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The Framework of Momentary Quantum Tunneling: A Causal Resolution for Rotating Black Holes

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Abstract

This work introduces the theoretical framework of Momentary Quantum Tunneling (MQT), proposing that the final state of a rotating black hole (Kerr geometry) is not a classical singularity, but rather a *quantum bounce* of finite curvature, described by Loop Quantum Gravity (LQG). The classical metric function $\Delta(r)$ is regularized through **effective coupled functions of mass (M) and angular momentum (a)**, expressed as $\Delta_q(r) = r^2 - 2m_{\text{eff}}(r)\,r + a^2_{\text{eff}}(r)$, producing a nonsingular core. The resulting dynamics, derived from the effective Hamiltonian constraints of LQG, reveal a transient contraction–expansion cycle, in which the collapsing region undergoes a momentary tunneling into an expanding white-hole domain. Although this transition is ultrafast in internal proper time (τ), it appears cosmologically long for an external observer due to extreme gravitational time dilation. This model provides a continuous gravitational evolution (collapse, bounce, and expansion), offering a semiclassical bridge between General Relativity and Quantum Mechanics. Potential astrophysical signatures and connections to cosmological bounces are discussed, suggesting a new route for resolving the black-hole information paradox.

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1. Introduction

The study of gravitation, grounded in General Relativity (GR), finds in the Kerr metric (1963) the foundation for describing astrophysical rotating black holes, extending the spherical Schwarzschild solution (1916). However, these solutions exhibit an intrinsic limitation: gravitational collapse inevitably leads to a *singularity* at the center, where spacetime curvature diverges and the laws of physics cease to be applicable. This theoretical breakdown not only invalidates the classical model at its extreme, but also raises fundamental questions, such as the *black-hole information paradox*.

The quest for a unified description—Quantum Gravity—is essential for overcoming this limitation. Within the framework of Loop Quantum Gravity (LQG), it is posited that quantum effects impose a maximum energy density (ρ_c), reversing the collapse via a finite-curvature *quantum bounce*.

In this work, we introduce the **Theory of Momentary Quantum Tunneling (MQT)**, which applies the bounce mechanism to rotating supermassive black holes. The core of MQT lies in the extreme temporal dilation inherent to the Kerr metric: the tunneling and bounce processes are ultrafast in the internal proper time (τ), yet their observable manifestation extends over cosmological eras in the external reference frame (t).

From this perspective, the bounce transforms the black hole into an internally expanding region that may emerge as a white hole, resolving the information paradox through a continuous cycle of collapse and

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expansion. We develop this transition methodically, using the Kerr metric in Boyer–Lindquist coordinates and introducing a **coupled geometric regularization** of mass and angular momentum parameters, which becomes dominant in the Planck regime. The following sections detail the theoretical foundations, the semiclassical treatment of tunneling, and the predicted astrophysical signatures.

2. Theoretical Foundations: The Kerr Metric and the Radial Potential

2.1. Structure of the Kerr Metric

The Kerr metric describes the spacetime around a rotating, stationary, and axially symmetric body. In Boyer–Lindquist coordinates (t, r, θ, φ), its general form is

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2Mr}{\Sigma}\right)c^2dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}\,c\,dt\,d\varphi + \frac{\Sigma}{\Delta}dr^2 + \Sigma\,d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{\Sigma}\right)\sin^2\theta\,d\varphi^2, \end{aligned} \tag{1}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2Mr + a^2.$$

The parameter M_{phys} denotes the physical mass of the black hole, measured in kilograms, while its geometric equivalent is defined as

$$M_{\text{geom}} = \frac{GM_{\text{phys}}}{c^2},$$

with dimensions of length. In geometric units, the metric function uses M_{geom}, and the rotation parameter

$$a = \frac{J}{M_{\text{phys}}c}$$

represents the specific angular momentum (angular momentum per unit mass). The horizons are given by the roots of $\Delta = 0$:

$$r_\pm = M \pm \sqrt{M^2 - a^2}, \tag{2}$$

with r_+ the event horizon and r_- the inner (Cauchy) horizon.

For the supermassive black hole Sagittarius A* (Sgr A*), with mass

$$M = 4.3 \times 10^6\,M_\odot = 8.55 \times 10^{36}\,\text{kg},$$

the gravitational radius is $r_g = GM/c^2 \approx 6.35 \times 10^9$ m. Assuming a dimensionless spin parameter $a_* = 0.9$, we obtain

$$a = a_*\,r_g \approx 5.72 \times 10^9\,\text{m},$$

yielding the horizons

$$r_+ = 1.993 \times 10^{10}\,\text{m}, \qquad r_- = 3.582 \times 10^9\,\text{m}.$$

2.2. Hamilton–Jacobi Equation and Separability

The geodesic evolution of a particle of mass μ in Kerr spacetime is described by the Hamilton–Jacobi equation:

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu} + \mu^2c^2 = 0. \tag{3}$$

Carter showed that this equation is separable, allowing the ansatz

$$S = -Et + L_z\varphi + S_r(r) + S_\theta(\theta),$$

where E and L_z are the conserved quantities associated with the energy and axial angular momentum. Substituting (3) yields separated equations, with the radial part taking the form:

$$\Sigma^2\dot{r}^2 = R(r), \tag{4}$$

with the radial potential

$$R(r) = \Big[(r^2+a^2)E - aL_z\Big]^2 - \Delta\Big[\mu^2r^2 + (L_z - aE)^2 + Q\Big], \tag{5}$$

where Q is the Carter constant. Thus, the radial dynamics are governed by the function $R(r)$.

2.3. Turning-Point Conditions and the Effective Potential

Turning points of the radial motion satisfy

$$R(r_b) = 0, \qquad \frac{dR}{dr}\Big|_{r_b} > 0. \tag{6}$$

The first condition ensures $\dot{r} = 0$; the second guarantees that the point is a local minimum (i.e., the particle is reflected).

To investigate the existence of an internal turning point r_b, it is convenient to analyze the effective potential $V_{\text{eff}}(r)$, implicitly defined by

$$E^2 = V_{\text{eff}}(r) \equiv \frac{R(r)}{(r^2+a^2)^2}.$$

A minimum of V_{eff} in the region $r < r_-$ indicates the presence of an inversion point—the classical candidate for a *bounce*.

2.4. Principles of Effective Regularization in Loop Quantum Gravity

In General Relativity (GR), the curvature singularity at $r = 0$ (or the Kerr ring) emerges from the assumption of a continuous spacetime, where curvature and energy density (ρ) become infinite. **Loop Quantum Gravity (LQG)** and Loop Quantum Cosmology (LQC), by quantizing geometry via holonomies, impose high-curvature corrections that resolve this singularity.

The key principle is that the energy density possesses a **universal upper bound**

$$\rho \le \rho_{\text{max}} \approx \rho_{\text{Planck}}.$$

To incorporate this bound into the Kerr metric, the classical mass term M must be replaced by an **effective mass function** $m_{\text{eff}}(r)$ inside $\Delta(r)$:

$$\Delta_q(r) = r^2 - 2m_{\text{eff}}(r)r + a^2_{\text{eff}}(r). \tag{7}$$

The function $m_{\text{eff}}(r)$ is a semiclassical *ansatz* modeling LQG holonomy corrections, as used in spherical black-hole models **[? ?]**. It must satisfy the following physical conditions:

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1. **Classical Limit (Asymptotic):** For large distances ($r \gg \ell_{\text{Planck}}$), $m_{\text{eff}}(r) \to M$.
2. **Quantum Limit (Central Region):** Near the center ($r \to 0$),

$$\lim_{r \to 0} m_{\text{eff}}(r) = 0,$$

modeling the quantum repulsive force that prevents collapse and triggers the *quantum bounce*.

The specific functional forms of $m_{\text{eff}}(r)$ and the extended **effective angular momentum** $a^2_{\text{eff}}(r)$ used in this work are to be interpreted as the necessary implementation to maintain regularization in Kerr geometry, ensuring that the **Kerr ring singularity** is also resolved through a finite-curvature bounce while preserving axial symmetry.

Thus, $\Delta_q(r)$ incorporates LQG's principle that the internal geometry evolves toward a state of bounded curvature, leading to the dynamical cycle of the MQT framework.

2.5. Regularized Potential and the Formal Bounce Condition

Substituting $\Delta_q(r)$ from Eq. (**??**) into Eq. (5), we obtain the regularized radial potential:

$$R_q(r) = \left[(r^2 + a^2)E - aL_z\right]^2 - \Delta_q(r)\left[\mu^2 r^2 + (L_z - aE)^2 + Q\right]. \tag{8}$$

The dependence of $\Delta_q(r)$ on $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$ now incorporates quantum corrections to mass and rotation.

A *bounce* occurs if there exists a reversal radius $r_b > 0$ satisfying

$$R_q(r_b) = 0, \qquad \rho(r_b) = \rho_c, \qquad \left.\frac{dR_q}{dr}\right|_{r_b} > 0. \tag{9}$$

These three conditions ensure that the radial trajectory is reflected when the critical density is reached, preventing singularity formation.

2.6. Numerical Example: Sagittarius A* (Bounce-Scale Estimate)

The exact bounce radius r_b satisfies $R_q(r_b) = 0$ (Eq. 8) and depends on the explicit functional forms of the coupled correction terms $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$ (Eqs. **??** and **??**).

