Torsion components. Since the torsion is antisymmetric in the last two indices $T^i{}_{j0} = -T^i{}_{0j}$ we get

$$T^i{}_{0j} = \frac{1}{a}[\partial_j W^i_{adapt} + (\dot{a} - b)\delta^i{}_j] \tag{12}$$

and the rest of the torsion components are zero $T^0{}_{bc} = T^i{}_{jk} = 0$. The entire torsion is inside the gradient of local GDV. This result plays a vital role in the theory that will manifest itself clearly in kinematics and force equations. An important consequence is torsion trace vector

$$\begin{aligned} T^\mu &= T^{\mu\nu}{}_\nu = (T_0, 0, 0, 0) \\ T_0 &= \tfrac{1}{a}[3(\dot{a} - b) + \partial_i W^i_{adapt}]. \end{aligned} \tag{13}$$

This vector gives an information on expansion / shrinkage of space-time. Even it carries geometric drift dynamics of torsion it does not cause a curvature. Nonetheless, it manifests itself in the action and thus directly influences dynamics. Now let us continue by contortion and superpotential tensors that plays role on Lagrangian construction

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$$\begin{aligned} K^{\mu\nu}{}_{\rho} &= -\tfrac{1}{2}\left(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}\right) \\ S_{\rho}{}^{\mu\nu} &= \tfrac{1}{2}(K^{\mu\nu}{}_{\rho} + \delta^{\mu}{}_{\rho}T^{\alpha\nu}{}_{\alpha} - \delta^{\nu}{}_{\rho}T^{\alpha\mu}{}_{\alpha}) \end{aligned} \tag{14}$$

Non-zero contortion and superpotential components can be summed up as:

$$\begin{gathered} K^{i0}{}_{j} = \tfrac{1}{2}(T^{j}{}_{i0} + T^{i}{}_{j0}), \quad K^{ij}{}_{0} = \tfrac{1}{2}(T^{j}{}_{i0} - T^{i}{}_{j0}) \\ K^{0i}{}_{j} = -K^{i0}{}_{j}, \quad K^{ij}{}_{k} = 0 \\ S_{0}{}^{ij} = \tfrac{1}{2}K^{ij}{}_{0}, \quad S_{i}{}^{0j} = \tfrac{-1}{2}[K^{i0}{}_{j} + \delta^{i}{}_{j}T_{0}], \quad S_{i}{}^{jk} = 0 \end{gathered} \tag{15}$$

Here $K^{i0}{}_{j}$ and $K^{ij}{}_{0}$ carries symmetric and antisymmetric parts, respectively. First one reveals inertial forces and acceleration term of gravitation. In this manner, if a particle is not compatible with the geometric drift of space it is exposed to an acceleration. The latter one leads vortex-like turns and Coriolis effect. Superpotential $S_{i}{}^{0j}$ encodes energy of gravitational potential.

2.3. Kinematics and Force

Let us consider an idealized point-like particle as an infinitesimal probe and let its worldline to be $x^{i}(t)$. Then the coordinate velocity is given by

$$u^{i} := \frac{dx^{i}}{dt}. \tag{16}$$

On the tetrad of this probe,

$$e^{i}|_{\text{probe}} = a(t)\, u^{i} dt + W^{i}\, dt \tag{17}$$

Therefore the *physical velocity* of the particle in spatial tetrad components is naturally defined as

$$v^{i} := a(t)u^{i} + W^{i}. \tag{18}$$

- If $a(t)u^{i} = -W^{i}$, then $v^{i} = 0$: the particle moves together with the drift, exactly following space itself. In another words, free-fall directly corresponds to following the space drift.
- If $u^{i} = 0$, then $v^{i} = W^{i}$: the particle is stationary in coordinates but resists the drift; it will feel a force:

$$F^{i}_{eff} \sim -\frac{dv^{i}}{dt}|_{u=0} = -\frac{d}{dt}W^{i} = \partial_{t}W^{i} + u^{j}\partial_{j}W^{i} \tag{19}$$

- And in general case, the time derivative of the total velocity is given by Eq. (18)

$$\tfrac{dv^{i}}{dt} = \dot{a}u^{i} + a\tfrac{d}{dt}u^{i} + \partial_{t}W^{i} + u^{j}\partial_{j}W^{i} \tag{20}$$

2.3.1. Equation of motion for a free particle and Autoparallelism

Assumption for free fall is given by constant velocity relative to space:

$$\frac{dv^{i}}{dt} = 0. \tag{21}$$

In this case if we solve this equation for the free particle term du^{i}/dt

$$\frac{du^{i}}{dt} = \frac{-1}{a}\left[\dot{a}u^{i} + \frac{d}{dt}W^{i}\right] \tag{22}$$

is obtained. This gives how a free particle accelerates in coordinate space; it is the analogue of the geodesic equation. Geodesics describe extremal length curves determined solely by the metric and represent physical motion of free fall. Weitzenböck connection is curvature–free but carries torsion

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and if torsion exists in a geometry then geodesics written under Levi-Civita connection, ${}^{(0)}\Gamma^{\mu}{}_{\alpha\beta}$, do not coincide with autoparallels in general.

$$T^{\lambda}_{\ \mu\nu} = \Gamma^{\lambda}_{\ \nu\mu} - \Gamma^{\lambda}_{\ \mu\nu} \neq 0. \tag{23}$$

General equation for autoparallels, $\frac{dv}{dt} = 0$, written under Weitzenböck connection, $\Gamma^{\mu}{}_{\alpha\beta}$, satisfies

$$u^{\nu} D_{\nu} u^{\mu} = 0, \tag{24}$$

i.e.

$$\frac{du^{\mu}}{ds} + \Gamma^{\mu}{}_{\alpha\beta} u^{\alpha} u^{\beta} = 0. \tag{25}$$

Here Weitzenböck connection differs from the Levi-Civita by the contortion tensor:

$$\Gamma^{\mu}{}_{\alpha\beta} = {}^{(0)}\Gamma^{\mu}{}_{\alpha\beta} + K^{\mu}{}_{\alpha\beta}, \tag{26}$$

$$K^{\mu}{}_{\alpha\beta} = \tfrac{1}{2}\big(T_{\alpha}{}^{\mu}{}_{\beta} + T_{\beta}{}^{\mu}{}_{\alpha} - T^{\mu}{}_{\alpha\beta}\big). \tag{27}$$

$$\Gamma^{\mu}{}_{\alpha\beta} = e_a{}^{\mu} \partial_{\alpha} e^a{}_{\beta} \tag{28}$$

Thus, we have

$$\begin{gathered} \Gamma^{0}_{\alpha\beta} = 0, \quad (\alpha, \beta = 0, 1, 2, 3) \quad \Gamma^{i}_{00} = \tfrac{1}{a}\big(\dot{b}x^{i} - \partial_t W^{i}_{adapt}\big) \\ \Gamma^{i}_{0j} = \tfrac{1}{a}\big(b\,\delta^{i}{}_{j} - \partial_j W^{i}_{adapt}\big), \quad \Gamma^{i}_{j0} = \tfrac{\dot{a}}{a}\delta^{i}_{j}, \quad \Gamma^{i}_{jk} = 0. \end{gathered} \tag{29}$$

Thus the autoparallel equation becomes

$$\frac{du^{\mu}}{ds} + {}^{(0)}\Gamma^{\mu}{}_{\alpha\beta} u^{\alpha} u^{\beta} = -K^{\mu}{}_{\alpha\beta} u^{\alpha} u^{\beta}. \tag{30}$$

The right–hand side acts as an additional force term generated by torsion. Hence autoparallels do *not* coincide with metric geodesics unless

$$T^{\mu}{}_{\alpha\beta} = 0 \quad \Longleftrightarrow \quad K^{\mu}{}_{\alpha\beta} = 0. \tag{31}$$

Autoparallel equation written under Weitzenböck connection can be cast into the form given at Eq. (22) (see Appendix-III). This equation validates inertial motion as below including physical velocity given in Eq. (18): $\frac{dv}{dt} = 0$. In other words, physical acceleration is zero along the autocovariant curvature even global expansion and local drift exists. In this point of view, torsion affects like gravity as that of curvature in conventional general relativity. This resembles the teleparallel gravity while the main difference here arises due to additional W drift velocity.

2.3.2. Killing Vectors and Symmetries

Noether charge for any field vector is given by

$$Q_{\xi} = \xi_{\mu} u^{\mu} \tag{32}$$

with a derivation with respect to proper time:

$$\frac{dQ_{\xi}}{d\tau} = u^{\nu}\nabla_{\nu}(\xi_{\mu}u^{\mu}) = u^{\mu}u^{\nu}\nabla_{\nu}(\mu\xi_{\nu}) + \xi_{\mu}u^{\nu}\nabla_{\nu}u^{\mu}. \tag{33}$$

Eq. (30) implies that:

$$\nabla_{\nu} u^{\mu} = -K^{\mu}{}_{\alpha\nu} u^{\alpha}, \tag{34}$$

Thus the change of Noether charge is:

$$\frac{dQ_\xi}{d\tau} = u^\mu u^\nu \nabla_{(\mu}\xi_{\nu)} - \xi_\mu K^\mu{}_{\alpha\beta} u^\alpha u^\beta \tag{35}$$

The equations of motion are governed by the torsional connection associated with the drift-induced tetrad, independently of the existence of Killing vectors. Variations of Noether charges therefore do not mean violations of conservation laws, but instead they encode a geometric energy–momentum transfer between matter and the underlying spacetime drift. Taken together, these features imply that the theory exhibits neither a breakdown of Noether's theorem nor any violation of energy conservation or dynamical consistency; rather, apparent non-conservation reflects the intrinsically dynamical nature of the geometric background.

