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Quantum Field Theory of Dirac Magnetic Monopoles and Their Interpretation as Cold Dark Matter WIMPs

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Abstract

Dirac magnetic monopoles are hypothetical elementary particles. By assuming their existence one can explain the quantization of electric charge, the August Kundt experiment, and the conservation of baryon and lepton number. Here I present a new nomenclature where I redefine isospin and hypercharge. By doing so I explain baryon and lepton number conservation as an effect of the electric-magnetic duality and the $U(1) \times U(1)$ gauge symmetry of quantum electromagnetodynamics. By using this method I predict the quantum numbers of an octet of magnetic monopoles. Another surprising result is that both leptons and quarks have nonzero magnetic isospin, a new quantum number. Moreover I show that Dirac magnetic monopoles can form low-mass bound states which are analogous to mesons, baryons, atoms, and molecules. I point out that these bound states could be the major component of cold dark matter. The PandaX Collaboration reported an excess of 4.3 events above the background in the PandaX-4T experiment. The best fit for this excess was obtained for a WIMP mass of 6 GeV. Here I show that both the mass and the interaction cross-section are compatible with bound states of Dirac magnetic monopoles.

Keywords: Einstein-Cartan theory; energy-momentum tensor; Lorentz transformations; gauge invariance; quantum field theories; magnetic monopoles; running coupling constant; weak energy condition; isospin; hypercharge; baryon number; lepton number; dyons; cold dark matter; dark matter; WIMP; weakly interacting massive particles

PACS: 11.30.Hv Flavor symmetries; 11.30.Ly Other internal and higher symmetries; 12.15.Cc Extensions of gauge or Higgs sector; 14.80.Hv Magnetic monopoles; 98.80.-k Cosmology

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1. Introduction

This paper presents radically new physical ideas which are far beyond the mainstream. In order to be understandable to students and physicists, I will start at a very elementary level. This paper is organized as follows. I will start with the two- and three-dimensional rotations of non-relativistic classical mechanics (ch. 2.2). I will continue with the spin rotations of non-relativistic quantum mechanics (ch. 2.3). This will be followed by the global Lorentz transformations of special relativity (ch. 2.4). This will be combined to the spin rotations of relativistic quantum mechanics (ch. 2.5). By analogy, this is applied to the local gauge transformations of quantum electrodynamics, quantum flavordynamics, and quantum chromodynamics (ch. 2.6). This is followed by the local Lorentz transformations of general relativity (ch. 2.7). I show that general relativity is not a generalization of special relativity (ch. 2.8). Moreover general relativity requires a symmetric energy-momentum tensor in contrast to quantum field theory which requires an asymmetric energy-momentum tensor. I show that both conditions can be satisfied if general relativity is extended to a (quantum) Einstein-Cartan theory which requires torsion as a gauge field and spin as a charge (ch. 2.8). This is analogous to isospin which is a pure quantum number in quantum mechanics, but the source of a gauge field in quantum field theory (ch. 2.9).

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Now I continue with radically new physical ideas. The reasons are that the standard theory of particle physics can explain neither the quantization of electric charge nor the conservation of baryon and lepton number. I will show that quantum field theories which include Dirac magnetic monopoles can explain both. I start with a simple proof of the famous Dirac quantization condition (ch. 3.2). I continue with a formulation of the quantum field theory of electric and magnetic charges (ideas in ch. 3.3, formalism in ch. 3.4). I suggest a desktop experiment to test this theory (ch. 3.5). By using the weak energy condition I show that the magnetic coupling constant and the fine-structure constant become of order unity at the Planck scale (ch. 3.7). This argumentation suggests that elementary magnetic monopoles have rest masses of the order of the Planck mass (ch. 3.7).

I will continue by generalizing the electric-magnetic duality for the weak and strong interactions (ch. 4). I regard strong and weak isospin (ch. 4.1) and argue why I am unhappy with these textbook definitions (ch. 4.2). I make a new definition of both isospin and hypercharge (ch. 4.3). By doing so I find that both baryon and lepton number are associated with hypercharge (ch. 4.4). I recall the electric-magnetic duality of ch. 3.3 (ch. 4.5) and extend it to the weak and strong interactions (ch. 4.6). By doing so I can explain the conservation of both baryon and lepton number (ch. 4.7). This procedure allows the determination of the quantum numbers of both the fermionic and bosonic elementary magnetic monopoles (ch. 4.8).

Leptons and quarks are bound to mesons, baryons, and atoms. I argue that chromomagnetic magnetic monopoles (called hanselons) are bound to analogous states (ch. 5.2), and I define their properties (ch. 5.3). Atoms of Dirac magnetic monopoles which consist of one gretelon and three hanselons could be the WIMP constituents of cold dark matter, because this can explain why there are much more baryons than anti-baryons in the universe, although the baryon number of the universe should be zero (ch. 5.4). By using this argumentation, I calculate the rest mass of the lightest bound state of Dirac magnetic monopoles (ch. 6.2) and its interaction cross-section with conventional matter (ch. 6.3). I argue that these WIMP bound states of Dirac magnetic monopoles have already been observed by the PandaX experiment for the search for dark matter WIMPs (ch. 6.5). I finish by pointing out that bound states of Dirac magnetic monopoles can be easily distinguished from other WIMP and dark matter candidates by examining their quantum numbers (ch. 6.6).

2. Standard Theory of Particle Physics

2.1. Introduction

I will present my argumentation in a didactic way. So I will start at a very elementary level. If not denoted otherwise then Latin indices run from 1 to 3, Greek indices run from 0 to 3. I will use the Einstein summation convention, where it is summed over all indices which appear twice. Moreover I will use the natural units

$$\hbar = c = \varepsilon_0 = 1 \tag{1}$$

where $\hbar = h/2\pi$ denotes the reduced Planck constant, c the speed of light, and ε_0 the electric field constant. Inner indices of matrices will be dropped.

I examine the groups which underly classical mechanics, non-relativistic quantum mechanics, special relativity, relativistic quantum mechanics, quantum electrodynamics, quantum flavourdynamics, quantum chromodynamics, and general relativity. This examination includes the rotations $SO(2)$ and $SO(3)$, the Pauli algebra, the Lorentz transformations, the Dirac algebra, and the $U(1)$, $SU(2)$, and $SU(3)$ gauge transformations. I argue that general relativity must be generalized to Einstein-Cartan theory, so that Dirac spinors can be described within the framework of gravitation theory.

2.2. Classical Mechanics

The space of classical mechanics is described by the three-dimensional Euclidian space. The scalar product of the three-vectors a_i and b_i is given by

$$a \cdot b = \delta_{ij} a_i b_j \tag{2}$$

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where
$$\delta_{ij} = \text{diag}\,(1,1,1) \tag{3}$$
denotes the Kronecker symbol. The vector product is given by
$$(a \times b)_k = \varepsilon_{ijk} a_i b_j \tag{4}$$
where ε_{ijk} denotes the totally anti-symmetric Levi-Civita symbol. The square of the infinitesimal line element is given by
$$ds^2 = \delta_{ij} dx_i dx_j \tag{5}$$
where dx_i denotes the infinitesimal coordinate difference of the two space-points x_i and y_i
$$dx_i = \lim_{y_i \to x_i} (y_i - x_i) \tag{6}$$
A rotation R around the rotation angle φ in the two-dimensional subspace is described by the orthogonal rotation group $SO(2)$
$$\begin{aligned} R &= \exp(-i\varphi D) \\ &= \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \in SO(2) \end{aligned} \tag{7}$$
where, because of the Euler equation
$$e^{-i\varphi} = \cos\varphi - i\sin\varphi \tag{8}$$
the generator of the rotation is
$$D = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{9}$$
A rotation R around the rotation angle φ_i in the three-dimensional space is described by the rotation group $SO(3)$
$$R = \exp(-i\varphi_i D_i) \in SO(3) \tag{10}$$
where the generators D_i of the rotation satisfy the commutator relation
$$[D_i, D_j] = i\varepsilon_{ijk} D_k \tag{11}$$
A representation of the generators of the $SO(3)$ group is
$$D_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{12}$$
$$D_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \tag{13}$$
$$D_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{14}$$
Note that the number of generators of the orthogonal groups is given by
$$\dim SO(n) = n(n-1)/2 \tag{15}$$

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and that the generators of $SO(n)$ are Hermitean.

2.3. Non-Relativistic Quantum Mechanics

Classical angular momentum is continuous. In quantum physics orbital angular momentum is quantized in units of $\hbar$ and intrinsic spin is quantized in units of $\hbar/2$. Intrinsic spin is described by the unitary group $SU(2)$. Its generators are the three Pauli matrices σ_i. An often used representation of the Pauli matrices is

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{16}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{17}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{18}$$

The commutator of the group $SU(2)$ is given by

$$[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k \tag{19}$$

The groups $SU(2)$ and $SO(3)$ are local isomorphic for angles $0 \leq \varphi < 2\pi$ where the group $SU(2)$ is covered by the group $SO(3)$. Note also that the commutators eq. (11) and eq. (19) differ by a factor of two. The anti-commutator is

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} \tag{20}$$

The multiplication of two three-vectors a_i and b_i is given by

$$\begin{aligned}(\sigma \cdot a)(\sigma \cdot b) &= (\sigma_i a_i)(\sigma_j b_j) \\ &= (\delta_{ij} + i\varepsilon_{ijk}\sigma_k)a_i b_j \\ &= a \cdot b + i\sigma \cdot (a \times b)\end{aligned} \tag{21}$$

According to non-relativistic quantum mechanics both spin and isospin are invariant under global transformations of the group $SU(2)$. If Ψ denotes a two-component Pauli-spinor, φ_i the three-component rotation angle vector, σ_i the Pauli matrices, and x a space-time point, then

$$\Psi\prime(x) = \exp(-i\varphi_i\sigma_i/2)\Psi(x) \tag{22}$$

If Ψ denotes a two-component iso-spinor, φ_i the three-component phase vector, and τ_i the Pauli matrices, then

$$\Psi\prime(x) = \exp(-i\varphi_i\tau_i/2)\Psi(x) \tag{23}$$

2.4. Special Relativity

The special theory of relativity is invariant under the semi-simple Poincare group. The parameters of its translational part are time and three-position. The parameters of its rotational part are the rotation angle three-vector and the three-component Lorentz boost.

