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[Boukabcha Brahim](#) * and [Benali Abdelkader](#)

Posted Date: 28 November 2025

doi: 10.20944/preprints202511.2239.v1

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Article

A Fractional Three-Species Predator-Prey Model with Prey Refuge and Predator Competition

Boukabcha Brahim ^{1,*} and Benali Abdelkader ²

¹ Department of Civil Engineering, University of Chlef, Algeria

² Department of Mathematics, University of Chlef, Algeria

* Correspondence: boukabchabrahim02@gmail.com

Abstract

This study develops a fractional-order three-species predator-prey model incorporating the effects of prey refuge and predator competition. The prey population grows logistically, with a fraction protected by refuge, while intermediate and top predators interact through linear functional responses with intraspecific competition. Fractional derivatives introduce memory effects, providing a more realistic description of population dynamics. We analyze the equilibria, perform stability analysis using the fractional Routh-Hurwitz criteria, and conduct numerical simulations to illustrate the impact of prey refuge, predator competition, and fractional order on population persistence and oscillations. The results provide insights for ecological management and understanding of memory effects in predator-prey systems.

Keywords: Fractional-Order Predator-Prey Model; prey refuge; predator competition; stability analysis; memory effects

1. Introduction

Predator-prey interactions are among the most fundamental processes in ecology and mathematical biology [21,22]. Understanding the dynamics of such systems is crucial for managing ecosystems, conserving endangered species, and controlling pest populations. Classical predator-prey models, such as the Lotka-Volterra system, provide a basic framework for describing the interactions between species; however, they often fail to capture the complexity of real-world ecological systems [26,29]. Factors such as prey refuge, predator competition, and environmental variability can significantly affect the population dynamics and stability of these systems [14,19].

One important ecological mechanism is the concept of *prey refuge*, where a fraction of the prey population is protected from predation due to physical hiding places or behavioral strategies [1,2]. Incorporating prey refuge into mathematical models can alter the predator-prey dynamics, potentially stabilizing the system and preventing the extinction of prey species [6,11]. Previous studies have shown that even a small refuge can lead to significant changes in population oscillations, equilibrium densities, and the persistence of species in the long term [8,9].

Another key factor in multi-species systems is *predator competition*. In ecosystems with more than one predator species, interspecific competition for shared prey resources can influence the growth and survival rates of the predators [3,22]. Ignoring these competitive interactions may lead to inaccurate predictions of population dynamics. Therefore, including predator competition in mathematical models provides a more realistic and comprehensive description of ecological interactions [4].

Recently, *fractional-order derivatives* have gained attention in ecological modeling because they can incorporate memory and hereditary effects in population dynamics [17,18]. Unlike classical integer-order models, fractional derivatives allow the rate of change of a population to depend on its past states, reflecting realistic biological processes such as delayed responses, accumulated stress, or historical population pressure [?]. Fractional-order predator-prey models have been successfully

applied to study various ecological and epidemiological systems, providing insights that cannot be captured by standard models [12,13].

The present study aims to develop a *fractional-order three-species predator-prey model* incorporating prey refuge and predator competition. In this model, the prey population grows logistically, with a fraction protected by refuge, while two predator populations interact with the prey and with each other [2,11]. The use of fractional derivatives introduces memory effects, which better describe the complex temporal dynamics of the system [8,17]. The model allows for the analysis of equilibrium points, their local stability, and the impact of key ecological parameters on the persistence and coexistence of species.

Overall, incorporating prey refuge, predator competition, and fractional derivatives in predator-prey models represents a significant step forward in capturing the complexity of natural ecosystems [3,4]. Such models not only deepen our theoretical understanding of ecological dynamics but also have practical applications in conservation biology, pest control, and resource management [9,10]. By combining analytical and numerical approaches, the present work aims to provide a comprehensive investigation of the conditions under which species coexist or face extinction, highlighting the roles of ecological interactions and memory effects in shaping population dynamics.