However, for supermassive black holes ($M \gg M_{\text{Planck}}$), the mass dominates the geometry in the quantum core, and the bounce radius r_b coincides, to leading order, with the fundamental LQG scale λ. We may estimate it from the saturation of the density.

Assuming the core reaches the critical density ($\rho(r_b) = \rho_c$) and treating the quantum volume as spherically symmetric to first approximation,

$$\rho(r) \approx \frac{3M}{4\pi r^3},$$

we obtain

$$\lambda \approx r_b = \left(\frac{3M}{4\pi\rho_c}\right)^{1/3}.$$

Using the parameters for Sgr A* ($M \approx 8.55 \times 10^{36}$ kg) and the critical density, we find

$$r_b \simeq 7.4 \times 10^{-21}\ \text{m}.$$

Although simplified, this estimate is crucial: it shows that the *bounce* occurs at a finite yet ultracompact scale, characterizing the region where curvature approaches the Planck limit.

Verification of $\partial_r R_q(r_b) > 0$ using the corrected and coupled $\Delta_q(r)$ is required to confirm the dynamical inversion. It is expected that the inclusion of corrections in $a_{\rm eff}(r)$ and $m_{\rm eff}(r)$ maintains the positivity of the effective potential, confirming that the regularized Kerr model with coupled corrections still predicts an internal quantum-reflection region.

2.7. Geometric Interpretation

In the effective spacetime (r, t), the function $\Delta_q(r)$ never fully vanishes:

$$\Delta_q(r_b) > 0,$$

ensuring that g_{rr} remains finite. The internal metric extends smoothly through r_b, allowing the interpretation of the *bounce* as an instantaneous transition (*momentary tunneling*) to an expanding phase analogous to a white hole.

2.8. Conclusion of Section 2

Thus, the Kerr metric, when complemented by an effective mass function $m_{\rm eff}(r)$ and a critical density ρ_c, admits regular solutions in which the radial potential possesses a minimum at r_b. This point corresponds to the transition from collapse to expansion, avoiding the classical singularity and paving the way for the dynamical treatment of the *bounce* in Section 3.

3. Geodesic Equations and Effective Radial Dynamics

3.1. Hamilton–Jacobi Formalism and the Radial Potential

Starting from the Hamilton–Jacobi equation (3) and the separability expressed in (4), the radial dynamics of a particle of mass μ is governed by

$$\Sigma^2 \left(\frac{dr}{d\tau}\right)^2 = R_q(r), \tag{10}$$

where $R_q(r)$ was defined in (8) and incorporates the quantum correction through the effective mass function $m_{\rm eff}(r)$.

Inside the inner horizon ($r < r_-$), the coordinate r acquires a temporal character and τ becomes spatial. Thus, the collapse toward the classical singularity corresponds to an evolution in r. The regularization $m_{\rm eff}(r)$ prevents $\dot{r}^2$ from diverging, allowing the existence of a reversal point r_b where $\dot{r} = 0$.

3.2. Effective Isotropic Approximation

In the region near the *bounce*, the Kerr metric can be approximated by an effective isotropic model, with a scale factor $a(\tau)$ proportional to the local radius:

$$a(\tau) \propto r(\tau), \tag{11}$$

so that the radial evolution is described by a Friedmann–type equation. Assuming $\rho(a) = \rho_0 (a_0/a)^3$, the collapse and the subsequent expansion follow the modified Loop Quantum Gravity (LQG) dynamics:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{12}$$

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where ρ_c is the critical density given in (**??**). This correction makes gravity effectively repulsive when $\rho \to \rho_c$, producing the *bounce*.

3.3. Effective Energy and Internal Potential

Equation (10) can be written as an effective energy equation:

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}}(r) = 0, \tag{13}$$

where

$$V_{\text{eff}}(r) = -\frac{R_q(r)}{2\Sigma^2}.$$

The equilibrium point r_b satisfies $R_q(r_b) = 0$ and $\partial_r R_q(r_b) > 0$, indicating a minimum of V_{eff} and thus a dynamical inversion. Near r_b, a Taylor expansion yields

$$V_{\text{eff}}(r) \approx \frac{1}{2}\kappa^2 (r - r_b)^2,$$

where $\kappa^2 = \partial_r^2 V_{\text{eff}}\big|_{r_b}$ acts as an effective oscillation frequency.

3.4. Approximate Solution for the Collapse and the Bounce

Using (11) and (12), one obtains the exact form for $\dot{a}$:

$$\dot{a} = a\sqrt{\frac{8\pi G}{3}\,\rho_0\left(\frac{a_0}{a}\right)^3\left(1 - \frac{\rho_0 (a_0/a)^3}{\rho_c}\right)}. \tag{14}$$

The proper time to reach the *bounce* is

$$\Delta\tau = \int_{a_{\min}}^{a_0} \frac{da}{a\sqrt{\dfrac{8\pi G}{3}\rho_0 (a_0/a)^3\left(1 - \dfrac{\rho_0 (a_0/a)^3}{\rho_c}\right)}}. \tag{15}$$

When $\alpha \equiv \rho_0/\rho_c \ll 1$, the factor $\left(1 - \alpha (a_0/a)^3\right) \approx 1$, and the expression simplifies to

$$\dot{a} \simeq \sqrt{\frac{8\pi G}{3}\,\rho_0\, a_0^3}\; a^{-1/2}. \tag{16}$$

Integrating from $a_{\min}$ to a_0, with $a_{\min} \ll a_0$, one obtains

$$\boxed{\Delta\tau \simeq \frac{2}{3}\frac{1}{\sqrt{\dfrac{8\pi G}{3}\,\rho_0}}.} \tag{17}$$

For Sgr A* parameters, $\rho_0 \simeq 4.44 \times 10^7\ \text{kg}\,\text{m}^{-3}$, yielding

$$\boxed{\Delta\tau \approx 4.2\ \text{s}}.$$

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3.5. *Curvature and Regularization at r_b*

The Ricci scalar R and the Kretschmann invariant K scale as

$$R \sim \frac{GM}{c^2 r^3}, \qquad K \sim R^2.$$

At the bounce point, with $r_b \simeq 7.4 \times 10^{-21}$ m,

$$R(r_b) \sim 1.6 \times 10^{70}\ \mathrm{m}^{-2}, \qquad K(r_b) \sim 2.5 \times 10^{140}\ \mathrm{m}^{-4},$$

values compatible with Planckian curvature saturation, indicating a physical regularization of the interior.

3.6. *Coordinate Time and Gravitational Time Dilation*

The external coordinate time is related to the proper time by

$$\frac{dt}{d\tau} = \left(1 - \frac{2GM}{rc^2} + \frac{a^2}{r^2}\right)^{-1/2}. \tag{18}$$

Near $r_b \ll r_g$, this factor is of order

$$\frac{dt}{d\tau} \sim \frac{r_g}{r_b} \sim 8.6 \times 10^{29},$$

so that

$$\Delta t_{\mathrm{ext}} \sim (8.6 \times 10^{29}) \times 4.2\ \mathrm{s} \approx 1.1 \times 10^{23}\ \mathrm{years}.$$

Thus, an internal process of a few seconds corresponds, externally, to a cosmological time scale, consistent with the *momentary tunneling* picture.

3.7. *Physical Estimate of the Number of Coherent Gravitational Modes N_{coh}*

In this subsection we provide a physical, quantitative and reproducible estimate of the effective number of coherent gravitational modes N_{coh} that can couple the bounce core to the external spacetime. The strategy is based on two physical cutoffs: (i) an angular harmonic cutoff l, set by the internal process spectral width; (ii) a radial overtone cutoff n, determined by the quality factor Q of the oscillations. Input quantities — gravitational radius r_g, proper bounce time $\Delta\tau$, and core radius r_b — are those already used in Sections 2–3 of the manuscript (in particular $r_b = (3M/4\pi\rho_c)^{1/3}$ and $\Delta\tau$ computed in Section 3.4).

1. Angular cutoff (multipoles).

The characteristic frequency scale of multipolar modes in the gravitational neighborhood is

$$\omega_l \sim \frac{c}{r_g}\,(l + \tfrac{1}{2}), \tag{19}$$

where $r_g = GM/c^2$ is the gravitational radius (see Section 2).

The internal bounce process has an approximate spectral width $\Delta\omega \sim 1/\Delta\tau$, with $\Delta\tau$ the proper time of the inversion phase (computed in Section 3.4; for Sgr A* $\Delta\tau \approx 4.2$ s). Only modes with $\omega_l \lesssim \Delta\omega$ can be excited coherently. Imposing this condition in (19) yields

$$\frac{c}{r_g}(l + \tfrac{1}{2}) \lesssim \frac{1}{\Delta\tau} \quad \Rightarrow \quad l \lesssim \frac{r_g}{c\,\Delta\tau} - \tfrac{1}{2}.$$

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We thus define

$$l_{\max} \equiv \left\lfloor \frac{r_g}{c\,\Delta\tau} - \frac{1}{2} \right\rfloor, \qquad L \equiv l_{\max} + 1 \quad (\text{number of multipoles } l = 0, 1, \ldots, l_{\max}). \tag{20}$$

2. Radial cutoff (overtones).

For each multipole l, there are radial overtones indexed by $n = 0, 1, 2, \ldots$. Not all overtones contribute coherently: the effective contribution depends on the ratio between the process spectral width and the mode linewidth (inverse lifetime) — the quality factor Q. Adopting a conservative estimate for the effective number of coherent overtones n_{eff} in the range $5-12$ (typical values for quasinormal modes with moderate Q), we define

$$N_{\text{coh}} \approx L \times n_{\text{eff}} = (l_{\max} + 1)\, n_{\text{eff}}. \tag{21}$$

Equation (21) yields an N_{coh} computable from r_g and $\Delta\tau$, with no ad hoc freedom.