The scalar product of the two four-vectors a^μ and b^μ is given by

$$a \cdot b = g_{\mu\nu}a^\mu b^\nu \tag{24}$$

where $g_{\mu\nu}$ denotes the metric tensor. It is

$$g^{\mu\nu} = g_{\mu\nu} \tag{25}$$

$$g^\mu{}_\nu = g_\nu{}^\mu = \delta^\mu_\nu = \text{diag}\,(1,1,1,1) \tag{26}$$

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In Minkowski coordinates the metric tensor is represented by

$$g_{\mu\nu} = \text{diag}\,(1, -1, -1, -1) \tag{27}$$

The square of the infinitesimal line element is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{28}$$

where dx^μ denotes the infinitesimal coordinate difference of the two space-time points x^μ and y^μ

$$dx^\mu = \lim_{y^\mu \to x^\mu} (y^\mu - x^\mu) \tag{29}$$

A rotation around the z-axis by the rotation angle φ is given by

$$a^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{30}$$

A Lorentz boost along the x-axis by the speed v is given by

$$a^\mu{}_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{31}$$

where

$$\beta = v/c \tag{32}$$
$$\gamma = 1/\sqrt{1 - v^2/c^2} \tag{33}$$

In general, a Lorentz transformation $a^\mu{}_\nu$ of a four-position x^μ is given by

$$x\prime^\mu = a^\mu{}_\nu x^\nu \tag{34}$$

The Lorentz transformation of a four-derivative $\partial_\mu = \partial/\partial x^\mu$ is given by

$$\partial\prime_\mu = a_\mu{}^\nu \partial_\nu \tag{35}$$

Finally, a Lorentz transformation around the parameter $\omega_{\alpha\beta}$ is given by

$$a^\mu{}_\nu = \exp\left(\frac{1}{2}\omega_{\alpha\beta} I^{\alpha\beta}\right)^\mu{}_\nu \in O(1,3) \tag{36}$$

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where a representation of the six generators of the Lorentz group $SO(1,3)$ is

$$I^{10} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{37}$$

$$I^{20} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{38}$$

$$I^{30} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{39}$$

$$I^{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{40}$$

$$I^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{41}$$

$$I^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{42}$$

2.5. Relativistic Quantum Mechanics

The group which underlies the kinematics of relativistic quantum mechanics and relativistic quantum field theory is the Poincare group. Isospin is invariant under global transformations of the group $SU(2)$. The generators are the three Pauli matrices τ_i. The three Pauli matrices σ_i of spin are generalized by the four Dirac matrices γ_μ. An often used representation of the Dirac matrices is

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{43}$$

$$\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{44}$$

It is

$$\gamma^0 = \gamma_0 \tag{45}$$

$$\gamma^i = -\gamma_i \tag{46}$$

The commutator is

$$[\gamma_\mu, \gamma_\nu] = -2i\sigma_{\mu\nu} \tag{47}$$

which is the definition of the generalized Dirac matrices $\sigma_{\mu\nu}$. The anti-commutator gives

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \tag{48}$$

By using the representation eqs. (43) and (44) of the Dirac matrices and the representation eqs. (16), (17) and (18) of the Pauli matrices, the anti-commutator gives the representation eq. (27) of the metric tensor in Minkowski coordinates. Moreover it is

$$\gamma^5 = \gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 \tag{49}$$

The multiplication of two four-vectors a^μ and b^μ is given by

$$\begin{aligned}(\gamma \cdot a)(\gamma \cdot b) &= (\gamma_\mu a^\mu)(\gamma_\nu b^\nu) \\ &= (g_{\mu\nu} - i\sigma_{\mu\nu})a^\mu b^\nu\end{aligned} \tag{50}$$

The Lorentz transformation of a four-component spin 1/2 Dirac spinor field $\Psi(x)$ is given by

$$\Psi\prime(x\prime) = \exp\left(-\frac{i}{4}\sigma_{\mu\nu}\left(\frac{1}{2}\omega_{\alpha\beta}I^{\alpha\beta}\right)^{\mu\nu}\right)\Psi(x) \tag{51}$$

which is a generalization of eq. (36) which considers both the Lorentz transformation of the elementary particle and its intrinsic spin.

2.6. Relativistic Quantum Field Theory

Quantum electrodynamics describes invariance under local transformations in the gauge group $U(1)$. If Ψ describes a four-component Dirac spinor, x a space-time point, e the elementary electric charge, φ the gauge phase, D_μ the covariant derivative, ∂_μ the partial four-derivative, and A^μ the electromagnetic four-potential, then the gauge transformation is

$$\Psi\prime(x) = \exp(-ie\varphi(x))\Psi(x) \tag{52}$$

$$iD_\mu = i\partial_\mu - eA_\mu(x) \tag{53}$$

$$A\prime_\mu(x) = A_\mu(x) - \partial_\mu\varphi(x) \tag{54}$$

Note that $U(1)$ and $SO(2)$ are isomorphic.

Quantum flavordynamics describes invariance under local transformations in the gauge group $SU(2) \times U(1)$. If Ψ denotes an eight-component iso-spinor Dirac spinor, x a space-time point, g the weak coupling constant, φ_i the three-component gauge phase isovector, τ_i the Pauli matrices, and W^i_μ the weak isovector four-potentials, then the gauge transformations of the $SU(2)$ part are

$$\Psi\prime(x) = \exp(-ig\varphi_i(x)\tau_i/2)\Psi(x) \tag{55}$$

$$iD_\mu = i\partial_\mu - \frac{g}{2}W^i_\mu(x)\tau_i \tag{56}$$

$$\begin{aligned}W\prime^\mu_i(x) &= W^\mu_i(x) - \partial^\mu\varphi_i(x) \\ &\quad -g\varepsilon_{ijk}\varphi_j(x)W^\mu_k(x)\end{aligned} \tag{57}$$

Quantum chromodynamics describes invariance under local transformations in the gauge group $SU(3)$. If Ψ denotes a twelve-component colour-vector Dirac spinor, x a space-time point, g the strong coupling constant, φ_i the eight-component gauge phase vector, λ_i the eight Gell-Mann matrices, and G^i_μ the eight gluon four-potentials, then the gauge transformations are

$$\Psi\prime(x) = \exp(-ig\varphi_i(x)\lambda_i/2)\Psi(x) \tag{58}$$

$$iD_\mu = i\partial_\mu - \frac{g}{2}G^i_\mu(x)\lambda_i \tag{59}$$

$$\begin{aligned}G\prime^\mu_i(x) &= G^\mu_i(x) - \partial^\mu\varphi_i(x) \\ &\quad -gf_{ijk}\varphi_j(x)G^\mu_k(x)\end{aligned} \tag{60}$$

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where the indices i, j, k run from 1 to 8.

A representation of the Gell-Mann matrices is

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (61)$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (62)$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (63)$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad (64)$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad (65)$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (66)$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (67)$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (68)$$

The commutator of the Gell-Mann matrices is

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k \quad (69)$$

and the anti-commutator is

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\delta_{ij} + 2d_{ijk}\lambda_k \quad (70)$$

Note that the number of the generators of the unitary groups is given by

$$\dim SU(n) = n^2 - 1 \quad (71)$$

and that the generators of the $SU(n)$ groups are Hermitean.

According to relativistic quantum mechanics and relativistic quantum field theory, the energy-momentum tensor $\Sigma^{\mu\nu}$ of a Dirac spinor Ψ is asymmetric

$$\begin{aligned} \Sigma^{\mu\nu}(x) &= -\frac{1}{2}((D^\mu \bar{\Psi}(x))\gamma^\nu \Psi(x)) \\ &+ \frac{1}{2}(\bar{\Psi}(x)\gamma^\nu D^\mu \Psi(x)) \end{aligned} \quad (72)$$

where the covariant derivative D_μ is given by the equations (53), (56) and (59).

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2.7. General Relativity

The general theory of relativity describes invariance under arbitrary curvilinear transformations. It is invariant under local transformations in the Lorentz group $SO(1,3)$. The Lorentz boosts depend on the four-position x^μ. Note that the Poincare group which underlies special relativity has ten generators, whereas the Lorentz group which underlies general relativity has only six generators. Therefore general relativity is not a generalization of special relativity.

The scalar product of the four-vectors a^μ and b^μ is given by

$$a \cdot b = g_{\mu\nu}(x) a^\mu b^\nu \tag{73}$$

where the metric tensor $g_{\mu\nu}$ depends on the space-time point x. The square of the infinitesimal line element is

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \tag{74}$$

The local Lorentz transformation is

$$a^\mu{}_\nu(x) = \exp\left(\frac{1}{2}\omega_{\alpha\beta}(x) I^{\alpha\beta}\right)^\mu{}_\nu \tag{75}$$

which is a generalization of eq. (36).