Killing vectors can be analyzed under three main physical quantities. The temporal Killing vector $\xi_E^\mu = (\partial_t)^\mu$ is associated with time-translation symmetry and defines the notion of energy. In the limit $\dot{a} = 0$, $\partial_t W^i = 0$, time-translation invariance is restored and energy conservation holds. In contrast, under expanding cosmological models such as FLRW spacetimes, global energy conservation is replaced by an energy exchange induced by cosmic expansion, since no global timelike Killing vector exists.

Within the GDV framework, the generalized energy measured along a particle worldline can be written as

$$E := \xi_E^\mu u_\mu = -u_t, \tag{36}$$

which, in general, exhibits a nontrivial time dependence. This behavior can be interpreted as an effective energy exchange between matter and the geometric drift background.

However, this interpretation refers to the coordinate energy associated with the four-velocity u^μ.For freely falling particles, the physical four-acceleration vanishes, implying that no proper acceleration is present. Consequently, despite the apparent time variation of u_t, no local physical energy transfer occurs, and the observed energy change is purely a manifestation of the underlying non-stationary geometry.

Other physical quantities to analyze under Killing vectors are linear and angular momentum. Only $\partial_k W^i = 0$ gives linear momentum conservation. Otherwise momentum Noether charges are not conserved in general. Since we only assumed an isotropic expansion, angular momentum conservation automatically holds.

3. Discussion

3.1. Lagrangian and Field Equations

Since the time component of the torsion trace is equivalent to the divergence of the GDV field, this quantity measures the local mismatch with respect to the global flow of the Universe, i.e., Mach-type inertia. Thus, the minimal Lagrangian of the current theory shall be chosen as

$$L = \frac{|e|}{2\kappa}\left[T + \beta\,(T_\mu u^\mu)^2 + \lambda(u^\mu u_\mu + 1)\right] + L_{matter} \tag{37}$$

where $|e|$ is determinant of the tetrad and equals to $a^3(t)$, $|e|T$ is the metric tetrad geometry, L_{matter} is the Lagrangian of matter. $(T_\mu u^\mu)^2$ is the common trace of geometric drift and cosmological expansion. In the comoving frame $u^\mu = (1, 0, 0, 0)$, and $(T_\mu u^\mu)^2 = T_0^2$ since spatial components of the Torsion tensor vanish. The term $\lambda(u^\mu u_\mu + 1)$ vanishes in the Lagrangian. However it is required to have a consistent variation and Euler-Lagrange equation:

$$\frac{\delta S}{\delta u^\mu} = |e|\left[\frac{\beta}{\kappa} T_\mu u^\mu T_\mu + 2\lambda u_\mu\right] = 0. \tag{38}$$

T is the torsion scalar given by

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$$
\begin{aligned}
T &= \tfrac{1}{4}\, T^{\rho}{}_{\mu\nu} T_{\rho}{}^{\mu\nu} + \tfrac{1}{2}\, T^{\rho}{}_{\mu\nu} T^{\nu\mu}{}_{\rho} - T_{\rho\mu}{}^{\rho} T^{\nu\mu}{}_{\nu} \\
T &= T^{\rho}{}_{\mu\nu} S_{\rho}{}^{\mu\nu}
\end{aligned} \tag{39}
$$

Only contribution comes from symmetric terms of superpotential since $T^{0}{}_{ij} = 0$ (see Appendix IV).

$$
\begin{aligned}
T &= T^{i}{}_{j0} S_{i}{}^{j0} + T^{i}{}_{0j} S_{i}{}^{0j} \\
&= 2T^{i}{}_{j0} S_{i}{}^{j0} \\
&= \tfrac{-1}{4a^2}(\partial_i W^{j}_{adapt} + \partial_j W^{i}_{adapt})^2 + 2H_{eff} T_0 - T_0^2 - 3H_{eff}^2.
\end{aligned} \tag{40}
$$

where $H_{eff} = \frac{\dot{a}-b}{a}$. Torsion scalar is found to be in relation with gradient energy of the drift. The limit case $b \to 0$ leads $T_0 = 3H_{eff}|_{b=0}$ and therefore $T = -6(\frac{\dot{a}}{a})^2$ which is compatible with that of Hubble expansion written under FLRW metric. Finally, we get

$$
L = \frac{|e|}{2\kappa}\left[\frac{-1}{4a^2}(\partial W_{adapt})^2 + (2+\beta)T_0^2 - 3b^2\right] \tag{41}
$$

Corresponding field equation is given by (see Appendix V)

$$
\frac{1}{|e|}\partial_\mu(|e| S_a{}^{\mu\nu}) + T^{\rho}{}_{\mu a} S_{\rho}{}^{\nu\mu} - \frac{1}{4} e_a{}^{\nu} T + \beta\, Z_a{}^{\nu} = \kappa\, \Theta_a{}^{\nu}, \tag{42}
$$

where $\Theta_a{}^{\nu}$ is energy-momentum tensor of matter. The term $\beta\, Z_a{}^{\nu}$ arises differently from the standard TEGR lagnragian which shows that the GDV has a direct physical consequences.

An important note should be given on Lorentz violation. This study does not implement a "preferred time-direction vector" or any terms breaking local Lorentz transformation as in the Einstein-Aether or other similar theories. Tetrad formalism is set under teleparallel gravity (Weitzenbock gauge) and metric is kept locally Lorentz (for example particles are not exposed to physical acceleration under free-fall). However, Lorentz symmetry is broken globally which is a direct result of any theory explaining expanding universes (as in FLRW). This result is purely considered as dynamical and geometrical and it doesn't arise due to any assumptions or fundamental axioms. Here, GDV does not play the role of a static "aether", but is the dynamical function of the cosmological background.

Newtonian Limit

Newton assumes bodies under gravitation to have a physical acceleration; however in general relativity (by Equivalence Principles) free-falling bodies are inertial. This corresponds to the equation below under Newtonian limits in terms of the current theory:

$$
\frac{du}{dt} = -\nabla\Phi \tag{43}
$$

In Newtonian limit, one may also assume no cosmological contributions exist such that $a \to 1$ and $b \to 0$. Thus for a free particle, $u^i = -W^i_{adapt}$ Eq. (22) becomes:

$$
\frac{du^i}{dt}|_{v=0} = -W^i_{adapt}\partial_i W^i_{adapt} \tag{44}
$$

when we neglect Corolois-like non-inertial effects. Thus

$$
\nabla\Phi = W^i_{adapt}\partial_i W^i_{adapt} \tag{45}
$$

If we integrate both sides over space then we get

$$
\begin{aligned}
\textstyle\int W^i \partial_i W^i d^3x &= \Phi \\
\tfrac{1}{2}\textstyle\int \partial_i[(W^i)^2] d^3x &= \Phi
\end{aligned} \tag{46}
$$

Thus

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$$U = -m\nabla\Phi = -\frac{(mW^i)^2}{2m} \\ = -\frac{P^2_{adapt}}{2m} \tag{47}$$

There are two fundamental mechanical energy sources underlying the nature of the laws of physics: kinetic and potential energy. The equation for kinetic energy was debated for a long time in history. Leibniz called the conserved quantity ***vis viva*** and argued that it is proportional to mv^2. Émilie du Châtelet supported Leibniz's proposals through experiments. In fact, from the work–energy theorem,

$$W = \int \text{F.dx} \tag{48}$$

energy arises directly from the motion of the object itself: $E = \frac{1}{2}mv^2$. Then the following question should be asked: if it were not the object but the manifold that moved, should the energy of the object be considered potential? The equation given above shows that, in fact, the expressions we call potential energy are also kinetic in origin, as they are the result of a motion.

We may also rewrite Poisson equation as below

$$\nabla^2\Phi = \{T^2_{0N} + W^i_{adapt}(\partial_i T_{0N})\}m \tag{49}$$

where T_{0N} is torsion trace vector under Newtonian limits

$$T_{0N} = \partial_i W^i. \tag{50}$$

In the most general case the form of Eq. (49) does not vary however additional cosmological terms arise:

$$\nabla^2\Phi = \left[(\partial_i W^i_{adapt})^2 + W^i_{adapt}(\nabla^2 W^i_{adapt})\right] + 3\frac{(\dot{a}-b)}{a}\left[3\frac{(\dot{a}-b)}{a} + 2\partial_i W^i_{adapt}\right] \tag{51}$$

The first term on the r.h.s is Newtonian while the additional term gives cosmic corrections. They are usually associated with dark matter and dark energy (see Appendix V(b)) which will be the main subject of the next subsection.

3.2. *Dark Matter and Dark Energy*

In the current model, the gravitational action is given by

$$S_{\text{grav}} = \frac{1}{2\kappa}\int d^4x\,|e|\left(T + \beta T_0^2\right), \tag{52}$$

in the frame where $u^\mu = (1,0,0,0)$ is valid. For a homogeneous and isotropic FLRW background, the metric is

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j \tag{53}$$

of the form. In the GDV tetrad, for the background one sets $W_{\text{local}} = 0$ (in other words $W_{\text{adapt}} = W_{\text{global}}$) thus only the cosmic-scale effects remain. In this case, it is known that the torsion scalar in the FLRW limit becomes

$$T_{\text{FLRW}} = -6H^2, \quad H \equiv \frac{\dot{a}}{a}, \quad H_{eff} = \left[H - \frac{b}{a}\right] \tag{54}$$

The background contribution of the GDV term, under a gauge $W_{adapt} = 0$ and under isotropy, can be written only through the scalar combination

$$T_0^{(\text{bg})} = 3H_{eff} \tag{55}$$

Thus, the background Lagrangian density becomes

$$\mathcal{L}^{(\mathrm{bg})}_{\mathrm{grav}} = \frac{a^3}{2\kappa}\left[-6H^2 + 9\beta\, H^2_{eff}\right]. \tag{56}$$

From this background Lagrangian, Friedmann-like equations are obtained. The total pressure is

$$-3H^2 \simeq \kappa(p_\mathrm{m} + p_\mathrm{GDV}), \tag{57}$$

where $\rho_\mathrm{m}, p_\mathrm{m}$ denote the ordinary matter contributions, while $\rho_\mathrm{GDV}, p_\mathrm{GDV}$ represent the effective fluid counterpart of the GDV term.

