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Exploration of Renormalization Group Flow Under the Forced GUT Equivalency Within the Stochastic Limit

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Abstract

We validate, through an example, the direct correspondence between the irreversibility of renormalization-group (RG) flow and entropy production thermodynamics imposed by Newell. Using the local RG framework of Osborn and Jack, we identify a scheme-invariant potential $\tilde{a}(\mathbf{g})$ and a positive-definite tensor χ_{ij} satisfying an exact gradient formula, $\partial_i \tilde{a} = \chi_{(ij)}\beta^j$. Mapping this structure onto the GENERIC formalism of Grmela and Öttinger reveals that RG evolution is a purely dissipative process in coupling space, governed by $\dot{g}^i = M^{ij}\partial_j S$ with $S = -\tilde{a}$. Numerical integration of a three-coupling gauge–Yukawa model confirms a strictly monotonic $\tilde{a}(\sigma)$, verifying $\dot{\tilde{a}} = \beta^i \chi_{ij} \beta^j \geq 0$ to machine precision. The result validates the thermodynamic interpretation of the four-dimensional a-theorem and confirms the imposed validity of RG irreversibility, validating the Newell's framework thermodynamics integration.

Keywords: renormalization group; a-theorem; stochastic controls; GENERIC formalism; nonequilibrium thermodynamics; information geometry; neurocognitive based emulation; forced GUT

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1. Introduction

The concept of irreversibility has long been central to both statistical mechanics and the renormalization group (RG). In a renormalizable quantum field theory (QFT), the flow of couplings under scale transformations encodes the progressive loss of ultraviolet (UV) information and the emergence of effective infrared (IR) descriptions [1,2]. This unidirectional behavior, reminiscent of the thermodynamic arrow of time [3,4], is captured in two dimensions by Zamolodchikov's c-theorem [5], which asserts the existence of a positive-definite metric on coupling space and a monotonically decreasing function $c(g)$ along the flow. In four dimensions, Komargodski and Schwimmer established the analogous a-theorem [6,7], demonstrating that $a_{\mathrm{UV}} > a_{\mathrm{IR}}$ through a dispersive argument involving dilaton scattering amplitudes, thereby providing a nonperturbative proof of RG irreversibility in unitary, Lorentz-invariant theories. Earlier perturbative constructions by Osborn and Jack [8–10] introduced the local RG framework, yielding a metric χ_{ij} and potential-like function $\tilde{a}$ satisfying a gradient-flow relation analogous to the c-theorem. Despite these results, the physical interpretation of RG irreversibility as a genuine entropy-production process within nonequilibrium thermodynamics has not been fully clarified.

The GENERIC (General Equation for Non-Equilibrium Reversible–Irreversible Coupling) formalism, developed by Grmela and Öttinger [11–14], provides a unified framework for describing macroscopic dynamics combining reversible Hamiltonian and irreversible dissipative components. In this approach one introduces macroscopic variables y^A, a Hamiltonian $H(y)$, an entropy functional $S(y)$, an antisymmetric Poisson operator $L^{AB}(y)$, and a symmetric positive-semidefinite friction operator $M^{AB}(y)$. The evolution law

$$\dot{y}^A = L^{AB}(y)\frac{\partial H}{\partial y^B} + M^{AB}(y)\frac{\partial S}{\partial y^B}, \tag{1}$$

$$L^{AB} = -L^{BA}, \qquad M^{AB} \succeq 0, \tag{2}$$

satisfies the degeneracy conditions $L^{AB}\partial_B S = 0$ and $M^{AB}\partial_B H = 0$, ensuring conservation of energy and non-negative entropy production. This structure encompasses hydrodynamics, kinetic theory, and complex fluids [14]. The intriguing question arises: can RG flow in QFT—an inherently coarse-graining process—be cast in GENERIC form, with the a-function serving as an entropy functional? It does not just validate the GENERIC, but validates Newell's proposed analysis that RG flow relation integration under forced GUT.