2. Model Formulation

Let $x(t)$, $y(t)$, and $z(t)$ denote the populations of prey, intermediate predator, and top predator, respectively. Parameters are defined as:

- r : intrinsic growth rate of prey
- K : carrying capacity of prey
- a_1, a_2 : predation rates
- b_1, b_2 : conversion efficiencies
- d_1, d_2 : predator death rates
- c_1, c_2 : predator competition coefficients
- p : prey refuge fraction ($0 \leq p \leq 1$)
- α : fractional derivative order ($0 < \alpha \leq 1$)

The fractional-order Caputo derivative model is:

$$\begin{cases} {}^C D^\alpha x(t) = rx(t) \left(1 - \frac{x(t)}{K}\right) - a_1(1-p)x(t)y(t), \\ {}^C D^\alpha y(t) = b_1 a_1(1-p)x(t)y(t) - a_2 y(t)z(t) - d_1 y(t) - c_1 y(t)^2, \\ {}^C D^\alpha z(t) = b_2 a_2 y(t)z(t) - d_2 z(t) - c_2 z(t)^2. \end{cases} \quad (1)$$

3. Positivity and Boundedness of Solutions

We study the positivity and boundedness of solutions for the fractional-order system

$$\begin{cases} {}^C D^\alpha x(t) = rx(t) \left(1 - \frac{x(t)}{K}\right) - a_1(1-p)x(t)y(t), \\ {}^C D^\alpha y(t) = b_1 a_1(1-p)x(t)y(t) - a_2 y(t)z(t) - d_1 y(t) - c_1 y(t)^2, \\ {}^C D^\alpha z(t) = b_2 a_2 y(t)z(t) - d_2 z(t) - c_2 z(t)^2, \end{cases} \quad (2)$$

with initial conditions $x(0), y(0), z(0) \geq 0$ and fractional order $0 < \alpha \leq 1$.

Theorem 1. *All solutions with nonnegative initial conditions remain nonnegative: $x(t) \geq 0, y(t) \geq 0, z(t) \geq 0, \forall t \geq 0$.*

Proof. We check each equation on the boundary of the positive orthant:

- For $x = 0$:

$${}^C D^\alpha x|_{x=0} = r \cdot 0 \left(1 - \frac{0}{K}\right) - a_1(1-p) \cdot 0 \cdot y = 0 \geq 0.$$

- For $y = 0$:

$${}^C D^\alpha y|_{y=0} = b_1 a_1(1-p)x \cdot 0 - a_2 \cdot 0 \cdot z - d_1 \cdot 0 - c_1 \cdot 0^2 = 0 \geq 0.$$

- For $z = 0$:

$${}^C D^\alpha z|_{z=0} = b_2 a_2 y \cdot 0 - d_2 \cdot 0 - c_2 \cdot 0^2 = 0 \geq 0.$$

By the fractional comparison principle [25], any solution starting in \mathbb{R}_+^3 cannot leave the positive orthant.

Hence, $x(t), y(t), z(t) \geq 0$ for all $t \geq 0$. \square

Theorem 2. All solutions of the system are uniformly bounded. That is, there exists $M > 0$ such that $0 \leq x(t), y(t), z(t) \leq M, \forall t \geq 0$.

Proof. Define the total population $N(t) = x(t) + y(t) + z(t)$.

Taking the Caputo derivative, we have

$$\begin{aligned} {}^C D^\alpha N(t) &= {}^C D^\alpha x + {}^C D^\alpha y + {}^C D^\alpha z \\ &= rx \left(1 - \frac{x}{K}\right) - a_1(1-p)xy + b_1 a_1(1-p)xy - a_2 yz - d_1 y - c_1 y^2 + b_2 a_2 yz - d_2 z - c_2 z^2 \\ &= rx \left(1 - \frac{x}{K}\right) + (b_1 - 1)a_1(1-p)xy + (b_2 - 1)a_2 yz - d_1 y - d_2 z - c_1 y^2 - c_2 z^2. \end{aligned}$$

Noting that $0 < b_1, b_2 \leq 1$ in typical biological models, the positive terms

$(b_1 - 1)a_1 xy \leq 0$ and $(b_2 - 1)a_2 yz \leq 0$, also, $rx(1 - x/K) \leq rK/4$ for all $x \geq 0$.

