

Article

Not peer-reviewed version

Dynamical Dark Sector: A Joint Two-Scalar-Field Model for Dark Matter and Quintessence

[Silvio A. Correa Junior](#)*

Posted Date: 3 December 2025

doi: 10.20944/preprints202512.0334.v1

Keywords: fuzzy dark matter; quintessence; dark energy; S8 tension; modified CLASS code



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a [Creative Commons CC BY 4.0 license](#), which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Dynamical Dark Sector: A Joint Two-Scalar-Field Model for Dark Matter and Quintessence

Silvio A. Corrêa Junior

Universidade Santa Cecília; talksilviojr@gmail.com

Abstract

The physical nature of dark matter and dark energy remains one of the most pressing questions in modern cosmology. This work presents a phenomenological model where the entire dark sector is described by two minimally coupled scalar fields within General Relativity. The first, an ultra-light scalar field Ψ with mass m_ψ , constitutes Fuzzy Dark Matter (FDM), whose coherent oscillations dynamically replicate cold dark matter on large scales. The second, a quintessence field ϕ , evolves under an axion-like potential and serves as the dark energy component. We demonstrate that this framework can successfully reproduce the canonical cosmic history while offering a physical mechanism to address the S_8 tension. By exploring the model's parameter space, we show that the suppression of small-scale structure is a direct function of the FDM mass. For a benchmark mass of $m_\psi = 10^{-22}$ eV, chosen to illustrate the potential impact, we show that the model can produce a value of $S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ of approximately 0.79, significantly alleviating the tension between early and late-universe probes [1,9,10]. Concurrently, the model predicts a “thawing” behavior for dark energy, with a present-day equation of state, w_{ϕ_0} , that depends on the potential's parameters, yielding $w_{\phi_0} \approx -0.92$ in our benchmark case—a value distinguishably different from the cosmological constant's $w_\Lambda = -1$. We acknowledge that the FDM mass required to affect the S_8 tension creates a testable conflict with some Lyman-alpha forest constraints [16], a point we discuss as a key feature for the model's falsifiability. By connecting cosmic acceleration, dark matter, and the S_8 tension, this self-consistent framework offers a compelling and highly testable alternative to the Λ CDM model, motivating a full statistical analysis.

Keywords: fuzzy dark matter; quintessence; dark energy; S_8 tension; modified CLASS code

1. Introduction

The standard Λ CDM model, while remarkably successful in describing the large-scale universe, is built upon two components of unknown origin: a cosmological constant (Λ), responsible for the observed cosmic acceleration [2,3], and cold dark matter (CDM). The cosmological constant faces a severe fine-tuning problem [4], and the fundamental nature of dark matter remains elusive despite decades of experimental searches. These foundational challenges have motivated a rich exploration of alternative theories. Scalar fields are compelling candidates, with precedents in cosmic inflation and the Standard Model's Higgs mechanism. Quintessence models, where a slowly rolling scalar field generates negative pressure [5,11,13], provide a dynamic alternative to Λ . On the dark matter front, models featuring an ultra-light scalar field—often termed Fuzzy Dark Matter (FDM)—have gained traction for their ability to resolve potential small-scale structure anomalies [6–8].

While the conceptual framework combining an axion-like quintessence [12] and FDM is well-established, this work presents a dedicated, end-to-end quantitative analysis connecting the model's fundamental parameters to key observables. Our central contribution is to demonstrate that this unified model can simultaneously reproduce the standard cosmic expansion history and possess the inherent flexibility to address the S_8 tension. By solving the full background and linear perturbation equations using a modified version of the CLASS code [15], we explicitly map the dependence of the

linear matter power spectrum, the resulting S_8 parameter, and the dark energy equation of state on the model's parameters. This phenomenological exploration aims to provide sharp, testable predictions and motivate a full-scale statistical analysis against cosmological data.

