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Article

On the Thermodynamic Foundations of Spacetime and Gravity

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Abstract

We introduce a theory in which gravity emerges as a thermodynamic phenomenon governed by a scalar field that sets the local rate of quantum evolution. Building on Jacobson's thermodynamic derivation of Einstein's equations and Verlinde's entropic gravity, this framework extends these ideas into a unified theory of spacetime thermodynamics. In the strong-field limit it reproduces General Relativity, while in weak-field and low-density environments it predicts modified gravitational dynamics that account for galaxy rotation curves and galaxy cluster mass discrepancies without invoking particle dark matter. On cosmological scales, the theory predicts an early epoch of emergent inflation without an inflaton field and a late-time evolving accelerating expansion driven by the gradual depletion of vacuum thermodynamic capacity, implying cyclic cosmic evolution. From first principles, the framework yields parameter-free predictions for the Hubble constant and the present matter density consistent with observations. We confront the theory with Pantheon+, Cosmic Chronometers, DESI DR2 BAO, and the CMB angular scale θ_* , and find that it provides a statistically preferred description of the data relative to Λ CDM, with $\Delta\text{BIC} \simeq -18.5$, resolving the Hubble Tension as an artifact of thermodynamic evolution. These results indicate that a thermodynamic origin of gravity and spacetime offers a coherent explanation of gravitational and cosmological phenomena.

Keywords: general relativity; MOND; Λ CDM; thermodynamics; entropy; quantum gravity; dark matter; dark energy; inflation; Hubble tension

1. Introduction

General Relativity and its cosmological extension, the Λ CDM model, represent a monumental achievement in modern physics. For over a century, this framework has successfully described the large-scale structure and evolution of the universe. Yet, this success is built upon two enigmatic pillars: dark matter and dark energy. These components, which together are said to constitute 95% of the universe's energy budget, remain entirely outside the Standard Model of particle physics. They are placeholders invoked to reconcile theory with observation, but they lack a fundamental physical explanation, forcing the model into a descriptive rather than explanatory role.

In recent years, this foundational uncertainty has been compounded by a series of observational crises. A persistent and sharpening discrepancy in measurements of the Hubble constant—the Hubble Tension [1]—now casts doubt on our understanding of the cosmic expansion itself. Simultaneously, the James Webb Space Telescope (JWST) is revealing galaxies in the dawn of time that are far too massive and mature to have formed within the standard model's timeline [2,3], presenting a significant challenge to our theories of structure formation. The Dark Energy Spectroscopic Instrument (DESI) has further reported evidence that dark energy may be evolving over cosmic time [4,5], contradicting the static cosmological constant of Λ CDM.

Together, these tensions suggest that dark matter and dark energy may not be missing substances but indications of a deeper principle—one rooted in the fundamental connection between spacetime, energy, and gravity.

1.1. Time, Gravity and Thermodynamics

The path to this theory may be illuminated by two insights of our current models. The first insight comes from Einstein's Special and General Relativity: motion, gravity, and time are deeply interconnected. Time dilation shows that mass-energy slows the passage of time. The local rate of time is set by the energy density at that point. This effect may not be a consequence of gravity, but its very origin.

The second insight arises from the intersection of gravity, quantum theory, and thermodynamics. In 1967, Andrei Sakharov [6] proposed the concept of induced gravity, in which spacetime curvature emerges as the macroscopic elasticity of the quantum vacuum, similar to the fluid mechanics. Richard Feynman similarly speculated that spacetime geometry could have a thermodynamic origin, with gravitational dynamics arising from statistical behavior at the microscopic level. Stephen Hawking's discovery of black hole radiation [7] and Jacob Bekenstein's formulation of black hole entropy [8] revealed a deep and unexpected thermodynamic structure underlying gravitational phenomena. In 1995, Theodore Jacobson demonstrated [9] that the Einstein field equations themselves can be derived from the Clausius relation, $\delta Q = TdS$, applied to local Rindler horizons, framing General Relativity as an equation of state. More recently, Erik Verlinde [10] has advanced the view that gravity is an entropic force emerging from the statistical behavior of microscopic information. In parallel, Mordehai Milgrom's Modified Newtonian Dynamics (MOND) [11,12], uncovered a characteristic acceleration scale, a_0 , which not only accounts for galactic rotation curves without dark matter but also intriguingly matches cosmological parameters such as the Hubble constant ($a_0 \sim cH_0$) [13].

These insights hints that gravitational anomalies may be thermodynamic or informational in origin, rather than the result of unseen matter or arbitrary modifications of Einstein's equations.

In this paper, we introduce the theory of gravity by modeling spacetime as a thermodynamic medium, grounded on the foundational principles of the invariance of the speed of light and the conservation of energy. Based on these axioms, we introduce two foundational principles. First, the Time-Energy Equivalence Principle posits that all forms of energy are manifestations of a single, underlying scalar field whose value represents the local rate of time. Second, the Law of Entropy Equilibrium states that the universe maintains a constant total entropy, dynamically balancing the disordering drive of heat (S_{chaos}) against the ordering drive of gravity (S_{order}).

From these two foundational principles, this paper constructs a complete theory of emergent gravity. With single thermodynamic response function, we will demonstrate that MOND emerges naturally from the thermodynamics of causal horizons, leading to a set of covariant field equations that are shown to be consistent with all classical tests of General Relativity. These equations provide a unified physical mechanism for galactic and galaxy clusters dynamics and evolving cosmic acceleration, eliminating the need for dark matter and a cosmological constant. We then reveal the shared quantum origin of the MOND acceleration scale (a_0) and the Hubble constant (H_0), deriving both from the first principles of quantum field theory and calculating their values in close agreement with observation. Subsequently, we develop the cosmological implications of the theory, showing how it provides a natural mechanism for an early inflationary epoch and predicts a cyclic universe. Next, we will show how this framework resolves a host of puzzles within the standard model, including the Hubble tension. Finally, the theory makes several falsifiable predictions, which can be tested with future observations.

2. The Time-Energy Equivalence Principle

This theory is built upon the foundational idea that gravity is not a fundamental force, but an emergent thermodynamic phenomenon. To build this case, we first recall the analogy proposed by Jacobson [9] in his derivation of the Einstein field equations from thermodynamics.

He likened gravitational waves in spacetime to sound waves in a gas—collective, statistical phenomena rather than fundamental fields. Just as the equations of sound describe the thermodynamics of countless molecular collisions, the Einstein equations may describe the statistical behavior of a deeper

medium. In this view, spacetime geometry is a macroscopic, emergent observable, with Einstein's field equations serving as an equation of state valid only under local thermodynamic equilibrium. Inspired by this, we take the next step: identifying the underlying medium as a scalar field whose defining property is the very flow of time.

To define a new spacetime medium, we start by reexamining Einstein's own equivalence principle. It states that the effects of gravity are locally indistinguishable from acceleration. From this, a universal connection emerges: acceleration causes time dilation (as seen in Special Relativity); gravity, being equivalent to acceleration, must therefore also cause time dilation; mass is the source of gravity, so it is fundamentally a source of time dilation; radiation, being a form of energy ($E = mc^2$), also gravitates and is therefore also a source of time dilation.

This reveals a universal rule: acceleration, mass, and radiation are all linked by their common ability to alter the local rate of time. This theory elevates this connection to a foundational principle: *The presence of any form of energy density within a region of spacetime is physically indistinguishable and equivalent to a localized slowing of the rate of time in that region.*

Also, the speed of light, c , is a universal constant for all observers. For c to be an intrinsic property of the universe, spacetime cannot be a passive void but must be a dynamic medium whose properties determine the speed of propagation. We call this medium the Timeflow field and it's modeled as a fundamental complex scalar field:

$$\Psi(x^\mu) = A(x^\mu)e^{i\phi(x^\mu)} \quad (1)$$

The physical phenomena of time and space emerge from the dynamics of this field's two scalar components: its amplitude A and its phase ϕ .

- Phase (ϕ): The Quantum "Tick" of Time.

The phase ϕ is a scalar field, and its gradient defines a fundamental 4-vector, the Timeflow 4-vector: $T_\mu(x^\mu) \equiv \partial_\mu \phi(x^\mu)$. This vector covariantly represents the flow of quantum phase in spacetime. The invariant scalar magnitude of this vector defines the local microscopic frequency, ω :

$$\omega(x^\mu) \equiv \sqrt{g^{\mu\nu}(x^\mu)T_\mu(x^\mu)T_\nu(x^\mu)} \quad (2)$$

This frequency ω is a true scalar, invariant for all observers. This scalar is proportional to the local energy density. A high energy density implies a large ω , which in turn manifests as a slow macroscopic passage of time. The duration of a macroscopic proper time interval ($d\tau$) is proportional to the period of this invariant oscillation: $d\tau \propto 1/\omega$.

- Amplitude (A): The Structural Fabric of Spacetime.

The amplitude of the field, $A(x^\mu) = |\Psi(x^\mu)|$, is a true scalar that represents the local integrity or density of the spacetime medium itself. A high concentration of energy excites the field to a higher amplitude A . We postulate that this amplitude defines the local spatial scale. The length of a macroscopic ruler (dL) is inversely proportional to the amplitude: $dL \propto 1/A$.

Now, we can state the core postulate of the field's dynamics. The invariance of the speed of light c is a fundamental property of the metric $g_{\mu\nu}$. Our theory postulates that this metric is constructed from Ψ such that the field's two scalar properties, ω and A , are dynamically locked in a constant ratio:

$$\frac{\omega(x^\mu)}{A(x^\mu)} = c \quad (3)$$

This ensures that time dilation (driven by ω) and length contraction (driven by A) are not independent phenomena, but are two inseparable and perfectly synchronized consequences of the field's response to energy.

The microscopic frequency corresponds to a slowing of macroscopic time. The perceived rate of time, which we call the Timeflow factor \mathcal{T} , is therefore inversely proportional to this underlying invari-

ant frequency. We define the dimensionless $\mathcal{T}(x^\mu)$ as the local macroscopic rate of time, normalized to the rate in a vacuum where the microscopic frequency is at its minimum ground-state value, ω_0 :

$$\mathcal{T}(x^\mu) \equiv \frac{\omega_0}{\omega(x^\mu)} \quad (4)$$

In this view, the continuous passage of time is an emergent property. In empty space, the field is in its low-energy ground state ($\omega = \omega_0$), and the rate of time is at its maximum ($\mathcal{T} = 1$). In a region of high energy, the field oscillates rapidly ($\omega > \omega_0$), which manifests as time flowing more slowly ($\mathcal{T} < 1$). The field's amplitude, $A(x^\mu)$, similarly defines the local spatial scale, such that the constancy of the speed of light emerges as a fundamental symmetry of the field, reflecting the invariant ratio between its microscopic frequency and amplitude Equation (3).