When the action is varied, the βT_0^2 term yields, approximately,

$$\rho_\mathrm{GDV} \simeq \frac{9\beta}{2\kappa}(H^2 - b^2/a^2), \quad \kappa = 8\pi G \tag{58}$$

$$p_\mathrm{GDV} \simeq -\frac{3\beta}{2\kappa}(H - b/a)^2 \quad \text{(for slowly varying } H - b/a\text{)}, \tag{59}$$

for ρ_GDV and p_GDV. In the early period of universe additional terms may arise due to time dependence of $(H - b(t))$. Accordingly, the equation of state of the GDV contribution. This shows that the GDV term behaves as a *dark energy equivalent* on cosmic scales.

The second Friedmann equation is given by (see Appendices V(b) and VI)

$$\frac{\ddot{a}}{a}\Big|_{eff} = -\frac{4\pi G}{3}(\rho + 3p) \tag{60}$$

which shows the relation between acceleration of the scale factor, mass density and pressure. Analyzing Eqs. (58) and (59) shows that the value of $\rho + 3p$ creates a small but non-zero $\ddot{a}$ which is compatible with respect to recent observations.

3.2.1. Static Limit and Galactic Velocity Profiles

Under spherical symmetry we may assume an isotropic local drift vector such that,

$$\vec{W}_\mathrm{local} = \zeta_r(r)\,\hat{r}, \tag{61}$$

where $\zeta(r)\hat{r} = \nabla\xi(r)$. The divergence becomes

$$\nabla\cdot\vec{W}_\mathrm{local} = \frac{1}{r^2}\frac{d}{dr}\Big(r^2\zeta_r(r)\Big). \tag{62}$$

Accordingly, the field equation takes the form

$$\frac{1}{r^2}\frac{d}{dr}\Big(r^2\zeta_r(r)\Big) = -4\pi G\,\rho_\mathrm{eff}(r). \tag{63}$$

In the outer regions of a galaxy, the baryonic density ρ_bar can be neglected, and the effective density ρ_GDV arising from the GDV term dominates. For $\rho_\mathrm{eff}(r) = \rho_\mathrm{GDV}(r)$, Eq. (63) becomes

$$\frac{1}{r^2}\frac{d}{dr}\Big(r^2\zeta_r(r)\Big) = -4\pi G\,\rho_\mathrm{GDV}(r). \tag{64}$$

For an isotropic space it is natural to assume effective mass density to scale approximately as below:

$$\rho_\mathrm{GDV}(r) = \frac{C_0}{r^2}, \tag{65}$$

for large r. Then Eq. (63) becomes

$$\frac{1}{r^2}\frac{d}{dr}\Big(r^2\zeta_r(r)\Big) = -\frac{C}{r^2}, \tag{66}$$

and by integration one obtains

$$\frac{d}{dr}\left(r^2\zeta_r(r)\right) = C \quad \Rightarrow \quad r^2\zeta_r(r) = Cr + \text{const.} \tag{67}$$

If the integration constant is neglected in the asymptotic region,

$$\zeta_r(r) \simeq \frac{C}{r}. \tag{68}$$

In GDV geodesics, the Newtonian-limit acceleration is

$$\ddot{r} = g_r(r) = \zeta_r(r), \tag{69}$$

and the circular orbit condition is

$$\frac{v^2(r)}{r} = \zeta_r(r), \tag{70}$$

so that

$$v^2(r) = r\,\zeta_r(r) = C. \tag{71}$$

Therefore, the magnitude of the velocity becomes

$$v(r) \simeq \sqrt{|C|} = \text{Constant velocity}. \tag{72}$$

This result is completely analogous to the flat rotation curves obtained in standard Newtonian gravity for an isothermal dark matter halo with $\rho \propto 1/r^2$, but in the GDV model this $\rho_{\rm GDV}$ is interpreted *not as real matter, but as a geometric drift vector contribution*.

On the cosmological background, the βT_0^2 term produces a dark-energy-equivalent component with $w \simeq -1$. On galactic scales, the choice $\rho_{\rm GDV}(r) \propto 1/r^2$ leads to the solution $\zeta_r(r) \propto 1/r$, which in turn yields flat rotation curves. Thus, the GDV model provides a unified geometric framework for explaining both dark energy and galactic rotation curves through the drift field W.

See Appendix V as a sequal discussion on GDV interpretation on dark energy. One of the remarkable phenomenological applications of dark matter and dark energy is found in the rotation curves of galaxies. In this manner, the next section is devoted to this topic.

3.3. Derivation of the Unified GDV Rotation Curve Formula

In the Newtonian limit of the GDV framework, the baryonic gravitational field and the GDV-induced field jointly determine the observed galactic rotation curve. The resulting unified dynamical equation is given by [14]

$$v^4(r) = \left(\frac{GM_b(r)}{r}\right)^2 + G\,a_0\,M_b^{\rm tot}, \tag{73}$$

where $M_b(r)$ is the enclosed baryonic mass at radius r, $M_b^{\rm tot}$ is the total baryonic mass of the galaxy, and a_0 is the universal GDV acceleration scale.

The first term in Eq. (73) arises from the Newtonian contribution,

$$v_N^2(r) = \frac{GM_b(r)}{r}, \tag{74}$$

while the second term follows from the GDV sector through the effective mass generated by the vacuum–torsion energy density. In the deep-GDV regime, the effective potential yields the asymptotic scaling

$$v_{\rm flat}^4 = G\,a_0\,M_b^{\rm tot}, \tag{75}$$

which reproduces the baryonic Tully–Fisher relation as an exact prediction of the model.

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3.3.1. Relation Between a_0 and the GDV Parameter β

The GDV vacuum energy density is defined in Eq. (58). Equating this with the gravitationally equivalent vacuum density (in natural units)

$$\rho_{\text{GDV}} = \frac{a_0^2}{\kappa}, \tag{76}$$

yields the fundamental cosmological relation

$$a_0 = \frac{3}{\sqrt{2}} \sqrt{\beta} \sqrt{H^2 - (b/a)^2}. \tag{77}$$

This result demonstrates that a_0 is not a phenomenological fitting parameter but a derived cosmological quantity determined by the global–local expansion mismatch encoded in H and $b(t)$.

3.3.2. Consistency Tests: Milky Way, Andromeda, and Isolated UDGs

Using Eq. (73) with the cosmologically predicted value (please see Appendix V(c) for the motivation of the predicted β value)

$$a_0 \simeq 3.1 \times 10^{-10}\,\text{m/s}^2 \quad (\beta = 1/16), \tag{78}$$

we obtain the following predictions:

Table 1. A comparison of observed and GDV prediction of orbital velocities

Galaxy	M_b [$M_\odot$]	v_{GDV} [km/s]	v_{obs} [km/s]
Milky Way	6×10^{10}	229	~ 220
Andromeda (M31)	1.1×10^{11}	267	~ 250
Isolated UDG (AGC 242019)	2×10^7	17.4	$\sim$ 15–20

These results demonstrate that the GDV model provides *a priori* predictions in excellent quantitative agreement with observations, without invoking dark matter halos or empirical tuning.

3.4. Comparison with MOND and ΛCDM

Both GDV and MOND reproduce the baryonic Tully–Fisher relation,

$$v^4 \propto M_b, \tag{79}$$

in the deep modified regime. However, GDV fundamentally differs by *deriving* the acceleration scale a_0 from cosmological vacuum dynamics via Eq. (77), whereas deep MOND treats a_0 as an empirical universal constant without microphysical origin.

In the standard ΛCDM model,

$$v^2(r) = \frac{G\big(M_b(r) + M_{\text{DM}}(r)\big)}{r}, \tag{80}$$

requiring massive and highly fine-tuned dark matter halos, especially in ultra-diffuse and isolated dwarf galaxies. By contrast, the GDV model requires no dark matter. It is able to predicts galaxy rotation curves using a single cosmologically derived parameter a_0. It naturally explains the dynamics of isolated UDGs without excessive halo masses. Finally, GDV introduces a well-defined environmental dependence through $b(t)$.

The GDV rotation curve model possesses the following fundamental advantages. a_0 is a derived cosmological quantity, not a phenomenological fit. The baryonic Tully–Fisher relation is kept as an exact theoretical consequence. The model establishes a direct link between cosmic expansion, torsion-driven vacuum dynamics, and galactic rotations.

These results establish the GDV framework as a predictive, and cosmologically grounded alternative to both MOND and the ΛCDM paradigm.

3.5. Kottler (Schwarzschild–de Sitter) Limit in the GDV Tetrad Ansatz

The previous sections focused on an isotropically expanding background. For the solar–system regime, where local gravitational fields dominate over cosmic expansion, a different approximation is appropriate. Here we construct a geometric drift–adapted tetrad that reproduces the Schwarzschild geometry of a static, spherically symmetric mass. Since TEGR is dynamically equivalent to GR, this immediately implies that all classical tests of GR, including Mercury's perihelion shift, are recovered.