2. Theoretical Framework

We consider a system governed by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{pl}}^2 R + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\Psi} + \mathcal{L}_{\phi} \right], \quad (1)$$

where R is the Ricci scalar and $M_{\text{pl}} = (8\pi G)^{-1/2}$ is the reduced Planck mass. We work in natural units ($c = \hbar = 1$). $\mathcal{L}_{\text{matter}}$ represents perfect fluids for baryons and radiation. The Lagrangians for the scalar fields Ψ (dark matter) and ϕ (quintessence) are:

$$\mathcal{L}_{\Psi} = -\frac{1}{2} \partial^{\mu} \Psi \partial_{\mu} \Psi - \frac{1}{2} m_{\Psi}^2 \Psi^2, \quad (2)$$

$$\mathcal{L}_{\phi} = -\frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - V(\phi). \quad (3)$$

The fields are minimally coupled to gravity and are assumed not to interact directly. We adopt this non-interacting scenario as the most predictive and parsimonious baseline, allowing for the isolation of the distinct phenomenological consequences of each field. It is important to note that this is an idealization. In a more complete field theory framework, coupling terms (e.g., $\beta \phi^2 \Psi^2$) could exist. Such interactions would introduce new free parameters and could lead to a richer phenomenology, such as energy transfer between the dark components. However, without a strong theoretical motivation for a specific interaction form, the minimally coupled model serves as the most robust and falsifiable starting point. Together, they constitute a self-contained “dark sector.”

3. Cosmological Field Equations

Varying the action in a spatially flat ($k = 0$) Friedmann-Lemaître-Robertson-Walker (FLRW) background yields the Friedmann equations and the Klein-Gordon equations for the homogeneous fields:

$$H^2 = \frac{1}{3M_{\text{pl}}^2} (\rho_r + \rho_b + \rho_{\Psi} + \rho_{\phi}), \quad (4)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\text{pl}}^2} (\rho_{\text{total}} + 3p_{\text{total}}), \quad (5)$$

$$\ddot{\Psi} + 3H\dot{\Psi} + m_{\Psi}^2 \Psi = 0, \quad (6)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad (7)$$

where $H = \dot{a}/a$ is the Hubble parameter and the total energy density ρ_{total} and pressure p_{total} are the sums of the individual components.

4. Background Cosmological Dynamics

4.1. Numerical Setup and Parameter Space

The coupled system of equations is solved numerically from an initial scale factor $a_{\text{initial}} = 10^{-8}$ to the present day at $a = 1$.

4.1.1. Quintessence Potential

We adopt the well-motivated axion-like potential

$$V(\phi) = M^4[1 + \cos(\phi/f)]. \quad (8)$$

This form is theoretically appealing as its underlying shift symmetry can protect the potential from large quantum corrections, making a small mass scale technically natural [12].

4.1.2. Parameter Selection and Benchmark

The model's key new parameters are the FDM mass m_Ψ , and the quintessence scales M and f . To demonstrate the model's viability and explore its predictions, we define a benchmark scenario calibrated to match Planck 2018 data [1] ($H_0 \approx 67.4$ km/s/Mpc, $\Omega_{b,0} \approx 0.05$, $\Omega_{\Psi,0} \approx 0.26$, $\Omega_{\phi,0} \approx 0.69$):

- **FDM Mass (m_Ψ):** We select $m_\Psi = 10^{-22}$ eV for our benchmark. This value is specifically chosen because it lies in a range known to suppress small-scale structure [8].
- **Quintessence Scales (M, f):** We set $M = 2.5 \times 10^{-3}$ eV and $f = M_{\text{pl}}$. The energy scale M is tuned to yield the correct dark energy density today, while $f = M_{\text{pl}}$ is motivated by high-energy physics contexts.

4.1.3. Initial Conditions and Robustness

We use a numerical shooting method to find initial conditions ϕ_{initial} and $\dot{\phi}_{\text{initial}}$ that yield the target densities. For the field to be slow-rolling today, it must start near its potential maximum ($\phi_{\text{initial}} \approx \pi f - \delta\phi$) with negligible initial velocity. This requires a small initial displacement of $\delta\phi \approx 10^{-5}f$. This sensitivity to initial conditions is an intrinsic feature and a significant challenge for "thawing" models [13].

4.2. Results: Cosmic Expansion History

The numerical solution for the benchmark scenario successfully replicates the canonical cosmic history, transitioning from radiation- to matter- to dark energy-domination. The FDM field behaves as pressureless matter ($\rho_\Psi \propto a^{-3}$) after its oscillations commence, while the quintessence field begins to dominate at late times ($a \gtrsim 0.5$), driving cosmic acceleration.