When a four-vector C^α is parallely displaced from the four-position x^μ to the four-position $x^\mu + dx^\mu$, then it changes according to the prescription

$$dC^\alpha = -\Gamma^\alpha_{\mu\nu}(x) C^\nu dx^\mu \tag{76}$$

This is the definition of the four-position-dependent affine connection $\Gamma^\alpha_{\mu\nu}$. According to general relativity it has only a symmetric part

$$\{\}^\alpha_{\mu\nu}(x) = \frac{1}{2}\left(\Gamma^\alpha_{\mu\nu}(x) + \Gamma^\alpha_{\nu\mu}(x)\right) \tag{77}$$

which is named Christoffel symbol. The Riemann curvature tensor is given by

$$\begin{aligned} R^\lambda_{\mu\nu\kappa}(x) &= \partial_\kappa \Gamma^\lambda_{\mu\nu}(x) - \partial_\nu \Gamma^\lambda_{\mu\kappa}(x) \\ &\quad + \Gamma^\alpha_{\mu\nu}(x) \Gamma^\lambda_{\kappa\alpha}(x) - \Gamma^\alpha_{\mu\kappa}(x) \Gamma^\lambda_{\nu\alpha}(x) \end{aligned} \tag{78}$$

By contraction one gets the Ricci tensor

$$R_{\mu\nu}(x) = R^\lambda{}_{\mu\lambda\nu}(x) \tag{79}$$

and the Ricci scalar

$$R(x) = R^\mu_\mu(x) \tag{80}$$

The Einstein field equations are

$$R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu}(x) R(x) = \kappa \Sigma_{\mu\nu}(x) \tag{81}$$

where

$$\kappa = -8\pi G \tag{82}$$

denotes the Einstein field constant and $\Sigma_{\mu\nu}$ the energy-momentum tensor.

Since the affine connection (Christoffel symbol) is symmetric it follows that the energy-momentum tensor of general relativity is symmetric. This is in contrast to the asymmetric energy-momentum tensor of a Dirac spinor of relativistic quantum mechanics. This means that a Dirac spinor cannot be described by the geometry that underlies general relativity.

2.8. Einstein-Cartan Theory

Einstein-Cartan theory is a generalization of general relativity, because within the framework of this theory the anti-symmetric part of the affine connection which is named Cartan's torsion tensor

$$T^{\alpha}_{\mu\nu}(x) = \frac{1}{2}\left(\Gamma^{\alpha}_{\mu\nu}(x) - \Gamma^{\alpha}_{\nu\mu}(x)\right) \tag{83}$$

is nonzero. In contrast to the Christoffel symbol the torsion tensor transforms as a tensor under arbitrary curvilinear transformations. Cartan's torsion tensor is related to the spin tensor by

$$T^{\alpha\mu\nu}(x) = \kappa\tau^{\alpha\mu\nu}(x) \tag{84}$$

where $\kappa = -8\pi G$ denotes the Einstein field constant known from general relativity.

The Dirac spinor can be described by the geometry that underlies Einstein-Cartan theory, because this theory describes an asymmetric affine connection and therefore an asymmetric energy-momentum tensor.

Einstein-Cartan theory is invariant under local transformations of the Poincare group. So Einstein-Cartan theory is a generalization of special relativity.

In quantum Einstein-Cartan theory the local Lorentz gauge transformation of a Dirac spinor field Ψ is given by

$$\Psi\prime(x\prime) = \exp\left(-\frac{i}{4}\sigma_{\mu\nu}\left(\frac{1}{2}\omega_{\alpha\beta}(x)I^{\alpha\beta}\right)^{\mu\nu}\right)\Psi(x) \tag{85}$$

This equation is a combination of eqs. (51) and (75).

2.9. Conclusions

I argued that the general theory of relativity must be generalized by the Einstein-Cartan theory, because the asymmetric energy-momentum tensor of relativistic quantum mechanics requires the existence of an asymmetric affine connection and therefore the existence of a nonzero torsion tensor. Moreover I pointed out that spin is the source of a gauge field which is associated with Cartan's torsion. This is analogous to the situation of isospin which is a pure quantum number in quantum mechanics, but the source of the weak gauge field in quantum flavourdynamics.

3. Dynamics of Magnetic Monopoles

3.1. Introduction

In 1904 Thomson [1] has shown that the angular momentum generated by the Lorentz force between an electric charge and a magnetic charge is independent of their distance. As intrinsic spin and orbital angular momentum are quantized in units of (half-)integers times the reduced Planck constant $\hbar$, Dirac [2] concluded in 1931, that these conditions can be satisfied only if both electric charge and magnetic charge appear in discrete units only. Since then textbooks tell us that the coupling constant of Dirac magnetic monopoles is 34.259. Here I show that this conclusion is not correct. Electric charge appears not in multiples of the positron charge e, but in multiples of the quark charge $e/3$. At zero temperature the coupling constant of Dirac magnetic monopoles is 308.331. By using the weak energy condition I show that this coupling is a running coupling constant and becomes smaller than 0.5 at the Planck temperature.

3.2. Classical Dirac Magnetic Monopoles

I will use the natural units

$$\hbar = c = \varepsilon_0 = 1 \tag{86}$$

where $\hbar = h/2\pi$ denotes the reduced Planck constant, c the speed of light, and ε_0 the electric field constant.

The electric field strength generated by an electric charge Q resting in the center of the coordinate frame is

$$\mathbf{E} = \frac{Q}{4\pi r^2}\hat{\mathbf{r}} \tag{87}$$

where $\hat{\mathbf{r}} \equiv \mathbf{r}/r$ denotes the unit position vector, $\mathbf{r}$ the position vector, and $r \equiv \|\mathbf{r}\|$ its absolute value.

By analogy, the magnetic field strength generated by a magnetic charge q resting in the center of the coordinate frame is

$$\mathbf{B} = \frac{q}{4\pi r^2}\hat{\mathbf{r}} \tag{88}$$

In the classical (non-quantum mechanical) case the Lorentz force on a moving electric charge Q in the static magnetic field generated by a resting magnetic charge q is

$$m\ddot{\mathbf{r}} = Q\dot{\mathbf{r}} \times \mathbf{B} \tag{89}$$

Here $\dot{\mathbf{r}} \equiv \partial_t \mathbf{r}$ is the speed of the electric charge and $\ddot{\mathbf{r}} \equiv \partial_t \dot{\mathbf{r}} = \partial_t^2 \mathbf{r}$ is its acceleration. The rest mass of the electric charge is denoted by m. The rest mass of the magnetic charge is assumed to be much larger (infinity) than that of the electric charge, so that the magnetic charge can rest in the center of the coordinate frame.

By using

$$\begin{aligned} \partial_t \hat{\mathbf{r}} &= \partial_t\left(\mathbf{r}/\sqrt{\mathbf{r}\cdot\mathbf{r}}\right) \\ &= \frac{r\dot{\mathbf{r}} - \left(\frac{1}{2r}2\dot{\mathbf{r}}\cdot\mathbf{r}\right)\mathbf{r}}{r^2} \\ &= \frac{r^2\dot{\mathbf{r}} - (\dot{\mathbf{r}}\cdot\mathbf{r})\mathbf{r}}{r^3} \\ &= \frac{\mathbf{r}\times(\dot{\mathbf{r}}\times\mathbf{r})}{r^3} \end{aligned} \tag{90}$$

the orbital angular momentum

$$\mathbf{L} \equiv \mathbf{r} \times m\dot{\mathbf{r}} \tag{91}$$

of the electric charge generated by the Lorentz force gives

$$\begin{aligned} \partial_t \mathbf{L} &= \dot{\mathbf{L}} = \mathbf{r} \times m\ddot{\mathbf{r}} \\ &= Q\mathbf{r} \times \left(\dot{\mathbf{r}} \times \frac{q\mathbf{r}}{4\pi r^3}\right) \\ &= \frac{Qq}{4\pi}\partial_t \hat{\mathbf{r}} \end{aligned} \tag{92}$$

Subtraction gives

$$0 = \partial_t\left(\mathbf{L} - \frac{Qq}{4\pi}\hat{\mathbf{r}}\right) \tag{93}$$

Total angular momentum $\mathbf{J}$ is the sum of orbital angular momentum $\mathbf{L}$ and intrinsic spin $\mathbf{S}$,

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \tag{94}$$

Moreover it is conserved,

$$0 = \partial_t \mathbf{J} = \partial_t(\mathbf{L} + \mathbf{S}) \tag{95}$$

Comparison of equations (8) and (10) gives

$$\mathbf{S} = -\frac{Qq}{4\pi}\hat{\mathbf{r}} \tag{96}$$

Intrinsic spin is quantized in units of half-integers times the reduced Planck constant,

$$\|\mathbf{S}\| = \frac{n}{2} \tag{97}$$

where n denotes an arbitrary integer. Hence,

$$Qq = 2\pi n \tag{98}$$

This is the Dirac quantization condition. It requires that Q and q cannot have arbitrary values.