To define the potential of this field, let's note the recurring role of the natural logarithm in linking microscopic properties to macroscopic potentials in physics. In statistical mechanics, the entropy of a system is given by Boltzmann's formula, $S = k_B \ln \Omega$, where the logarithm connects the number of microscopic states (Ω) to the macroscopic entropy.

Following this principle, we define the Timeflow thermodynamic potential, $\chi(x^\mu)$, as the natural logarithm of the Timeflow field, as this provides the most natural connection between the underlying field and its macroscopic potential:

$$\chi(x^\mu) \equiv \ln(\mathcal{T}(x^\mu)) \quad (5)$$

The mathematical expression of the total energy-momentum tensor of the universe could be expressed as the energy-momentum tensor of the Timeflow Potential field itself:

$$T_{\mu\nu}^{(\text{total})} \equiv T_{\mu\nu}^{(\chi)} \quad (6)$$

This implies that what we perceive as "matter" and "spacetime" are not separate entities, but different manifestations of this single, underlying field. For the purpose of constructing a workable physical model, we decompose the total energy of the Timeflow field into two distinct components:

1. The Matter Component ($T_{\mu\nu}^{(m)}$): This represents the energy of ordered, self-sustaining, localized configurations of the Timeflow field. These highly-condensed, persistent "solitons" are what we observe as particles of matter.
2. The Vacuum Component ($T_{\mu\nu}^{(v)}$): This represents the energy of the field's potential and gradients between the localized matter configurations. It is the energy of the medium itself—the "elastic tension" of the field as it is stretched and compressed by the presence of matter. This is the component responsible for what is traditionally perceived as vacuum energy.

Therefore, our foundational principle can be expressed as a decomposition:

$$T_{\mu\nu}^{(\text{total})} = T_{\mu\nu}^{(\chi)} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(v)} \quad (7)$$

The terms $T_{\mu\nu}^{(m)}$ and $T_{\mu\nu}^{(v)}$ (where v now refers to the vacuum/background component) are not fundamentally different substances, but a practical separation of the total field energy into its condensed and diffuse parts. From this perspective, all energy in the universe is ultimately a manifestation of a single conserved quantity: the rate at which the quantum state of the universe evolves.

3. The Law of Entropy Equilibrium

If Timeflow is a quantum field, it should obey the laws of thermodynamics. The First Law of Thermodynamics states that energy cannot be created or destroyed, but only transferred or transformed from one form to another. The classical Second Law of Thermodynamics asserts that the entropy of an isolated system cannot decrease [14].

We consider the entire universe as a single, isolated thermodynamic system. The total energy content of this system is described by the Timeflow field. According to the First Law, the change in the internal energy of an isolated system is zero ($\Delta U = 0$).

The First Law is expressed as:

$$\Delta U = Q - W \quad (8)$$

where Q is the heat added to the system and W is the work done by the system. This leads to a fundamental statement of energy balance:

$$Q = W \quad (9)$$

This implies that over any period of cosmic evolution, the total heat energy absorbed or redistributed within the system must equal the total work done by the system. If the Timeflow field represents all energy in the universe and satisfies the most fundamental First Law universally, the Second Law could be satisfied only if heat and work are balanced.

The Second Law is experimentally valid for thermodynamic systems in which gravitational effects are negligible. In such contexts, energy flows primarily through heat and radiation, and the law correctly describes the tendency of energy to spread, equilibrate, and become less available for work.

However, as emphasized by Roger Penrose [15], gravity fundamentally alters the entropy dynamics of the universe. Unlike thermal processes, gravitational interaction concentrates energy, reduces local entropy, and creates ordered configurations. In the interplay of heat and gravity, the universe does not evolve monotonically toward disorder. Instead, it is governed by a dynamic balance between two opposing entropic drives:

- The Drive for Chaos ($S_{\text{chaos}} > 0$): This is the standard thermodynamic entropy of the system, representing the field's kinetic energy, or "heat." It is a measure of the microscopic disorder associated with the field's temporal oscillations. This disorder is fundamentally quantified by the microscopic frequency (ω) of the Timeflow field. A higher frequency corresponds to more rapid phase evolution, and thus higher entropy. The physically consistent definition, following the Boltzmann framework, is:

$$S_{\text{chaos}} \equiv k_B \ln\left(\frac{\omega}{\omega_0}\right) \quad (10)$$

where ω_0 is the ground-state frequency of the vacuum. The relationship to heat is given by the Clausius relation:

$$\delta Q = T dS_{\text{chaos}} \quad (11)$$

- The Drive for Order ($S_{\text{order}} < 0$): This is the negentropy of the system, representing the field's potential energy stored in gravitational structures. It is a measure of structural information ($I_{\text{structure}}$) and order. This order is fundamentally quantified by the microscopic amplitude (A) of the Timeflow field. A higher amplitude corresponds to a more energetic, gravitationally bound, and structurally ordered state. The definition is therefore:

$$S_{\text{order}} \equiv -k_B \cdot I_{\text{structure}} = -k_B \ln\left(\frac{A}{A_0}\right) \quad (12)$$

where A_0 is the ground-state amplitude of the vacuum. The work done to create this structure is given by:

$$\delta W = -T dS_{\text{order}} \quad (13)$$

The negative sign signifies that as the system does positive work to build structure ($\delta W > 0$), its entropy, S_{order} , must decrease, signifying a greater degree of order.

We then calculate the total entropy that must obey this structural rule. The total entropy is the sum of the chaotic and orderly components:

$$S_{\text{total}} = S_{\text{chaos}} + S_{\text{order}} \quad (14)$$

Substituting the definitions for chaos and order entropy:

$$S_{\text{total}} = k_B \ln\left(\frac{\omega}{\omega_0}\right) - k_B \ln\left(\frac{A}{A_0}\right) \quad (15)$$

Using the properties of logarithms, we can combine these terms:

$$S_{\text{total}} = k_B \ln\left(\frac{\omega/\omega_0}{A/A_0}\right) = k_B \ln\left(\frac{\omega}{A} \cdot \frac{A_0}{\omega_0}\right) \quad (16)$$

Now, we apply the invariance of the speed of light axiom from Eq. Equation (3). Since $\omega/A = \omega_0/A_0$, the term inside the logarithm is identically 1:

$$S_{\text{total}} = k_B \ln\left(\frac{\omega_0}{A_0} \cdot \frac{A_0}{\omega_0}\right) = k_B \ln(1) \quad (17)$$

This yields the final result:

$$S_{\text{total}} = 0 \quad (18)$$

Therefore, to satisfy the First Law of Thermodynamics and the invariance of the speed of light, we propose a more fundamental principle: *The total entropy of the universe is dynamically balanced by an opposing process of structuring and dissipation, such that the sum of chaos and order entropy remains zero.*

$$S_{\text{total}} = S_{\text{chaos}} + S_{\text{order}} = 0 \quad (19)$$

The dynamics of this evolution are governed by a single, universal non-linear response function, $\mu(x)$, which dictates how the field responds to gradients in both space and time. This response is characterized by a single fundamental constant, the Timeflow critical temperature, T_0 .

This non-linear symmetry provides an explanation of why ordered energy configurations (S_{order}) have an inherent tendency to evolve towards greater complexity, progressing through a hierarchy of emergent structures. In this picture, the growth of complexity is not a violation of thermodynamics, but rather a manifestation of the balance condition, where increases in local order are compensated by corresponding increases in surrounding chaos, preserving the total entropy.

With this framework we can make a deeper explanation for the observed arrow of time and the Second Law. The relentless tendency of gravity to form structure means that the structural information of the universe is a non-decreasing function of time.

The law of entropy equilibrium is thus stated as an equality between the thermodynamic entropy of the universe, S_{chaos} , and the total information encoded in its gravitational structure, $I_{\text{structure}}$. Substituting these definitions into the equilibrium law Equation (19) yields:

$$S_{\text{chaos}} = k_B \cdot I_{\text{structure}} \quad (20)$$

This directly implies the classical Second Law of Thermodynamics as an emergent consequence of structure formation.

$$\frac{dI_{\text{structure}}}{dt} \geq 0 \quad \implies \quad \frac{dS_{\text{chaos}}}{dt} \geq 0$$

The observed increase in thermodynamic entropy is thus driven by the continuous writing of information into the cosmic web by gravity.

4. Entropy Equilibrium Function

To define the dynamics of entropy equilibrium, we begin with the First Law of Thermodynamics applied to the total energy content of the Timeflow field. The internal energy density is conserved and

is composed of two distinct thermodynamic contributions: the work done to create structure and the heat dissipated into the vacuum.

$$E_{\text{total}} = W + Q \quad (21)$$

We identify these thermodynamic quantities with the components of the total stress-energy tensor $T_{\mu\nu}^{(\chi)}$ defined in Eq. 7:

Work (W): represents the energy expended to create stable, ordered gravitational structures (solitons). This corresponds to the matter component ($T_{\mu\nu}^{(m)}$). From the definition of order entropy Equation (13), the work is given by the energetic cost of negentropy:

$$W \equiv E_{\text{order}} \propto T_{00}^{(m)} = - \int T dS_{\text{order}} = T |S_{\text{order}}| \quad (22)$$

(Since $S_{\text{order}} < 0$, the energy stored in structure is positive).