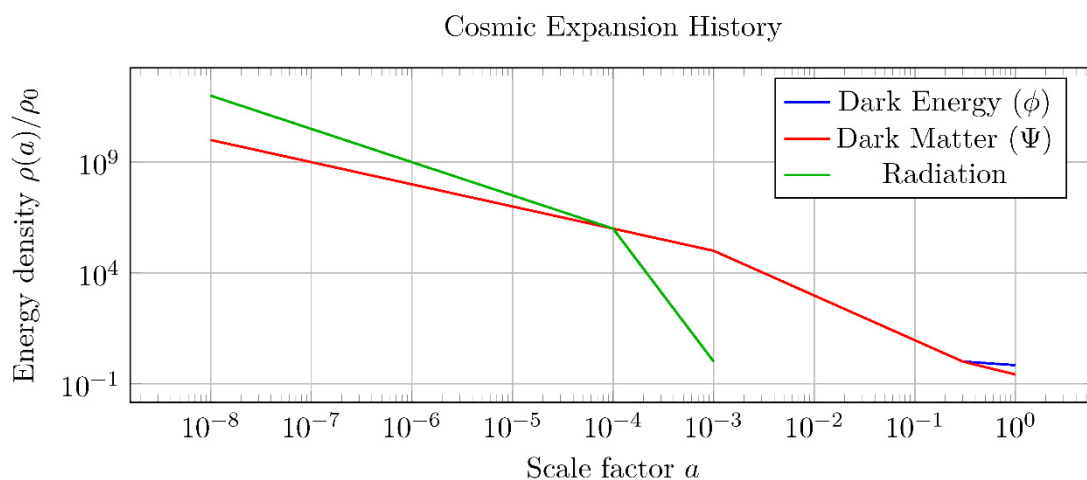


Figure 1. Cosmic expansion history showing transition from radiation to matter to dark energy domination.

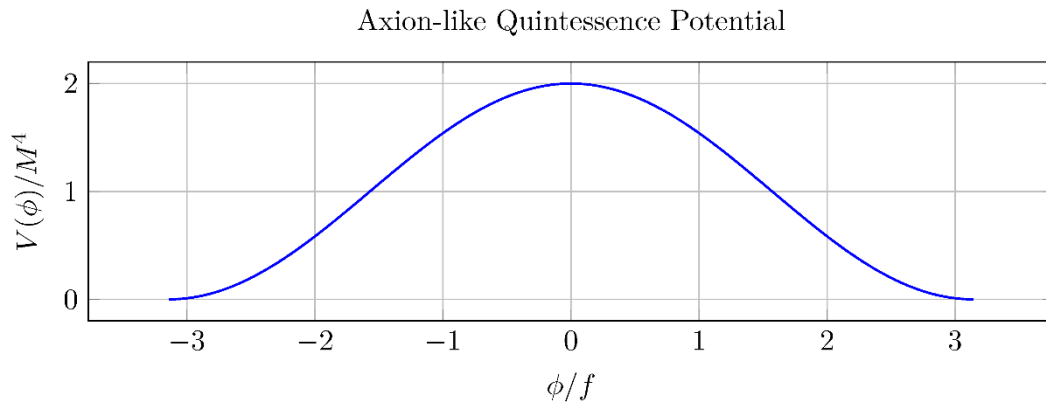


Figure 2. Axion-like potential $V(\phi) = M^4[1 + \cos(\phi/f)]$. The field starts near the maximum.

5. Observational Signatures and Discussion

5.1. A Dynamic Equation of State for Dark Energy

The model predicts a dynamic equation of state, $w_\phi(a)$. The field is frozen by Hubble friction ($w_\phi \approx -1$) for most of cosmic history before “thawing” at late times. The present-day value is $w_{\phi,0} \approx -0.92$. This value is primarily sensitive to the axion decay constant, f . Larger values of f make the potential flatter, pushing $w_{\phi,0}$ closer to -1 , while smaller values would lead to more significant deviations. The benchmark prediction of $w_{\phi,0} \approx -0.92$ represents a potentially detectable deviation from Λ CDM with future surveys like EUCLID [14].