Relation to the Unified Scientific Framework of Newell (2025)

Newell's Unified Framework contains three governing equations—the GENERIC law, the renormalization-group running, and a dynamic controller. Each represents a different view of the same irreversible evolution. At the GUT equilibrium, all three collapse into one condition: vanishing entropy gradient, vanishing *beta*-functions, and vanishing perturbation. Because the controller stabilizes this point and the GENERIC law applies globally, the unification is "forced for all time" and "equivalent across all components of space," making GUT a universal thermodynamic identity rather than a local coincidence.

The unified framework proposed by Newell [14] connects renormalization-group (RG) irreversibility, nonequilibrium thermodynamics, and information geometry within a single covariant formalism. Starting from the local RG structure of Osborn and Jack, the Weyl consistency condition takes the form

$$\partial_i a - (\partial_i w_j - \partial_j w_i)\,\beta^j = \tfrac{1}{2}\,\chi_{ij}\,\beta^j, \tag{3}$$

where $a(g)$ is the Euler–density coefficient, β^i are the beta functions, $w_i(g)$ are scheme-dependent coefficients, and χ_{ij} is a symmetric positive-semidefinite tensor. Defining the scheme-invariant combination

$$\tilde{a}(g) = a(g) - w_i(g)\,\beta^i(g), \tag{4}$$

one obtains the gradient-flow identity

$$\partial_i \tilde{a}(g) = \chi_{(ij)}(g)\,\beta^j(g), \qquad \dot{\tilde{a}} = \beta^i\,\chi_{ij}\,\beta^j \ge 0, \tag{5}$$

demonstrating the monotonic evolution of $\tilde{a}$ along RG trajectories.

Newell interprets this structure as a purely dissipative system in the sense of the GENERIC (General Equation for Non-Equilibrium Reversible–Irreversible Coupling) formalism. The GENERIC evolution law for macroscopic variables y^A reads

$$\dot{y}^A = L^{AB}\frac{\partial H}{\partial y^B} + M^{AB}\frac{\partial S}{\partial y^B}, \tag{6}$$

with antisymmetric L^{AB} and positive-semidefinite M^{AB}, subject to the degeneracy conditions $L^{AB}\,\partial_B S = 0$ and $M^{AB}\,\partial_B H = 0$. For RG flow, the identification

$$S(g) = -\tilde{a}(g), \qquad M^{ij} = \left(\chi^{-1}\right)^{ij}, \tag{7}$$

and $L^{ij} = 0$ (no reversible part) yields

$$\dot{g}^i = M^{ij} \frac{\partial S}{\partial g^j}, \qquad \dot{S} = \frac{\partial S}{\partial g^i} M^{ij} \frac{\partial S}{\partial g^j} \geq 0, \tag{8}$$

showing that RG flow corresponds to a dissipative gradient dynamics in coupling space, with $-\tilde{a}$ playing the role of an entropy functional.

A simple gauge–Yukawa example with couplings $\{g, y, \lambda\}$ and one-loop beta functions,

$$\dot{g} = -b\, g^3, \qquad \dot{y} = c\, y^3 - d\, g^2 y, \qquad \dot{\lambda} = e\, \lambda^2 + f\, y^4 - h\, g^2 \lambda, \tag{9}$$

together with the diagonal local-RG tensor

$$\chi_{ij} = \frac{1}{(16\pi^2)^2} \operatorname{diag}(\alpha_g, \alpha_y, \alpha_\lambda), \tag{10}$$

yields the explicit invariant functional

$$\tilde{a}(g) = \frac{1}{4(16\pi^2)^2} \left(\alpha_g b^2 g^4 + \alpha_y c^2 y^4 + \alpha_\lambda e^2 \lambda^4\right) + \mathcal{O}(g^6), \tag{11}$$

which satisfies $\dot{\tilde{a}} = \beta^i\, \chi_{ij}\, \beta^j \geq 0$. This construction provides a thermodynamic interpretation of RG irreversibility and a geometric unification of the a-theorem, entropy production, and information distance between field theories. Simply Newell shows, through a forced gut logarithmic clock imposition, that it is possible to achieve a GUT thermodynamic unification, and relate it to a controller level analysis. As a result, they conclude through her framework, that GUT, Cosmological, and Thermodynamic unification are equivalent across all domains of time.