Heat (Q): represents the energy existing as microscopic fluctuations in the field background. This corresponds to the vacuum component ($T_{\mu\nu}^{(v)}$). From the Clausius relation, the heat is given by the vacuum entropy:

$$Q \equiv E_{\text{chaos}} \propto T_{00}^{(v)} = \int T dS_{\text{chaos}} = TS_{\text{chaos}} \quad (23)$$

We now define the Entropy Equilibrium function μ as the thermodynamic efficiency of the spacetime medium. It represents the fraction of the total field energy that performs useful Work (Matter) rather than dissipating as Heat (Vacuum).

$$\mu \equiv \frac{W}{E_{\text{total}}} = \frac{W}{W + Q} \quad (24)$$

Substituting the Clausius relations derived above:

$$\mu = \frac{T |S_{\text{order}}|}{T (|S_{\text{order}}| + S_{\text{chaos}})} \quad (25)$$

Since the temperature T cancels out we can also express this purely in terms of entropy:

$$\mu = \frac{|S_{\text{order}}|}{|S_{\text{order}}| + S_{\text{chaos}}} \quad (26)$$

4.1. The Wave-Mechanical Formulation

This thermodynamic partition finds its microscopic origin in the wave mechanics of the Timeflow field. The field Ψ at any point in spacetime is a quantum superposition of the ordered signal (Matter) and the background noise (Vacuum).

$$\Psi_{\text{total}} = \Psi_{\text{order}} + \Psi_{\text{chaos}} \quad (27)$$

The vacuum is defined as the ground state of the field, oscillating at the fundamental frequency ω_0 with amplitude A_{vac} :

$$\Psi_{\text{chaos}}(t) = A_{\text{vac}} e^{-i\omega_0 t} \quad (28)$$

The intensity of this wave represents the irreducible background energy density of the vacuum (Q):

$$|\Psi_{\text{chaos}}|^2 \equiv \rho_{\text{chaos}} \quad (29)$$

Matter represents a localized excitation of the field, carrying structural information. The intensity of this wave represents the local energy density of mass-energy (W):

$$|\Psi_{\text{order}}|^2 \equiv \rho_{\text{order}} \quad (30)$$

The total energy density of the system is determined by the square of the total superposition. This naturally includes an interference term dependent on the relative phase difference $\Delta\phi$ between the matter and vacuum states:

$$|\Psi_{\text{total}}|^2 = |\Psi_{\text{order}}|^2 + |\Psi_{\text{chaos}}|^2 + 2|\Psi_{\text{order}}||\Psi_{\text{chaos}}|\cos(\Delta\phi) \quad (31)$$

We can now re-derive the efficiency function μ purely in terms of wave mechanics. It is the ratio of the Ordered Intensity (Signal) to the Total Intensity (Superposition):

$$\mu = \frac{|\Psi_{\text{order}}|^2}{|\Psi_{\text{total}}|^2} = \frac{\rho_{\text{order}}}{\rho_{\text{order}} + \rho_{\text{chaos}} + 2\sqrt{\rho_{\text{order}}\rho_{\text{chaos}}}\cos(\Delta\phi)} \quad (32)$$

This definition closes the loop with the unified Timeflow field postulate Equation (7). The total stress-energy tensor $T_{\mu\nu}^{(\chi)}$ is simply the matter tensor renormalized by the local wave-mechanical state of the vacuum:

$$T_{\mu\nu}^{(\chi)} = \frac{T_{\mu\nu}^{(m)}}{\mu} \quad (33)$$

5. Temperature and Acceleration

The background "Drive for Chaos" is identified with the temperature of the cosmological event horizon. In an accelerating universe described by a Hubble constant H_0 , this horizon has a characteristic Gibbons-Hawking temperature.

The temperature of the de Sitter cosmological horizon is given by:

$$T_{GH} = \frac{\hbar c}{2\pi k_B R_H} \quad (34)$$

where R_H is the Hubble radius, $R_H \approx c/H_0$. We define the theory's Critical Temperature (T_0) as this fundamental background temperature.

$$T_0 \equiv T_{GH} = \frac{\hbar c}{2\pi k_B (c/H_0)} \quad (35)$$

This simplifies to a direct relationship between the critical temperature and the rate of cosmic expansion:

$$T_0 = \frac{\hbar H_0}{2\pi k_B} \quad (36)$$

The Unruh effect [16] establishes a fundamental equivalence between acceleration and temperature for any observer.

$$T_{\text{Unruh}} = \frac{\hbar a}{2\pi k_B c} \quad (37)$$

A simple equivalence of these two temperatures ($T_{\text{Unruh}} = T_{GH}$) would lead to the relation $a_0 = cH_0$. However, this fails to account for the crucial geometric difference between the two phenomena. The Unruh effect describes the temperature experienced by a linearly accelerating observer, while the Gibbons-Hawking temperature describes the thermal properties of a spherically symmetric cosmic horizon.

To determine the critical acceleration a_0 , we must couple the local Unruh effect to the global boundary conditions of the universe. In Quantum Field Theory, a thermal state is characterized by periodicity in imaginary (Euclidean) time. For a system bounded by a horizon, this periodicity corresponds to a fundamental wavelength determined by the size of the causal boundary. For the cosmological horizon, the fundamental vacuum wavelength is therefore given by the horizon circumference:

$$\lambda_{\text{vac}} = 2\pi R_H \quad (38)$$

The vacuum ground state is a standing wave resonance of the Timeflow field bounded by this horizon. The acceleration a_0 corresponds to the minimum energy excitation that fits this boundary condition. The characteristic Rindler length $L_a = c^2/a$ of this minimum acceleration must match the fundamental wavelength of the vacuum:

$$L_{a_0} = \lambda_{vac} \quad (39)$$

Substituting the geometric definitions ($L_{a_0} = c^2/a_0$ and $R_H = c/H_0$):

$$\frac{c^2}{a_0} = 2\pi \left(\frac{c}{H_0} \right) \quad (40)$$

Solving for a_0 , we derive the MOND acceleration scale as a geometric consequence of the field's confinement:

$$a_0 = \frac{cH_0}{2\pi} \quad (41)$$

This establishes the 2π factor as a result of the cyclic topology of the vacuum boundary.

This background energy density provides a universal "floor" value. Any gravitational interaction with an acceleration significantly above this floor behaves classically Newtonian, while any interaction that falls to the level of this floor enters the modified (MOND) regime.

6. Emergence of MOND

Having defined the Entropy Equilibrium function μ in terms of thermodynamic quantities, and established the fundamental acceleration scales via the Unruh and Gibbons-Hawking temperatures, we now derive the explicit form of the gravitational response function.

To connect entropy terms $|S_{\text{order}}|$ and S_{chaos} to kinematic observables, we apply the Holographic Principle. The active degrees of freedom (entropy capacity) on a causal horizon are excited by the energy of the horizon's thermal bath. Assuming the system is in local thermodynamic equilibrium (equipartition), the entropy magnitude is proportional to the characteristic temperature:

$$|S| \propto k_B T \quad (42)$$

The ordered entropy corresponds to the information accessible to a local observer accelerating through the field. This is determined by the local Unruh temperature, which is driven by the total effective acceleration a :

$$|S_{\text{order}}| \propto T_{\text{Unruh}} = \frac{\hbar a}{2\pi k_B c} \quad (43)$$

The chaotic entropy corresponds to the irreducible information background of the universe. This is determined by the Critical Vacuum Temperature (T_0), which acts as a thermal floor driven by the fundamental acceleration a_0 :

$$S_{\text{chaos}} \propto T_0 = \frac{\hbar a_0}{2\pi k_B c} \quad (44)$$

We substitute these kinematic identifications into the definition of the Entropy Equilibrium function (Eq. 26). The physical constants (\hbar, c, k_B) cancel out, leaving a pure ratio of temperatures:

$$\mu = \frac{|S_{\text{order}}|}{|S_{\text{order}}| + S_{\text{chaos}}} = \frac{T_{\text{Unruh}}}{T_{\text{Unruh}} + T_0} \quad (45)$$

Substituting the accelerations:

$$\mu \left(\frac{a}{a_0} \right) = \frac{a}{a + a_0} = \frac{a/a_0}{1 + a/a_0} \quad (46)$$

Surprisingly, this function looks exactly like simple MOND interpolating function.

The effective change in the horizon's entropy, dS_{eff} , is the standard Bekenstein-Hawking entropy change [8], [7], dS_{BH} , multiplied by the fraction of degrees of freedom available to participate in the thermodynamic exchange:

$$dS_{eff} = dS_{BH} \cdot \mu\left(\frac{a}{a_0}\right) \quad (47)$$

Applying the Clausius relation, $\delta Q = TdS$, to the horizon:

$$\delta Q_{source} = T_{eff} \cdot dS_{eff} = T_{eff} \cdot dS_{BH} \cdot \mu\left(\frac{a}{a_0}\right) \quad (48)$$

From the Verlinde's thermodynamic gravity derivation [10], we know that the heat flow from the matter source, δQ_{source} , is what generates the Newtonian force, F_N . We also know that the product of the effective temperature and the standard entropy change, $T_{eff} \cdot dS_{BH}$, generates the effective force that the observer feels, F_{eff} . Therefore, the thermodynamic relation simplifies to a direct relation between the forces:

$$F_N = F_{eff} \cdot \mu\left(\frac{a}{a_0}\right) \quad (49)$$

Dividing by the test mass m , we arrive at the fundamental relationship between the Newtonian gravitational field (g_N) and the effective field (a), we find exactly a MOND formula:

$$g_N = a \cdot \mu\left(\frac{a}{a_0}\right) \quad (50)$$

The force law in Eq. Equation (50) implicitly defines the effective gravitational coupling, G_{eff} . By writing the fields in terms of the source mass M and distance R :

$$g_N = \frac{GM}{R^2} \quad \text{and} \quad a = \frac{G_{eff}M}{R^2} \quad (51)$$

Substituting these into the force law:

$$\frac{GM}{R^2} = \frac{G_{eff}M}{R^2} \cdot \mu\left(\frac{a}{a_0}\right) \quad (52)$$

Solving for G_{eff} yields its final form:

$$G_{eff} = \frac{G}{\mu(a/a_0)} \quad (53)$$

Here we can see that this non-linear modification to gravity is inevitable consequence of the Law of Entropy Equilibrium Equation (19).

7. Field Equations

Having derived the effective gravitational coupling G_{eff} , we now generalize this result to a covariant theory. We achieve this by applying our derived G_{eff} to the thermodynamic argument of Jacobson [9], which shows that the Einstein Field Equations are the equation of state for spacetime.

Jacobson's derivation considers an arbitrary point in spacetime and the local Rindler horizon perceived by an accelerating observer. The Clausius relation, $\delta Q = TdS$, is applied to this local horizon.

The heat flux δQ is identified with the energy of matter crossing the horizon, as given by the flux of the matter stress-energy tensor, $T_{\mu\nu}^{(m)}$. The temperature T is the local Unruh temperature. The entropy change dS is assumed to be proportional to the change in the horizon's area, dA , with the standard gravitational constant G as the proportionality factor: $dS = \frac{k_B c^3}{4\hbar G} dA$.

Demanding that this relation holds for all local observers forces the geometry of spacetime to obey the Einstein Field Equations: $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{(m)}$.

This theory modifies this derivation at the final step. The entropy of the horizon is not classical; it is determined by the fraction of available microstates, μ Equation (26).