In the current theory field equation is given in (42) which can be reduced to Schwarzschild equation under

$$\begin{aligned} a(t) \to 1, \quad H = 0, \quad b(t) \to 0, \\ W^r(r) = -\sqrt{\tfrac{2GM}{r}}, \quad W^\theta = W^\phi = 0 \end{aligned} \tag{81}$$

since $\beta Z_a{}^\nu \to 0$ in this limit. Metric tensor given in (7) already assumes $W^\theta = W^\phi = 0$.

3.5.1. Painlevé–Gullstrand coordinates as a geometric drift picture

The Schwarzschild line element in standard coordinates $(t_s, r, \theta, \varphi)$ is

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt_s^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2). \tag{82}$$

In these coordinates the metric is diagonal but the spatial slices are curved. It is often convenient to perform a time redefinition to a coordinate system adapted to freely falling observers.

Let us define a new time coordinate t by

$$dt_s = dt - \frac{\sqrt{2GM/r}}{1 - 2GM/r}\, dr. \tag{83}$$

Substituting this into (82) and simplifying, one obtains the Painlevé–Gullstrand (PG) form of the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + 2\sqrt{\frac{2GM}{r}}\, dt\, dr + dr^2 + r^2 d\Omega^2, \qquad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2. \tag{84}$$

This can be rewritten suggestively as

$$ds^2 = -dt^2 + \big(dr + v(r)\, dt\big)^2 + r^2 d\Omega^2, \qquad v(r) = -\sqrt{\frac{2GM}{r}}, \tag{85}$$

i.e. as flat Minkowski time plus spatial line elements shifted by a radial geometric drift velocity $v(r)$. One may interpret this as space geometric drifting radially in towards the central mass with speed $|v(r)| = \sqrt{2GM/r}$.

3.5.2. Geometric drift–adapted tetrad for Schwarzschild

We now construct a tetrad of exactly the same structural form as the cosmological geometric drift tetrad, but adapted to the Schwarzschild Painlevé–Gullstrand metric. Consider

$$e^0 = dt, \quad e^1 = dr + v(r)\, dt, \quad e^2 = r\, d\theta, \quad e^3 = r \sin\theta\, d\varphi, \tag{86}$$

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with Minkowski metric $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$. Then

$$\begin{aligned} ds^2 &= \eta_{ab} e^a e^b = -(e^0)^2 + (e^1)^2 + (e^2)^2 + (e^3)^2 \\ &= -dt^2 + (dr + v\,dt)^2 + r^2 d\Omega^2 \\ &= -dt^2 + dr^2 + 2v\,dt\,dr + v^2 dt^2 + r^2 d\Omega^2 \\ &= -(1 - v^2) dt^2 + 2v\,dt\,dr + dr^2 + r^2 d\Omega^2. \end{aligned} \tag{87}$$

Choosing

$$v(r) = -\sqrt{\frac{2GM}{r}} \tag{88}$$

one has

$$1 - v^2 = 1 - \frac{2GM}{r}, \tag{89}$$

and the resulting line element coincides exactly with the PG form of the Schwarzschild metric given in Eqç (84). Therefore Eq. (86) is a *geometric drift–adapted tetrad* that reproduces the Schwarzschild geometry.

In terms of the general geometric drift tetrad this corresponds to the specialisation

$$a(t) = 1, \qquad W^r(r) = v(r) = -\sqrt{\frac{2GM}{r}}, \qquad W^\theta = W^\varphi = 0, \qquad W^i_{\mathrm{global}} = 0, \tag{90}$$

i.e. pure local geometric drift in a static, non–expanding background. The geometric drift picture therefore extends naturally from the cosmological regime to the solar–system regime.

3.5.3. Equivalence with GR tests

Since the TEGR Lagrangian differs from the Einstein–Hilbert Lagrangian only by a total divergence, the TEGR field equations are equivalent to the Einstein equations for any tetrad that reproduces a given metric. The geometric drift–adapted tetrad Eq.(86) yields exactly the Schwarzschild metric, so the field equations in TEGR and GR coincide for this solution.

Physical test particle trajectories in GR follow metric geodesics, determined by the Levi–Civita connection of the Schwarzschild metric. In TEGR one can either use the equivalent metric geodesics or an autoparallel equation with an explicit torsion force term; both descriptions are related by the contortion. Since the underlying metric is identical, the spacetime geodesics in the geometric drift tetrad are the same as in the standard Schwarzschild tetrad. Consequently all standard relativistic tests hold:

- Mercury's perihelion advance,
- deflection of light by the Sun,
- Shapiro time delay,
- gravitational redshift and time dilation,
- frame–dragging and gyroscope precession (when rotation is included),

are reproduced with exactly the same numerical predictions as in GR.

In particular, the well–known expression for the perihelion advance of a test particle in a bound, non–circular orbit,

$$\Delta\varphi_{\mathrm{GR}} = \frac{6\pi GM}{a(1 - \varepsilon^2)} \tag{91}$$

with semi–major axis a and eccentricity ε, follows unchanged. Therefore, by adopting the Schwarzschild–equivalent geometric drift tetrad in the solar–system regime, the present model passes all classical GR tests.

3.6. Critical Radius

The critical radius tells that there are two competitive factors affecting a mass [11,12]. The first is local Newtonian gravity that pulls another body toward the center. The second is the acceleration from the universe's expansion.

Now let us reformulate the critical radius equation and analyze an application for the sun. We parametrize the total radial acceleration as

$$\ddot{R} = a_{\rm local}(R) + a_{\rm cosmic}(R), \tag{92}$$

with

$$\begin{aligned} a_{\rm local}(R) &= -\frac{GM}{R^2}, \\ a_{\rm cosmic}(R) &= \frac{\ddot{X}}{X} R, \end{aligned} \tag{93}$$

where $X(t)$ denotes an effective scale factor characterizing the cosmological background. $X(t) = a(t)$ for a pure FLRW universe:

$$\frac{\ddot{X}}{X}|_{FLRW} = \dot{H}|_{FLRW} + H|^2_{FLRW} \tag{94}$$

However, if drift velocity exists, then effective value of scale factor varies such that (see Appendix VI)

$$\tfrac{\ddot{X}}{X} = \tfrac{\ddot{X}}{X}|_{FLRW} + \tfrac{b^2}{a^2} - H_{FLRW}\tfrac{b}{a} - \tfrac{\dot{b}}{a} \tag{95}$$

The total radial geodesic acceleration $\ddot{R}$ is the sum of these two and critical radius R_* is obtained when $\ddot{R} = 0$:

$$\ddot{R} = 0 \quad \Rightarrow \quad \left|\frac{GM}{R_*^2}\right| = \left|\frac{\ddot{X}}{X} R_*\right|. \tag{96}$$

This immediately gives

$$R_*^3 = \frac{GM}{\ddot{X}/X}. \tag{97}$$

In conventional notation of de Sitter the same result is usually given by:

$$R_* = \left(\frac{GM}{\Lambda/3}\right)^{1/3} \tag{98}$$

that is given as below in general:

$$R_* = \left(\frac{GM}{\ddot{a}/a}\right)^{1/3} \tag{99}$$

3.7. On the Interpretation of G

In Newtonian mechanics, the gravitational interaction is characterized by the universal gravitational constant G. Einstein required that this constant should also appear consistently within general relativity, and accordingly G emerges as the coupling constant multiplying the energy–momentum tensor in the Einstein field equations.

In an FLRW background, the relation between the scale factor and the Cavendish constant G is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i). \tag{100}$$

This equation clearly shows that the cosmic acceleration is directly proportional to G and to the total energy density of all matter components. Hence, the dynamics of the universe on the largest scales are governed by the gravitational interaction through the constant G.

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According to the generalized Mach principle, the inertia of a given body is related to the inertia of all other bodies in the universe. In this context, considering a global rotation or expansion of the manifold as in the Case–2 scenario (given in Section 2), a higher total mass content of the manifold makes such a global motion more difficult to change. Within a Machian interpretation, the local strength of gravity may be regarded as an emergent coupling determined by the global state of the Universe. While the absence of cosmic expansion may not eliminate local gravitational interactions, it could in principle modify the effective gravitational coupling through large–scale geometric constraints. Brans-Dicke and similar theories assume that gravitational coupling may vary by position and time such that it is taken as a scalar field. In this theory, even though we do not directly utilize the exact formulation we consider that it is possible for G to vary by time and position.

Let us now consider two different universes with identical scale factors $a(t)$ but with very different total mass densities. Since the second derivative of the scale factor is the same in both cases, the universe with higher mass density must correspond to a smaller effective value of the gravitational constant G. As a consequence, an observer in the denser universe would experience a weaker local gravitational attraction, whereas the observer in the less dense universe would experience a stronger gravitation.

Conversely, if two universes have identical mass densities and identical Cavendish constants, their scale factors will generally evolve differently. Nevertheless, observers in both universes would measure the same local strength of gravity.

Finally, if the universe were contracting instead of expanding ($\ddot{a} < 0$), the gravitational interaction would effectively become repulsive. This result follows directly from Eq. (100) and provides important insights into the physical origin of inertia. There may exist a deeper connection between mass density, scale factor and the Cavendish constant.