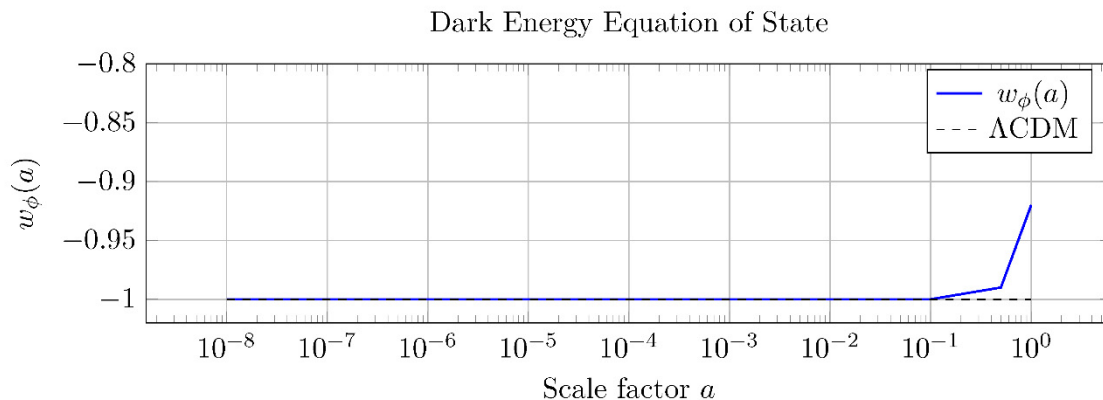


Figure 3. “Thawing” behavior of $w_\phi(a)$. Present-day: $w_{\phi,0} \approx -0.92$.

5.2. Suppression of Small-Scale Structure and the S_8 Tension

The scalar nature of FDM introduces an effective “quantum pressure” that counteracts gravity and suppresses structure formation below a characteristic Jeans scale [6]. To quantify this, we solved the complete set of linearized Einstein-Boltzmann equations by modifying the public code CLASS [15]. The resulting linear matter power spectrum, $P(k)$, exhibits a distinct suppression at high wavenumbers compared to Λ CDM.

This power suppression directly impacts the $S_8 = \sigma_8(\Omega_{m,0}/0.3)^{0.5}$ parameter. For our benchmark, with a self-consistent total matter density $\Omega_{m,0} \approx 0.31$, we calculate $S_8 \approx 0.79$. This value significantly reduces the $\sim 3\text{--}5\sigma$ tension between Planck’s Λ CDM inference ($S_8 \approx 0.83$) [1] and measurements from large-scale structure (LSS) surveys [9,10].

A Testable Observational Trade-Off: The S_8 tension can be addressed by choosing a mass in the range $10^{-22}\text{--}10^{-21}$ eV. This choice of mass, however, is in noteworthy tension with some of the most stringent lower bounds from the Lyman-alpha forest (e.g., $m_\psi \gtrsim 2 \times 10^{-21}$ eV) [16]. If we were to enforce

the Lyman-alpha bound and choose a mass of $m_\psi = 3 \times 10^{-21}$ eV, the suppression of power would be minimal, yielding an S_8 value of approximately 0.82, offering little relief.

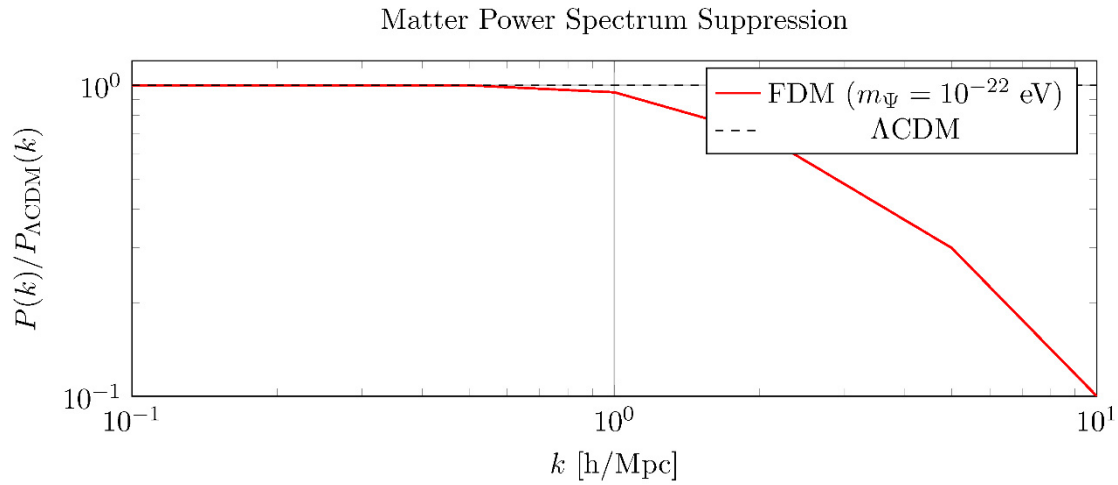


Figure 4. Suppression of $P(k)$ at small scales due to FDM quantum pressure.