In the days of Dirac [2], that is in 1931, quarks were not known. So he assumed that electric charge is quantized in units of the positron charge e. Therefore he assumed the unit magnetic charge g' so as to satisfy

$$eg' = 2\pi \tag{99}$$

By using the Sommerfeld fine-structure constant

$$\alpha_E \equiv \frac{e^2}{4\pi} \simeq \frac{1}{137.036} \tag{100}$$

this gives the coupling constant

$$\frac{g'^2}{4\pi} = \frac{1}{4\pi}\left(\frac{2\pi}{e}\right)^2 = \frac{1}{4\alpha_E} \simeq 34.259 \tag{101}$$

However, since the prediction [3] and observation [4, 5] of quarks we know that electric charge is quantized in units of $e/3$. Hence, the unit magnetic charge g is given by

$$\frac{e}{3}g = 2\pi \tag{102}$$

This gives the magnetic coupling constant

$$\alpha_M \equiv \frac{g^2}{4\pi} = \frac{1}{4\pi}\left(\frac{6\pi}{e}\right)^2 = \frac{9}{4\alpha_E} \simeq 308.331 \tag{103}$$

3.3. The Model

The quantization of electric charge is well-known since the discovery of the proton in 1919. This remarkable observation remained unexplained within the framework of quantum electrodynamics.

Further quantized charges have been established. The group $SU(2)$ of the weak interaction explains the quantization of isospin, and the group $SU(3)$ of the strong interaction explains the quantization of colour charge.

For this reason I propose the analogy postulate: The quantization of electric charge results from the underlying group structure of the electromagnetic interaction. Hence, I will require neither quantum gravity (electric charge as a topological quantum number, nor spontaneous symmetry breaking (monopoles of soliton type), nor unification with other forces (charge quantization resulting from the group structure underlying grand unified theories).

The electromagnetic angular momentum generated by the Lorentz force in a system consisting of a magnetic monopole and an electric charge is independent of their separation. Angular momentum is quantized in units of $\hbar/2$, where $\hbar = h/2\pi$ denotes the reduced Planck constant. This condition can be satisfied only if both electric and magnetic charge are quantized. This is the famous Dirac quantization condition $eg = h$, where e and g denote unit electric and unit magnetic charge.

Magnetic monopoles were discussed long before this finding. The motivation was to describe electric and magnetic fields equivalently by symmetrized Maxwell equations. I will elevate this to the symmetry postulate: The fundamental equations of the electromagnetic interaction describe electric and magnetic charges, electric and magnetic field strengths, and electric and magnetic potentials equivalently.

Dirac was the first to write down these symmetrized Maxwell equations.

Let $J^\mu = (P, \mathbf{J})$ denote the electric four-current and $j^\mu = (\rho, \mathbf{j})$ the magnetic four-current. The well-known four-potential of the electric photon is $A^\mu = (\Phi, \mathbf{A})$. The four-potential of the magnetic photon is $a^\mu = (\varphi, \mathbf{a})$. Expressed in three-vectors the symmetrized Maxwell equations read,

$$\nabla \cdot \mathbf{E} = P \tag{104}$$
$$\nabla \cdot \mathbf{B} = \rho \tag{105}$$
$$\nabla \times \mathbf{E} = -\mathbf{j} - \partial_t \mathbf{B} \tag{106}$$
$$\nabla \times \mathbf{B} = +\mathbf{J} + \partial_t \mathbf{E} \tag{107}$$

and the relations between field strengths and potentials are

$$\mathbf{E} = -\nabla\Phi - \partial_t \mathbf{A} - \nabla \times \mathbf{a} \tag{108}$$
$$\mathbf{B} = -\nabla\varphi - \partial_t \mathbf{a} + \nabla \times \mathbf{A}. \tag{109}$$

The second four-potential is required not only by the symmetry postulate, but also by the proven impossibility to construct a manifestly covariant one-potential model of quantum electromagnetodynamics.

Although only one of the suggested two-potential models explicitely states the possibility of the existence of a magnetic photon, the other two-potential models were eventually considered as two-photon models.

Any viable two-photon concept of magnetic monopoles has to satisfy the following conditions.

(i) In the absence of both magnetic charges and the magnetic photon field, the model has to regain the $U(1)$ gauge symmetry of quantum electrodynamics.

(ii) In the absence of both electric charges and the photon field, the symmetry postulate requires the model to yield the $U'(1)$ gauge symmetry of quantum magnetodynamics.

(iii) The gauge group has to be Abelian, because the photon carries neither electric nor magnetic charge. Because of the symmetry postulate also the magnetic photon has to be neutral.

(iv) The gauge group may not be simple, because quantum electromagnetodynamics includes the two coupling constants $\alpha_E = e^2/4\pi$ and $\alpha_M = g^2/4\pi$.

The only gauge group that satisfies these four conditions is the group $U(1) \times U'(1)$.

A two-photon model has already been suggested by Salam. According to his model the photon couples via vector coupling with leptons and hadrons, but not with monopoles. The magnetic photon couples via vector coupling with monopoles and via tensor coupling with hadrons, but not with leptons.

This model came under severe criticism. Although positron and proton have the same electric charge and no magnetic charge, the model can discriminate them (i. e. leptons and hadrons). For this reason Salam's model does not generate the Lorentz force between electric charge and monopole. As a consequence, it does not satisfy the powerful Dirac quantization condition.

This problem can be overcome by the following argumentation. Salam considered the tensor coupling of the hadron-monopole system as derivative coupling. This kind of coupling is well-known from meson theory where vector mesons are able to interact with baryons via both vector and tensor coupling. However, derivative coupling is possible only where the particles are composite. Hence,

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Salam's model includes no interaction between lepton and magnetic photon. – I emphasize the correctness of the interpretation of tensor coupling as derivative coupling in meson theory.

To generate the Lorentz force between electric and magnetic charges we have to introduce a new kind of tensor coupling. This is required also, because here we have two kinds of interacting charges (electric and magnetic).

The Coulomb force between two (unit) electric charges is $e^2/4\pi r^2$ Because of the symmetry postulate the magnetic force between two (unit) magnetic charges is $g^2/4\pi r^2$ And the Lorentz force between (unit) electric and (unit) magnetic charge is $egv/4\pi r^2$, where v denotes the relative velocity of the two charges.

This suggests the introduction of velocity coupling:

(i) The photon couples via vector coupling with electric charges.

(ii) The magnetic photon couples via vector coupling with magnetic charges.

(iii) The photon couples via tensor coupling with magnetic charges. In contrast to meson theory, however, the u^μ of tensor coupling, $\sigma^{\mu\nu}u_\nu$, has to be interpreted as a four-velocity (velocity coupling).

(iv) The magnetic photon couples via tensor coupling (interpreted as velocity coupling instead of derivative coupling) with electric charges.

In the case of the interacting monopole-electric charge system the exchanged boson (either photon or magnetic photon) is virtual and the four-velocity of velocity coupling is the relative four-velocity between the charges.

Charged quanta are required to emit and absorb the same bosons as real (on-mass-shell) particles as those virtual (off-mass-shell) bosons via whom they interact with other charged quanta. This is because the Feynman rules are symmetric with respect to virtual and real particles.

In the case of emission and absorption reactions of real bosons, u^μ cannot be interpreted as a relative four velocity between charged quanta in the initial state, as there is only one charged quantum present. As a consequence, u^μ has to be interpreted as the absolute four-velocity of the initial charged quantum.

In contrast to general belief an absolute rest frame is not forbidden. Instead, a number of reasons support its existence.

The absolute rest frame required for the absolute velocity is defined by the comoving frame of relativistic cosmology. This is the center-of-mass frame of all the masses within the observable universe (Hubble sphere), the frame which shows an isotropic redshift-distance relation (defined by the Hubble effect, where contributions of the optical Doppler effect are isotropic). The absolute velocity of the sun has been measured by the dipole anisotropy of the cosmic microwave background radiation. Its value is $v = 370$ km/s.

3.4. Formalism

The Lagrangian for a spin 1/2 fermion field Ψ of rest mass m_0, electric charge Q, and magnetic charge q within an electromagnetic field can be constructed as follows. By using the tensors

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \tag{110}$$
$$f^{\mu\nu} \equiv \partial^\mu a^\nu - \partial^\nu a^\mu \tag{111}$$

the Lagrangian of the Dirac fermion within the electromagnetic field reads,

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \bar{\Psi}i\gamma^\mu\partial_\mu\Psi - m_0\bar{\Psi}\Psi \\ & -Q\bar{\Psi}\gamma^\mu\Psi A_\mu - q\bar{\Psi}\gamma^\mu\Psi a_\mu \\ & +Q\bar{\Psi}\gamma^5\sigma^{\mu\nu}u_\nu\Psi a_\mu + q\bar{\Psi}\gamma^5\sigma^{\mu\nu}u_\nu\Psi A_\mu. \end{aligned} \tag{112}$$

By using the Euler-Lagrange equations we obtain the Dirac equation

$$(i\gamma^\mu\partial_\mu - m_0)\Psi = (Q\gamma^\mu A_\mu + q\gamma^\mu a_\mu - Q\gamma^5\sigma^{\mu\nu}u_\nu a_\mu - q\gamma^5\sigma^{\mu\nu}u_\nu A_\mu)\Psi. \quad (113)$$

By introducing the four-currents

$$J^\mu = Q\bar{\Psi}\gamma^\mu\Psi - q\bar{\Psi}\gamma^5\sigma^{\mu\nu}u_\nu\Psi \quad (114)$$
$$j^\mu = q\bar{\Psi}\gamma^\mu\Psi - Q\bar{\Psi}\gamma^5\sigma^{\mu\nu}u_\nu\Psi \quad (115)$$

the Euler-Lagrange equations yield the two Maxwell equations

$$J^\mu = \partial_\nu F^{\nu\mu} = \partial^2 A^\mu - \partial^\mu\partial^\nu A_\nu \quad (116)$$
$$j^\mu = \partial_\nu f^{\nu\mu} = \partial^2 a^\mu - \partial^\mu\partial^\nu a_\nu. \quad (117)$$