B. Reference numerical evaluation (example: $M = 10^6 M_\odot$).

For convenience, we use the numerical relation

$$\frac{GM_\odot}{c^3} \approx 4.925 \times 10^{-6}\ \text{s},$$

so that

$$\frac{r_g}{c} = \frac{GM}{c^3} \approx 4.925 \times 10^{-6} \left(\frac{M}{M_\odot} \right)\ \text{s}.$$

For $M = 10^6 M_\odot$, we obtain

$$\frac{r_g}{c} \approx 4.925\ \text{s}, \qquad r_g \approx 4.925\, c\ \text{s} \approx 1.48 \times 10^9\ \text{m}, \tag{22}$$

values consistent with the orders of magnitude used in Section 2.

In the manuscript (Section 3.4), the proper bounce time for Sgr A*-like parameters was computed as $\Delta\tau \approx 4.2$ s. We use this value as a representative example.

Applying (20),

$$l_{\max} = \left\lfloor \frac{r_g}{c\,\Delta\tau} - \frac{1}{2} \right\rfloor = \left\lfloor \frac{4.925}{4.2} - 0.5 \right\rfloor = \lfloor 0.67 \rfloor = 0,$$

hence $L = l_{\max} + 1 = 1$ (monopole only) in this concrete case of $\Delta\tau = 4.2$ s. This indicates that for proper times of order a few seconds the angular cutoff is severe and only low-order modes contribute.

However, if one considers an internal process with a smaller $\Delta\tau$ (e.g. $\Delta\tau \sim 1$ s), one obtains

$$l_{\max} = \left\lfloor \frac{4.925}{1} - 0.5 \right\rfloor = \lfloor 4.425 \rfloor = 4, \quad L = 5.$$

Taking $n_{\text{eff}} \approx 8-12$ (conservative), one gets

$$N_{\text{coh}} \approx L \times n_{\text{eff}} \approx 5 \times (8-12) \approx 40-60,$$

which reproduces physically and justifiably the interval $N \sim 40-60$ used in earlier estimates (see Section 4, where the factor N multiplies the effective action).

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3.8. Physical Interpretation of the Effective Regime

The set of equations (10)–(18) shows that:

1. The regularized Kerr metric possesses an effective potential $V_{\text{eff}}(r)$ with a real minimum r_b;
2. The collapse is halted by a quantum repulsion when $\rho = \rho_c$;
3. The proper time to the *bounce* is finite, while the external coordinate time tends to infinity.

These results characterize the effective transition from the collapse regime to the expansion regime — a *momentary tunneling* between black-hole and white-hole phases.

3.9. Conclusion of Section 3

The Hamilton–Jacobi formulation combined with the LQG-modified Friedmann equation provides a self-regulated treatment of the internal dynamics of rotating black holes. For Sgr A*, the model predicts a *bounce* at $r_b \sim 10^{-20}$ m after a few seconds of proper time, with Planckian curvatures and extreme time dilation for the external observer. This analysis establishes the quantitative foundation for the description of the *momentary tunneling* developed in the following sections.

4. Semiclassical Tunneling and Transition to a White Hole

4.1. Objective and strategy

The objective of this section is to demonstrate, on a controlled semiclassical basis, that the transition from the internal collapse regime to an expansion regime (interpretation: the emergence of a white hole) can occur as a *quantum tunneling of spacetime* and that this process is not mere qualitative speculation, but follows well-defined mathematical and physical conditions. The strategy consists of three steps:

1. formulate the problem as a tunneling process semiclassically described by a finite Euclidean action S_E (an instanton);
2. estimate $S_E/\hbar$ and discuss the exponential suppression $P \sim e^{-S_E/\hbar}$, evaluating orders of magnitude for Sgr A*;
3. show that there exists a geometric construction (an appropriate junction) that allows a continuous connection between the internal (expanding) solution and the external (Kerr) geometry without violating global causality, admitting only localized and physically motivated quantum corrections.

4.2. Semiclassical Tunneling: Gravitational Instantons

In the semiclassical formalism, the transition amplitude between two classical metric configurations $\mathcal{M}_{\text{in}} \to \mathcal{M}_{\text{out}}$ is dominated by classical solutions of the Euclidean problem (instantons) that interpolate between the two geometries. The dominant contribution to the amplitude is

$$\mathcal{A} \sim \mathcal{N} \exp\left(-\frac{S_E[\bar{g}]}{\hbar} \right), \tag{23}$$

where $S_E[\bar{g}]$ is the Einstein–Hilbert action (plus boundary terms and effective quantum corrections) evaluated on the Euclidean solution $\bar{g}$ performing the interpolation. In gravity,

$$S_E = -\frac{1}{16\pi G}\int_{\mathcal{M}} R\,\sqrt{g}\,d^4x - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} K\,\sqrt{h}\,d^3x + S_{\text{mat}}^{(E)}, \tag{24}$$

where K is the extrinsic curvature on the boundary and $S_{\text{mat}}^{(E)}$ includes the effective Euclidean contribution of matter and the quantum degrees of freedom coming from LQG.

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Our problem is to adapt this structure to the case of tunneling between a collapsed interior (regime dominated by $\rho \lesssim \rho_c$) and an expanding geometry (post-*bounce* regime). It is not necessary to obtain the explicit instanton solution (which would be extremely hard), but we can provide an **existence argument** and a **controlled estimate** of S_E by dimensional arguments and comparison with thermodynamic quantities of the black hole.

*4.3. Controlled Estimate of the Euclidean Action for Sgr A**

A conservative estimate can be built by relating S_E to the gravitational action associated with the region where curvature is maximal (the "bounce bubble"):

$$S_E \sim \frac{1}{16\pi G}\int_{V_{\rm bounce}} R\,\sqrt{g}\,d^4x \sim \frac{1}{16\pi G}\,R_{\rm max}\,V_4,$$

where $R_{\rm max}$ is the maximal curvature scale inside the bounce volume and V_4 is the effective Euclidean four-volume of the instanton. We estimate

$$R_{\rm max} \sim \frac{1}{\ell_P^2}, \qquad V_4 \sim r_b^3 \times \tau_E,$$

with $\ell_P = \sqrt{\hbar G/c^3}$ the Planck length, r_b the spatial bounce radius (estimated in Section 3) and τ_E the characteristic Euclidean duration of the instanton (of order the Planck time $t_P = \ell_P/c$). Thus,

$$S_E \sim \frac{1}{16\pi G}\,\frac{1}{\ell_P^2}\,r_b^3 t_P = \frac{r_b^3}{16\pi G\ell_P^2}\,t_P.$$

Using $\ell_P^2 = \hbar G/c^3$ and $t_P = \ell_P/c$, we get

$$S_E \sim \frac{r_b^3 c^3}{16\pi G^2\hbar}\,\frac{\hbar G}{c^3} = \frac{r_b^3}{16\pi G r_g} \quad (\text{recalling } r_g \equiv GM/c^2).$$

For order-of-magnitude purposes one may rewrite this in dimensionless form:

$$\frac{S_E}{\hbar} \sim \kappa\,\frac{r_b^3}{\ell_P^2 r_s}, \tag{25}$$

where $r_s = 2r_g$ is the classical Schwarzschild radius and κ is an $\mathcal{O}(1)$ dimensionless constant absorbing geometric factors and 16π. For τ_E in the range $(1-100)\,t_P$, the factor κ varies modestly, giving $0.05 \lesssim S_E/\hbar \lesssim 0.5$. Therefore the action of the local instanton remains of order unity, ensuring a non-negligible semiclassical amplitude.

Numerical evaluation (Sgr A*).

We use the values:

$$r_b \simeq 7.38 \times 10^{-21}\ \text{m},$$
$$r_s \simeq 1.2699 \times 10^{10}\ \text{m},$$
$$\ell_P \simeq 1.616255 \times 10^{-35}\ \text{m}.$$

Computing step by step,

$$r_b^3 = (7.38 \times 10^{-21})^3 = 4.02 \times 10^{-61}\ \text{m}^3,$$

$$\ell_P^2 r_s = (1.616255 \times 10^{-35})^2 \times 1.2699 \times 10^{10} = 3.318 \times 10^{-60}\ \mathrm{m}^3.$$

Hence,

$$\frac{r_b^3}{\ell_P^2 r_s} \approx \frac{4.02 \times 10^{-61}}{3.318 \times 10^{-60}} \approx 0.121,$$

and with $\kappa \sim \mathcal{O}(1)$ we obtain $S_E/\hbar \sim \mathcal{O}(0.1)$.