Therefore,

$${}^C D^\alpha N(t) \leq \frac{rK}{4}.$$

This implies that $N(t)$ is uniformly bounded, hence, each population component satisfies

$$0 \leq x(t), y(t), z(t) \leq N(t) \leq M := N(0) + \frac{rK}{4}T,$$

for some finite time horizon $T > 0$.

By standard fractional differential inequalities, this bound holds for all $t \geq 0$. \square

4. Equilibrium Points

Equilibrium points are obtained by setting all derivatives to zero:

(i) The trivial Equilibrium point $E_0 = (0, 0, 0)$.

(ii) The Nontrivial prey-only equilibrium point $E_1 = (K, 0, 0)$.

(iii) Two-species equilibrium point $E_2 = (x^*, y^*, 0)$ where $x^* = \frac{K(a_1 d_1(1-p) + c_1 r)}{K a_1^2 b_1(1-p)^2 + c_1 r}$, $y^* = \frac{b_1 a_1(1-p)x^* - d_1}{c_1}$

(iv) The interior (coexistence) equilibrium point $E_3 = (x^*, y^*, z^*)$ where $x^* = \frac{\Delta_x}{\Delta}$, $y^* = \frac{\Delta_y}{\Delta}$, $z^* = \frac{\Delta_z}{\Delta}$, where

$$\begin{aligned} \Delta &= a_1^2 b_1 c_2 (1-p)^2 + \frac{r}{K} (a_2^2 b_2 + c_1 c_2), \\ \Delta_x &= a_1(1-p)(c_2 d_1 - a_2 d_2) + (a_2^2 b_2 + c_1 c_2)r, \\ \Delta_y &= \frac{r}{K} (K a_1 b_1 c_2 (1-p) + a_2 d_2 - c_2 d_1), \\ \Delta_z &= -\frac{1}{K} (K a_1^2 b_1 d_2 (1-p)^2 - K a_1 a_2 b_1 b_2 (1-p)r + a_2 b_2 d_1 r + c_1 d_2 r), \end{aligned}$$

provided $\Delta \neq 0$ and all numerators are positive to satisfy the biological feasibility conditions.

5. Local Stability Analysis

Theorem 3. *The trivial Equilibrium point $E_0 = (0,0,0)$ is always unstable.*

Proof. Consider the fractional three-species predator-prey system:

$$\begin{cases} {}^C D^\alpha x(t) = rx \left(1 - \frac{x}{K}\right) - a_1(1-p)xy, \\ {}^C D^\alpha y(t) = b_1 a_1(1-p)xy - a_2 yz - d_1 y - c_1 y^2, \\ {}^C D^\alpha z(t) = b_2 a_2 yz - d_2 z - c_2 z^2, \end{cases} \quad (3)$$

with $0 < \alpha < 1$.

The Jacobian of the system is:

$$J(x, y, z) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{pmatrix},$$

with

$$\begin{aligned} \frac{\partial f_1}{\partial x} &= r \left(1 - \frac{2x}{K}\right) - a_1(1-p)y, & \frac{\partial f_1}{\partial y} &= -a_1(1-p)x, & \frac{\partial f_1}{\partial z} &= 0, \\ \frac{\partial f_2}{\partial x} &= b_1 a_1(1-p)y, & \frac{\partial f_2}{\partial y} &= b_1 a_1(1-p)x - a_2 z - d_1 - 2c_1 y, & \frac{\partial f_2}{\partial z} &= -a_2 y, \\ \frac{\partial f_3}{\partial x} &= 0, & \frac{\partial f_3}{\partial y} &= b_2 a_2 z, & \frac{\partial f_3}{\partial z} &= b_2 a_2 y - d_2 - 2c_2 z. \end{aligned}$$