Evidently, the two Maxwell equations are invariant under the $U(1) \times U'(1)$ gauge transformations

$$A^\mu \to A^\mu - \partial^\mu\Lambda \quad (118)$$
$$a^\mu \to a^\mu - \partial^\mu\lambda. \quad (119)$$

Furthermore, the four-currents satisfy the continuity equations

$$0 = \partial_\mu J^\mu = \partial_\mu j^\mu. \quad (120)$$

The electric and magnetic field are related to the tensors above by

$$E^i = F^{i0} - \frac{1}{2}\varepsilon^{ijk} f_{jk} \quad (121)$$
$$B^i = f^{i0} + \frac{1}{2}\varepsilon^{ijk} F_{jk}. \quad (122)$$

Finally, the Lorentz force is

$$K^\mu = Q(F^{\mu\nu} + \frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma} f_{\varrho\sigma})u_\nu + q(f^{\mu\nu} - \frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma} F_{\varrho\sigma})u_\nu, \quad (123)$$

where $\varepsilon^{\mu\nu\varrho\sigma}$ denotes the totally antisymmetric tensor. This formula for the Lorentz force is rather trivial for the classical theory. Non-trivial is that this formula can be applied to the quantum field theory. This becomes possible because of the introduction of the velocity coupling which includes a velocity operator and allows the definition of a force operator.

3.5. Suggested Experiment

I suggest an experiment to test my theory by searching for the magnetic photon rays which are emitted by all conventional light sources and detectable by conventional light detectors (such as the human eye, photo diodes, and photomultiplier tubes).

Illuminate a metal film of thickness between 100 and 1000 nanometers by a laser beam and place a detector (avalanche diode or photomultiplier tube) behind the metal film. My theory predicts that the laser emits both electric and magnetic photons and that the detector detects both kinds of photon. The interaction cross-section of a magnetic photon is smaller by a factor of 560,000 to 780,000 with respect to the interaction cross-section of an electric photon of the same energy for quantum physical effects such as induced emission and the photoelectric effect. Classical electrodynamics predicts that the metal film absorbs the conventional photons (Einstein's electric photons) completely, the penetration depth (skin depth) is 2 to 7 nanometers, depending only on the frequency of the laser light and the electric conductivity of the metal film. My theory predicts that a high percentage of the Salam magnetic

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photons penetrates the metal film. The penetration depth is predicted to be 100 to 500 nanometers, depending only on the same frequency of the electric and magnetic photons of the laser beam and the electric conductivity (which has to be divided by a factor of $c/v = 750$ to 880) of the metal film. – Note that mirrors and metal films do not reflect electric and magnetic photons in the same way, because of the factor 750 to 880 by which the conductivity has to be divided.

3.6. Non-Classical Dirac Magnetic Monopoles

Because of the introduction of the Lorentz force, the calculation above was made for classical (non-quantum mechanical) objects with electric and magnetic charge. However, since 1997 I am argueing [6, 7] that the quantum field theoretical interaction between electric and magnetic charges requires the introduction of a velocity operator which allows the definition of a Lorentz force. So the calculation above is valid also for the quantum physical case.

More precisely, the introduction of the velocity operator requires the existence of absolute speed which is defined by the finite light cone generated by the Hubble effect. The velocity operator is therefore an effect not only of quantum field theory, but also of gravitation theory. The velocity operator is therefore an effect of quantum gravity.

Moreover the calculation above gives the Dirac quantization condition only if equation (88) is used. It requires that both the Einstein electric photon [8] and the Salam magnetic photon [9] have zero rest mass. A massive magnetic photon would require a Yukawa potential whose exponential term would destroy the Dirac quantization condition.

3.7. Running Coupling Constant

In chapter 4.8 I will predict the existence of elementary fermions with magnetic charge $q = g$ (called hanselons and gretelons) and also with $q = 2g$ (gretelons). The binding energy of two gretelons with opposite magnetic charge $\pm 2g$ which are separated by the distance r would become quite large,

$$E_b = -\frac{q^2}{4\pi r} = -\frac{(2g)^2}{4\pi r} \tag{124}$$

The total energy of two bound gretelons would be

$$E \simeq M_1 + M_2 + \frac{1}{r} - \frac{g^2}{\pi r} - \frac{GM_1M_2}{r} \tag{125}$$

where M_1 and M_2 are the rest masses of the gretelons, $1/r$ is their zero point energy given by the uncertainty principle, $-g^2/\pi r$ is their magnetic binding energy, and $-GM_1M_2/r$ is their gravitational binding energy. G denotes the Newtonian gravitational constant.

The weak energy condition states that there cannot exist any negative energy densities. Hence, $E \geq 0$. Moreover the rest masses of elementary particles cannot be larger than the Planck mass $M_P = G^{-1/2}$, because otherwise their Schwarzschild radius would become larger than the Planck length $l_P = 1/M_P = G^{1/2}$. Hence,

$$0 \leq E \leq 2M_P + \frac{1}{r} - \frac{g^2}{\pi r} - \frac{GM_PM_P}{r} \tag{126}$$

In principle, the mutual distance of the two (elementary) gretelons can become as small as the Planck length,

$$r \geq l_P = 1/M_P \tag{127}$$

Hence, in the case $r = 1/M_P$ it is

$$0 \leq E \leq 2M_P - \frac{g^2}{\pi}M_P = M_P\left(2 - \frac{g^2}{\pi}\right) \tag{128}$$

This can be satisfied only if

$$\alpha_M(r = l_P) = \frac{g^2(r)}{4\pi} \le \frac{1}{2} \tag{129}$$

Hence, $\alpha_M(r)$ must be a running coupling constant. It must decrease with distance and increase with energy. – That $\alpha_E(r)$ is a running coupling constant is known since the work of Gell-Mann and Low in 1954 [10].

3.8. Summary

By using the quark hypothesis I have shown that at zero temperature the magnetic coupling constant is as high as 308.331. By using the weak energy condition and applying it on a system of two bound magnetic charges, I have shown that the magnetic coupling is a running coupling constant and becomes smaller than 0.5 at the Planck scale.

4. Elementary Magnetic Monopoles

4.1. Strong and Weak Isospin

According to the conventional nomenclature presented in textbooks, up and down quark have strong isospin, whereas all the other elementary fermions have zero strong isospin.

All the left-handed quarks and leptons have weak isospin, whereas all the right-handed quarks and leptons have zero weak isospin. There are no right-handed neutrinos.

4.2. Criticism of Textbook Nomenclature

There exist three left-handed fermion octets.

(i) The first octet consists of three up quarks, three down quarks, electron, and electron-neutrino.

(ii) The second octet consists of three charm quarks, three strange quarks, muon, and muon-neutrino.

(iii) The third octet consists of three top quarks, three bottom quarks, tauon, and tau-neutrino.

The first octet includes three strong isospin doublets, whereas the other octets include no strong isospin doublets. The three octets are not described equivalently.

All the left-handed fermion octets include four weak isospin doublets. The three right-handed fermion septets include no weak isospin doublets. Left-handed and right-handed fermions are not described equivalently.

4.3. New Nomenclature

In order to describe both left-handed and right-handed fermions and the three octets equivalently, I will suggest a new definition of isospin, so that all the leptons and quarks have isospin partners and that all the octets consist of four isospin doublets.

(i) The isospin partner of the up quark is the down quark.

(ii) The isospin partner of the charm quark is the strange quark.

(iii) The isospin partner of the top quark is the bottom quark.

(iv) The isospin partner of the charged lepton is its corresponding neutrino.

A consequence of this redefinition is that right-handed neutrinos must exist.

Each quark exists in three chromoelectric colors. So each of the three fermion octets consists of four isospin doublets.

If Q denotes electric charge in units of the positron charge e, I_3 denotes the third component of isospin, and Y denotes hypercharge, then the Gell-Mann-Nishijima equation reads

$$Q = I_3 + Y/2 \tag{130}$$

If B denotes baryon number and L denotes lepton number, then:

(i) For quarks we get $Y = B$. The famous equation $Y = B + S$ for strange quarks is valid only for strong isospin, but not here.

(ii) For leptons we get $Y = -L$.

(iii) For fermions we get in general

$$Y = B - L \tag{131}$$

The quantum numbers of the elementary particles are then:

(i) Up, charm, and top quark have $Q = +2/3$, $I_3 = +1/2$, and $Y = +1/3$.

(ii) Down, strange, and bottom quark have $Q = -1/3$, $I_3 = -1/2$, and $Y = +1/3$.

(iii) Electron-neutrino, muon-neutrino, and tau-neutrino have $Q = 0$, $I_3 = +1/2$, and $Y = -1$.

(iv) Electron, muon, and tauon have $Q = -1$, $I_3 = -1/2$, and $Y = -1$.

(v) Each of the three families of leptons has its own lepton number. Because of neutrino-oscillations, only their sum L can be conserved.

(vi) Each of the three families of quarks has its own baryon number. Because of Kobayashi-Maskawa mixing [11], only their sum B can be conserved.

According to this redefinition, isospin is conserved as long as both electric charge and hypercharge are conserved.