The thermodynamic law we enforce is therefore:

$$\delta Q = T dS_{eff} = T d\left(\mu \cdot \frac{k_B c^3}{4\hbar G} A\right) \quad (54)$$

When this modified law is required to hold for all observers, the resulting equation of state for spacetime is no longer the standard Einstein Field Equation. Instead, it becomes a modified field equation where the coupling between geometry and matter is state-dependent:

$$G_{\mu\nu} = \frac{8\pi G}{\mu \cdot c^4} T_{\mu\nu}^{(m)} \quad (55)$$

This can be rearranged into a more suggestive form, which we take as the static thermodynamical equation of gravity of the theory:

$$\mu \cdot G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{(m)} \quad (56)$$

Here, the argument of μ is implicitly a function of the local geometry, as the effective entropy is determined by the curvature of spacetime.

To better understand the physical content of this equation, we can express it in the standard form of General Relativity by defining an effective stress-energy tensor for the vacuum part of total energy of Timeflow field, $T_{\mu\nu}^{(\chi)}$ Equation (7), such that:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu}^{(\chi)}\right) = \frac{8\pi G}{c^4} \left(T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(v)}\right) \quad (57)$$

By comparing this with Equation (56), we can directly solve for $T_{\mu\nu}^{(v)}$:

$$T_{\mu\nu}^{(v)} = \left(\frac{1-\mu}{\mu}\right) T_{\mu\nu}^{(m)} \quad (58)$$

Alternatively, substituting $T_{\mu\nu}^{(m)}$, we find its expression in terms of pure geometry:

$$T_{\mu\nu}^{(v)} = \frac{c^4}{8\pi G} (1-\mu) G_{\mu\nu} \quad (59)$$

This tensor represents the energy, momentum, and pressure of the effective scalar degree of freedom. The conservation of this tensor, $\nabla^\mu T_{\mu\nu}^{(v)} = 0$, is guaranteed by the Bianchi identity [17] ($\nabla^\mu G_{\mu\nu} \equiv 0$) and the conservation of matter ($\nabla^\mu T_{\mu\nu}^{(m)} = 0$), leading to the same consistency condition as before:

$$(\nabla^\mu \mu) G_{\mu\nu} = 0 \quad (60)$$

This completes the derivation of the full covariant framework.

8. Consistency with General Relativity

The Timeflow Gravity (TG) field equation Equation (56) modifies the Einstein Field Equations by introducing the state-dependent thermodynamic function $\mu(a/a_0)$. A crucial requirement for any viable theory of gravity is that it must reproduce the well-tested predictions of General Relativity (GR) in the appropriate limit, particularly within the Solar System.

The consistency of TG with GR is ensured by the behavior of the μ function in the high-acceleration regime. The behavior of the theory is determined by the ratio $x = a/a_0$, where a is the local gravitational acceleration and $a_0 \approx 1.2 \times 10^{-10} \text{ m/s}^2$ is the critical acceleration scale. The Solar System is a high-acceleration environment ($a \gg a_0$, or $x \gg 1$).

The derived interpolating function Equation (46) is:

$$\mu(x) = \frac{x}{1+x} \quad (61)$$

In the strong-field limit, the function rapidly approaches unity:

$$\lim_{x \rightarrow \infty} \mu(x) = 1 \quad (62)$$

Consequently, the effective gravitational constant converges to the Newtonian constant G :

$$\lim_{a \gg a_0} G_{eff} = G \quad (63)$$

In the high-acceleration limit, the Timeflow Gravity field equations become indistinguishable from the standard Einstein Field Equations of General Relativity.

We can quantify the deviation from GR by examining the effective gravitational constant G_{eff} . In the high-acceleration regime ($x \gg 1$):

$$G_{eff} = G \frac{1+x}{x} = G \left(1 + \frac{1}{x}\right) = G \left(1 + \frac{a_0}{a}\right) \quad (64)$$

The relative deviation from GR is characterized by the small parameter $\epsilon = a_0/a$. We evaluate this deviation for the classical tests of GR.

1. Perihelion Precession of Mercury: This test probes the gravitational field at Mercury's orbit. The Sun's acceleration at this distance is approximately $a_M \approx 0.0396 \text{ m/s}^2$. The corresponding ratio is:

$$x_M = \frac{a_M}{a_0} \approx \frac{0.0396}{1.2 \times 10^{-10}} \approx 3.3 \times 10^8 \quad (65)$$

The deviation from GR is extremely small: $\epsilon_M = 1/x_M \approx 3.0 \times 10^{-9}$.

2. Deflection of Light and Shapiro Delay: These tests probe the field near the surface of the Sun, where the acceleration is $a_{Sun} \approx 274 \text{ m/s}^2$. The corresponding ratio is:

$$x_{Sun} = \frac{a_{Sun}}{a_0} \approx \frac{274}{1.2 \times 10^{-10}} \approx 2.28 \times 10^{12} \quad (66)$$

The deviation from GR is negligible: $\epsilon_{Sun} = 1/x_{Sun} \approx 4.4 \times 10^{-13}$.

This demonstrates that TG naturally satisfies all Solar System constraints. The modifications to gravity proposed by this theory only become significant in the ultra-low acceleration regimes found on galactic and cosmological scales, ensuring consistency with established gravitational experiments in high-density environments.

9. Galaxy Rotation

The true power of this framework lies in its ability to generate MOND-like phenomenology from its core principles. Here we derive the weak-field limit, explain the crucial role of the environment, and show how the theory naturally accounts for its own applicability at different scales.

We begin with the derived equation Equation (56). Our goal is to find the non-relativistic, static limit of this equation, which must be consistent with the thermodynamically derived force law relating the effective acceleration (\vec{a}) to the Newtonian acceleration sourced by baryonic matter (\vec{g}_N):

$$\vec{g}_N = \vec{a} \cdot \mu \left(\frac{|\vec{a}|}{a_0} \right) \quad (67)$$

By expressing these accelerations as gradients of their respective potentials ($\vec{a} = -\nabla\Phi$ and $\vec{g}_N = -\nabla\Phi_N$) and taking the divergence of the entire equation, we can formulate a field theory. Using the standard Poisson equation, $\nabla \cdot \vec{g}_N = -4\pi G\rho_m$, we arrive at:

$$\nabla \cdot \left[\mu \left(\frac{|\nabla\Phi|}{a_0} \right) \nabla\Phi \right] = 4\pi G\rho_m \quad (68)$$

This is the well-known AQUAL [18] formulation of MOND. It demonstrates that the complex covariant field equations of Timeflow Gravity naturally reduce to the successful MOND phenomenology in the appropriate limit.

A key prediction of this theory, arising from the non-linearity of Equation (68), is the External Field Effect (EFE) [19]. This effect states that the internal dynamics of a gravitational system depend not only on its internal mass but also on the background gravitational field in which it is embedded.

Consider a small dwarf galaxy with low internal acceleration, \vec{g}_{int} , orbiting a massive host galaxy that provides a large, nearly constant external field, \vec{g}_{ext} . The total acceleration experienced by a star in the dwarf is $\vec{g}_{total} = \vec{g}_{int} + \vec{g}_{ext}$. The behavior of the μ function is determined by the magnitude of this total field, $|\vec{g}_{total}|$.

If the external field is strong such that $|\vec{g}_{ext}| \gg a_0$, then it follows that $|\vec{g}_{total}| \gg a_0$, even if the internal acceleration is weak ($|\vec{g}_{int}| \ll a_0$). In this scenario, the entire system is forced into the high-acceleration regime where $\mu \approx 1$. Consequently, the internal dynamics of the dwarf galaxy will be purely Newtonian, and no MONDian effects (i.e., no need for dark matter) would be observed. The MONDian enhancement of gravity is reduced by the strong external field. This explains why some satellite galaxies appear to have less dark matter than expected—they are simply in a regime where the modified dynamics are suppressed by their environment.

9.1. The Tully-Fisher Relation and RAR

The Timeflow framework, by naturally producing MOND-like dynamics in the weak-field limit, provides a direct theoretical origin for the observed scaling laws in spiral galaxies: the Baryonic Tully-Fisher Relation (BTFR) [20] and the Radial Acceleration Relation (RAR) [21]. These empirical laws are inevitable consequences of the theory's fundamental force law.

The BTFR is the observed tight correlation between a galaxy's total baryonic mass (M_b) and its asymptotic flat rotation velocity (v_f), empirically found to be $M_b \propto v_f^4$. In the Timeflow theory, this arises directly from the deep-MOND limit.

For a test particle (like a star) in a stable circular orbit at a large radius in a galaxy, the effective gravitational acceleration is given by the centripetal acceleration, $a = v_f^2/r$. In the outer regions of galaxies, this acceleration is far below the critical scale, $a \ll a_0$. In this low-acceleration regime, our derived force law from Equation (50) simplifies to:

$$g_N \approx \frac{a^2}{a_0} \quad (69)$$

The Newtonian gravitational field, g_N , is sourced by the total baryonic mass of the galaxy, $g_N = GM_b/r^2$. Equating the two expressions for g_N :

$$\frac{GM_b}{r^2} = \frac{(v_f^2/r)^2}{a_0} = \frac{v_f^4}{a_0 r^2} \quad (70)$$

The radial distance term, r^2 , cancels from both sides, leaving a direct relationship between the galaxy's mass and its flat rotation velocity:

$$GM_b = \frac{v_f^4}{a_0} \implies M_b = \frac{1}{Ga_0} v_f^4 \quad (71)$$

This is precisely the Baryonic Tully-Fisher Relation, with the correct fourth-power dependence. The theory thus provides a first-principles explanation for this fundamental law of galactic dynamics.

The RAR is the tight empirical correlation observed between the acceleration felt by stars (g_{obs}) and the acceleration that can be accounted for by the visible baryonic matter alone (g_N). This relation is a direct restatement of the theory's central force law.

By identifying the effective acceleration (a) with the total observed acceleration (g_{obs}), and the Newtonian acceleration (g_N) with the acceleration sourced by baryons, our fundamental force law from Equation (50) can be written as:

$$g_N = g_{obs} \cdot \mu \left(\frac{g_{obs}}{a_0} \right) \quad (72)$$

This equation provides a unique, one-to-one mapping between the baryonic acceleration and the observed acceleration, which is exactly what the RAR shows. The specific functional form of $\mu(x) = x/(1+x)$ derived in Equation (46) dictates the precise shape of the RAR, which has been shown to be an excellent fit to observational data. The RAR is therefore the most direct observational evidence for the non-linear thermodynamic response of the Timeflow field that underpins the entire theory.

10. Unifying Galaxies and Clusters

A longstanding challenge for modified gravity theories is the "Cluster Problem." While theories like MOND successfully predict galactic rotation curves, they typically underpredict the dynamical mass of galaxy clusters, often requiring residual dark matter.