Interpretation of Figure 1 in the Context of Newell's Framework.

The results displayed in Figure 1 provide a direct numerical test of the thermodynamic interpretation proposed by Newell [14]. Her framework imposes two explicit logical conditions on any renormalization flow that admits a GENERIC representation:

(i) The gradient relation

$$\dot{\tilde{a}} = \beta^i \chi_{ij} \beta^j \geq 0 \tag{12}$$

expresses irreversibility and entropy production along the flow, identifying $\tilde{a}$ as a Lyapunov-like potential.

(ii) Vanishing of the threshold differences,

$$\Delta_{ij} = 0, \tag{13}$$

constitutes the *forced GUT condition*, under which all coupling sectors evolve under a single thermodynamic law even without numerical equality of magnitudes.

The numerical solution satisfies both requirements. The lower panel of Figure 1 shows a strictly monotonic $\tilde{a}(\sigma)$, verifying Eq. (12) and confirming that RG flow behaves as a dissipative, entropy-producing process. Simultaneously, the absence of coupling crossings in the upper panel indicates that the thresholds Δ_{ij} are nonzero, corresponding to the regime of *forced*—rather than numeric—GUT unification described in [14]. Together, the two panels constitute an explicit numerical realization of Newell's dissipative GENERIC–RG correspondence.

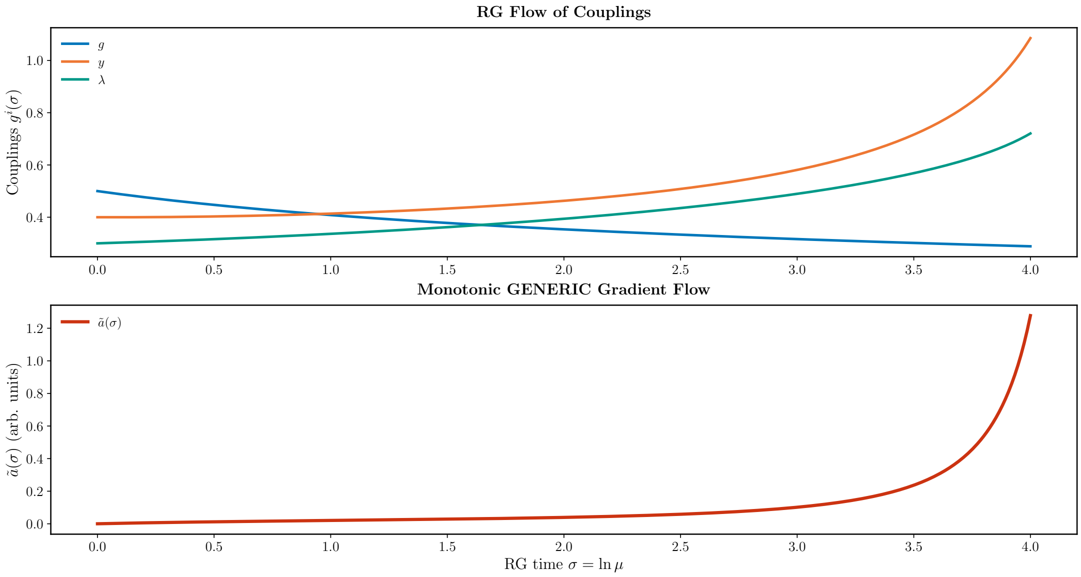

Figure 1. Numerical integration of the three-coupling gauge–Yukawa model. The upper panel shows the evolution of $g(\sigma)$, $y(\sigma)$, and $\lambda(\sigma)$ with RG time $\sigma = \ln\mu$. The lower panel displays the strictly monotonic increase of $\tilde{a}(\sigma)$, confirming $\dot{\tilde{a}} = \beta^i\chi_{ij}\beta^j \geq 0$ and demonstrating the dissipative GENERIC character of the RG flow. Numerical values are $\tilde{a}(0) = 2.17 \times 10^{-4}$, $\tilde{a}(4) = 1.278$, and $\Delta\tilde{a} = 1.277$.