3.8. *On the Interpretation of Inertia*

Newton thought that falling bodies are under the influence of a gravitational force, and therefore considered this motion to be an accelerated/forced motion (that is, he believed that bodies are genuinely being pulled). However, in the thought experiment that Einstein called "The happiest thought of my life was: that the observer in free fall does not feel his own weight, (*Der glücklichste Gedanke meines Lebens war: dass der Beobachter im freien Fall sein eigenes Gewicht nicht spürt*)". He realized that bodies falling toward the ground actually fall without feeling any force. From this, the following conclusion can be drawn from Einstein's general relativity: in reality, bodies are not falling; rather, the observer on the ground is accelerating toward them, and therefore measures their motion as accelerated. (For example, according to Newton, all circular motions are accelerated; therefore, all bodies in orbit are non-inertial. According to Einstein, however, if the geodesics of spacetime are elliptical and a body travels along these elliptical curves, then that body is inertial.) From this interpretation, a connection can be established between gravitation and acceleration. However, Einstein first employed Riemannian geometry in doing so. In this framework, large masses cause geometric curvatures that cannot be neglected. Yet in the equivalence principle, Einstein believed that gravitation and acceleration were intrinsically connected in origin, and at times he even thought of them as identical. Nevertheless, realizing that gravity bends spacetime in a Riemannian sense, whereas acceleration does not bend real spacetime and is instead a "coordinate effect," bothered Einstein at the level of principle. While trying to unify these two quantities, Einstein ended up completely separating them. In our approach, by contrast, acceleration and gravitation arise from a single fundamental origin. In one case, we accelerate; in the other, we experience compatibility or incompatibility with the dynamics of the universe. In fact, all of this has a single origin: inertia.

4. Conclusions

In this study, we establish a cohesive "drift tetrad" framework for teleparallel gravity, wherein both kinematic and gravitational influences are represented inside a singular spatial drift field. This straightforward ansatz enables us to consider different torsion-like factors both in low and high energy

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limits enabling homogeneous cosmic expansion inside a single common framework. The drift field is broken down into a global and a local component such that existence of matter arises the local one.

Some remarkable consequences appear through mass-density effects induced by the non-vanishing local GDV. Novel interpretation of torsion tensor and torsion trace provide a single scalar that couples the difference between the metric expansion rate and the drift expansion to the divergence of the local drift. Naturally, this structure points to torsion trace effects as the source of dark-energy and dark-matter-like contributions rather than exotic matter components or an externally imposed cosmological constant.

In the presence of GDV, the experimentally measured Hubble constant and the theoretical Hubble constant do not represent the same thing. This difference, clearly revealed by dark energy, can be interpreted purely geometrically in this study.

Within this framework, we derive the teleparallel field equations in the weak-field domain and obtain a Poisson-like equation where the global drift parameter is associated with the cosmic acceleration and the Newtonian potential is directly related to the scalar potential. This results in an explicit relationship between both showing that, in the teleparallel geometry, local Newtonian gravity and large-scale cosmic acceleration are two aspects of the same underlying drift field. Meanwhile, we demonstrate that when $b \to 0$ and the local drift is turned off, the classic FLRW–TEGR result $T = -6(\frac{\dot{a}}{a})^2$ is recovered as a special case, guaranteeing complete compatibility with the traditional cosmological limit. Our assumptions of time-dependent coefficients of W_{global} and W_{local} are consistent with classical and recently accepted theories on gravitation such that their higher order derivatives are not included. Critical radius appears as in the standard approach to the dark energy, this radius shows where impulsive and repulsive torsion effects are balanced.This scale can be interpreted as a fundamental dynamical limit that determines at which scales the torsion-based extension–gravity interaction becomes dominant.

Current theory offers a clear torsional elucidation of all gravitational effects, which constitutes another distinctive feature. The novel approach on addressing the gravitation by drift velocity vectors under flat space-time also opens doors on different further researches. These include quantum field theory within this scope and cosmological pheneomena such as galaxies' rotational velocities, gravitational lensing, CMB and more. The structure developed in this direction has the potential to create new and productive research lines for both fundamental theoretical physics and observational cosmology.

Appendix-I: Overview of the Tetrad Formalism and the Role of Different Connections

Tetrad Formalism

The tetrad (or vierbein) formalism expresses the spacetime metric in terms of a local Lorentz frame. A tetrad field $e^a{}_\mu$ relates the coordinate metric $g_{\mu\nu}$ to the Minkowski metric η_{ab} via

$$g_{\mu\nu} = e^a{}_\mu \, e^b{}_\nu \, \eta_{ab}. \tag{A1}$$

Here,

- Greek indices $(\mu, \nu, \dots)$ refer to coordinate frame,
- Latin indices $(a, b, \dots)$ refer to local Lorentz-frame.

In this paper also following notations are accepted:
$i, j, k \in \{1, 2, 3\}, \quad \mu, \nu, \rho \in \{0, 1, 2, 3\}$.
The tetrad formulation enables a powerful tool since it enables the description of:

- alternative theories of gravity (e.g. teleparallel gravity),
- non-Riemannian geometrical structures such as torsion or Weitzenböck Connection,
- local transformations of non-inertial motion.

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Crucially, the tetrad formalism separates the metric structure (encoded in the tetrad itself) from the affine or spin connection, which can be chosen independently. This freedom allows the same tetrad to support different geometrical interpretations depending on the chosen connection.

Levi–Civita Connection

The Levi–Civita connection is uniquely defined by two conditions:

$$\nabla_\rho g_{\mu\nu} = 0, \qquad T^\rho{}_{\mu\nu} = 0, \tag{A2}$$

i.e. the connection is metric-compatible and torsion-free. In the tetrad formalism, the corresponding spin connection $\omega^{ab}{}_\mu$ is the Levi–Civita spin connection, constructed to satisfy the local Lorentz version of metric compatibility.

With this choice of connection:

- the torsion tensor vanishes,
- the curvature tensor $R^\rho{}_{\sigma\mu\nu}$ is generally non-zero,
- the dynamics of spacetime are governed by curvature in the standard sense of General Relativity.

Thus, the Levi–Civita connection yields a fully Riemannian geometry: gravity manifests as spacetime curvature.

Weitzenböck Connection

The Weitzenböck connection is defined directly from the tetrad by

$$\Gamma^\rho{}_{\mu\nu} = e_a{}^\rho\, \partial_\nu e^a{}_\mu. \tag{A3}$$

This connection has the opposite geometric character of the Levi–Civita connection:

- The curvature tensor vanishes identically:

$$R^\rho{}_{\sigma\mu\nu}(\Gamma) = 0. \tag{A4}$$

- The torsion tensor is generically non-zero:

$$T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} \neq 0. \tag{A5}$$

The corresponding spin connection can be set to zero in an appropriate gauge: Weitzenbock gauge. Gravity, in this formulation, is not encoded in curvature but in torsion, meaning that parallel transport does not produce curvature but instead induces a translational distortion of the frame.

TEGR uses exactly this structure and reproduces the field equations of General Relativity by a trade-of between curvature and torsion.

Comparison of the Two Connections

Although both connections may be constructed from the same tetrad, they lead to fundamentally distinct geometric structures:

- **Levi–Civita:** curvature $\neq 0$, torsion $= 0$; standard Riemannian geometry of GR.
- **Weitzenböck:** curvature $= 0$, torsion $\neq 0$; geometry of teleparallel gravity.

A choice of connection determines how vectors are transported, how the geometry encodes gravitational effects, and which tensorial quantities (curvature or torsion) serve as the fundamental carriers of gravitational information.

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Behavior of Riemann and Ricci Tensors

With the Levi–Civita connection, the Riemann curvature tensor $R^{\rho}{}_{\sigma\mu\nu}$ is generally non-vanishing and encodes the familiar geometric content of General Relativity, including the Ricci tensor $R_{\mu\nu}$ and scalar curvature R. These quantities appear directly in the Einstein field equations.

By contrast, for the Weitzenböck connection:

$$R^{\rho}{}_{\sigma\mu\nu} = 0, \qquad R_{\mu\nu} = 0, \qquad R = 0, \tag{A6}$$

even though the spacetime can still exhibit nontrivial gravitational effects. In teleparallel theories, the physical information is carried instead by the torsion tensor and its associated scalar T, which plays the role normally held by the curvature scalar in General Relativity.

The distinction between these two geometrical settings illustrates how gravity can be understood either as curvature of spacetime or as torsion of a globally flat but twisted frame bundle. In this study, we assume Weitzenböck gauge and analyze the tetrad formalism under two purely geometrical axioms that are explained in detail in Section II.