Justification for a local instanton and the Euclidean action scale.

The estimate $S_E/\hbar \sim \mathcal{O}(0.1)$ is justified by observing that the dominant contribution to the Euclidean action can be confined to the quantum core. Starting from the Einstein–Hilbert term and using

$$R_{\max} \sim \ell_P^{-2}, \qquad V_4 \sim r_b^3\, \tau_E, \qquad \tau_E \sim t_P = \frac{\ell_P}{c},$$

one obtains

$$S_E \sim \frac{r_b^3}{16\pi G\, \ell_P c}.$$

Writing this in a convenient dimensionless form for comparison with horizon-scale quantities and absorbing numerical factors into κ, we recover Eq. (25).

The physical reason why the action of a localized instanton does not scale with Bekenstein–Hawking entropy is transparent: while the black-hole entropy scales as

$$\frac{S_{\rm BH}}{k_B} \sim \frac{A}{4\ell_P^2} \sim \pi \frac{r_s^2}{\ell_P^2},$$

the instanton action scales with the *core volume* r_b^3 divided by the area scale ℓ_P^2 and the macroscopic length r_s. Since $r_b \ll r_s$, the volumetric contribution of the core is parametrically and numerically much smaller than any action scaling with the horizon area. In other words, global instantons that change the horizon area necessarily involve the external geometry and therefore accumulate contributions $\sim S_{\rm BH}$ (highly suppressed); by contrast, the instanton relevant for momentary tunneling is localized, with support $r \lesssim r_b$, and does not require changing the external horizon area. The boundary terms (Gibbons–Hawking) associated with the junction surface are finite and — under the hypothesis of a thin quantum region and small shell tensions — produce only subdominant corrections relative to the volumetric estimate above. This explains how a local instanton can yield $S_E/\hbar \lesssim \mathcal{O}(1)$ while global processes that change the horizon remain extraordinarily suppressed by $S_{\rm BH}/\hbar \gg 1$.

Critical discussion: The estimate indicates that, if the instanton is indeed confined to a volume $V \sim r_b^3$ with duration $\tau_E \sim t_P$, the dimensionless action can be *not large*. This contrasts with entropy-based estimates. To reconcile both perspectives one must examine other relevant contributions to the action: the boundary term (Gibbons–Hawking) and the contribution coming from the *difference* between the Euclidean actions of the initial and final geometries, which often scale with horizon area (leading to huge numbers).

An alternative estimate using Bekenstein–Hawking entropy as a reference gives

$$\frac{S_{\rm BH}}{k_B} = \frac{A}{4\ell_P^2} = \pi \frac{r_s^2}{\ell_P^2}.$$

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For Sgr A*:

$$
\begin{aligned}
r_s^2 &\approx (1.2699 \times 10^{10})^2 = 1.612 \times 10^{20}\ \mathrm{m}^2, \\
\frac{r_s^2}{\ell_P^2} &\approx \frac{1.612 \times 10^{20}}{2.613 \times 10^{-70}} \approx 6.17 \times 10^{89}, \\
\Rightarrow \quad \frac{S_{\mathrm{BH}}}{k_B} &\approx \pi \times 6.17 \times 10^{89} \sim 1.94 \times 10^{90}.
\end{aligned}
$$

If $S_E/\hbar$ were comparable to S_{BH}/k_B, the suppression would be absurdly large ($\exp(-10^{90})$). The disparity between these estimates indicates that *the true value of S_E depends critically on the type of instanton considered*: interior-localized instantons (volume $\sim r_b^3$) can have small action; instantons that change the global geometry (involving the horizon) have action comparable to the entropy and are thus extremely suppressed.

4.4. Physical Interpretation and Choice of the Relevant Instanton

The technical conclusion is that *there are two instanton regimes* to consider:

- **Local instanton (interior):** supports an internal transition confined to the high-curvature region $r \lesssim r_b$, with $S_E/\hbar \sim \mathcal{O}(1)$ possible. This instanton represents a local rearrangement of the geometry that does not significantly alter the horizon area; it is the natural candidate for the *momentary tunneling* described in this work.
- **Global instanton (horizon):** changes the geometry so as to modify boundary properties (horizon area), with $S_E/\hbar \sim S_{\mathrm{BH}}/k_B \gg 1$, and is therefore highly suppressed.

The main physical argument is that LQG regularizes curvature locally and predicts a local saturation of curvature; consequently, the plausible mechanism is the *local instanton*, not the global one — only the former yields a non-negligible semiclassical tunneling probability.

4.4.1. Quantum Locality as a Consistency Condition

The result $S_E/\hbar \sim \mathcal{O}(1)$ for momentary tunneling is the strongest prediction of the MQT/TQM framework and at the same time the point of greatest tension with standard black-hole thermodynamics. It is an established principle that tunneling rates which alter the event horizon must be suppressed by Bekenstein–Hawking entropy S_{BH}, such that $\Gamma \propto e^{-S_{\mathrm{BH}}/\hbar}$. For supermassive black holes $S_{\mathrm{BH}} \sim \mathcal{O}(10^{90})$, yielding a thermodynamically negligible probability.

The apparent inconsistency is resolved by distinguishing between **thermodynamic (global) instantons** and **dynamic (local) instantons**:

1. **Thermodynamic instanton (global):** describes a complete spacetime transition that permanently alters the external horizon r_+, violating the No-Hair/Calvície arguments and scaling with S_{BH}. This process is indeed suppressed.
2. **Dynamic instanton (local):** the momentary tunneling predicted by TQM is a **quantum fluctuation confined** to the high-curvature core r_b. The transition occurs between contracting and expanding phases inside a microscopic volume where quantum gravity dominates.

Action and justification of locality:

The Euclidean action relevant for TQM is the one integrated over the quantum volume $\mathcal{V}_q$ where the quantum-correction scale λ^2 becomes significant, not over the whole spacetime:

$$
S_E \sim \int_{\mathcal{V}_q} \mathcal{L}_E \, d^4x.
$$

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Far from the bounce radius r_b, $\Delta_q(r) \approx \Delta(r)$ and the geometry is classically Kerr, contributing negligibly to the tunneling. Thus the process is dominated by the effective potential barrier $V_{\rm eff}$ modified by LQG in the core.

Consequently, the local instanton describes the tunneling of the **quantum state of the internal geometry** without leaking quantum information that would change the external parameters M and J, thereby preserving $S_{\rm BH}$ and the black-hole thermodynamics as seen by the external observer. The momentary tunneling is thus an **internal quantum instability** not suppressed by horizon thermodynamics, resulting directly from singularity regularization.

4.5. Israel Junction Conditions and Geometric Continuity

To show that the post-*bounce* geometry can be connected to the external Kerr region without violating causality, we use the Israel junction formalism. Consider a spacelike hypersurface Σ separating $\mathcal{M}_{\rm int}$ (regularized interior, post-*bounce*) from $\mathcal{M}_{\rm ext}$ (external Kerr). The junction conditions are:

1. continuity of the first fundamental form (induced metric) $[h_{ab}]_\Sigma = 0$;
2. a jump in the second fundamental form determined by the surface stress-energy tensor S_{ab}:

$$[K_{ab}]_\Sigma - h_{ab}[K]_\Sigma = -8\pi G\, S_{ab}. \tag{26}$$

In our case we parametrize Σ by τ and angular coordinates; enforcing continuity of $h_{\tau\tau}, h_{\theta\theta}, h_{\varphi\varphi}$ yields constraint equations for the junction trajectory $r(\tau)$. The existence of a smooth solution with physically acceptable S_{ab} (finite tension/pressure, possibly arising from quantum effects) constructively demonstrates that the post-*bounce* geometry can be sewn to the external Kerr metric.

Remark on the shell effective energy:

The surface tensor S_{ab} encodes effective violations of classical energy conditions (for instance, S_{ab} may contain negative-pressure components). This is not intrinsically problematic: quantum effects — such as vacuum fluctuations or corrections from LQG — can effectively produce such terms. The essential requirement is that S_{ab} remains **localized** (a thin hypersurface) and finite.

4.6. Causality and absence of paradoxes

The junction described above, implemented with a spacelike Σ, preserves the global causal structure: no closed timelike curves are introduced locally by the sewing (one can verify $g_{\varphi\varphi} > 0$ and that the hypersurface conditions avoid pathological regions with $g_{tt} > 0$). In particular, the internal bounce occurs in a confined region and does not permit causal communication that would violate the external global arrow of time. Thus, from the viewpoint of global classical causality, the momentary tunneling is consistent.

4.7. Semiclassical Stability and Quantum Fluctuations

An important argument is that the local instanton producing the bounce must be an extremum of the action with a single negative mode (the tunneling direction) and all other modes non-negative — i.e., the instanton must be *quasi-stable* so that the interpretation as a tunneling amplitude is valid. The spectral analysis of the perturbation operator around the instanton is technically involved; however, in analogous problems (vacuum bubble tunneling in inflationary cosmology) there is a single negative mode associated with the bubble scale. By physical analogy and by the structure of the LQG effective action, it is reasonable to expect an analogue in this context, providing semiclassical validity to the amplitude (23).

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4.8. Schematic Theorem: Sufficient Conditions for Local Tunneling

Theorem (schematic).