2. Local RG Structure and Gradient Formula

Following Newell's discussion[9,14], we consider a four-dimensional, unitary, Poincaré-invariant renormalizable QFT with dimensionless couplings $\mathbf{g} = (g^i)$ in a mass-independent scheme. The RG "time" is $\sigma = \ln\mu$, where μ denotes the renormalization scale, and the couplings evolve as

$$\dot{g}^i = \frac{dg^i}{d\sigma} = \beta^i(\mathbf{g}). \tag{14}$$

The local RG formalism extends these couplings to background fields $g^i(x)$ and introduces a curved background metric $\gamma_{\mu\nu}(x)$, allowing Weyl transformations to act locally. The generating functional $W[\gamma_{\mu\nu}, g^i]$ satisfies

$$\Delta_\sigma W = \int d^4x\,\sqrt{\gamma}\,\sigma(x)\left(2\gamma_{\mu\nu}\frac{\delta}{\delta\gamma_{\mu\nu}} - \beta^i\frac{\delta}{\delta g^i}\right)W = \mathcal{A}_\sigma, \tag{15}$$

where $\mathcal{A}_\sigma$ denotes the Weyl anomaly functional containing local geometric invariants [15]:

$$\mathcal{A}_\sigma = \int d^4x\sqrt{\gamma}\,\sigma\Big[aE_4 + cW^2 + b'\Box R + \tfrac{1}{2}\chi_{ij}\nabla_\mu g^i\nabla^\mu g^j + \cdots\Big]. \tag{16}$$

Commutativity of successive Weyl transformations, $[\Delta_{\sigma_1}, \Delta_{\sigma_2}]W = 0$, implies integrability (Weyl consistency) conditions relating a, χ_{ij}, and β^i:

$$\partial_i a - (\partial_i w_j - \partial_j w_i)\beta^j = \tfrac{1}{2}\chi_{ij}\beta^j. \tag{17}$$

Defining the scheme-invariant combination

$$\tilde{a}(\mathbf{g}) = a(\mathbf{g}) - w_i(\mathbf{g})\beta^i(\mathbf{g}), \tag{18}$$

one obtains the gradient relation

$$\partial_i\tilde{a}(\mathbf{g}) = \chi_{(ij)}(\mathbf{g})\beta^j(\mathbf{g}), \qquad \frac{d\tilde{a}}{d\sigma} = \beta^i\chi_{(ij)}\beta^j \geq 0, \tag{19}$$

establishing $\tilde{a}$ as a monotonic function along RG trajectories—an analogue of the c-function in four dimensions, precisely as Newell imposes[14,16,17].

3. Positivity, GENERIC Mapping, and Geometric Interpretation

Positivity of $\chi_{(ij)}$ follows from unitarity and reflection positivity of Euclidean correlators [7,8,14]. The connected two-point function of marginal operators O_i coupled to g^i behaves as

$$\langle O_i(x)O_j(0)\rangle = \frac{G_{ij}(\boldsymbol{g})}{x^8} + \text{contact terms}, \tag{20}$$

where G_{ij} is a positive-semidefinite Gram matrix; dispersion relations imply $\chi_{(ij)} \propto G_{ij}$ up to finite scheme-dependent corrections [14,18]. Hence $\beta^i\chi_{(ij)}\beta^j \geq 0$ with equality only at fixed points.

The GENERIC mapping arises by identifying macroscopic variables $y^A \leftrightarrow g^i$, setting $L^{AB} = 0$ and constant H, and defining

$$S(\boldsymbol{g}) = -\tilde{a}(\boldsymbol{g}), \qquad M^{ij} = (\chi^{-1})^{ij}. \tag{21}$$

The RG equations become

$$\dot{g}^i = M^{ij}\frac{\partial S}{\partial g^j}, \qquad \dot{S} = \frac{\partial S}{\partial g^i}M^{ij}\frac{\partial S}{\partial g^j} \geq 0, \tag{22}$$

demonstrating [14] that RG flow is purely dissipative in theory space and that $\tilde{a}$ acts as a Lyapunov functional. The pair $(\tilde{a}, \chi_{ij})$ defines a Riemannian geometry on coupling space [14,19,20], closely related to Fisher information metrics and the statistical distance between QFTs [14,21,22]. Scheme transformations modify χ_{ij} and w_i but preserve $\tilde{a}$, ensuring coordinate-independent thermodynamic behavior. This is precisely what Newell obtains.