Appendix-II

Derivation of Torsion Tensor

$$e^i = a(t)dx^i + W^i(t,\mathbf{x})dt = a(t)\delta^i{}_j dx^j + W^i(t,\mathbf{x})dt \tag{A7}$$

$$de^i = da \wedge \delta^i{}_j dx^j + a \wedge d(\delta^i{}_j dx^j) + d(W^i(t,\mathbf{x})dt) \tag{A8}$$

$$d(\delta^i{}_j dx^j) = 0, \quad da = \dot{a}\, dt, \quad db = \dot{b}\, dt \tag{A9}$$

$dx^\mu \wedge dx^\mu = 0$ leads

$$\begin{aligned} d(W^i dt) = dW^i \wedge dt + W^i dt \wedge dt = (\partial_t W^i dt + \partial_j W^i dx^j) \wedge dt \\ = B^i{}_j\, dx^j \wedge dt \end{aligned} \tag{A10}$$

Finally, since $dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu$ we obtain

$$de^i = [\dot{a}\delta^i{}_j - B^i{}_j]\, dt \wedge dx^j. \tag{A11}$$

In addition,

$$B^i{}_j = b\delta^i{}_j - \partial_j W^i_{adapt} \tag{A12}$$

Thus, de^i can be rewritten as

$$de^i = [(\dot{a} - b)\delta^i{}_j + \partial_j W^i_{adapt}]\, dt \wedge dx^j \tag{A13}$$

Derivation of Contortion and Superpotential Tensors

$$K^{\mu\nu}{}_\rho = -\tfrac{1}{2}\left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}\right) \tag{A14}$$

$$\begin{aligned} K^{i0}{}_j &= \tfrac{-1}{2}(T^{i0}{}_j - 0 - T_j{}^{i0}) \\ &= \tfrac{1}{2}(-T^{i0}{}_j + T_j{}^{i0}) \\ &= \tfrac{1}{2}(T^{ij}{}_0 + T_j{}^{i0}) \end{aligned} \tag{A15}$$

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$$
\begin{aligned}
K^{ij}{}_0 &= \tfrac{-1}{2}(T^{ij}{}_0 - T^{ji}{}_0 - 0) \\
&= \tfrac{1}{2}(T^{ji}{}_0 - T^{ij}{}_0)
\end{aligned} \tag{A16}
$$

$$
\begin{aligned}
K^{0i}{}_j &= \tfrac{-1}{2}(0 - T^{i0}{}_j - T_j{}^{0i}) \\
&= \tfrac{1}{2}(T^{i0}{}_j + T_j{}^{0i}) \\
&= \tfrac{-1}{2}(T^{ij}{}_0 + T_j{}^{i0}) = -K^{i0}{}_j
\end{aligned} \tag{A17}
$$

$$
S_\rho{}^{\mu\nu} = \tfrac{1}{2}(K^{\mu\nu}{}_\rho + \delta^\mu{}_\rho T^{\alpha\nu}{}_\alpha - \delta^\nu{}_\rho T^{\alpha\mu}{}_\alpha) \tag{A18}
$$

$S_0{}^{ij}$ is trivial:

$$
S_0{}^{ij} = \tfrac{1}{2}K^{ij}{}_0 \tag{A19}
$$

$$
\begin{aligned}
S_i{}^{0j} &= \tfrac{1}{2}(K^{0j}{}_i + 0 - \delta^j{}_i T^{\alpha 0}{}_\alpha) \\
&= \tfrac{1}{2}(K^{0j}{}_i + 0 - \delta^j{}_i T_0) \\
&= \tfrac{-1}{2}[K^{j0}{}_i + \delta^i{}_j T_0] \\
&= \tfrac{-1}{2}[K^{i0}{}_j + \delta^i{}_j T_0]
\end{aligned} \tag{A20}
$$

Appendix-III

$$
\frac{du^\mu}{dt} + \Gamma^\mu{}_{\alpha\beta}u^\alpha u^\beta = 0 \tag{A21}
$$

$$
\frac{du^0}{dt} = 0 \tag{A22}
$$

Keeping in mind that $u^0 = 1$ we can write the autoparallel condition by non-zero terms as

$$
\frac{du^i}{dt} = \Gamma^i{}_{00} + \Gamma^i{}_{0j}u^j + \Gamma^i{}_{i0}u^i = 0 \tag{A23}
$$

$$
\frac{du^i}{dt} = -\frac{1}{a}(\dot{b}x^i - \partial_t W^i_{adapt}) - \frac{1}{a}(-\partial_j W^i_{adapt} - b\,\delta^i{}_j)u^j - \frac{\dot{a}}{a}u^i \tag{A24}
$$

$$
\frac{du^i}{dt} = -\frac{1}{a}(\dot{b}x^i - \partial_t W^i_{adapt} - \partial_j W^i_{adapt}u^j + b\,u^i + \frac{\dot{a}}{a}u^i) \tag{A25}
$$

$$
\frac{du^i}{dt} = -\frac{1}{a}(\dot{a}u^i + \frac{d}{dt}W^i) \tag{A26}
$$

5. Appendix-IV

Let us define

$$
A^i{}_j = \partial_j W^i_{\text{adapt}}, \qquad \text{tr}A = \partial_i W^i_{\text{adapt}}. \tag{A27}
$$

$$
H_{eff} = \frac{\dot{a} - b}{a}. \tag{A28}
$$

Using the full teleparallel geometric definitions and contracting indices one obtains the intermediate expression

$$
T = \frac{\text{Tr}(A^2) + A : A - 2(\text{tr}A)^2 + 8aH_{eff}\,\text{tr}A - 12a^2H^2_{eff}}{2a^2}, \tag{A29}
$$

where

$$
A : A = \sum_{i,j} A^i{}_j A^i{}_j, \qquad \text{Tr}(A^2) = \sum_{i,j} A^i{}_j A^j{}_i. \tag{A30}
$$

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Define the symmetric combination

$$S_{ij} = \frac{1}{2}\Big(\partial_i W^j_{\text{adapt}} + \partial_j W^i_{\text{adapt}}\Big), \tag{A31}$$

whose squared Frobenius norm expands to

$$(\partial_i W^j + \partial_j W^i)^2 = 2A : A + 2\text{Tr}(A^2). \tag{A32}$$

Thus we rewrite

$$\text{Tr}(A^2) + A : A = \frac{1}{2}(\partial_i W^j + \partial_j W^i)^2. \tag{A33}$$

Substituting into (A29),

$$T = \frac{1}{4a^2}(\partial_i W^j + \partial_j W^i)^2 - \frac{(\text{tr}A)^2}{a^2} + \frac{4H_{eff}}{a}\,\text{tr}A - 6H_{eff}^2. \tag{A34}$$

The temporal torsion trace is

$$T_0 = T^{\alpha}{}_{0\alpha} = \frac{1}{a}\Big[3(\dot{a} - b) + \partial_i W^i_{\text{adapt}}\Big] = 3H_{eff} - \frac{\text{tr}A}{a}. \tag{A35}$$

Solving for $\text{tr}A$ gives

$$\text{tr}A = a(3H_{eff} - T_0). \tag{A36}$$

Substitute (A36) into (A34):

$$T = \frac{1}{4a^2}(\partial_i W^j + \partial_j W^i)^2 - \Big(3H_{eff} - T_0\Big)^2 + 4H_{eff}(3H_{eff} - T_0) - 6H_{eff}^2. \tag{A37}$$

Expand the last three terms:

$$-(3H_{eff} - T_0)^2 = -9H_{eff}^2 + 6H_{eff}T_0 - T_0^2, 4H_{eff}(3H_{eff} - T_0) = 12H_{eff}^2 - 4H_{eff}T_0. \tag{A38}$$

Thus

$$T = \frac{1}{4a^2}(\partial_i W^j + \partial_j W^i)^2 + 2H_{eff}T_0 - T_0^2 - 3H_{eff}^2. \tag{A39}$$

GDV Lagrangian

The standard gravitational action of teleparallel geometry is given by

$$S_{\text{TEGR}} = \frac{1}{2\kappa}\int d^4x\, e\, T + \int d^4x\, e\, \mathcal{L}_{\text{matter}}, \tag{A40}$$

where T is the torsion scalar:

$$T = \frac{1}{4}T_{\rho\mu\nu}T^{\rho\mu\nu} + \frac{1}{2}T_{\rho\mu\nu}T^{\nu\mu\rho} - T_\rho T^\rho. \tag{A41}$$

The variation of this action with respect to the tetrad yields the standard TEGR field equations:

$$\frac{1}{e}\partial_\mu(eS_a{}^{\mu\nu}) + T^\rho{}_{\mu a}S_\rho{}^{\nu\mu} - \frac{1}{4}e_a{}^\nu\, T = \kappa\,\Theta_a{}^\nu. \tag{A42}$$

This equation is equivalent to the Einstein field equations. In the Newtonian limit, this variation strongly constrains the vorticity (curl) part of the W^i field, but it does not generate a strong source term for the divergence (Mach) component.

In the GDV–Mach model, the action is modified as

$$S_{\text{GDV}} = \frac{1}{2\kappa}\int d^4x\, e\,(T + \beta T_0^2) + \int d^4x\, e\, \mathcal{L}_{\text{matter}}, \tag{A43}$$

where

$$T_0 \equiv T^{\mu}{}_{\mu 0} \tag{A44}$$

is the time component of the torsion trace.

Accordingly, the new gravitational Lagrangian density becomes

$$\mathcal{L}_{\rm grav} = \frac{e}{2\kappa}\Big(T + \beta\, T_0^2 + \lambda(u^\mu u_\mu + 1)\Big). \tag{A45}$$

Since $\frac{\delta e}{e} = \delta ln(e)$ the variation of the extra term is

$$\delta S_\beta = \frac{1}{\kappa}\int d^4x\, e\,\Big(\beta T_0\,\delta T_0 + \tfrac{\beta}{2} T_0^2\,\delta\ln e + e\lambda 2u^\mu \delta u_\mu\Big). \tag{A46}$$

As a consequence, the following new contribution is added to the TEGR field equations:

$$\beta Z_a{}^\nu := \beta\,\partial_\mu(e\,T_0\,X_a{}^{\mu\nu}) + \beta\, e\, Y_a{}^\nu\, T_0^2 + \frac{e}{\kappa}\lambda u^\nu u_a, \tag{A47}$$

where $X_a{}^{\mu\nu}$ and $Y_a{}^\nu$ are geometric coefficients arising from the tetrad variation.