The Timeflow Gravity framework resolves this by identifying the wave-mechanics mechanism behind the Entropy Equilibrium function μ Equation (32). Therefore, the observational difference between Galaxies and Clusters is identified as a phase transition between coherent and thermal wave dynamics.

10.1. The Coherent Phase (Galaxies)

Rotationally supported systems (Spiral Galaxies). In dynamically "cold" systems, the motion of stars and gas is ordered. The matter wavefunction maintains a constant phase relationship ($\Delta\phi \approx \text{const}$) with the vacuum background.

In the low-acceleration regime ($|\Psi_{order}| \ll |\Psi_{chaos}|$), the interference term dominates. Assuming constructive interference driven by the ordering principle:

$$|\Psi_{tot}|^2 \approx 2|\Psi_{order}||\Psi_{chaos}| \quad (73)$$

Substituting this into Equation (32), the efficiency scales as the square root of the intensity ratio:

$$\mu_{coh} \approx \frac{|\Psi_{order}|^2}{2|\Psi_{order}||\Psi_{chaos}|} \propto \sqrt{\frac{g_N}{a_0}} \quad (74)$$

This yields the MOND force law:

$$g_{obs} = \frac{g_N}{\mu} \propto \sqrt{g_N a_0} \quad (75)$$

This coherent response produces the characteristic $1/r$ falloff of the potential (since $g \propto 1/r$), naturally generating the flat rotation curves observed in spiral galaxies.

10.2. The Thermal Phase (Clusters)

Pressure supported systems (Galaxy Clusters) are dynamically "hot," with high velocity dispersion ($\sigma_v \sim 1000$ km/s) and no dominant rotational plane. This chaotic motion causes rapid phase fluctuations, decoupling the matter from the vacuum.

The interference term averages to zero ($\langle \cos \Delta\phi \rangle = 0$). The total energy is the result of incoherent addition:

$$|\Psi_{tot}|^2 = |\Psi_{order}|^2 + |\Psi_{chaos}|^2 \quad (76)$$

In the low-acceleration regime, the vacuum term dominates ($|\Psi_{tot}|^2 \approx |\Psi_{chaos}|^2$). The efficiency becomes:

$$\mu_{therm} \approx \frac{|\Psi_{order}|^2}{|\Psi_{chaos}|^2} \propto \frac{g_N}{a_0} \quad (77)$$

Since $g_{obs} = g_N/\mu$, the field becomes approximately constant:

$$g_{obs} \approx a_0 \quad (78)$$

A constant acceleration field a_0 implies a gravitational source that grows with area ($g \propto M/r^2 \implies M \propto r^2$). We interpret this as the vacuum acting as a fluid with a fundamental surface density Σ_{vac} , derived from the critical acceleration:

$$\Sigma_{vac} \equiv \frac{a_0}{2\pi G} \approx 0.286 \text{ kg/m}^2 \quad (79)$$

We define the holographic mass, M_{holo} , as this vacuum density integrated over the projected area of the cluster's virial sphere ($A = \pi R_{vir}^2$). The total effective mass is:

$$M_{eff} = M_{bar} + \Sigma_{vac}(\pi R_{vir}^2) = M_{bar} + \frac{a_0 R_{vir}^2}{2G} \quad (80)$$

This linear addition of mass distinguishes clusters from the MONDian square-root amplification of galaxies.

We test this prediction against the Coma Cluster, a typical rich cluster with a virial radius of $R_{vir} \approx 2 \text{ Mpc} (\approx 6.17 \times 10^{22} \text{ m})$. The predicted Holographic Mass is:

$$\begin{aligned} M_{holo} &\approx 0.286 \text{ kg/m}^2 \times \pi(6.17 \times 10^{22} \text{ m})^2 \\ &\approx 3.4 \times 10^{45} \text{ kg} \\ &\approx 1.7 \times 10^{15} M_{\odot} \end{aligned}$$

This prediction matches the dynamically inferred mass of the Coma Cluster ($\sim 1 - 2 \times 10^{15} M_{\odot}$) without requiring any particle dark matter. The theory predicts that roughly 90% of the cluster's effective mass is this holographic vacuum contribution, consistent with the observed baryon fraction of $f_b \sim 0.1$.

Also, this phase-dependent response provides a natural resolution to the Bullet Cluster [22], [23], observations. In this system, the gravitational lensing center is spatially separated from the baryonic gas following a high-velocity collision. Within the Timeflow framework, this is a manifestation of a dynamic phase transition. During the collision, the intergalactic gas is shocked and randomized, transitioning into a high-entropy thermal phase where phase decoherence suppresses its gravitational amplification. Conversely, the stars and galaxies remain in a collisionless, coherent phase, maintaining their constructive interference with the vacuum. Consequently, the lensing signal—driven by phase coherence—follows the ordered baryonic component (stars) rather than the chaotic component (gas), producing the apparent separation without the need for dark matter particles.

11. Holographic Partition

To determine the global matter density Ω_m , we examine the partition of the Timeflow field's energy between its ordered (matter) and chaotic (vacuum) components. We start with the definition of the Entropy Equilibrium function μ Equation (26):

$$\mu = \frac{|S_{\text{order}}|}{|S_{\text{order}}| + S_{\text{chaos}}} = \frac{|S_{\text{order}}|}{S_{\text{total}}}$$

This function μ represents the thermodynamic efficiency of the spacetime medium—the fraction of the total field capacity that is successfully converted into ordered structure. Cosmologically, this corresponds directly to the matter density parameter: $\Omega_m \equiv \mu$.

We determine the values of these entropy terms from the topology of the complex field $\Psi(x) = Ae^{i\phi}$.

The total thermodynamic capacity of the field is defined by the complete gauge symmetry of the vacuum ground state. As a complex scalar field, the vacuum manifold is the unit circle (S^1) on the complex plane. The total information capacity is the complete path length of the thermal phase loop:

$$S_{\text{total}} \propto \oint_{S^1} |d\Psi| = 2\pi \quad (81)$$

This represents the maximum available degrees of freedom in the system ($|S_{\text{order}}| + S_{\text{chaos}}$).

The entropy of order corresponds to the formation of linear, causal structures (matter). Geometrically, this requires a linear translation of the field amplitude A across the phase space. The maximum linear structure that can be inscribed within the vacuum's fundamental domain is the diameter of the unit circle.

$$|S_{\text{order}}| \propto \int_{D^1} |dA| = 2 \quad (82)$$

This represents the maximum structural information the field can sustain before wrapping around the gauge symmetry.

Substituting these geometric measures into the efficiency equation:

$$\Omega_m = \mu = \frac{|S_{\text{order}}|}{S_{\text{total}}} = \frac{2}{2\pi} = \frac{1}{\pi} \quad (83)$$

This yields a purely theoretical prediction for the matter and vacuum density of the universe:

$$\Omega_m \approx 0.3183 \quad (84)$$

$$\Omega_{\text{vac}} = 1 - \frac{1}{\pi} \approx 0.6817 \quad (85)$$

This geometric prediction is in remarkable agreement with precision cosmological measurements. The Planck 2018 results [24] measured the matter density parameter to be:

$$\Omega_m^{\text{Planck}} = 0.315 \pm 0.007 \quad (86)$$

Our theoretically derived $\Omega_m = 1/\pi \approx 0.318$ lies well within the 1σ confidence interval of the empirical data. This resolves the "coincidence problem" [25] (why is the density of vacuum energy today of the same order of magnitude as the density of matter) and suggests that the Dark Energy is simply the necessary phase capacity required to support the linear amplitude of matter.

12. Deriving the a_0 and the H_0

The fundamental principle of this theory is that the dynamics of the universe are governed by a complex scalar field $\Psi(x^\mu)$. Therefore, macroscopic cosmological parameters, such as the Hubble

constant, and galactic dynamical scales, such as the MOND acceleration, are not arbitrary constants but emergent properties of the field's quantum stability.

We model the stabilization of the Ψ field as a quantum tunneling process from a primordial, unstable "false vacuum" to the stable true vacuum. The rate of this tunneling establishes the fundamental "tick" of the vacuum, ω_0 .

In non-perturbative Quantum Field Theory, the decay rate per unit volume for a vacuum transition is given by the Coleman–Callan formula [26]:

$$\Gamma = C e^{-S_E/\hbar} \quad (87)$$

where S_E is the Euclidean action of the instanton solution. In gauge theories, the action scales inversely with the coupling constant of the interaction, $S_E/\hbar \propto 1/\alpha$, where α is the fine-structure constant.

We must determine the prefactor C , which represents the effective flux of vacuum fluctuations. Dimensional analysis provides the fundamental rate: the vacuum fluctuates at the Planck frequency, $\omega_{Pl} = 1/t_{Pl}$. However, treating the vacuum as a perfect continuum implies an unphysically infinite energy density. We resolve this by treating the vacuum as a quantum dielectric.

The vacuum creates virtual particle–antiparticle pairs at the Planck rate $1/t_{Pl}$. For a virtual fluctuation to contribute to the global metric evolution (i.e., to become a "real" unit of proper time), it must cross the coherence threshold. This requires a self-interaction loop (vacuum polarization). In quantum electrodynamics (QED), the probability amplitude for a vacuum fluctuation to couple to the field is governed by the fine-structure constant α . Physically, α represents the "opacity" of the vacuum to its own fluctuations. Therefore, the effective flux C is the fundamental frequency scaled by the probability coefficient:

$$C = \omega_{Pl} \cdot P(\text{interaction}) = \frac{1}{t_{Pl}} \cdot \alpha. \quad (88)$$

Thus, the prefactor is derived from the dielectric properties of the quantum vacuum:

$$C = \frac{\alpha}{t_{Pl}}. \quad (89)$$

Combining the efficiency factor and the exponential suppression, we identify the fundamental frequency of the bare vacuum, ω_0 :

$$\omega_0 \equiv \frac{\alpha}{t_{Pl}} e^{-1/\alpha} \quad (90)$$

12.1. Calculating the H_0

This microscopic frequency manifests macroscopically as the expansion rate of the vacuum. As established in Equation (41), the conversion from the local frequency of a linear observer to the global frequency of a spherical horizon involves the geometric factor 2π . We therefore define the theoretical vacuum Hubble rate, H_{vac} , as the expansion rate of a universe dominated entirely by this scalar field potential (Ω_{vac}):

$$H_{vac} = 2\pi \cdot \omega_0 = \frac{2\pi\alpha}{t_{Pl}} e^{-1/\alpha} \quad (91)$$

We calculate the numerical values using the 2022 CODATA [27] values for the fundamental constants: $t_{Pl} \approx 5.391247 \times 10^{-44}$ s; $\alpha \approx 1/137.036$; $e^{-1/\alpha} \approx 3.062 \times 10^{-60}$.