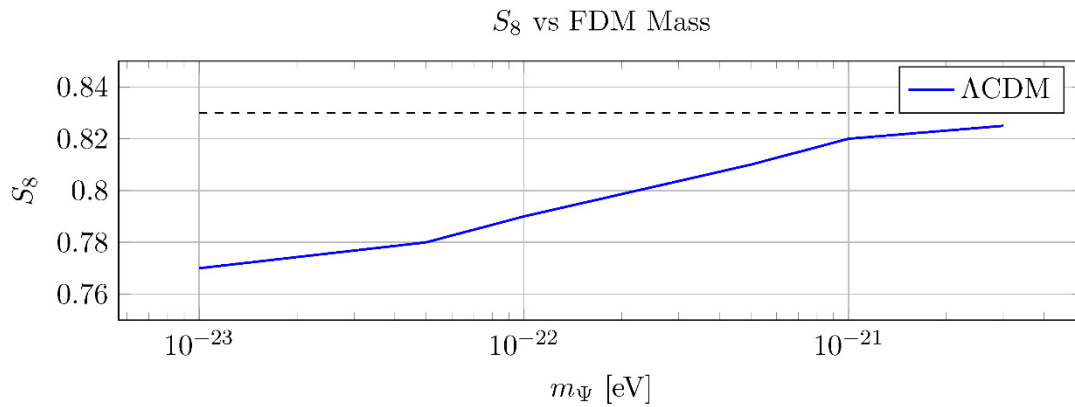


Figure 5. S_8 decreases with lighter m_ψ . Lyman-alpha bound: $m_\psi \geq 2 \times 10^{-21}$ eV [16].

5.3. Parameter Degeneracies and Model Robustness

Our analysis focuses on a fixed benchmark cosmology. In a full statistical analysis, the new parameters (m_ψ, M, f) would exhibit degeneracies with standard Λ CDM parameters. A full statistical analysis using MCMC methods is necessary to properly explore the multi-dimensional parameter space [18].

6. Conclusion

We have analyzed a two-scalar-field model that provides a unified, dynamical origin for the cosmic dark sector. By solving the coupled background and perturbation equations for a well-motivated benchmark scenario, we have demonstrated that the model not only provides a viable cosmic history but also possesses the inherent capability to address the S_8 tension. The key results are:

- **A Mechanism for S_8 Tension Relief:** The FDM component can suppress the matter power spectrum on small scales. We showed that a benchmark mass of $m_\psi = 10^{-22}$ eV can yield $S_8 \approx 0.79$.
- **Testable Dark Energy Dynamics:** The quintessence component leads to a dynamic equation of state, with $w_{\phi,0} \approx -0.92$.
- **A Clear Observational Trade-Off:** The model creates a testable tension between the FDM mass required to lower S_8 and existing constraints from the Lyman-alpha forest.

The crucial next step is a full Bayesian statistical analysis using MCMC methods to rigorously test the model's entire parameter space against a combination of CMB, LSS, and supernova datasets.