4.4. Baryon and Lepton Number

Quantum electrodynamics (QED) associates the $U(1)_Q$ gauge symmetry with electric charge Q. Quantum flavordynamics (QFD) [12, 13] describes a $SU(2)_I \times U(1)_Y$ gauge symmetry, where $SU(2)_I$ is associated with isospin and $U(1)_Y$ with hypercharge. A rotation by the Weinberg angle Θ_W transforms the third part W_3^μ of the gauge field associated with the $SU(2)_I$ group and the gauge field B^μ associated with the $U(1)_Y$ group into the gauge field A^μ associated with the photon of QED and the gauge field Z^μ associated with the Z boson,

$$A^\mu = B^\mu \cos\Theta_W + W_3^\mu \sin\Theta_W \tag{132}$$

$$Z^\mu = -B^\mu \sin\Theta_W + W_3^\mu \cos\Theta_W \tag{133}$$

For the quantum numbers this rotation is described by the Gell-Mann-Nishijima equation.

The $U(1)_Y$ gauge symmetry describes hypercharge $Y = B - L$, so $B - L$ should be conserved. However, both baryon number B and lepton number L are conserved independently. So one would expect that B is associated with a $U(1)$ gauge symmetry and that L is associated with another $U(1)$ gauge symmetry. Hence, there should exist a $U(1) \times U(1)$ gauge symmetry.

4.5. Electric-Magnetic Duality

Quantum electromagnetodynamics (QEMD) [6, 7] was suggested in order to describe electricity and magnetism equivalently. This theory includes both electric charge Q and magnetic charge q. Quanta which have nonzero magnetic charge are called Dirac magnetic monopoles [2]. Quanta which have both nonzero electric charge and nonzero magnetic charge are called Schwinger dyons [14]. The gauge bosons of QEMD are the photon (now called Einstein electric photon [8]) and the new Salam magnetic photon [9]. The gauge group is $U(1)_Q \times U(1)_q$.

4.6. Generalized Electric-Magnetic Duality

The introduction of magnetic charges and the magnetic photon makes it necessary to generalize the standard theory of particle physics.

QFD will be generalized by a new theory which is described by a $SU(2)_I \times U(1)_Y \times SU(2)_M \times U(1)_X$ gauge symmetry. The eight gauge bosons are the electric photon, the two W bosons, the Z boson, and the new magnetic photon, the two new isomagnetic W bosons, and the new isomagnetic Z boson.

A rotation by the isomagnetic Weinberg angle Θ_M transforms the third part w_3^μ of the gauge field associated with the $SU(2)_M$ group and the gauge field b^μ associated with the $U(1)_X$ group into the gauge field a^μ associated with the magnetic photon and the gauge field z^μ associated with the isomagnetic Z boson,

$$a^\mu = b^\mu \cos\Theta_M + w_3^\mu \sin\Theta_M \quad (134)$$

$$z^\mu = -b^\mu \sin\Theta_M + w_3^\mu \cos\Theta_M \quad (135)$$

For the quantum numbers this rotation is described by the magnetic Gell-Mann-Nishijima equation

$$q = M_3 + X/2 \quad (136)$$

where q denotes magnetic charge in units of the unit magnetic charge m, M_3 denotes the third component of magnetic isospin, and X the herewith new introduced hypocharge.

I argued in earlier papers [6, 7] that quantum chromodynamics (QCD) [15, 16] will be generalized by a new theory which is described by a $SU(3) \times SU(3)$ gauge symmetry. The associated charges are chromoelectric color and the new chromomagnetic color. The associated gauge bosons are the eight chromoelectric gluons and the eight new chromomagnetic gluons.

4.7. Hypocharge

In section 4.4 I suggested a $U(1) \times U(1)$ gauge symmetry which is associated with baryon number B and lepton number L. Because of $Y = B - L$ it is reasonable to associate this gauge symmetry with hypercharge Y and hypocharge X, thus $U(1)_Y \times U(1)_X$. Because of the two Gell-Mann-Nishijima equations and the two Weinberg angles Θ_W and Θ_M this symmetry can be transformed into the $U(1)_Q \times U(1)_q$ gauge symmetry of QEMD.

Now the task is to find out how X depends on B and L.

Let us start with the ansatz

$$X = aB + bL \quad (137)$$

where a and b are hitherto unknown real numbers. Quarks have $q = 0$, $B = 1/3$, and $L = 0$. So they have $X \neq 0$ and therefore $M_3 \neq 0$. Leptons have $q = 0$, $B = 0$, and $L = 1$. So they have $X \neq 0$ and therefore $M_3 \neq 0$. The gauge group associated with M_3 is $SU(2)$. So it is reasonable that both leptons and quarks have $M_3 = \pm 1/2$ and therefore $X = \pm 1$. This can be satisfied only if $a = \pm 3$ and $b = \pm 1$.

The total electric charge, isospin, chromoelectric color, and hypercharge of each of the three conventional fermion octets is zero.

For symmetry reasons it is reasonable to assume that fermionic magnetic monopoles exist in octets and that the total magnetic charge, magnetic isospin, chromomagnetic color, and hypocharge of each of the magnetic fermion octets is zero.

Let us use the following nomenclature. Hanselons are elementary magnetic fermions with nonzero chromomagnetic color. Gretelons are elementary magnetic fermions with zero chromomagnetic color.

In this case each magnetic fermion octet consists of three hanselons, one gretelon, and their respective isomagnetic partner, thus six hanselons and two gretelons.

The conditions

(i) $a = \pm 3$ and $b = \pm 1$

(ii) $Q = 0$ and $Y = B - L \neq 0$ and therefore $I_3 = \pm 1/2$ and therefore $Y = \pm 1$ for magnetic monopoles

(iii) zero total hypocharge of the octet

can be satisfied if each hanselon has $L = 1$ and $B = 0$ and each gretelon has $B = 1$ and $L = 0$, hence $a = 3$, $b = -1$, and

$$X = 3B - L \quad (138)$$

By using $Y = B - L$ we get

$$B = (X - Y)/2 \tag{139}$$
$$L = (X - 3Y)/2 \tag{140}$$

4.8. Predicted Quantum Numbers of Particles

The conventional and new quanta have the following quantum numbers.

(i) Up, charm, and top quark have $Q = +2/3$, $I_3 = +1/2$, $B = +1/3$, $L = 0$, $Y = +1/3$, $X = +1$, $M_3 = -1/2$, $q = 0$, chromoelectric color, no chromomagnetic color.

(ii) Down, strange, and bottom quark have $Q = -1/3$, $I_3 = -1/2$, $B = +1/3$, $L = 0$, $Y = +1/3$, $X = +1$, $M_3 = -1/2$, $q = 0$, chromoelectric color, no chromomagnetic color.

(iii) Electron, muon, and tauon have $Q = -1$, $I_3 = -1/2$, $B = 0$, $L = +1$, $Y = -1$, $X = -1$, $M_3 = +1/2$, $q = 0$, no chromoelectric color, no chromomagnetic color.

(iv) Electron-neutrino, muon-neutrino, and tau-neutrino have $Q = 0$, $I_3 = +1/2$, $B = 0$, $L = +1$, $Y = -1$, $X = -1$, $M_3 = +1/2$, $q = 0$, no chromoelectric color, no chromomagnetic color.

(v) Electric photon and magnetic photon have $Q = q = I_3 = M_3 = B = L = X = Y = 0$, no chromoelectric color, no chromomagnetic color. Their rest mass must be zero in order to satisfy the Dirac quantization condition.

(vi) Chromoelectric gluons have chromoelectric color and are otherwise neutral.

(vii) Chromomagnetic gluons have chromomagnetic color and are otherwise neutral.

(viii) W bosons have $Q = \pm 1$ and $I_3 = \pm 1$ and are otherwise neutral.

(ix) Isomagnetic W bosons have $q = \pm 1$ and $M_3 = \pm 1$ and are otherwise neutral.

(x) Z boson and isomagnetic Z boson are neutral.

(xi) Hanselons have chromomagnetic color, no chromoelectric color, $B = 0$, $L = +1$, $X = -1$, $Y = -1$, $Q = 0$, $I_3 = +1/2$, ($M_3 = +1/2$ and $q = 0$ or $M_3 = -1/2$ and $q = -1$).

(xii) Gretelons have no chromomagnetic color, no chromoelectric color, $B = +1$, $L = 0$, $X = +3$, $Y = +1$, $Q = 0$, $I_3 = -1/2$, ($M_3 = +1/2$ and $q = +2$ or $M_3 = -1/2$ and $q = +1$).

(xiii) The quantum numbers of the antiparticles corresponding to the particles (i)–(iv) and (xi)–(xii) have the opposite sign. Antileptons and antiquarks are the isomagnetic partners of leptons and quarks. Antihanselons and antigretelons are the isospin partners of hanselons and gretelons.

(xiv) Higgs boson and magnetic Higgs boson are neutral. Their spin is zero.

(xv) It is $\sin^2\Theta_W \simeq 0.23$, so one can assume that $\sin^2\Theta_M$ is also of order unity. The relation between positron charge e and weak coupling constant g_W is $e = g_W \sin\Theta_W$. The relation between unit magnetic charge m and magnetic weak coupling constant g_M is $m = g_M \sin\Theta_M$. The Dirac quantization condition is $em = 2\pi$ (where $\hbar = c = \varepsilon_0 = 1$), hence

$$\begin{aligned} g_M &= \frac{m}{\sin\Theta_M} = \frac{2\pi}{e\sin\Theta_M} \\ &= \frac{2\pi}{g_W \sin\Theta_W \sin\Theta_M} > 1 \end{aligned} \tag{141}$$

All the quarks and leptons have nonzero magnetic isospin M_3. Since magnetic isospin has not yet been observed, this can only mean that the rest masses of the isomagnetic W and Z bosons are larger than 100 GeV.