From Equation (128), we first find H_0 in SI units:

$$\begin{aligned} H_0 &\approx \frac{2\pi \cdot (1/137.036)}{5.391247 \times 10^{-44} \text{ s}} \cdot (3.062 \times 10^{-60}) \\ &\approx (8.504 \times 10^{41} \text{ s}^{-1}) \cdot (3.062 \times 10^{-60}) \approx 2.604 \times 10^{-18} \text{ s}^{-1} \end{aligned}$$

Converting to standard cosmological units (1 Mpc $\approx 3.0857 \times 10^{19}$ km):

$$\begin{aligned} H_0 &\approx (2.604 \times 10^{-18} \text{ s}^{-1}) \cdot (3.0857 \times 10^{19} \text{ km/Mpc}) \\ &\approx 80.36 \text{ km/s/Mpc} \end{aligned}$$

This value represents the pure expansion rate of the vacuum in the absence of any load.

Our universe is not just vacuum, it contains matter. As derived in the holographic partition Equation (83), the vacuum energy density is geometrically restricted to a fraction of the total capacity:

$$\Omega_{\text{chaos}} = 1 - \frac{1}{\pi} \approx 0.6817 \quad (92)$$

According to the Friedmann relationship ($H \propto \sqrt{\rho}$), the observed expansion rate H_0 is the vacuum rate modulated by the square root of the active vacuum density. The presence of matter effectively acts as a "load" on the vacuum, scaling the expansion rate by the factor $\sqrt{\Omega_{\text{chaos}}}$:

$$H_0 = H_{\text{vac}} \sqrt{\Omega_{\text{chaos}}} \quad (93)$$

Substituting the values:

$$H_0 \approx 80.36 \times \sqrt{0.6817} \approx 80.36 \times 0.8256 \approx 66.35 \text{ km/s/Mpc} \quad (94)$$

This theoretical prediction is in excellent agreement with the Planck 2018 [24] CMB measurement of $H_0 = 67.4 \pm 0.5$ km/s/Mpc. It suggests that the tension between local ($H_0 \sim 73$) and cosmic ($H_0 \sim 67$) measurements may arise because local measurements in voids probe closer to the bare H_{vac} , while global measurements average over the matter-damped bulk.

12.2. Calculating the a_0

Finally, we address the galactic acceleration scale, a_0 . Unlike the expansion rate H_0 , which is a global average damped by matter density, the critical acceleration a_0 represents a fundamental phase transition threshold of the vacuum field itself. It is determined by the properties of the bare vacuum, not the matter-filled average.

Therefore, a_0 is derived directly from the unmodulated vacuum rate H_{vac} via the horizon resonance condition ($a_0 = cH_{\text{vac}}/2\pi$):

$$a_0 = c \cdot \omega_0 = \frac{c \cdot H_{\text{vac}}}{2\pi} \quad (95)$$

Substituting the calculated value of $H_{\text{vac}} \approx 80.36$ km/s/Mpc ($2.60 \times 10^{-18} \text{ s}^{-1}$):

$$a_0 \approx \frac{(3.00 \times 10^8) \cdot (2.60 \times 10^{-18})}{1} \approx 1.24 \times 10^{-10} \text{ m/s}^2 \quad (96)$$

This theoretical value provides a remarkable match with the empirical record. While early MOND calibrations often cited $a_0 \approx 1.2 \times 10^{-10} \text{ m/s}^2$, precision studies of gas-rich galaxies—which provide the cleanest test of the acceleration law due to their negligible stellar-mass uncertainties—have refined this constant. Specifically, McGaugh [28] utilized the Baryonic Tully-Fisher Relation (BTFR) to determine an empirical value of:

$$a_0 = 1.24 \pm 0.14 \times 10^{-10} \text{ m/s}^2 \quad (97)$$

The fact that Timeflow Gravity derives the value of 1.24 strictly from the geometry of the vacuum horizon and the speed of light (c) suggests that a_0 is the scale at which the matter-wave frequency matches the cosmic horizon frequency. This resolves the long-standing mystery of why $a_0 \approx cH_0$, identifying it as a fundamental frequency of the Timeflow field.

Therefore, starting from first principles of quantum vacuum decay, with this framework we can unify the galactic scale (a_0) and cosmological scale (H_0) from pure constants of nature (α, t_{pl}, π), showing that they both stem from the same underlying quantum process.

13. Friedmann Equation

In the standard cosmological model, the expansion of the universe is described by the time evolution of a scale factor $a(t)$, governed by the Friedmann equations [29]. This expansion is often framed as a stretching of space itself, but from the perspective of Timeflow Gravity, the phenomenon has a more direct interpretation: energy density, cosmic expansion, and time dilation are representations of the same thermodynamic reality.

Special Relativity teaches that energy, time, and space are inseparably linked. A moving clock runs slower relative to a stationary observer, and moving rulers contract. In this theory, we extend this principle to the cosmos: just as velocity in SR changes the relationship between time and space for a moving observer, cosmic energy density changes the relationship between time and space for the universe as a whole.

In this view, the scale factor $a(t)$ is simply the inverse of the global timeflow rate $\mathcal{T}(t)$. In a denser early universe, proper time ran more slowly everywhere. As the universe evolves, the changing energy density alters the global rate of time, which we perceive as the expansion of space. The principle is simple and intuitive: dark energy is like the "heat" generated by the "work" of cosmic matter. As the work dies down with cosmic evolution, the heat it produces must also dissipate. This is in stark contrast to Λ CDM, where dark energy is a constant background temperature that never changes.

Here we derive the cosmological field equations by applying the covariant TG framework to a homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) [30] spacetime.

$$ds^2 = -c^2 dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \quad (98)$$

This leads directly to the standard Friedmann constraint:

$$H^2 = \frac{8\pi G}{3} \rho_{\text{total}} \quad (99)$$

Based on the law of entropy equilibrium Equation (19), we define the total cosmic energy density as the incoherent sum of the bulk and boundary intensities:

$$\rho_{\text{total}}(a) = \rho_{\text{order}}(a) + \rho_{\text{chaos}}(a) \quad (100)$$

The "Order" component consists of the localized excitations of the field (matter and radiation) that exist within the bulk spatial volume. Their evolution is governed by standard 3D geometry. As the scale factor a increases, the volume V increases as a^3 . Thus, the density of the contents dilutes volumetrically (where the a^{-4} term for radiation includes redshift):

$$\rho_{\text{order}}(a) = \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} \quad (101)$$

The "Chaos" component represents the vacuum energy—the potential of the Timeflow field itself. In the standard Λ CDM model, this is assumed to be a constant (Ω_Λ). However, in our holographic partition model (Section 11), the vacuum is not an infinite constant source, but a finite cyclic reservoir.

Since the Timeflow field Ψ evolves linearly along the causal ray (1D), the depletion of the vacuum potential is linear with the scale factor. Using the partition ratio derived in Equation (83) ($\Omega_{\text{order}} = 1/\pi$), the vacuum density evolution is given by the depletion of the total capacity:

$$\rho_{\text{chaos}}(a) = 1 - a \cdot \Omega_{\text{order}} = 1 - \frac{a}{\pi} \quad (102)$$

Substituting these two components back into the normalized Friedmann relation ($H \propto \sqrt{\rho}$), we arrive at the master equation for cosmic evolution:

$$H(a) = H_0 \sqrt{\left(\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right) + \left(1 - \frac{a}{\pi}\right)} \quad (103)$$

This equation unifies the physics of the "contents" (matter scaling volumetrically) with the physics of the "container" (vacuum scaling linearly).

14. Emergent Inflation

One of the significant puzzles in modern cosmology is the "Horizon Problem"—the observation that distant regions of the universe have identical temperatures despite apparently never having been in causal contact. The standard solution is Cosmic Inflation, a hypothetical period of exponential expansion in the very early universe. In the standard model, this requires introducing a new, arbitrary scalar field (the "inflaton") and a specific potential to drive it.

In Timeflow Gravity, inflation is an emergent property of the vacuum dynamics at $a \approx 0$. It arises naturally from the master equation without invoking any new fields or parameters.

Consider the state of the universe at the onset of the cycle, prior to the formation of matter or radiation. In this "False Vacuum" state, the Order density is zero ($\Omega_m = \Omega_r = 0$).

Substituting these conditions into the master equation Equation (103):

$$H(a) = H_0 \sqrt{0 + \left(1 - \frac{a}{\pi}\right)} \quad (104)$$

In the limit where the universe is very small ($a \ll \pi$), the vacuum depletion term is negligible ($1 - a/\pi \approx 1$). The equation simplifies to:

$$H(a) \approx H_0 = \text{const} \quad (105)$$

A constant Hubble parameter, $H(t) = \dot{a}/a = H_0$, describes a de Sitter universe. The solution to this differential equation is an exponential function:

$$a(t) = a_i e^{H_0 t} \quad (106)$$

This result demonstrates that the default kinematic state of the Timeflow field—in the absence of matter—is exponential inflation. This phase solves the Horizon Problem naturally. Since the expansion is driven by the global vacuum potential, the entire volume is causally connected to the single quantum state of the field, establishing the uniform temperature observed in the CMB today.

In standard inflation, the expansion must stop via a "slow-roll" decay of the inflaton field. In this theory, the exit is triggered by the Quantum Tunneling event derived in Equation (90).

The exponential expansion continues until the probability of vacuum decay reaches unity. At this moment, the vacuum tunnels from the pure state to the matter-filled state. Energy is transferred from the vacuum potential (Chaos) to localized excitations (Order). Mathematically, this appears as the instantaneous injection of the radiation term Ω_r :

$$\Omega_r : 0 \rightarrow \Omega_{r,0} \quad (107)$$

At this moment, the a^{-4} term in the master equation instantly overwhelms the constant vacuum term. The expansion switches from exponential ($e^{H_0 t}$) to the power-law deceleration of the radiation era ($t^{1/2}$). This phase transition is what we observe as the Big Bang.

15. The Big Bang and Structure Formation

Following the emergent inflation of the Planck epoch, the universe enters a long phase dominated by the thermodynamics of its contents. In this regime ($0 < a \ll 1$), the "Order" density terms (Ω_r/a^4 and Ω_m/a^3) are effectively infinite compared to the vacuum background. The "Chaos" potential term ($1 - a/\pi \approx 1$) exists but is energetically negligible.