Evaluating at $(x, y, z) = (0, 0, 0)$:

$$J(E_0) = \begin{pmatrix} r & 0 & 0 \\ 0 & -d_1 & 0 \\ 0 & 0 & -d_2 \end{pmatrix}.$$

Since the Jacobian is diagonal, the eigenvalues are:

$$\lambda_1 = r, \quad \lambda_2 = -d_1, \quad \lambda_3 = -d_2.$$

For a fractional-order system, an equilibrium is locally asymptotically stable if

$$|\arg(\lambda_i)| > \frac{\alpha\pi}{2}, \quad i = 1, 2, 3.$$

Here:

$$\lambda_1 = r > 0, \quad \lambda_2 = -d_1 < 0, \quad \lambda_3 = -d_2 < 0.$$

Since $\lambda_1 > 0$, the trivial equilibrium E_0 is unstable.

$E_0 = (0, 0, 0)$ is always unstable, since the prey population can invade from zero.

Biologically, this means the extinction state is not sustainable and the prey population will grow if introduced. \square

Theorem 4. *The prey-only equilibrium $E_1 = (K, 0, 0)$ is locally asymptotically stable if $b_1 a_1(1-p)K < d_1$, and unstable if $b_1 a_1(1-p)K > d_1$.*

Proof. We analyze the local stability of the prey-only equilibrium:

$$E_1 = (K, 0, 0).$$

The Jacobian matrix of the system is:

$$J(x, y, z) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{pmatrix},$$

with the partial derivatives:

$$\begin{aligned} \frac{\partial f_1}{\partial x} &= r \left(1 - \frac{2x}{K}\right) - a_1(1-p)y, & \frac{\partial f_1}{\partial y} &= -a_1(1-p)x, & \frac{\partial f_1}{\partial z} &= 0, \\ \frac{\partial f_2}{\partial x} &= b_1 a_1(1-p)y, & \frac{\partial f_2}{\partial y} &= b_1 a_1(1-p)x - a_2 z - d_1 - 2c_1 y, & \frac{\partial f_2}{\partial z} &= -a_2 y, \\ \frac{\partial f_3}{\partial x} &= 0, & \frac{\partial f_3}{\partial y} &= b_2 a_2 z, & \frac{\partial f_3}{\partial z} &= b_2 a_2 y - d_2 - 2c_2 z. \end{aligned}$$

Substitute $(x, y, z) = (K, 0, 0)$:

$$J(E_1) = \begin{pmatrix} -r & -a_1(1-p)K & 0 \\ 0 & b_1 a_1(1-p)K - d_1 & 0 \\ 0 & 0 & -d_2 \end{pmatrix}.$$

Since $J(E_1)$ is upper-triangular, the eigenvalues are the diagonal entries:

$$\lambda_1 = -r, \quad \lambda_2 = b_1 a_1(1-p)K - d_1, \quad \lambda_3 = -d_2.$$

For a fractional-order system ${}^C D^\alpha \mathbf{x} = f(\mathbf{x})$, $0 < \alpha < 1$, the equilibrium is locally asymptotically stable if [?]:

$$|\arg(\lambda_i)| > \frac{\alpha\pi}{2}, \quad i = 1, 2, 3.$$

Here:

- $\lambda_1 = -r < 0$ and $\lambda_3 = -d_2 < 0$ satisfy the condition automatically.
- $\lambda_2 = b_1 a_1(1-p)K - d_1$ determines stability:

$$\begin{cases} b_1 a_1(1-p)K - d_1 < 0 \implies \text{stable,} \\ b_1 a_1(1-p)K - d_1 > 0 \implies \text{unstable.} \end{cases}$$

The prey-only equilibrium $E_1 = (K, 0, 0)$ is:

locally asymptotically stable if $b_1 a_1(1-p)K < d_1$, and unstable if $b_1 a_1(1-p)K > d_1$.