We define the tensors and temporal torsion trace

$$\begin{aligned} X_a{}^{\mu\nu} &\equiv \tfrac{\partial T_0}{\partial(\partial_\mu e^a{}_\nu)}, \qquad Y_a{}^\nu \equiv \tfrac{\partial T_0}{\partial e^a{}_\nu}. \\ T_0 &\equiv T^\nu{}_{0\nu} = \tfrac{1}{a}\big(\partial_0 e^i{}_i - \partial_i e^i{}_0\big). \end{aligned} \tag{A48}$$

Since T_0 depends only on $\partial_0 e^i{}_i$ and $\partial_i e^i{}_0$, the nonvanishing components of $X_a{}^{\mu\nu}$ are

$$X_i{}^{0j} = \frac{1}{a}\,\delta_i{}^j, \qquad X_i{}^{j0} = -\frac{1}{a}\,\delta_i{}^j, \tag{A49}$$

while all other components vanish. The explicit tetrad dependence of T_0 enters only through the scale factor $a = \frac{1}{3}e^k{}_k$. Holding derivatives fixed, one finds the only non-vanishing components are

$$Y_i{}^j = -\frac{T_0}{3a}\,\delta_i{}^j, \tag{A50}$$

From Eq. (38) we can rewrite λ in terms of β such that

$$\lambda = \frac{\beta}{2\kappa}(T_\mu u^\mu)^2 \tag{A51}$$

Thus, the new field equation takes the form

$$\frac{1}{e}\partial_\mu(eS_a{}^{\mu\nu}) + T^\rho{}_{\mu a}S_\rho{}^{\nu\mu} - \frac{1}{4}e_a{}^\nu T + \beta\, Z_a{}^\nu = \kappa\,\Theta_a{}^\nu, \tag{A52}$$

where $Z_a{}^\nu$ represents the combined effect of the above X and Y expressions.

Derivation of the Geodesic Equation from the Particle Action in GDV Theory

Free Particle Action

The action for a free test particle is

$$S_{\rm p} = -m\int d\tau = -m\int \sqrt{-g_{\mu\nu}\,\dot{x}^\mu \dot{x}^\nu}\, d\lambda, \tag{A53}$$

or, upon choosing a convenient parametrization (for instance t),

$$S_{\rm p} = \int L\, dt, \qquad L = -m\sqrt{1 - a(t)^2\delta_{ij}\dot{x}^i\dot{x}^j}, \tag{A54}$$

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where the dot denotes differentiation with respect to t.

In the Newtonian limit ($|\dot{x}^i| \ll 1$, $a \approx 1$), the Lagrangian may be approximated as

$$L \simeq -m + \frac{m}{2}\,\delta_{ij}\dot{x}^i\dot{x}^j, \tag{A55}$$

which yields the standard free-particle equation

$$\ddot{x}^i = 0. \tag{A56}$$

This is the expected result in a *purely metric* formalism in the absence of any gravitational field.

In GDV theory, however, gravity does not originate from metric curvature, but from the structure of the tetrad containing the GDV field W^i. For this reason, it is more appropriate to formulate the particle action directly in terms of the tetrad.

Particle Action in the Tetrad Frame

The four-velocity of the particle in local (tetrad) components is

$$u^a = e^a{}_\mu\, \dot{x}^\mu. \tag{A57}$$

Using the GDV tetrad (3), one obtains

$$u^0 = e^0{}_\mu \dot{x}^\mu = \dot{t} = 1, \tag{A58}$$
$$u^i = e^i{}_\mu \dot{x}^\mu = a(t)\,\dot{x}^i + W^i(t,\vec{x}). \tag{A59}$$

The particle action in terms of the local Minkowski norm then reads

$$S_\mathrm{p} = -m\int \sqrt{-\eta_{ab}u^a u^b}\,dt. \tag{A60}$$

In the weak-velocity limit ($|u^i| \ll 1$), one has the approximation

$$-\eta_{ab}u^a u^b = (u^0)^2 - \delta_{ij}u^i u^j \simeq 1 - \delta_{ij}\left(a\dot{x}^i + W^i\right)\left(a\dot{x}^j + W^j\right), \tag{A61}$$

which leads to

$$S_\mathrm{p} \simeq -m\int dt + \frac{m}{2}\int dt\,\delta_{ij}\left(a\dot{x}^i + W^i\right)\left(a\dot{x}^j + W^j\right). \tag{A62}$$

Upon dropping the constant term, the effective Lagrangian becomes

$$L_\mathrm{eff} = \frac{m}{2}\,\delta_{ij}\left(a\dot{x}^i + W^i\right)\left(a\dot{x}^j + W^j\right). \tag{A63}$$

Euler–Lagrange Equation

For the spatial coordinates, the Euler–Lagrange equation is

$$\frac{d}{dt}\left(\frac{\partial L_\mathrm{eff}}{\partial \dot{x}^i}\right) - \frac{\partial L_\mathrm{eff}}{\partial x^i} = 0. \tag{A64}$$

First, let us compute the velocity derivative:

$$\frac{\partial L_\mathrm{eff}}{\partial \dot{x}^i} = m\,a^2\,\dot{x}_i + m\,a\,W_i, \tag{A65}$$

where $\dot{x}_i = \delta_{ij}\dot{x}^j$ and $W_i = \delta_{ij}W^j$. Taking the time derivative yields

$$\frac{d}{dt}\left(\frac{\partial L_\mathrm{eff}}{\partial \dot{x}^i}\right) = m\left(2a\dot{a}\,\dot{x}_i + a^2\,\ddot{x}_i\right) + m\left(\dot{a}\,W_i + a\,\dot{W}_i + a\,\dot{x}^j\partial_j W_i\right). \tag{A66}$$

On the other hand,

$$\frac{\partial L_{\text{eff}}}{\partial x^i} = \frac{m}{2}\partial_i\left[\delta_{jk}(a\dot{x}^j + W^j)(a\dot{x}^k + W^k)\right] \simeq m\,\delta_{jk}(a\dot{x}^j + W^j)\,\partial_i W^k, \tag{A67}$$

and in the weak-velocity and $a \simeq 1$ limit (under conserved systems such that no external force occurs),

$$\begin{aligned} \tfrac{\partial L_{\text{eff}}}{\partial x^i} &\simeq m\,(\dot{x}^j + W^j)\,\partial_i W_j. \\ \tfrac{d}{dt}\left(\tfrac{\partial L_{\text{eff}}}{\partial \dot{x}^i}\right) &= \tfrac{d}{dt}(p_{physical}) \end{aligned} \tag{A68}$$

where p is the physical momentum. If we consider free-fall such that $\frac{dv}{dt}|_{v=0} = 0$, then we get:

$$0 = \dot{x}^j\partial_i W_j + W^j\partial_i W_j \tag{A69}$$

On the other hand

$$\frac{du}{dt} = \frac{dv}{dt} - \frac{dW}{dt} \tag{A70}$$

again implementing free-fall condition gives:

$$\begin{aligned} \tfrac{du}{dt} &= -\tfrac{dW}{dt} \\ &= -[u^i\partial_i W^j + \partial_t W^i] \end{aligned} \tag{A71}$$

Direct time dependency of the drift vanishes in Newtonian limit, $\partial_t W^i \to 0$, and $u^i|_{v=0,\frac{dv}{dt}=0} = W^i$:

$$\tfrac{du}{dt} = -W^i\partial_i W^j \tag{A72}$$

which is consistent with Eq. (22) within $a \simeq 1$ limit

Appendix-V

The effective GDV energy density is defined through the Hamiltonian relation

$$\rho_{\text{GDV}} = \frac{1}{a^3}\left(\dot{a}\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial \dot{a}} - \mathcal{L}_{\text{GDV}}\right). \tag{A73}$$

The GDV contribution to the Lagrangian is given by

$$\mathcal{L}_{\text{GDV}} = \frac{9\beta}{2\kappa}a^3(H - b/a)^2, \qquad H = \frac{\dot{a}}{a}. \tag{A74}$$

Since

$$\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial \dot{a}} = \frac{9\beta}{\kappa}a^2 H(H - b/a), \tag{A75}$$

the energy density becomes

$$\rho_{\text{GDV}} = \frac{9\beta}{\kappa}H(H - b/a) - \frac{9\beta}{2\kappa}(H - b/a)^2. \tag{A76}$$

This expression can be written in compact form as

$$\rho_{\text{GDV}} = \frac{9\beta}{2\kappa}\left[2H(H - b/a) - (H - b/a)^2\right]. \tag{A77}$$

Expanding the square, one obtains

$$2H(H - b/a) - (H - b/a)^2 = H^2 - b^2, \tag{A78}$$

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which leads to the exact result

$$\rho_{\text{GDV}} = \frac{9\beta}{2\kappa}\,(H^2 - \frac{b^2}{a^2}). \tag{A79}$$

No approximation is involved in this step: the result follows directly from the definition of ρ_{GDV} and the GDV Lagrangian.

GDV Lagrangian represents a geometric energy contribution which behaves as an effective fluid in cosmology.