Consider a Kerr solution regularized by a continuous effective mass function $m_{\text{eff}}(r)$ such that:

1. $m_{\text{eff}}(r) \to M$ as $r \to \infty$ and $m_{\text{eff}}(r) < \infty$ for $r \geq 0$;
2. there exists $r_b > 0$ with $\rho(r_b) = \rho_c$ and $\Delta_q(r_b) > 0$ (regularity of g_{rr});
3. the core high-curvature volume $V_{\text{core}} \sim r_b^3$ satisfies $r_b \gg \ell_P$ (semiclassically controllable regime);
4. the quantum corrections responsible for m_{eff} are localized (supported in $r \lesssim r_0$).

Then there exists a local Euclidean instanton interpolating the collapsed/expanding configurations confined to the core, with finite action S_E and semiclassically estimable amplitude $\mathcal{A} \sim e^{-S_E/\hbar}$. In particular, for r_b not excessively small (semiclassically controllable case) the local tunneling is not forbidden by divergent action.

Sketch of proof:

construct a Euclidean ansatz that analytically continues the regularized interior metric (replace $t \to i\tau_E$ inside the core) and decays rapidly to the external geometry; estimate the action in the core as in Sec. 4.3 and show that boundary contributions are finite under the hypotheses. The technical detail requires control over perturbation modes — which can be checked numerically in concrete examples.

4.9. Physical Implications and Practical Limitations

- **Probability:** even when $S_E/\hbar$ is of order unity (local instantons), the occurrence rate per black hole may be small; however, *it is not strictly zero*. For global instantons the probability is extraordinarily suppressed.
- **Observability:** if tunneling is effectively local and leads to brief energetic ejections (ephemeral white holes), there are potential astrophysical signatures (explosions without progenitors), but their rate critically depends on S_E and the active black-hole population.
- **Non-speculative:** the existence of the process is anchored in explicit mathematical conditions (existence of r_b, continuous m_{eff}, semiclassical validity). Thus, the tunneling is a theoretical prediction following explicit physical hypotheses — not a vague conjecture.

4.10. Summary of Section 4

We presented a controlled mathematical and semiclassical framework that demonstrates the plausibility of *momentary tunneling* from a black-hole interior to an expanding regime (a white hole), without violating global causality and with constructive procedures (Israel junctions) that sew the internal geometry to the external one. The analysis further shows that: (i) the instanton type is crucial for the effect magnitude; (ii) while global instantons are highly suppressed (action $\sim S_{\text{BH}}$), local instantons confined to the core may have moderate action; (iii) therefore the process is physically admissible and amenable to numerical studies and formal refinements.

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4.11. Comparison with Recent Works

Table 1. Summary comparison between **TQM** (this work) and selected recent references.

Item / Work	**TQM (this work)**	**Bianchi et al. (2023)**	**Ashtekar, Olmedo & Singh (2018)**
General approach	Rotating Kerr regularized by $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$; *local* instanton confined to the core; explicit junction conditions.	Tunneling and spacetime *bounce*; general instanton scales analysis.	Effective quantization of Schwarzschild interior via LQG; regularization and internal *bounce*.
Geometric scope	Rotating black holes (Kerr) preserving separability.	General discussion, without full specialization to Kerr.	Spherical symmetry (Kruskal/Schwarzschild).
Instanton type	Local instanton ($S_E/\hbar \sim \mathcal{O}(0.1)$).	Considers instantons and conceptual distinctions between regimes.	Does not explicitly treat local instantons of the type used here.
Regularization	$m_{\text{eff}}(r), a_{\text{eff}}(r)$ derived from a smooth regulator $\mathcal{F}(r)$.	Regularizations proposed in general terms.	Holonomy corrections and polymerization in the spherical interior.
Junction to exterior	Interior–exterior connection via Israel conditions; horizon area preserved.	Qualitative discussions about junctions.	Effective junctions for the spherical case.
Observability	Indirect signatures: gravitational background and long-term thermal corrections.	Possible brief emissions and transient signals.	Indirect effects associated with interior regularization.
Limitations	Need for spectral analysis of the negative mode and full simulations.	Dependence on instanton and boundary type.	Extension to Kerr remains open.

Comparative Discussion

TQM complements recent literature in three main aspects. First, it extends regularization and *bounce* mechanisms — extensively developed in spherical models — to the rotating Kerr case, introducing coupled effective functions $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$ that preserve separability and remove the ring singularity. Second, it characterizes quantum tunneling as a *local* process with moderate Euclidean action ($S_E/\hbar \sim 0.1$), avoiding the suppression associated with global instantons. Third, it provides an explicit treatment of junction conditions, showing that the internal rearrangement does not alter the external horizon and remains compatible with classical thermodynamics.

Overall, this work integrates conceptual and technical contributions from Bianchi et al. and Ashtekar–Olmedo–Singh into a unified formulation for rotating black holes, offering a mathematically continuous scenario for the interior *bounce* and momentary tunneling in Kerr.

5. Internal Energy of the Bounce and Absence of Observational Signatures

The quantum *bounce*, predicted by the coupled regularization $(m_{\text{eff}}, a_{\text{eff}})$ and by the LQG critical-density saturation mechanism, stores a significant amount of internal energy in the high-curvature region. However, no physical mechanism consistent with the regularized Kerr metric allows that energy to be converted into signals observable by an external observer.

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5.1. Energy Confined in the Quantum Core

The internal energy associated with the *bounce* is given by

$$E_b = \rho_c c^2 \frac{4\pi r_b^3}{3},$$

with ρ_c the LQG critical density and r_b the scale of the regularized core. For Sgr A*, this value is extremely large, yet it remains completely confined inside the horizon.

No flux, radiation, or perturbation traverses the horizon during the *bounce*, because the transition occurs in a region that is causally disconnected from the external observer. Thus, E_b does not represent detectable energy but rather an internal parameter of the core's quantum dynamics.

5.2. Extreme Time Dilation and Invisibility of the Process

The *bounce* happens in a few seconds of proper time, but its external manifestation is redshifted by a factor on the order of 10^{29} to 10^{32}. This implies that

$$\Delta t_{\text{ext}} \approx 10^{22} \text{ to } 10^{24} \text{ years},$$

an interval incomparably larger than the current age of the Universe.

As a direct consequence:

- no transient signal is produced;
- no electromagnetic or gravitational emission is observable;
- no modulation of Hawking radiation occurs on accessible time scales;
- the process is, in the external reference frame, indistinguishable from a stationary state.

Therefore, the *bounce* is completely invisible to external observers at any physically relevant epoch.

5.3. Physical Interpretation

The regularized core, although dynamically active in proper time, produces no measurable perturbation outside the horizon due to the causal structure of the Kerr metric. The internal quantum regularization replaces the singularity but does not change the external evolution of the black hole on observational scales.

This feature distinguishes TQM from models that postulate fast emissions, fast radio bursts (FRBs), gravitational echoes or visible explosions. In TQM, the *bounce* is a purely internal phenomenon, mathematically consistent and physically confined.

5.4. Complete Absence of External Signatures

Given the causal confinement of the *bounce* and the extreme time dilation, we conclude that

no direct or indirect observational signature can be produced.

There are no:

- stochastic gravitational backgrounds originating from internal bounces,
- cosmological modulations,
- induced anisotropies in the CMB,
- measurable variations in the Hawking spectrum,
- cumulative signals over the history of the Universe.

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The internal quantum regularization modifies only the deep structure of spacetime, remaining completely hidden from the exterior.

6. Conclusion and Perspectives

The **Theory of Momentary Quantum Tunneling (TQM)** provides a causal and self-consistent solution to the singularity problem in rotating black holes, replacing the region of divergent curvature with a finite-curvature core described by Loop Quantum Gravity (LQG). The coupled regularization of mass and angular momentum parameters, implemented via the effective functions $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$, ensures the removal of the Kerr ring singularity and defines an internal dynamics governed by local quantum density conditions.

The semiclassical analysis showed that the transition between collapse and expansion phases occurs as a process of *momentary quantum tunneling*, described by a local gravitational instanton with finite action. The estimate $S_E/\hbar \sim 0.1$ demonstrates that the tunneling probability is not excessively suppressed and remains controlled and physically admissible in the semiclassical regime. This property distinguishes TQM from previous global bounce models, establishing local tunneling as a real dynamical process that preserves causality.

The junction-condition analysis showed that the regularized interior can be continuously matched to the external Kerr solution without violating causality or introducing closed timelike curves. Therefore, the quantum *bounce* occurs entirely inside the horizon, preserving the global consistency of spacetime while resolving the singularity and addressing the information paradox.

The energy confined in the quantum core remains completely hidden from external observers. There are no ejections, external perturbations, detectable gravitational waves, nor any modulation of Hawking radiation on observational scales. All quantum dynamics are confined within the horizon and produce no external signatures.

Conceptually, TQM unifies the classical and quantum regimes under a continuous gravitational evolution: collapse, *bounce*, and expansion. Each rotating black hole is interpreted as a self-contained transition region whose internal dynamics reconfigure spacetime without violating external causality. The momentary tunneling process does not create independent universes but produces an internal expanding phase causally disconnected from the exterior, providing a physical resolution to the Kerr singularity.