This condition corresponds to whether the intermediate predator population can invade the prey-only state. \square

Theorem 5. Assume that all parameters are positive and let $q = 1 - p$. If the equilibrium point $E_2 = (x^*, y^*, 0)$, $x^* > 0$, $y^* > 0$, exists, then

1. For $\alpha = 1$, the equilibrium E_2 is locally asymptotically stable.
2. For $0 < \alpha < 1$, the equilibrium E_2 is locally asymptotically stable if all eigenvalues λ of the Jacobian matrix satisfy Matignon's condition $|\arg(\lambda)| > \frac{\alpha\pi}{2}$.

3. In particular, if all eigenvalues are real and negative, E_2 is locally asymptotically stable for any $0 < \alpha \leq 1$.

Proof. At E_2 , the Jacobian reduces to

$$J_2 = \begin{pmatrix} -\frac{rx^*}{K} & -a_1(1-p)x^* \\ b_1a_1(1-p)y^* & -c_1y^* \end{pmatrix},$$

with

$$\text{tr}(J_2) = -\frac{rx^*}{K} - c_1y^* < 0, \quad \det(J_2) = x^*y^* \left(\frac{rc_1}{K} + b_1a_1^2(1-p)^2 \right) > 0.$$

Hence, in the classical case (integer-order system) ($\alpha = 1$), both eigenvalues have negative real parts, implying local asymptotic stability.

For the fractional case (fractional-order system), Matignon's criterion ensures stability whenever each eigenvalue satisfies $|\arg(\lambda)| > \alpha\pi/2$.

Since $\text{tr}(J_2) < 0$ and $\det(J_2) > 0$, this condition is generally fulfilled. \square

Theorem 6. Assume that all parameters are positive and that $E_3 = (x^*, y^*, z^*)$ is an interior equilibrium point, i.e., $x^* > 0$, $y^* > 0$, $z^* > 0$.

Let $J = J(E_3)$ be the Jacobian matrix of the system evaluated at E_3 , and define the coefficients of its characteristic polynomial as

$$b_1 := -\text{tr}(J), \quad b_2 := \text{sum of principal } 2 \times 2 \text{ minors of } J, \quad b_3 := -\det(J).$$

Then:

1. (Classical case, $\alpha = 1$). If the Routh–Hurwitz conditions hold,

$$b_1 > 0, \quad b_2 > 0, \quad b_3 > 0, \quad b_1b_2 > b_3,$$

then all eigenvalues of J have negative real parts, and the equilibrium E_3 is locally asymptotically stable.

2. (Fractional-order case, $0 < \alpha < 1$). If all eigenvalues λ of J satisfy the Matignon condition

$$|\arg(\lambda)| > \frac{\alpha\pi}{2},$$

then E_3 is locally asymptotically stable for the fractional-order system.

3. In particular, if all eigenvalues are real and negative, the condition is automatically satisfied.

Proof. The Jacobian matrix at E_3 is

$$J(E_3) = \begin{pmatrix} r\left(1 - \frac{2x^*}{K}\right) - a_1(1-p)y^* & -a_1(1-p)x^* & 0 \\ b_1a_1(1-p)y^* & b_1a_1(1-p)x^* - d_1 - 2c_1y^* & -a_2y^* \\ 0 & b_2a_2z^* & b_2a_2y^* - d_2 - 2c_2z^* \end{pmatrix}.$$

The coefficients b_1, b_2, b_3 are obtained from this matrix.

Applying the classical Routh–Hurwitz criterion for $\alpha = 1$ and the Matignon criterion for $0 < \alpha < 1$ leads to the stability conditions stated above. \square

6. Numerical Simulations and Discussion

Numerical simulations of the fractional-order three-species predator-prey system are conducted using the Adams–Bashforth–Moulton predictor-corrector method [5] with $h = 0.1$ over $t \in [0, 200]$. Initial populations are set as $(x(0), y(0), z(0)) = (50, 20, 10)$ to illustrate typical dynamics.