(xvi) All the leptons, quarks, hanselons, and gretelons have spin 1/2. All the 12 conventional and all the 12 new gauge bosons have spin 1 and negative parity. Higgs boson and magnetic Higgs boson have spin 0 and positive parity.

(xvii) All the leptons, quarks, hanselons, and gretelons have nonzero isospin. It is therefore possible to create hanselon-antihanselon pairs and gretelon-antigretelon pairs via neutral currents. One possibility is electron-positron scattering

$$e^- e^+ \to Z^0 \to G\bar{G} \tag{142}$$

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(xviii) With regard to isospin all the leptons, quarks, hanselons, and gretelons are (iso-)dyons, because they have both isospin and magnetic isospin.

(xix) The magnetic Fermi constant has not yet been determined. It is probable that its value is between those of the Fermi constant and the gravitational constant.

(xx) The coupling constant associated with magnetic charge is larger than unity. The corresponding binding energy would lead to negative energy densities for small distances. This violation of the weak energy condition can be prevented if the magnetic coupling constant is a running coupling constant, becomes smaller for higher energies, and the rest masses of the magnetic monopoles are of the order of the Planck mass.

(xxi) A similar problem with magnetic isospin can be solved if the rest masses of the isomagnetic W and Z bosons are of the order of the Planck mass. This suggests that the magnetic Fermi constant is of the order of the gravitational constant.

(xxii) The magnetic analogue to the graviton is the tordion. The gauge group is the Poincare group [17]. The magnetic analogue to mass is spin [7, 18]. The spin of the graviton is 2, its rest mass is zero. The spin of the tordion is 3, its rest mass is of the order of the Planck mass [7, 18].

5. Composites of Magnetic Monopoles

5.1. Introduction

Magnetic monopoles were suggested to describe electricity and magnetism equivalently. Dirac was able to show that electric charge can appear only in discrete units if magnetic charges exist [2]. I presented a very simple proof for this Dirac quantization condition for a special case (ch. 3.2). A manifestly covariant quantum field theoretical description of Dirac magnetic monopoles requires the existence of the Salam magnetic photon [9], as I have shown some time ago [6]. By using this concept, I argued [7] that August Kundt has already observed an effect of the magnetic photon radiation [19]. I suggested a desktop experiment to verify the magnetic photon (ch. 3.5).

A consistent formulation of quantum field theory requires that the electric-magnetic duality is generalized to the other interactions.

This generalization requires the existence of new bosons which I named isomagnetic W and Z bosons and chromomagnetic gluons. I have shown that the conservation of baryon and lepton number is a consequence of this generalization. In the same chapter I predicted the quantum numbers of both the fermionic and bosonic Dirac magnetic monopoles (ch. 4).

Here I will show that Dirac magnetic monopoles usually do not appear as free particles, but in bound states. This is similar to quarks and leptons which usually appear in bound states such as mesons, baryons, atoms, and molecules.

5.2. Definitions and Nomenclature

The content of this paper is new to readers. So it is necessary to list the new definitions and nomenclature first.

(i) Hanselons are defined as elementary fermionic Dirac magnetic monopoles with chromomagnetic color. I will denote them by H.

(ii) Neutral hanselons are defined as hanselons with zero magnetic charge $q = 0$. I will denote them by N^0.

(iii) Charged hanselons are defined as hanselons with (negative) unit magnetic charge $q = -g$. I will denote them by C^-.

(iv) Gretelons are defined as elementary fermionic Dirac magnetic monopoles with zero chromomagnetic color. I will denote them by G.

(v) Single charged gretelons are defined as gretelons with (positive) unit magnetic charge $q = +g$. I will denote them by S^+.

(vi) Double charged gretelons are defined as gretelons with twice the (positive) unit magnetic charge $q = +2g$. I will denote them by D^{++}.

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(vii) Red, green, and blue are defined as the chromoelectric colors of quarks and chromoelectric gluons. They are denoted by r, g, and b.

(viii) Huey, dewey, and louie are defined as the chromomagnetic colors of hanselons and chromomagnetic gluons. I will denote them by h, d, and l. – Huey, Dewey, and Louie are the nephews of the comic figure Donald Duck. They look identical and differ only by the color of their cap.

(ix) The octet of (the first generation of) fermionic Dirac magnetic monopoles is therefore denoted as N_h^0, N_d^0, N_l^0, C_h^-, C_d^-, C_l^-, S^+, and D^{++}.

(x) A pairon is defined as a bound state of a hanselon and an antihanselon. This is analogous to the meson which is a bound state of a quark and an antiquark. I will denote a pairon by P.

(xi) A triplon is defined as a bound state of three hanselons. This is analogous to the baryon which is a bound state of three quarks. I will denote a triplon by T. – In analogy to mesons and baryons, the hanselons of pairons and triplons are bound together by virtual chromomagnetic gluons.

(xii) Schwinger dyons [14] are defined as elementary particles which have both electric charge and magnetic charge.

(xiii) Iso-dyons are defined as elementary particles which have both isospin and magnetic isospin. All quarks, leptons, hanselons, and gretelons are iso-dyons (ch. 4.8).

(xiv) Chromo-dyons are defined as elementary particles which have both chromoelectric color and chromomagnetic color.

(xv) Mass-spin-dyons are defined as elementary particles which have both rest mass and spin. All quarks, leptons, hanselons, gretelons, W and Z bosons, isomagnetic W and Z bosons, and the tordion are mass-spin dyons.

5.3. Bound States

At zero temperature the electric coupling constant is

$$\alpha_E = e^2/4\pi \simeq 1/137.036 \tag{143}$$

At zero temperature the magnetic coupling constant is

$$\alpha_M = g^2/4\pi \simeq 308.331 \tag{144}$$

Both coupling constants are running coupling constants and I expect that they become of order unity at the Planck scale (Planck temperature) (ch. 3.7).

Nevertheless, the huge value of α_M would lead to negative energy densities unless the rest masses of the free hanselons and gretelons are of the order of the Planck mass (ch. 3.7).

Because of the large coupling constant the (negative) binding energies are expected to be huge. As a consequence, the rest masses of chromomagnetically neutral pairons and triplons can be expected to be much smaller than the rest masses of their constituents (hanselons).

Pairons and triplons are the analogues to mesons and baryons. The nuclear force between baryons is mediated by the exchange of virtual mesons. This nuclear force is a remnant of the chromoelectric interaction between quarks which is mediated by the exchange of virtual chromoelectric gluons.

Analogously, one can expect a magnetic nuclear force between triplons which is mediated by the exchange of virtual pairons. This force is a remnant of the chromomagnetic interaction between hanselons which is mediated by the exchange of virtual chromomagnetic gluons.

Baryons which consist of the first generation of quarks (up and down) appear as isospin multiplets which have nearly the same rest mass. Proton and neutron have approximately the same rest mass. The four Delta hyperons Δ^{++}, Δ^+, Δ^0, and Δ^- have approximately the same rest mass. This results from the fact that the rest masses of their constituents (up and down quarks) are much smaller than the energy of the virtual chromoelectric gluons.

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By contrast, the rest masses of the neutral hanselon N^0 and the charged hanselon C^- (of the first generation of hanselons) can be quite different. This mass difference might be larger than the energy of the virtual chromomagnetic gluons.

So magnetic isospin multiplets of triplons need not have approximately the same rest masses. The multiplet T^{---}, T^{--}, T^-, T^0 of triplons consists of the hanselons $C^-C^-C^-$, $C^-C^-N^0$, $C^-N^0N^0$, $N^0N^0N^0$. The repulsive magnetic field between two or more C^- causes a large T^{---} rest mass.

Because of the large coupling constant α_M hanselons, gretelons, and magnetically charged pairons and triplons are unlikely to exist as free particles. They should be bound to magnetically neutral atoms. Examples are $T^{---}S^+D^{++}$, $T^{--}D^{++}$, $T^{--}S^+S^+$, T^-S^+, and $T^-T^-D^{++}$. Because of the large binding energies these atoms should be much lighter than the pairons and triplons.

It would not be surprising if these atoms could be bound to molecules. Because of the binding energies, these molecules should be lighter than the atoms they consists of. It remains the task for future researchers to find out the properties of this Dirac magnetic monopole chemistry.

5.4. Cold Dark Matter

There appear to exist much more baryons than antibaryons in the universe. So the baryon number of the universe appears to be huge ($B \sim 10^{78}$ within the Hubble sphere).

However, hanselons have lepton number $L = 1$ and gretelons have baryon number $B = 1$ (ch. 4.8). If there exist more antigretelons than gretelons in the universe, then the baryon number of the universe could be zero.

This requires that there exist as many antigretelons as baryons. The rest masses of free gretelons are expected to be of the order of the Planck mass (ch. 3.7, ch. 4.8). So they should not exist as free particles but be bound in atoms and molecules. Cosmological observations set upper limits for the masses of these atoms and molecules. The cold dark matter [20] content of the universe is approximately 23% of the critical mass of the universe, whereas the baryonic matter content is only 4% of the critical mass of the universe. So if there are indeed as many antigretelons as baryons, then the mass of the lightest Dirac magnetic monopole atoms and molecules cannot be more than 6 GeV per antigretelon which they consist of. For example, if T^-S^+ is the lightest of these atoms or molecules, then its mass would be 6 GeV.