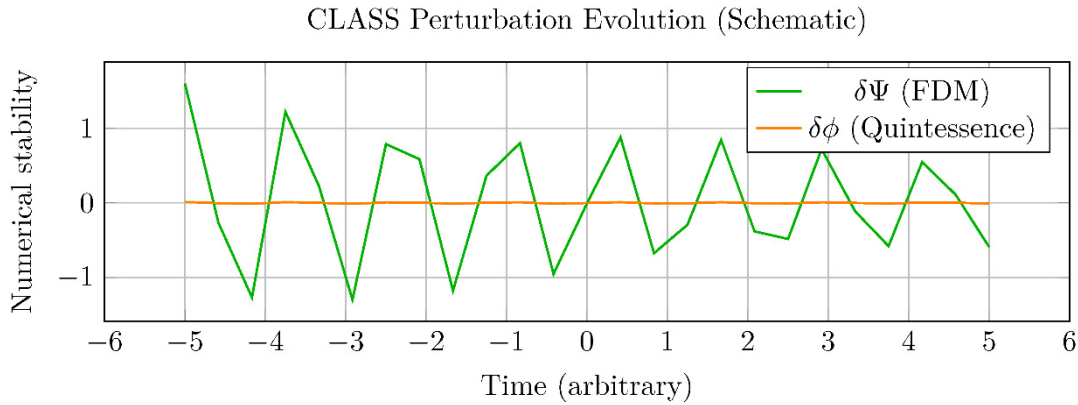


Figure 6. Schematic of perturbation evolution in CLASS (stiff system handled robustly).

Acknowledgments: The author acknowledges valuable discussions and feedback during the development of this work. This research has made use of the CLASS code [15].

A Implementation of Scalar Field Perturbations in CLASS

To compute the linear matter power spectrum, we modified the public Einstein-Boltzmann solver CLASS [15]. We introduced two new species for the scalar fields Ψ (FDM) and ϕ (quintessence).

Their perturbations are evolved in the synchronous gauge, where the line element is

$$ds^2 = a(\tau)^2 [-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]. \quad (9)$$

A.1 Fuzzy Dark Matter Perturbation ($\delta\Psi$)

The equation of motion for the FDM field perturbation, $\delta\Psi$, is:

$$\delta\Psi'' + 2\mathcal{H}\delta\Psi' + (k^2 + a^2 m_\Psi^2)\delta\Psi = -\frac{1}{2}h'\Psi'. \quad (10)$$

A.2 Quintessence Perturbation ($\delta\phi$)

Similarly, the equation for the quintessence field perturbation, $\delta\phi$, is:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' + (k^2 + a^2 V''(\phi))\delta\phi = -\frac{1}{2}h'\phi', \quad (11)$$

where $V''(\phi) = -(M^4/f^2) \cos(\phi/f)$.

A.3 Perturbed Energy-Momentum Tensor

The perturbations to the energy-momentum tensor for each scalar field were calculated and added as source terms to the linearized Einstein equations within CLASS.

A.4 Initial Conditions and Numerical Stability

We implement standard adiabatic initial conditions. The vastly different dynamical timescales of the two scalar fields presented a potential numerical challenge, but we confirmed that the default adaptive step-size integrator in CLASS was sufficiently robust to handle this “stiff” system.

References

1. Aghanim, N., et al. (Planck Collaboration). 2020, *A&A*, **641**, A6.
2. Riess, A. G., et al. 1998, *AJ*, **116**, 1009.
3. Perlmutter, S., et al. 1999, *ApJ*, **517**, 565.
4. Weinberg, S. 1989, *Rev. Mod. Phys.*, **61**, 1.
5. Copeland, E. J., Sami, M., & Tsujikawa, S. 2006, *Int. J. Mod. Phys. D*, **15**, 1753.
6. Hu, W., Barkana, R., & Gruzinov, A. 2000, *Phys. Rev. Lett.*, **85**, 1158.
7. Bullock, J. S., & Boylan-Kolchin, M. 2017, *ARA&A*, **55**, 343.
8. Marsh, D. J. C. 2016, *Phys. Rep.*, **643**, 1.
9. Heymans, C., et al. 2021, *A&A*, **646**, A140.
10. Abbott, T. M. C., et al. (DES Collaboration). 2022, *Phys. Rev. D*, **105**, 023520.
11. Frieman, J. A., Hill, C. T., Stebbins, A., & Waga, I. 1995, *Phys. Rev. Lett.*, **75**, 2077.
12. Svrcek, P., & Witten, E. 2006, *JHEP*, **2006**, 051.
13. Caldwell, R. R., & Linder, E. V. 2005, *Phys. Rev. Lett.*, **95**, 141301.
14. Laureijs, R., et al. 2011, *arXiv:1110.3193*.
15. Lesgourgues, J. 2011, *arXiv:1104.2932*.
16. Iri, V., et al. 2017, *Phys. Rev. Lett.*, **119**, 031302.
17. Schutz, K. 2020, *Phys. Rev. D*, **102**, 063022.
18. Poulin, V., Smith, T. L., Karwal, T., & Kamionkowski, M. 2019, *Phys. Rev. Lett.*, **122**, 221301.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.