Immediately after the Big Bang ($a \ll 10^{-4}$), the energy density of the universe is dominated by relativistic particles (photons and neutrinos). In this limit, the radiation term Ω_r/a^4 overwhelms all other contributions in the master equation Equation (103):

$$\frac{\Omega_r}{a^4} \gg \frac{\Omega_m}{a^3} \gg \left(1 - \frac{a}{\pi}\right) \quad (108)$$

Consequently, the expansion rate simplifies to:

$$H(a) \approx H_0 \sqrt{\frac{\Omega_r}{a^4}} = \frac{H_0 \sqrt{\Omega_r}}{a^2} \quad (109)$$

This aligns with standard cosmology, predicting a rapid deceleration of the expansion. From the Timeflow perspective, this era represents the high-frequency regime. The energy density is so immense that the local Timeflow frequency ω is orders of magnitude higher than the vacuum baseline ω_0 . Space expands rapidly because the high energy density forces a rapid relaxation of the spacetime medium. Integrating $H = \dot{a}/a \propto a^{-2}$ yields the classic time-dependence:

$$a(t) \propto t^{1/2} \quad (110)$$

As the universe expands ($10^{-4} < a < 0.7$), the radiation density drops sharply (a^{-4}) due to the combined effects of volumetric dilution and redshift. The matter density, which dilutes only volumetrically (a^{-3}), eventually overtakes radiation. This crossover marks the beginning of the era of structure formation.

In this regime, the Friedmann equation Equation (103) simplifies to:

$$H(a) \approx H_0 \sqrt{\frac{\Omega_m}{a^3}} = \frac{H_0 \sqrt{\Omega_m}}{a^{1.5}} \quad (111)$$

This deceleration is gentler than the radiation phase. The integrated growth law becomes:

$$a(t) \propto t^{2/3} \quad (112)$$

This recovers the standard solution for the matter dominated era.

Crucially, during this epoch the "Chaos" term ($1 - a/\pi$) begins to play a subtle role. While still sub-dominant to matter, the vacuum potential is slowly depleting. In the standard Λ CDM model, the vacuum energy is a constant background that suddenly becomes relevant. In Timeflow Gravity, the vacuum energy is a dynamic reservoir that is being consumed by the expansion. The formation of galaxies and clusters occurs because the gravitational "Order" of matter is locally strong enough to overcome the global expansion. However, as a approaches unity, the matter density dilutes below the threshold of the background vacuum density, setting the stage for the current era of acceleration.

16. Evolving Expansion

As the universe expands toward the present scale ($a \approx 1$), the matter density dilutes sufficiently that it no longer dominates the energy budget. The "Chaos" term—the residual vacuum potential—emerges as the primary driver of cosmic evolution. This marks the transition from the decelerating, structure-forming universe to the accelerating, vacuum-dominated universe we observe today.

In the current epoch, the radiation contribution is negligible ($\Omega_r \approx 0$). The master equation Equation (103) is governed by the interplay between the diminishing matter density and the depleting vacuum potential:

$$H(a) \approx H_0 \sqrt{\frac{\Omega_m}{a^3} + \left(1 - \frac{a}{\pi}\right)} \quad (113)$$

In the standard Λ CDM model, Ω_m and Ω_Λ are free parameters determined by fitting observational data. In Timeflow Gravity, however, these values are fixed by the holographic partition Equation (83). Recall that the matter density is geometrically constrained to $\Omega_m = 1/\pi$. Substituting this constant into the equation yields a parameter-free prediction for the expansion history:

$$H(a) = H_0 \sqrt{\frac{1}{\pi a^3} + 1 - \frac{a}{\pi}} \quad (114)$$

The shape of the expansion curve $H(a)$ is entirely determined by the geometry of the cycle (π), with no adjustable parameters for Dark Energy or Matter density.

A key observational test for any dark energy model is the equation of state parameter, w , defined by the ratio of pressure to density: $w = P/\rho$. In the standard Λ CDM model, the vacuum is a cosmological constant, implying a fixed value of $w = -1$. In Timeflow Gravity, the vacuum density is dynamic: $\rho_{\text{vac}}(a) = 1 - a/\pi$. We can derive the effective $w(a)$ using the continuity equation for cosmic fluids:

$$\dot{\rho}_{\text{vac}} + 3H(\rho_{\text{vac}} + P_{\text{vac}}) = 0 \quad (115)$$

The time derivative of the density is:

$$\dot{\rho}_{\text{vac}} = \frac{d\rho_{\text{vac}}}{da} \cdot \frac{da}{dt} = \left(-\frac{1}{\pi}\right) \cdot (aH) \quad (116)$$

Substituting this into the continuity equation:

$$-\frac{aH}{\pi} + 3H(\rho_{\text{vac}} + P_{\text{vac}}) = 0 \quad (117)$$

Dividing by $3H$ and solving for pressure P_{vac} :

$$P_{\text{vac}} = \frac{a}{3\pi} - \rho_{\text{vac}} \quad (118)$$

The equation of state parameter $w(a) = P_{\text{vac}}/\rho_{\text{vac}}$ is thus:

$$w(a) = \frac{a/(3\pi)}{1 - a/\pi} - 1 = \frac{a}{3(\pi - a)} - 1 \quad (119)$$

This result yields a definitive prediction: Dark Energy is not constant. At $a \rightarrow 0$: $w \rightarrow -1$. The early vacuum behaves like a cosmological constant. At Present ($a = 1$): Substituting $\pi \approx 3.1416$:

$$w(1) = -1 + \frac{1}{3(\pi - 1)} \approx -1 + \frac{1}{6.42} \approx -0.84 \quad (120)$$

This result identifies the vacuum as a "thawing" quintessence field. The acceleration of the universe is less aggressive than a cosmological constant and is naturally braking as the cycle proceeds.

The theoretical prediction of $w(1) \approx -0.84$ provides a physical basis for the recent findings of evolving Dark Energy by the DESI Collaboration [5]. Their primary result using the DESI + CMB + Pantheon+ samples ($w_0 = -0.838 \pm 0.055$) is almost identical to the value derived here from the depletion of the vacuum. This suggests that what is being observed as a 'departure from Λ CDM' is actually the natural prediction of the TG framework

17. The Big Stop and Cyclic Universe

Standard cosmology offers a bleak "Heat Death" for the universe, where expansion continues eternally, eventually tearing apart all causal structures. Timeflow Gravity predicts a radically different fate. The presence of the geometric term $(1 - a/\pi)$ in the fundamental field equation implies that the universe is not an open system, but a bounded cycle.

As derived in the Friedmann equation Equation (103), the vacuum energy density is not constant but is a finite potential that depletes as the universe expands. This depletion imposes a geometric limit on the scale factor a .

$$\rho_{chaos}(a) = 1 - \frac{a}{\pi} \quad (121)$$

This density must remain non-negative for the field to remain physical. Consequently, the scale factor cannot exceed the critical value $a_{max} = \pi$.

$$a(t) \leq \pi \quad (122)$$

As $a \rightarrow \pi$, the vacuum energy—the engine of expansion—vanishes entirely. The unified Friedmann equation at this limit becomes:

$$H(a \rightarrow \pi) = H_0 \sqrt{\frac{\Omega_m}{\pi^3} + 0} \approx H_0 \sqrt{\frac{1/3.14}{31}} \approx 0.1H_0 \quad (123)$$

While there is a small residual inertial expansion due to matter, the capacity of the spacetime medium to support further linear extension is exhausted. The universe reaches its maximum volume and facing the "Big Stop".

Once the expansion halts ($\dot{a} = 0$), the dynamics are dominated by the gravitational attraction of the remaining structure and the negative pressure of the over-extended vacuum. The equation of motion reverses:

$$H(t) < 0 \quad (124)$$

The universe enters a contraction phase. As a decreases, the vacuum potential recharges $(1 - a/\pi)$ increases), and the matter density concentrates (Ω_m/a^3) .

This contraction leads inevitably to a "Big Crunch". As $a \rightarrow 0$, the energy density spikes, eventually reaching the Planck scale. In standard General Relativity, this results in a singularity. In Timeflow Gravity, however, the universe is treated as a quantum field.

When the contracting universe reaches the critical energy density of the false vacuum, the conditions for the quantum tunneling event derived in Equation (90) are re-established. The field is reset to its high-energy metastable state, back to the inflation.

Thus, the universe breathes in a self-sustaining cycle, driven eternally by the immutable geometry of the field itself.

18. Cosmological Tests

To ensure a rigorous comparison, we define the following models based on their respective treatments of the vacuum energy density and gravitational coupling.

1. Λ CDM (The Baseline): This model assumes a constant vacuum energy density (the cosmological constant, Λ). The normalized Hubble function is given by:

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda} \quad (125)$$

where $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$. It utilizes 5 free parameters in our fit: $\{H_0, \Omega_m, M, r_d, m_\nu\}$.

2. w CDM: This model relaxes the assumption of a constant vacuum energy, introducing the equation of state (EOS) parameter w . The vacuum density evolves as $\rho_{de} \propto (1+z)^{3(1+w)}$. It adds one

degree of freedom $\{w\}$, allowing us to test if the data prefers a "phantom" ($w < -1$) or "thawing" ($w > -1$) dark energy profile.

We define the normalized Hubble function $E(z) \equiv H(z)/H_0$ for the Timeflow model based on the master equation derived in Eq. 103. Including radiation (Ω_r) and the time-dependent vacuum density $\rho_{\text{vac}}(a) = 1 - a/\pi$, the expansion history is given by:

$$E(z)_{\text{TG}} = \sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{\text{vac}}(z)} \quad (126)$$

where the vacuum density evolves as:

$$\Omega_{\text{vac}}(z) = 1 - \frac{1}{\pi(1+z)} \quad (127)$$

We test four variations of this framework to evaluate its sensitivity to data and theory:

1. TG Free: This version treats H_0 and Ω_m as free parameters, identical to the Λ CDM fit. It tests whether the functional form of the TG modulation provides a better fit to the expansion history than the standard model when given the same statistical flexibility.

2. TG Theory: This is the most restrictive and economical version. All cosmological parameters are fixed to their theoretically derived values:

$$\Omega_{\text{order}} = \frac{1}{\pi} \approx 0.3183, \quad H_0 = H_{\text{vac}} \sqrt{1 - \Omega_{\text{order}}}, \quad H_{\text{vac}} = 2\pi \cdot \omega_0 = \frac{2\pi\alpha}{t_{\text{Pl}}} e^{-1/\alpha} \quad (128)$$

The only parameters varied are the nuisance variables $\{M, r_d, m_v\}$. A high performance in this model indicates that the theory's geometric priors align naturally with the physical state of the universe.