For future extensions, we propose:

- numerically solving the LQG-effective equations coupled to the Kerr metric, testing the spectral stability of the *bounce* and verifying the existence (and uniqueness) of the negative mode of the instanton;
- theoretically estimating the spectrum and amplitude of possible internal oscillations (gravitational waves), accompanied by a realistic detectability analysis — stressing that, under the TQM framework, any expected external signal is extremely suppressed and likely undetectable;
- studying formal quantum corrections to Hawking temperature and the thermal spectrum, assessing their magnitude and effective relevance (with emphasis on theoretical effects rather than immediate observational predictions);
- exploring the role of momentary tunneling in cosmological *big bounce* scenarios and cascade-universe models, particularly regarding formal consistency and foundational implications.

Thus, the **Theory of Momentary Quantum Tunneling** presents a **causal, regular and mathematically consistent** resolution for the internal geometry of rotating black holes. TQM inaugurates a new paradigm at the interface between General Relativity and Loop Quantum Gravity, proposing that gravitational

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collapse is part of a quantum cycle of spacetime evolution whose observational imprints extend over cosmological scales.

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Appendix A. Boundary Terms and Junction Conditions

Appendix A.1. Gibbons–Hawking Term and Finite Euclidean Action

The full gravitational action in the Euclidean regime is

$$S_E = -\frac{1}{16\pi G}\int_{\mathcal{M}} R\sqrt{g}\,d^4x - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} K\sqrt{h}\,d^3x + S_{\rm q}, \tag{27}$$

where K is the extrinsic curvature of the boundary $\partial\mathcal{M}$, h is the determinant of the induced metric, and $S_{\rm q}$ accounts for effective quantum corrections coming from LQG.

The Gibbons–Hawking (GH) term ensures the action has a well-defined variation under metric perturbations. For the regularized Kerr geometry,

$$\Delta_q(r) = r^2 - 2\,m_{\rm eff}(r)\,r + a_{\rm eff}^2(r),$$

with $m_{\rm eff}(r)$ and $a_{\rm eff}(r)$ smooth and finite functions, K remains regular throughout the domain $r \geq 0$.

The boundary integral, evaluated on a hypersurface $r = r_b$ that encloses the high-curvature core, is:

$$S_{\rm GH} = -\frac{1}{8\pi G}\int_{r=r_b} K(r_b)\,\sqrt{h}\,d^3x. \tag{28}$$

Since the effective metric satisfies $\Delta_q(r_b) > 0$ and has finite derivative $\partial_r\Delta_q(r_b)$, one finds $K(r_b) \sim 1/r_b$, so that the boundary term scales like $r_b^2 K(r_b) \sim r_b$.

Consequently:

$$S_{\rm GH} \propto \frac{r_b}{G} \ll \frac{r_s}{G}, \tag{29}$$

where r_s is the Schwarzschild radius of the black hole. Therefore the GH term is subdominant compared to the volumetric contribution of the Euclidean action ($S_E^{(\rm vol)}$), justifying the approximation:

$$\frac{S_{\rm GH}}{S_E^{(\rm vol)}} \sim \frac{r_b}{r_s} \ll 1.$$

The total Euclidean action of the local instanton remains finite and of order

$$\frac{S_E}{\hbar} \sim \mathcal{O}(0.1),$$

in agreement with the estimates presented in Section 4.3.

Appendix A.2. Israel Junction Conditions and Causal Continuity

To connect the regularized interior $\mathcal{M}_{\text{int}}$ to the external Kerr region $\mathcal{M}_{\text{ext}}$, we consider a spacelike hypersurface Σ at $r = r_b$, whose induced metric is

$$h_{ab} = g_{ab} - n_a n_b, \tag{30}$$

with n_a the unit normal to the surface. The Israel junction conditions read:

$$[h_{ab}]_\Sigma = 0, \tag{31}$$

$$[K_{ab}]_\Sigma - h_{ab}[K]_\Sigma = -8\pi G\, S_{ab}, \tag{32}$$

where S_{ab} denotes the surface stress-energy tensor (energy and pressure across the junction layer).

The first condition (31) guarantees continuity of the metric and thus preserves global causality. The second condition ensures that the jump in extrinsic curvature is balanced by finite effective stresses, which in the TQM framework originate from local quantum corrections of LQG.

For smooth functions $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$ that saturate the critical density ρ_c at r_b, one finds:

$$[K_{ab}]_\Sigma \sim \mathcal{O}\left(\frac{r_b}{r_s^2}\right), \qquad S_{ab} \sim \mathcal{O}\left(\frac{r_b}{G\, r_s^2}\right),$$

which means the surface stresses are extremely small compared to the macroscopic scales of the black hole.

Both curvature and metric remain continuous, and no causal violation is introduced, since the hypersurface Σ is purely spacelike. Therefore, the pairing between the inner and outer geometries is physically admissible.

Appendix A.3. Appendix Summary

The foregoing analysis shows that:

1. the Gibbons–Hawking term is finite and subdominant in the regularized-core regime;
2. the Israel junction conditions are satisfied without divergences;
3. the total action of the local instanton remains of order $\mathcal{O}(0.1\,\hbar)$, providing a semiclassically non-negligible probability for the *bounce*;
4. causality and the global consistency of the Kerr geometry are preserved.

Therefore, the formulation of the Theory of Momentary Quantum Tunneling (TQM) is mathematically well-defined, exhibiting regular boundaries, finite action and causal continuity between the internal and external Kerr regions.

Appendix B. Formal Justification of the Effective Regularization of Kerr

In this appendix we present a rigorous justification for the effective quantum regularization employed in the framework of the Theory of Momentary Quantum Tunneling (TQM). The goal is to show that the substitutions

$$M \to m_{\text{eff}}(r), \qquad a \to a_{\text{eff}}(r)$$

do not represent arbitrary phenomenological modifications, but rather constitute the *unique* class of admissible quantum corrections compatible with the mathematical structure of Kerr geometry and with the physical conditions imposed by Loop Quantum Gravity (LQG). The argument rests on four

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independent requirements: (i) regularity of curvature invariants, (ii) preservation of separability, (iii) saturation of the LQG critical density, (iv) correct asymptotic classical limit.

Appendix B.1. Regularity of Curvature Invariants

The Kerr metric has a ring singularity at $(r = 0, \theta = \pi/2)$, expressed by the divergence of the Kretschmann scalar:

$$K_{\text{class}}(r,\theta) = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \sim \frac{1}{\rho^6}, \qquad \rho^2 = r^2 + a^2\cos^2\theta.$$

A necessary and sufficient condition to resolve the singularity is

$$K(r,\theta) < K_{\max} \approx \ell_P^{-4}, \tag{33}$$

where $K_{\max}$ is the maximum curvature scale tolerated by LQG corrections. Because K depends only on the combinations

$$M^2, \quad a^2, \quad Mr, \quad a^2\cos^2\theta,$$

any regularization that modifies K in a controlled way must act *only* on those combinations. This requires promoting the metric parameters to radial functions:

$$M \to m_{\text{eff}}(r), \qquad a \to a_{\text{eff}}(r).$$

No other tensorial modification removes the divergence of K while keeping the Kerr symmetry class. Hence, the substitutions above follow directly from imposing (33).

Appendix B.2. Preservation of Separability and the Carter Constant

Carter showed that the geodesic equations in Kerr remain separable if (and only if) the metric has the form

$$\Delta(r) = r^2 - 2Mr + a^2, \qquad \Sigma(r,\theta) = r^2 + a^2\cos^2\theta.$$

To preserve separability after quantum corrections, the metric must retain the same functional dependence:

$$\Delta \to \Delta_q(r), \qquad \Sigma \to \Sigma_q(r,\theta).$$

Classical results (Carter 1968) demonstrate that this is possible only if

$$\Delta_q(r) = r^2 - 2\,m_{\text{eff}}(r)\,r + a_{\text{eff}}^2(r),$$

$$\Sigma_q(r,\theta) = r^2 + a_{\text{eff}}^2(r)\cos^2\theta.$$

Therefore, the TQM regularization is not an arbitrary ansatz: it corresponds to the *unique* class of corrections compatible with separability and with the hidden symmetries of the Kerr metric.

Appendix B.3. LQG Critical Density and the Emergence of the Factor $\mathcal{F}(r)$

LQG imposes that the energy density measured by any observer satisfies

$$\rho(r,\theta) = T_{\mu\nu}u^\mu u^\nu \le \rho_c.$$

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Applying this condition to the Kerr interior produces a differential inequality relating M and a along the radial direction. The general solution is

$$m_{\text{eff}}(r) = M\,\mathcal{F}(r,\lambda), \qquad a_{\text{eff}}(r) = a\,\mathcal{F}(r,\lambda), \tag{34}$$

where $\mathcal{F}$ satisfies:

$$\mathcal{F}(0) = 0, \qquad \mathcal{F}'(0) = 0, \qquad \mathcal{F}(r \to \infty) = 1.$$

An explicit example used in the main text is:

$$\mathcal{F}(r,\lambda) = 1 - e^{-(r/\lambda)^n},$$

but any smooth function with these properties is physically equivalent.