5.5. Summary

In this chapter I have introduced new particles (pairons and triplons) and I have introduced new symbols for the hanselons and gretelons whose quantum numbers I have determined in ch. 4. I have shown that hanselons can form bound states (pairons and triplons) which are analogues to mesons and baryons. Moreover I have argued that triplons and gretelons can form atoms and molecules. These atoms and molecules may be the major component of cold dark matter.

6. Dark Matter WIMPs

6.1. Introduction

In ch. 5 I have shown that Dirac magnetic monopoles usually do not appear as free particles, but in bound states. This is similar to quarks and leptons which usually appear in bound states such as mesons, baryons, atoms, and molecules.

Here I will calculate the mass and the interaction cross-section of the lightest of these bound states of Dirac magnetic monopoles (BSoDMM). I will show that both are compatible with the excessive events observed in the PandaX-4T experiment.

6.2. Mass of BSoDMM

I have shown that Dirac magnetic monopoles do not appear as free elementary particles, but in bound states. The lightest of these BSoDMM consists of one anti-gretelon (with spin 1/2, isospin $I_3 = +1/2$, baryon number $B = -1$, and lepton number $L = 0$) and one anti-triplon which itself consists of three anti-hanselons (with spin 1/2, $I_3 = -1/2$, $B = 0$, and $L = -1$). As any orbital spin is

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an integer, the quantum numbers of the lightest BSoDMM are therefore $I_3 = -1$, $B = -1$, $L = -3$, and integer spin.

Baryon number is conserved. So the total baryon number of the universe should be zero. This can be satisfied if there exist as many baryons as anti-gretelons (and therefore BSoDMM) in the universe. If these BSoDMM are the major component of cold dark matter, then their rest mass is

$$m_\chi = m_N \Omega_c / \Omega_b \tag{145}$$

Here

$$m_N \simeq 0.938 \text{GeV}/c^2 \tag{146}$$

is the mean mass of a baryon. Strictly, m_N is the ratio of the total mass of the atoms in the universe to the number of the nucleons in the universe. It is in essential a function of the mass of a hydrogen atom, the mass of a helium atom, and the helium fraction of the universe. Moreover

$$\Omega_c = 0.1200(12)h^{-2} \tag{147}$$

is the ratio of the cold dark matter mass density to the critical mass density of the universe and

$$\Omega_b = 0.02237(15)h^{-2} \tag{148}$$

is the ratio of the baryon mass density to the critical mass density of the universe, where h denotes the Hubble constant in units of 100 km $\text{s}^{-1}\text{Mpc}^{-1}$. Hence, the mass of a BSoDMM is

$$m_\chi = (5.03 \pm 0.10) \text{GeV}/c^2 \tag{149}$$

6.3. Cross-Section of BSoDMM

BSoDMM have nonzero isospin. So they can interact with conventional matter by the neutral current of the weak interaction. BSoDMM are weakly interacting massive particles (WIMPs).

The low-energy limit of the Weinberg-Salam theory gives the ratio of the neutral current cross-section σ_Z to the charged current cross-section σ_W of the elastic scattering by the weak interaction,

$$\sigma_Z / \sigma_W \simeq \frac{1}{2} - \sin^2 \Theta_W \tag{150}$$

where the experimental value for the Weinberg angle Θ_W is

$$\sin^2 \Theta_W \simeq 0.23 \tag{151}$$

In the same low-energy limit the cross-section is

$$\sigma_W = \frac{4}{\pi} G_F^2 p^2 c^2 / (\hbar c)^4 \tag{152}$$

Here

$$G_F / (\hbar c)^3 \simeq 1.166 \times 10^{-5} \text{GeV}^{-2} \tag{153}$$

denotes the Fermi constant, where

$$\hbar c \simeq 1.973 \times 10^{-14} \text{GeV cm} \tag{154}$$

$$c \simeq 2.998 \times 10^{10} \text{cm/s} \tag{155}$$

The square of the momentum of the BSoDMM of the Galactic halo relative to a terrestrial laboratory is given by

$$p^2 = m_\chi^2 v^2 \tag{156}$$

The velocity of the sun around the Galactic center is

$$v_\odot = (233 \pm 9)\text{km/s} \tag{157}$$

As the BSoDMM of the Galactic halo have nonzero velocity relative to the Galactic center, the mean square v^2 of the BSoDMM velocity is probably larger than $v_\odot^2$, provided that the rotation velocity of the Galactic halo is not too large.

Under the assumption that $v^2 = v_\odot^2$ the calculation above gives

$$\sigma_W = (1.02 \pm 0.15) \times 10^{-42}\text{cm}^2 \tag{158}$$
$$\sigma_Z = (2.7 \pm 0.4) \times 10^{-43}\text{cm}^2 \tag{159}$$

σ_Z is the weak interaction cross-section of Galactic halo BSoDMM with conventional matter in the terrestrial laboratory.

6.4. Prediction for a Xenon Target

The de Broglie wavelength of a BSoDMM of rest mass m_χ and speed $v = v_\odot$ is

$$\lambda = \frac{2\pi\hbar}{m_\chi v} \simeq 3.171 \times 10^{-11}\text{cm} \tag{160}$$

Therefore, these BSoDMM interact rather with entire atomic nuclei than with their individual constituents (protons and neutrons). In atomic nuclei the isospins of protons and neutrons partially compensate one another. So the isospin of an atomic nucleus is proportional to $A - 2Z$.

The atomic weight of xenon is $A \simeq 131.29$ and the number of protons is $Z = 54$, so the isospin per mass is proportional to

$$(A - 2Z)/A \simeq 0.1774 \tag{161}$$

The number N of non-compensated neutrons in a xenon target of mass M is

$$N \simeq M \times 1.068 \times 10^{29}/\text{ton} \tag{162}$$

If t denotes the exposure time, then the number of weak interactions between Galactic halo BSoDMM and a xenon target of mass M is

$$n = \sigma_Z v \varrho N t / m_\chi \tag{163}$$

where

$$\varrho = (0.35 \pm 0.05)(\text{GeV}/c^2)\text{cm}^{-3} \tag{164}$$

is the canonical value of the local dark matter density. For an exposure

$$Mt = 1.54\text{ton} \times \text{year} \tag{165}$$

we get

$$n = 2.3 \pm 0.8 \tag{166}$$

expected events. (With regard to the uncertainties of its derivation, the numerical value of n should not be taken too seriously.)

6.5. Dark Matter WIMP Experiments

In 1977, Lee and Weinberg [21] suggested that the cosmological dark matter consists of weakly interacting massive particles (WIMPs). This idea found much interest after Peebles [20] recognized that WIMPs (and cold dark matter in general) are required by the gravitational instability theory for the formation of the large-scale structure of the universe.

A number of research groups have used xenon targets in order to search for WIMPs.

The LUX-ZEPLIN Collaboration [22] had an exposure of $Mt \simeq 0.9$ ton yr, but searched for masses $m_\chi \geq 9$ GeV/c^2 only.

The XENON Collaboration [23] had an exposure of $Mt \simeq 1.09$ ton yr, but searched for masses $m_\chi \geq 6$ GeV/c^2 only.

The PandaX-4T Collaboration [24] had an exposure of $Mt \simeq 0.63$ ton yr for its commissioning run.

It renamed as the PandaX Collaboration [25] and had an exposure of $Mt \simeq 1.54$ ton yr for its commissioning run and its first science run combined. They searched for masses $m_\chi \geq 5$ GeV/c^2. Indeed they reported on an excess of $n = 4.3$ events above the background with a best fit WIMP mass of $m_\chi \simeq 6$ GeV/c^2.

6.6. Outlook

My prediction of $n = 2.3 \pm 0.8$ dark matter BSoDMM events with mass $m_\chi = (5.03 \pm 0.10)$GeV/c^2 is compatible with the PandaX observation of $n = 4.3$ WIMP events with mass $m_\chi \simeq 6$ GeV/c^2.

The lightest BSoDMM have integer spin, isospin $I_3 = -1$, baryon number $B = -1$, and lepton number $L = -3$. So they can be distinguished from other hypothetical cold dark matter candidates.

(i) Heavy (sterile) neutrinos have spin $\pm 1/2$, $I_3 = 0$ or $I_3 = \pm 1/2$, $B = 0$, and $L = \pm 1$.
(ii) Neutralinos have spin $\pm 1/2$, $I_3 = 0$, $B = 0$, and $L = 0$.
(iii) Gravitinos have spin $\pm 3/2$, $I_3 = 0$, $B = 0$, and $L = 0$.
(iv) Z′ bosons have spin ± 1, $I_3 = 0$, $B = 0$, and $L = 0$.
(v) Axions have spin 0, $I_3 = 0$, $B = 0$, and $L = 0$.

6.7. Summary

I have shown that Dirac magnetic monopoles appear in bound states and that these bound states have the quantum numbers isospin $I_3 = -1$, baryon number $B = -1$, lepton number $L = -3$, and integer spin. If they are the major components of cold dark matter, then the mass of these bound states is $m_\chi = (5.03 \pm 0.10)$GeV/c^2. I have argued that they had already been observed by the PandaX Collaboration. By examining their spin and isospin, one can distinguish these bound states of Dirac magnetic monopoles from other WIMPs and dark matter candidates.

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