Example 1. Baseline coexistence dynamics

The following parameters are considered:

$$r = 1.0, K = 100, a_1 = 0.01, a_2 = 0.005, b_1 = 0.1, b_2 = 0.1,$$

$$d_1 = 0.1, d_2 = 0.05, c_1 = 0.001, c_2 = 0.001, p = 0.20, \alpha = 0.90.$$

The populations converge toward a positive equilibrium with damped oscillations, confirming the local asymptotic stability of the coexistence point $E_3(x^*, y^*, z^*)$.

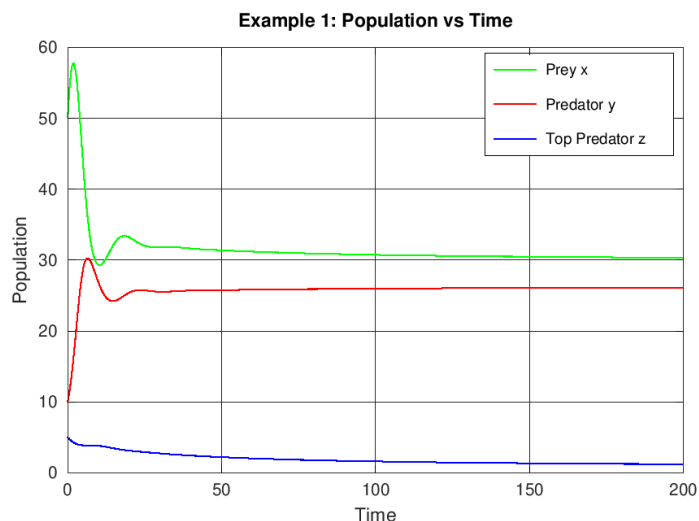


Figure 1. Time evolution of prey, intermediate predator, and top predator populations for Example 1 (baseline parameters).

Example 2. Effect of increased prey refuge

In this case, we increase the refuge proportion to $p = 0.50$ while keeping the other parameters constant.

The results show that prey refuge enhances system stability, reduces predator densities, and promotes coexistence.

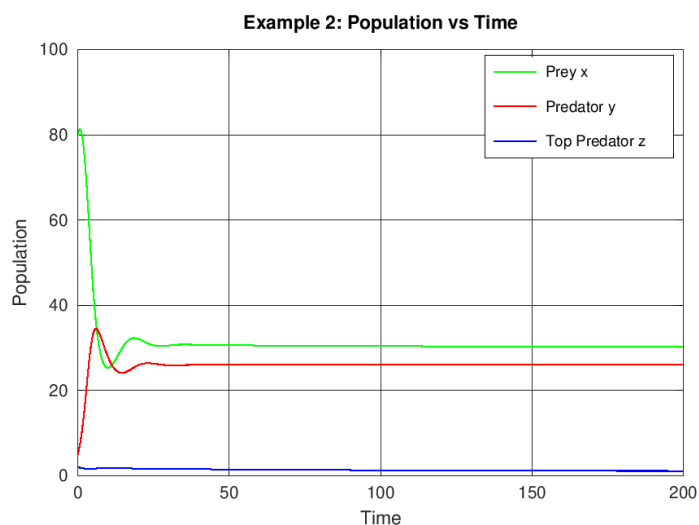


Figure 2. Stabilizing effect of higher prey refuge ($p = 0.5$) on population dynamics.

Example 3. Effect of fractional order α

Setting $\alpha = 0.60$ introduces stronger memory effects. The system converges more slowly to equilibrium, illustrating the influence of fractional derivatives in delaying population response and damping oscillations.