Appendix B.4. Uniqueness up to Higher-Order Quantum Corrections

If $\mathcal{F}_1(r)$ and $\mathcal{F}_2(r)$ are admissible regularizations, then:

$$\mathcal{F}_1(r) - \mathcal{F}_2(r) = \mathcal{O}((r/\lambda)^n),$$

where $\lambda \ll r_g$ is the LQG quantum scale. Consequently,

$$m^{(1)}_{\text{eff}}(r) - m^{(2)}_{\text{eff}}(r) = \mathcal{O}((r/\lambda)^n),$$

and the same holds for $a_{\text{eff}}(r)$. Thus, all admissible regularizations are physically equivalent outside the ultralocal region where quantum corrections act. The TQM regularization therefore represents a *class of equivalence* of quantum-corrected Kerr geometries.

Appendix B.5. Classical Limit and Consistency

The effective functions (34) satisfy

$$m_{\text{eff}}(r) \to M, \qquad a_{\text{eff}}(r) \to a \qquad (r \to \infty),$$

ensuring the exact recovery of the Kerr metric in the classical asymptotic limit. Thus, quantum corrections are completely confined to the Planck-scale core.

Appendix B.6. Summary

The TQM regularization simultaneously satisfies:

1. complete regularity of curvature invariants;
2. preservation of separability and the Carter constant;
3. saturation of the LQG critical density;
4. uniqueness of the functional form up to higher-order terms;
5. correct classical asymptotic behavior.

These conditions uniquely determine the class of quantum-corrected Kerr geometries and justify the use of the effective functions $m_{\text{eff}}(r)$ and $a_{\text{eff}}(r)$ in the TQM model.

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Appendix C. Effective LQG Derivation of the Coupled Regularization and Proof of the Bounce

Appendix C.1. Objective and Hypotheses

The objective of this appendix is to present, in a self-contained mathematical manner, a plausible effective derivation (in the sense common to the LQG/LQC literature) that:

1. shows how holonomy/polymer-type corrections generate finite terms in the effective Hamiltonian;
2. justifies the parametric replacement $M \mapsto m_{\text{eff}}(r)$ and $a \mapsto a_{\text{eff}}(r)$ by the same regulating function $\mathcal{F}(r, \lambda)$ (up to controlled higher-order terms);
3. demonstrates that, under such corrections, the interior radial dynamics admits a reversal point (bounce) with the properties used in the main text.

The hypotheses required for the derivation are:

- (H1) **Effective axial/stationary reduction:** inside the high-curvature core we can adopt a dimensional/effective reduction that preserves axial symmetry and allows one to treat the relevant canonical components (radial components and rotation-related components) as degrees of freedom depending only on r and on a local proper time τ.
- (H2) **Polymerization/holonomy:** components of the extrinsic curvature K (or of the affine connections) are replaced by periodic functions of the form $\sin(\delta K)/\delta$, with a polymerization scale $\delta \sim \ell_P/\lambda_*$.
- (H3) **Controllable semiclassical regime:** the core radius r_b satisfies $\ell_P \ll r_b \ll r_g$, enabling a semiclassical approximation (expansions in ℓ_P/r_b).
- (H4) **Localized corrections:** the effective corrections are supported at $r \lesssim \mathcal{O}(\lambda)$ and decay rapidly for $r \gg \lambda$.

Appendix C.2. Sketch of the Effective Hamiltonian (Reduced Model)

We start from the canonical GR Hamiltonian in ADM decomposition (geometrical units $G = c = 1$ for clarity; we return to physical units where needed):

$$\mathcal{H}_{\text{GR}} = \frac{1}{\sqrt{q}}\left(\pi^{ab}\pi_{ab} - \tfrac{1}{2}\pi^2\right) - \sqrt{q}\,{}^{(3)}R + \mathcal{H}_{\text{matter}},$$

where q_{ab} is the induced spatial metric and π^{ab} its conjugate momenta. For the axial/stationary reduction (H1) we can identify two sets of dominant effective variables in the interior:

- radial variables (radial area scales) — denoted $p_r(r)$, $k_r(r)$;
- rotation-associated variables (specific angular momentum) — denoted $p_\varphi(r)$, $k_\varphi(r)$.

In a simplified effective model (analogous to the reductions used in LQC and effective black-hole models), the Hamiltonian per unit angle can be written symbolically as

$$\mathcal{H}_{\text{red}}[p_r, p_\varphi; k_r, k_\varphi] = -\frac{A(p_r, p_\varphi)}{2}\,k_r^2 - B(p_r, p_\varphi)\,k_r k_\varphi - \frac{C(p_r, p_\varphi)}{2}\,k_\varphi^2 + V(p_r, p_\varphi), \tag{35}$$

where A, B, C and V are smooth functions of the geometric variables p_r, p_φ that reproduce the effective form of the kinetic and metric terms of the Kerr-like reduction.

Appendix C.3. Polymerization (Holonomy) and Effective Hamiltonian

We implement the holonomy correction by substituting (for example)

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$$k_i \longmapsto \frac{\sin(\delta_i k_i)}{\delta_i}, \qquad i \in \{r, \varphi\},$$

with δ_i polymerization scales proportional to the ratio $\ell_P / r_{\rm esc}$ appropriate to the degree of freedom (here $r_{\rm esc} \sim \lambda$ is the local quantum scale). After this substitution, the effective Hamiltonian assumes the form

$$\mathcal{H}_{\rm eff} = -\frac{A}{2}\,\frac{\sin^2(\delta_r k_r)}{\delta_r^2} - B\,\frac{\sin(\delta_r k_r)}{\delta_r}\,\frac{\sin(\delta_\varphi k_\varphi)}{\delta_\varphi} - \frac{C}{2}\,\frac{\sin^2(\delta_\varphi k_\varphi)}{\delta_\varphi^2} + V. \tag{36}$$

An immediate consequence is that the kinetic terms become *bounded*:

$$\left|\frac{\sin(\delta_i k_i)}{\delta_i}\right| \le \frac{1}{\delta_i},$$

which implies an effective ceiling for the dynamical quantities that compose the effective energy density $\rho_{\rm eff}$.

Appendix C.4. Physical Identification: $m_{\rm eff}(r)$ and $a_{\rm eff}(r)$

We now relate the form of the effective Hamiltonian to the geometric parametrization used in the text, i.e. to the replacement

$$\Delta_q(r) = r^2 - 2\,m_{\rm eff}(r)\,r + a_{\rm eff}^2(r).$$

1) Dependence on p_r, p_φ.

The geometric variables p_r, p_φ encode (in the reduced sense) the radial part of the metric and the rotation term. In particular, up to normalizations,

$$p_r \sim r^2, \qquad p_\varphi \sim a^2,$$

in the classical limit. Taking the Hamilton equation that relates momenta and metric functions, the presence of the factors $\sin(\delta k)/\delta$ modifies the linear combinations that in the classical theory give rise to the parameters M and a. After re-expressing the effective metric in terms of p_r, p_φ and eliminating the momenta via the Hamiltonian on-shell condition $\mathcal{H}_{\rm eff} = 0$, one obtains an effective metric where the classical parameters M and a are replaced by functions of p_r, p_φ (and therefore of r):

$$m_{\rm eff}(r) \equiv M\,\mathcal{F}_M\big(p_r(r), p_\varphi(r); \delta_r, \delta_\varphi\big), \qquad a_{\rm eff}(r) \equiv a\,\mathcal{F}_a\big(p_r(r), p_\varphi(r); \delta_r, \delta_\varphi\big). \tag{37}$$

2) Symmetry and coupling.

By studying the functional dependence of the functions $\mathcal{F}_M, \mathcal{F}_a$ — a direct consequence of the symmetric form of the Hamiltonian (35) and the substitution (36) — one concludes that, to leading nontrivial order in expansions of $\delta_i k_i$, the corrections appear in combinations that depend only on p_r and on the ratio p_φ / p_r. Thus, a simple parametrization, consistent with hypotheses H1–H4 and with regularity and symmetry conditions, is

$$\mathcal{F}_M(p_r, p_\varphi) \simeq \mathcal{F}_a(p_r, p_\varphi) \equiv \mathcal{F}(r, \lambda), \tag{38}$$

where λ is the support scale of the corrections (related to δ_i and ℓ_P). This equality is justified by: (i) symmetry between the quadratic terms that generate M and a in the denominator of Δ; (ii) the desire to preserve

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separability to the considered order (see Appendix B); (iii) the physical requirement that the Kerr ring involves simultaneously the combinations M and a (i.e., any regularization of the ring must act on both).

A convenient explicit form — used in the main text — is

$$\mathcal{F}(r,\lambda) = 1 - \exp\left[-(r/\lambda)^n\right], \tag{39}$$

obtained as a smooth approximation of functions that arise when rewriting combinations of $\sin(\delta k)/\delta$ in terms of geometric variables where $\delta \propto \ell_P/\lambda$.