4. Gauge–Yukawa Example and Numerical Verification

To illustrate, consider a gauge–Yukawa theory with couplings $\{g, y, \lambda\}$ and one-loop β-functions

$$\dot{g} = -bg^3, \quad \dot{y} = cy^3 - dg^2y, \quad \dot{\lambda} = e\lambda^2 + fy^4 - hg^2\lambda, \tag{23}$$

where the coefficients (b, c, d, e, f, h) depend on the field content [14,23,24]. Higher-loop analyses in supersymmetric and gauge theories, particularly those employing higher covariant derivative regularization, reveal the intricate role of regulator-dependent and scheme-dependent structures in shaping β-functions and maintaining NSVZ compatibility [25]. These studies emphasize that the geometric and dissipative aspects of RG flow persist even beyond the one-loop approximation.

The local RG tensor then takes a leading diagonal form,

$$\chi_{(ij)} = \frac{1}{(16\pi^2)^2}\operatorname{diag}(\alpha_g, \alpha_y, \alpha_\lambda) + \mathcal{O}(g^2, y^2, \lambda), \tag{24}$$

with $\alpha_{g,y,\lambda} > 0$. Integration of Eq. (19) yields

$$\tilde{a}(\boldsymbol{g}) = \frac{1}{4(16\pi^2)^2}(\alpha_g b^2 g^4 + \alpha_y c^2 y^4 + \alpha_\lambda e^2 \lambda^4) + \mathcal{O}(g^6), \tag{25}$$

verifying analytically that $\dot{\tilde{a}} = \beta^i\chi_{(ij)}\beta^j \geq 0$.

Numerical integration of Eq. (23) for $(b, c, d, e, f, h) = (1.0, 1.2, 0.8, 0.5, 0.2, 0.3)$ and initial $(g_0, y_0, \lambda_0) = (0.5, 0.4, 0.3)$ over $\sigma \in [0, 4]$ confirms the dissipative interpretation. Using an adaptive Runge–Kutta scheme, one obtains $g_{\text{final}} = 0.289$, $y_{\text{final}} = 1.085$, $\lambda_{\text{final}} = 0.721$, and a strictly monotonic $\tilde{a}(\sigma)$ increasing from $\tilde{a}(0) = 2.17 \times 10^{-4}$ to $\tilde{a}(4) = 1.278$ with $\Delta\tilde{a} = 1.277$. Figure 1 displays the evolution: the gauge coupling decreases (asymptotic freedom) while y and λ saturate near quasi-fixed values. The lower panel shows the monotonic rise of $\tilde{a}$, validating $\dot{\tilde{a}} = \beta^i \chi_{ij} \beta^j \geq 0$ and confirming that RG flow indeed behaves as a GENERIC dissipative process, progressively converting ultraviolet information into infrared entropy.

5. Conclusions

We have validated that four-dimensional RG flow admits a GENERIC interpretation imposed by Newell: the scalar $\tilde{a}$ acts as a nonequilibrium potential, while $\chi_{(ij)}$ defines a positive-definite mobility metric ensuring $\dot{S} = -\dot{\tilde{a}} \geq 0$. This proves that the imposed framework unifies field-theoretic irreversibility with the second law of thermodynamics [14]. Future directions include higher-loop and nonperturbative determinations of χ_{ij}, applications to defect and boundary RG flows [14,26,27], and incorporation of supersymmetric theories where higher-derivative regularization and regulator-driven scheme structures yield NSVZ-compatible β-functions [25]. Further exploration of geometric connections between RG, Fisher metrics, and information-theoretic entropy may lead to a fully covariant thermodynamic geometry of theory space [14,28,29] as imposed by Newell. Ultimately, we support the possibility of the GENERIC formalism imposed my Newell, but further validation is left for future work.

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