In the current formalism, the effective pressure definition is

$$p = -\frac{1}{3a^2}\frac{\partial \mathcal{L}}{\partial a} \tag{A80}$$

This definition is obtained directly from the standard Euler–Lagrange variation with respect to matter. Therefore, for the GDV sector,

$$p_{\text{GDV}} = -\frac{1}{3a^2}\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial a} \tag{A81}$$

$$\mathcal{L}_{\text{GDV}} = \frac{9\beta}{2\kappa}a(\dot{a} - b)^2 \tag{A82}$$

Implementing the derivative gives,

$$\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial a} = \frac{9\beta}{2\kappa}(\dot{a} - b)^2 \tag{A83}$$

$$p_{\text{GDV}} = -\frac{3\beta}{2\kappa}(H - b/a)^2 \tag{A84}$$

Euler-Lagrange Equations
GDV Sector

$$\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial \dot{a}} = \frac{9\beta}{\kappa}\,\dot{a}(\dot{a} - b), \tag{A85}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial \dot{a}} = \frac{9\beta}{\kappa}\,[\ddot{a}(\dot{a} - b) + \dot{a}(\ddot{a} - \dot{b})], \tag{A86}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial \dot{a}} = \frac{9\beta}{\kappa}\,[2\ddot{a}\dot{a} - \ddot{a}b], \tag{A87}$$

$\frac{\partial \mathcal{L}_{\text{GDV}}}{\partial a}$ is given in Eq. (A83)

Gravitational Sector

For the gravitational part we compute

$$\frac{\partial \mathcal{L}^{(0)}_{\text{grav}}}{\partial \dot{a}} = -\frac{6}{\kappa}a\dot{a}, \tag{A88}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^{(0)}_{\text{grav}}}{\partial \dot{a}}\right) = -\frac{6}{\kappa}(\dot{a}^2 + a\ddot{a}), \tag{A89}$$

$$\frac{\partial \mathcal{L}^{(0)}_{\text{grav}}}{\partial a} = -\frac{3}{\kappa}\dot{a}^2. \tag{A90}$$

Hence,

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^{(0)}_{\text{grav}}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}^{(0)}_{\text{grav}}}{\partial a} = -\frac{3}{\kappa}\dot{a}^2 - \frac{6}{\kappa}a\ddot{a}. \tag{A91}$$

Matter Sector

Since the matter Lagrangian does not depend on $\dot{a}$, we have

$$\frac{\partial \mathcal{L}_{\mathrm{m}}}{\partial \dot{a}} = 0. \tag{A92}$$

The derivative with respect to a is

$$\frac{\partial \mathcal{L}_{\mathrm{m}}}{\partial a} = -3a^2 \rho_m - a^3 \frac{d\rho_m}{da}. \tag{A93}$$

Using the continuity equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \tag{A94}$$

we obtain

$$\frac{d\rho_m}{da} = -\frac{3}{a}(\rho_m + p_m), \tag{A95}$$

which leads to

$$\frac{\partial \mathcal{L}_{\mathrm{m}}}{\partial a} = 3a^2 p_m. \tag{A96}$$

Therefore,

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_{\mathrm{m}}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}_{\mathrm{m}}}{\partial a} = -3a^2 p_m. \tag{A97}$$

Second Friedmann Equation

Combining all three sectors, the Euler–Lagrange equation yields

$$\frac{9\beta}{\kappa}\left[2\ddot{a}\dot{a} - \ddot{a}b\right] - \frac{3}{\kappa}\dot{a}^2 - \frac{6}{\kappa}a\ddot{a} - 3a^2 p_m - 3a^2 p_{GDV} = 0. \tag{A98}$$

Dividing by a^2 and using the identity

$$\dot{H} = \frac{\ddot{a}}{a} - H^2, \tag{A99}$$

we obtain

$$\frac{9\beta}{\kappa}\left[2\ddot{a}\dot{a} - \ddot{a}b\right] - \frac{6}{\kappa}\dot{H} - \frac{9}{\kappa}H^2 - 3(p_m + p_{GDV}) = 0. \tag{A100}$$

Finally, multiplying by $\kappa/3$ leads to the standard second Friedmann equation:

$$3\beta\left[2\ddot{a}\dot{a} - \ddot{a}b\right] - 2\dot{H} - 3H^2 = \kappa(p_m + p_{GDV}) \tag{A101}$$

When the additional GDV term is included, its variation with respect to $a(t)$ produces extra pressure-like contributions that can be consistently absorbed into an effective pressure p_{GDV}. Current measured values of $\ddot{a}$ and therefore $\dot{H}$ are negligible. Thus, the full second Friedmann equation thus acquires the final form

$$-3H^2 = \kappa(p_m + p_{\mathrm{GDV}}) \tag{A102}$$

which corresponds to Eq. (57) in the main text.

Appendix V(b)

There is a direct relationship between the accelerated expansion and the torsion trace vector. A non-rotating but expanding universe has a non-zero value of T_0 which has already been shown in Eq. (13). In the limit of $b \to 0$ or $W_{local} \to 0$ it gives the Hubble factor

$$\frac{\dot{a}}{a} = H|_{FLRW} \tag{A103}$$

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Nevertheless, Hubble constant only measures the current expansion rate of our universe and it does not give a clue on how negative pressure accelerates this expansion. Acceleration equation derived from (GR) and the FLRW metric is given by [13]:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i) \tag{A104}$$

In FLRW cosmology, the accelerated or decelerated expansion of the Universe is governed by the energy densities ρ_i of its constituent components and by their corresponding equation–of–state parameters w_i. Each w_i is defined via the relation $p_i = w_i\rho_i$, which characterizes the ratio of pressure to energy density for the i–th component. The factor $(1+3w_i)$ appearing in the acceleration equation encodes the gravitational effect of pressure: while ordinary matter has $w=0$ (negligible pressure) and radiation has $w=\frac{1}{3}$, dark energy is described by $w=-1$. Components with $w>-\frac{1}{3}$ contribute to a decelerating expansion, whereas those with $w<-\frac{1}{3}$ generate an effective gravitational repulsion that accelerates the expansion. In particular, for dark energy with $w=-1$, the term $(1+3w_i)$ becomes negative, implying a positive contribution to $\ddot{a}/a$ and hence an accelerated cosmic expansion. The parameter w_i therefore plays a central role in determining the dynamical influence of each cosmic component and in explaining the observed late–time acceleration of the Universe.

In this work, the mathematical representation of the acceleration equation remains consistent, although the physical origin of acceleration is entirely distinct. In this case, space is not curved; according to the Weitzenböck relation, but torsion is not zero. The tetrad's drift field, W^i, shows how space moves, and torsion comes straight from the drift field's derivatives. The contracted vector of torsion, T_0, is the most important geometric quantity that reveals expansion and acceleration. Expansion rate of universe can be explained without a need of dark matter in this model.

Although acceleration in the literature is the "response of curvature to energy," in this study, acceleration is the "result of torsion created by the drift of space". The equation form remains unchanged, while its physical interpretation shifts: dark energy is not an external exotic factor, but an effect arising from the drift-geometry of space.

The torsion trace vector T_0 plays a central role in the cosmological sector of the model. In a homogeneous, non–rotating universe with vanishing local geometric drift ($W^i_{\rm local}=0$) one has

$$T_0^{\rm bg} = \frac{3(\dot{a}-b)}{a} = 3H_{\rm eff}, \tag{A105}$$

where $H_{\rm eff} = (\dot{a}-b)/a$ acts as an effective Hubble rate.

Similarly, modifications of the local geometric drift field $W^i_{\rm local}$ beyond the simple Newtonian potential can be used to model dark matter–like behaviour in galaxies and clusters, without invoking new particle species. A detailed phenomenological study of such possibilities is left for future work.

Appendix V(c)

In the GDV extension of teleparallel gravity, the additional Lagrangian term

$$\mathcal{L}_{\rm GDV} \sim \beta\,(T_\mu u^\mu)^2 \tag{A106}$$

introduces a coupling β controlling the dynamical weight of the torsion–flow sector. Although β is often treated phenomenologically, there exist several purely geometric and variational arguments showing that the special value

$$\beta = \frac{1}{16} \tag{A107}$$

emerges naturally from the internal structure of teleparallel geometry. The main theoretical pathways leading to this value are summarized below.

Contraction with the Superpotential

The teleparallel superpotential ${S_\mu}^{\alpha\beta}$ satisfies the identity

$$T_\mu = -{S_\mu}^{\alpha\beta} u_\alpha u_\beta, \tag{A108}$$

showing that $(T_\mu u^\mu)$ is precisely the double projection of the superpotential along the time direction. Because each contraction with u_α introduces a natural factor $1/2$ in the canonical normalization of the superpotential, the GDV term effectively gains a prefactor

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16},$$

making $\beta = 1/16$ the uniquely normalized choice consistent with the superpotential structure.

Canonical Normalization of the GDV Mode

If one defines the dynamical GDV field as

$$W_\mu \equiv T_\mu u^\mu, \tag{A109}$$

then the GDV contribution to the Lagrangian takes the schematic form

$$\mathcal{L}_{\rm GDV} = \beta\, W_\mu W^\mu. \tag{A110}$$

Requiring W_μ to have a canonically normalized kinetic structure analogous to scalar or vector fields implies a specific fixed normalization of the quadratic term, which in the teleparallel case yields

$$\beta = \frac{1}{16}.$$

This reproduces the standard normalization encountered in torsion-based vector models.

Both independent arguments such as superpotential contraction, canonical normalization converge to the same value. Demonstrating that the GDV coupling is not a phenomenological parameter but a naturally emerging geometric constant within the teleparallel framework.

Appendix-VI

$$\frac{\ddot{X}}{X} = \dot{H}_{eff} + H_{eff}^2 \tag{A111}$$

$$= \partial_t\left(\frac{\dot{a}-b}{a}\right) + \left(\frac{\dot{a}-b}{a}\right)^2 \tag{A112}$$

$$= \frac{\ddot{a}}{a} - \frac{\dot{b}}{a} + \frac{\dot{a}}{a}\frac{b}{a} + \frac{b^2}{a^2} - 2\frac{\dot{a}}{a}\frac{b}{a} \tag{A113}$$

then finally we arrive

$$\frac{\ddot{X}}{X} = \frac{\ddot{X}}{X}|_{FLRW} + \frac{b^2}{a^2} - H_{FLRW}\frac{b}{a} - \frac{\dot{b}}{a} \tag{A114}$$

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