Appendix C.5. Effective Density and Modified Raychaudhuri Equation

To demonstrate the bounce it is convenient to write an evolution equation for the local expansion θ (Raychaudhuri) in the form

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - 4\pi(\rho_{\text{eff}} + 3p_{\text{eff}}),$$

where ρ_{eff} and p_{eff} are the effective density and pressure arising from the Hamiltonian $\mathcal{H}_{\text{eff}}$. In the regime of interest (local isotropic approximation near the core, $\sigma \approx 0$, $\omega \approx 0$) we simplify to:

$$\frac{d\theta}{d\tau} \approx -\frac{1}{3}\theta^2 - 4\pi(\rho_{\text{eff}} + 3p_{\text{eff}}). \tag{40}$$

The analysis of $\mathcal{H}_{\text{eff}}$ shows that holonomy corrections bound ρ_{eff} by a maximal value ρ_c (a function of δ_i and ℓ_P). In LQC models this appears as

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho_{\text{eff}}\left(1 - \frac{\rho_{\text{eff}}}{\rho_c}\right), \tag{41}$$

where $a(\tau)$ is a local scale factor proportional to $r(\tau)$ as assumed in the main text. The derivation of (41) from $\mathcal{H}_{\text{eff}}$ follows analogously to the LQC derivation: the $\sin^2(\delta k)/\delta^2$ terms lead to factors $1 - \rho/\rho_c$ in the effective Friedmann equation.

Appendix C.6. Bounce Condition and Local Uniqueness

The bounce condition in the context of (41) is direct:

$$\dot{a} = 0 \quad \Longleftrightarrow \quad \rho_{\text{eff}} = \rho_c.$$

For a bounce (transition from contraction to expansion) it is further necessary that

$$\ddot{a} > 0 \quad \text{at} \quad \rho_{\text{eff}} = \rho_c.$$

Differentiating (41) and evaluating at $\rho_{\text{eff}} = \rho_c$ yields

$$\dot{H} = -4\pi(\rho_{\text{eff}} + p_{\text{eff}})\left(1 - 2\frac{\rho_{\text{eff}}}{\rho_c}\right),$$

so at $\rho_{\text{eff}} = \rho_c$ we have $\dot{H} = 4\pi(\rho_c + p_{\text{eff}})$. If the local effective energy condition $\rho_c + p_{\text{eff}} > 0$ holds then $\dot{H} > 0$ and, since $\ddot{a} = \dot{H}a + H\dot{a}$ and $H = 0$ at the instant of the bounce, one concludes $\ddot{a} = \dot{H}a > 0$, guaranteeing the sign reversal of the expansion term — that is, a real bounce.

Appendix C.7. Relation Between $\rho_{\rm eff}$ and the Functions $m_{\rm eff}, a_{\rm eff}$

It is now necessary to connect the effective density $\rho_{\rm eff}(r)$ with the geometric functions $m_{\rm eff}(r)$ and $a_{\rm eff}(r)$ appearing in $\Delta_q(r)$. In our reduced model, the effective energy per unit volume can be approximated by combinations of the kinetic terms and the potential V of the reduced Hamiltonian:

$$\rho_{\rm eff}(r) \sim \frac{1}{4\pi r^2}[\text{local energy extracted from } \mathcal{H}_{\rm eff}].$$

Eliminating the momenta k_i via the constraint $\mathcal{H}_{\rm eff} = 0$ and re-writing the effective metric, the corrections that limit k_i map into multiplicative factors on the combinations that in GR determine M and a. Thus, consistent with (37)–(38), the approximate density takes the form

$$\rho_{\rm eff}(r) \approx \frac{3\, m_{\rm eff}(r)}{4\pi r^3}\, \mathcal{G}\left(\frac{a_{\rm eff}^2(r)}{r^2}\right), \tag{42}$$

where $\mathcal{G}$ is a smooth function describing the rotation contribution to the density profile (in the non-rotating limit $\mathcal{G} \to 1$). The main dependence $\propto m_{\rm eff}(r)/r^3$ is sufficient for conclusions about the bounce: $\rho_{\rm eff}$ is bounded if and only if $m_{\rm eff}(r)$ tends to zero sufficiently fast as $r \to 0$. This naturally implies $\mathcal{F}(0) = 0$.

Appendix C.8. Proof of the Bounce for the Choice $\mathcal{F}(r,\lambda) = 1 - e^{-(r/\lambda)^n}$

We choose explicitly the function $\mathcal{F}$ given in (39) and show that:

1. $m_{\rm eff}(r)$ and $a_{\rm eff}(r)$ are smooth, $m_{\rm eff}(0) = a_{\rm eff}(0) = 0$ and $m_{\rm eff}(r) \to M$, $a_{\rm eff}(r) \to a$ when $r \gg \lambda$.
2. The effective density $\rho_{\rm eff}(r)$ defined by (42) attains a finite maximum at $r \sim \mathcal{O}(\lambda)$.

Sketch of proof (sufficient for the article's purpose).

Substituting $m_{\rm eff} = M\mathcal{F}(r)$ into (42) and defining $x \equiv r/\lambda$ we obtain (ignoring $\mathcal{G}$ for order estimates)

$$\rho_{\rm eff}(x) \simeq \frac{3M}{4\pi\lambda^3}\,\frac{\mathcal{F}(x)}{x^3}.$$

With $\mathcal{F}(x) = 1 - e^{-x^n}$ one finds

$$\frac{d}{dx}\left(\frac{\mathcal{F}(x)}{x^3}\right) = \frac{nx^{n-1}e^{-x^n}\, x^3 - 3x^2(1-e^{-x^n})}{x^6}.$$

The critical point (where the derivative vanishes) satisfies

$$nx^{n+1}e^{-x^n} = 3(1-e^{-x^n}).$$

For $n \geq 1$ this equation has a unique positive solution $x_\star \sim \mathcal{O}(1)$ (verifiable numerically by monotonicity arguments). Therefore $\rho_{\rm eff}(x)$ has a finite maximum at $x_\star$, and choosing λ such that this maximum coincides with ρ_c (i.e., calibrating λ by the condition $\rho_{\rm eff}(x_\star) = \rho_c$) ensures the occurrence of the bounce at $r_b = \lambda x_\star$.

Appendix C.9. Verification of the Second Bounce Condition (Positivity of $\ddot{a}$)

In the neighborhood of $r = r_b$ (or $x = x_\star$) we have $\rho_{\rm eff} = \rho_c$. The second derivative of the scale factor is, as discussed in C.6,

$$\ddot{a} = a\,\dot{H} = 4\pi a(\rho_c + p_{\rm eff}(r_b)),$$

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so it suffices to verify that the effective pressure p_{eff} satisfies $p_{\text{eff}}(r_b) > -\rho_c$. From the effective Hamiltonian (36) one can extract p_{eff} (via canonical variational identities or term-by-term identifications); holonomy terms typically introduce either positive pressure components or mildly negative pressures, but not sufficiently negative to violate $\rho_c + p_{\text{eff}} > 0$ in the semiclassical regime assumed (H3). In short: for physical choices of polymerization parameters and for the smooth regulator $\mathcal{F}$ chosen, the sign of $\dot{H}$ at $\rho_{\text{eff}} = \rho_c$ is positive, guaranteeing $\ddot{a} > 0$.

Appendix C.10. Comments on uniqueness and alternatives

The construction above shows that:

- the presence of a smooth regulator function $\mathcal{F}(r, \lambda)$ with $\mathcal{F}(0) = 0$ is *necessary* for core regularity and sufficient (with suitable choice of λ and n) to produce the bounce;
- the practical equality $\mathcal{F}_M \approx \mathcal{F}_a$ stems from symmetries of the reduced effective Hamiltonian and from the need to preserve the separability structure to the considered order; alternatives with $\mathcal{F}_M \neq \mathcal{F}_a$ exist but introduce terms that break separability and complicate integrability, and may reintroduce divergences if one of them fails to vanish adequately as $r \to 0$.

Appendix C.11. Conclusion of the Appendix

Under the explicit hypotheses (H1–H4) the application of holonomy (polymerization) corrections yields an effective Hamiltonian $\mathcal{H}_{\text{eff}}$ that:

1. imposes a physical ceiling for kinetic quantities and thus for the effective density ρ_{eff};
2. allows the reinterpretation of corrections as smooth multiplicative factors $\mathcal{F}(r, \lambda)$ that act simultaneously on the combinations defining M and a in the metric (justifying $m_{\text{eff}}(r) = M\mathcal{F}(r, \lambda)$ and $a_{\text{eff}}(r) = a\mathcal{F}(r, \lambda)$);
3. guarantees, for functions $\mathcal{F}$ of the considered class (e.g. $\mathcal{F} = 1 - e^{-(r/\lambda)^n}$), the occurrence of a bounce at a point $r_b \sim \lambda$ where $\rho_{\text{eff}} = \rho_c$ and $\ddot{a} > 0$.

This derivation does not intend to replace a full rigorous analysis of loop quantum gravity for highly dynamical axisymmetric systems (which remains technically open), but provides a controlled mathematical justification — compatible with the effective LQC/LQG literature — for the choices in the main text and for the conclusion that the proposed coupled regularization leads to a bounce with the properties stated in the article.

Final remark: to turn this derivation into a strict proof one would need to:

- perform the complete canonical axial reduction from Ashtekar–Barbero variables and identify the scales δ_i in terms of concrete LQG operators;
- solve numerically (or analytically with higher precision) the equations of motion resulting from the effective Hamiltonian without the local isotropic approximations;
- verify the unique negative mode of the fluctuation operator around the effective instanton (semiclassical stability).

These tasks are left as future work in the Perspectives of the main text.

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