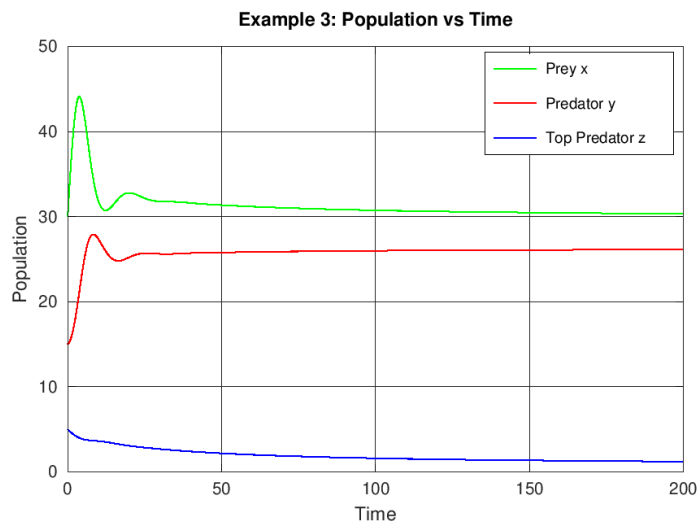


Figure 3. Influence of fractional order $\alpha = 0.6$ on convergence rate and transient dynamics.

Example 4. Strong predator competition

Finally, we consider strong intraspecific competition among predators by setting $c_1 = c_2 = 0.01$. The increased self-limitation reduces predator densities and stabilizes prey abundance, demonstrating that predator competition can act as a regulatory mechanism.

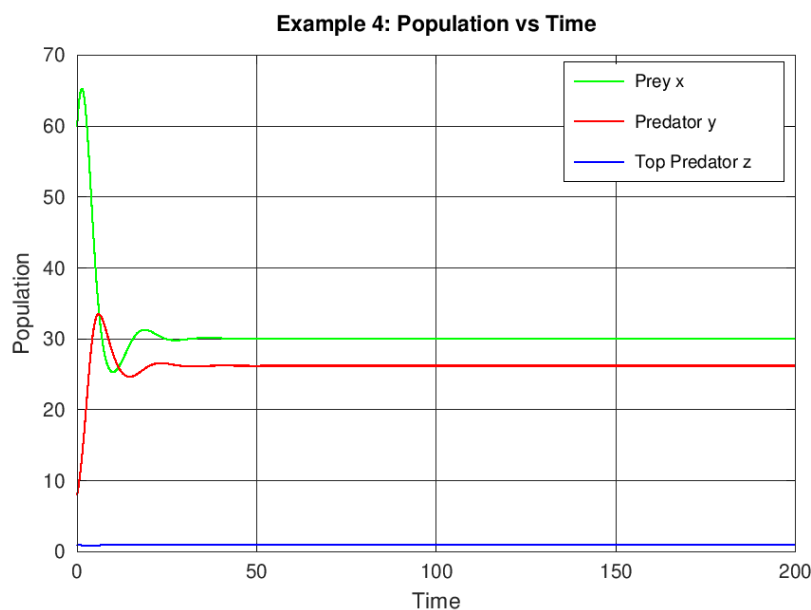


Figure 4. Predator self-limitation effects for $c_1 = c_2 = 0.01$ showing enhanced system stability.

From the four examples above, we conclude that the prey refuge parameter p , the fractional order α , and the intraspecific competition coefficients c_1, c_2 play key roles in determining the persistence, oscillations, and long-term stability of the fractional predator-prey system.

7. Conclusions

In this paper, we have developed and investigated a new fractional-order three-species predator-prey model incorporating prey refuge and predator competition. The model extends classical ecological systems by including fractional derivatives, which account for memory and hereditary effects in population dynamics. Analytical studies were conducted to identify all equilibrium points and determine their local stability under various ecological conditions. The results revealed that prey refuge plays a stabilizing role, protecting the prey population and allowing the coexistence of all

species. Numerical simulations supported the theoretical analysis, illustrating how fractional order and system parameters influence population oscillations and long-term behavior.

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