3. TG Late (Late-Universe Probe): To investigate the Hubble tension, this model is fitted exclusively to late-time datasets (Pantheon+ SN and Cosmic Chronometers). By excluding the BAO and CMB constraints, we determine the "local" preference of the TG expansion history.

4. TG CMB (High-Redshift Probe): This configuration focuses on the angular scale of the sound horizon θ_* . It fixes Ω_m to the theoretical value ($1/\pi$) and varies only H_0 to find the best fit for the CMB acoustic peaks. The comparison between H_0 in TG Late and TG CMB serves as a direct test of the model's ability to resolve the Hubble tension.

Table 1. **Parameter** Freedom and Vacuum Treatment Summary. This table categorizes the models used in this study based on their statistical degrees of freedom and theoretical assumptions regarding the vacuum. The Λ CDM and w CDM models serve as the standard empirical benchmarks using a static cosmological constant or a constant equation-of-state (w). The TG Free and TG Late/CMB models are used to test the sensitivity of the Timeflow Gravity (TG) framework when allowing specific parameters to vary against individual datasets (Supernovae/Cosmic Chronometers vs. the CMB). Finally, TG Theory represents the "Zero-Parameter" master equation, where all cosmological values are fixed to first-principles geometric identities derived from π and the fine-structure constant α .

Model	Free Cosmological Parameters	Vacuum Treatment
Λ CDM	$\{H_0, \Omega_m\}$	Constant (Λ)
w CDM	$\{H_0, \Omega_m, w\}$	Constant w
TG Free	$\{H_0, \Omega_m\}$	Dynamic $\mu(a)$ modulation
TG Theory	None (Fixed to π, α)	Dynamic $\mu(a)$ modulation
TG Late	$\{H_0, \Omega_m\}$	SN + CC Likelihood only
TG CMB	$\{H_0\}$	CMB θ_* Likelihood only

18.1. Datasets and Cosmological Parameters

The statistical viability of Timeflow Gravity is assessed through a joint likelihood analysis. Each dataset is mapped to the model via specific cosmological kernels derived from the normalized Hubble function $E(z)$ and the comoving distance $D_M(z)$.

18.1.1. Type Ia Supernovae: Pantheon+

We utilize the Pantheon+ compilation [31], which consists of 1,701 light curves corresponding to 1,550 unique Type Ia supernovae. For our cosmological constraints, we consider the redshift range $0.01 < z < 2.26$. The lower bound is implemented to minimize the impact of local peculiar velocities, which can introduce significant noise in the low-redshift expansion rate. The theoretical distance modulus μ_{th} is calculated via the luminosity distance $d_L(z)$:

$$\mu_{th}(z) = 5 \log_{10} \left[(1+z) \int_0^z \frac{c dz'}{H_0 E(z')} \right] + 25 + M \quad (129)$$

The likelihood χ_{SN}^2 is computed using the full 1701×1701 covariance matrix, which accounts for both statistical uncertainties and systematic correlations across different survey subsets.

18.1.2. Baryon Acoustic Oscillations: DESI DR2

The Baryon Acoustic Oscillations (BAO) provide a standard ruler calibrated by the sound horizon r_d at the end of the drag epoch. We implement the latest constraints from the DESI DR2 [5] sample, utilizing both isotropic and anisotropic measurements. The isotropic volume distance $D_V(z)$ is defined as:

$$D_V(z) = [z D_M^2(z) \frac{c}{H(z)}]^{1/3} \quad (130)$$

For anisotropic observations, we utilize the transverse comoving distance $D_M(z)$ and the Hubble distance $D_H(z) = c/H(z)$. The BAO χ^2 is calculated by comparing the observed ratios $\{D_V/r_d, D_M/r_d, D_H/r_d\}$ against the model predictions:

$$\chi_{BAO}^2 = \sum_{ij} \Delta y_i (C_{BAO}^{-1})_{ij} \Delta y_j \quad (131)$$

where Δy is the difference vector between the model and data, and C_{BAO} is the covariance matrix provided by the DESI collaboration, which accounts for the correlations between different distance markers at the same redshift.

18.1.3. Cosmic Chronometers (CC)

We utilize 30 independent measurements of the Hubble parameter $H(z)$ derived from the differential aging of passively evolving galaxies [32]. This method estimates the expansion rate directly via the relation:

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt} \quad (132)$$

Because this approach does not rely on the cosmic distance ladder or a specific pre-recombination sound horizon, it provides a crucial model-independent anchor for the $E(z)$ function. The contribution to the likelihood is computed as a diagonal χ^2 :

$$\chi_{CC}^2 = \sum_i \frac{[H_{obs}(z_i) - H_{model}(z_i)]^2}{\sigma_{H,i}^2} \quad (133)$$

18.1.4. CMB Angular Scale

To ensure compatibility with the early universe without performing a full CMB power spectrum analysis, we implement the angular scale of the sound horizon θ_* , which is one of the most precisely measured quantities in cosmology. It is defined as:

$$\theta_* = \frac{r_s(z_*)}{D_M(z_*)} \quad (134)$$

where $z_* \approx 1090$ is the redshift of last scattering. The sound horizon r_s is integrated from the early universe ($z \rightarrow \infty$) down to z_* using the sound speed $c_s(z) = c/\sqrt{3(1 + 3\rho_b/4\rho_\gamma)}$. The χ^2_{CMB} is then:

$$\chi^2_{\text{CMB}} = \frac{[\theta_{*,\text{model}} - \theta_{*,\text{obs}}]^2}{\sigma_\theta^2} \quad (135)$$

where we adopt the Planck 2018 [24] value $\theta_{*,\text{obs}} = 0.010411 \pm 0.000003$.

18.1.5. Model Selection

Given that the Timeflow Gravity variations range from two-parameter fits to zero-parameter theoretical predictions, simple χ^2 comparisons are insufficient for model selection. We utilize the Bayesian Information Criterion (BIC) to penalize models for unnecessary complexity:

$$BIC = k \ln(N) + \chi^2_{\text{min}} \quad (136)$$

where k is the number of free parameters and N is the number of data points. A model is considered statistically superior if it significantly reduces the χ^2 without a proportional increase in k .

According to the Kass and Raftery scale [33], a difference of $|\Delta BIC| > 10$ relative to the baseline (Λ CDM) constitutes decisive evidence in favor of the model with the lower score.

18.2. Results and Interpretation

The statistical results presented in Table 2 indicate a decisive shift in model preference when Information Criteria are applied. While the TG Free model provides the absolute lowest χ^2 (720.3), the TG Theory model represents the most robust description of the data. By fixing the matter density to the geometric prior $\Omega_m = 1/\pi$ and H_0 to the value derived from the Planck-scale vacuum thermodynamics, the model avoids the dimensionality penalty associated with free parameters. This results in a ΔBIC of -18.8 relative to Λ CDM, which constitutes decisive evidence for the TG framework.

Table 2. A comparative analysis across distance-ladder (Pantheon+ SN), expansion-geometry (BAO), and cosmic-clock (CC) datasets. Λ CDM and w CDM represent standard empirical baselines requiring several free parameters. TG Theory represents the zero-parameter solution, where H_0 and Ω_m are fixed to first-principles geometric identities ($66.35 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $1/\pi$, respectively). Notably, TG Theory achieves a ΔBIC of -18.8 relative to Λ CDM, providing "decisive evidence" in favor of the Timeflow framework under Bayesian model selection. While Λ CDM requires shifting H_0 toward ≈ 68 to minimize χ^2 , the TG variations demonstrate that a consistent $H_0 \approx 66.3$ can satisfy the late-universe expansion history and the CMB acoustic scale (θ_*) simultaneously, effectively resolving the Hubble tension through geometric phase modulation rather than parameter tuning.

Metric	Λ CDM	w CDM	TG Free	TG Theory	TG Late	TG CMB
H_0 ($\text{km s}^{-1} \text{ Mpc}^{-1}$)	67.73	66.59	66.23	66.35	66.29	67.04
Ω_m	0.307	0.302	0.303	0.318	0.302	0.318
$\sum m_\nu$ (eV)	0.139	0.000	0.002	0.136	N/A	N/A
r_d (Mpc)	148.8	149.0	149.4	147.4	N/A	N/A
Total χ^2	728.3	720.8	720.3	724.3	711.4	N/A
> SN χ^2	701.0	695.5	695.5	696.7	695.5	N/A
> BAO χ^2	12.0	9.6	9.0	11.4	N/A	N/A
> CC χ^2	15.3	15.7	15.9	16.2	15.9	N/A
BIC Score	765.3	765.2	757.3	746.5	N/A	N/A
ΔBIC (vs. Λ CDM)	0.0	-0.1	-8.0	-18.8	N/A	N/A
Age (Gyr)	13.82	13.90	13.95	13.73	13.95	13.59
q_0 (Deceleration)	-0.539	-0.444	-0.393	-0.363	-0.396	-0.363
$100\theta_*$	1.0314	1.0295	1.0272	1.0358	1.0269	1.0411

The analysis of the Hubble constant across different data subsets (TG Late and TG CMB) provides a resolution to the ongoing "Hubble tension" [1]. The value inferred from the CMB sector alone

($H_0 = 67.04$) and the value derived from local late-time probes ($H_0 = 66.29$) differ by approximately 1%, a significant reduction compared to the 5-6% tension found in the standard model. This suggests that the time-dependent modulation of the vacuum density, $\rho_{vac}(a)$, correctly accounts for the evolution of the expansion rate between the epoch of recombination and the present day.

Furthermore, the deceleration parameter $q_0 \approx -0.36$ in the TG Theory model represents a distinct departure from the Λ CDM value of -0.54 . This shallower acceleration is compensated by a slightly different early-universe expansion history, as evidenced by the $100\theta_*$ values. The proximity of the TG Theory's θ_* (1.0358) to the observed value (1.0411) without free-parameter adjustment suggests that the geometric coupling of the vacuum to the matter density captures a fundamental aspect of cosmic topology.

These results suggest that the Timeflow Gravity framework is a statistically preferred description of the available cosmological data.

19. Predictions

A scientific theory must be testable, and its predictions must be capable of being confronted with observation in a way that allows for clear falsification. The Timeflow Gravity framework leads to several distinct, quantitative predictions that differ from both Λ CDM and other modified gravity models. Here I highlight three such predictions that are particularly accessible to current and near-future observational programs.

19.1. The Environmental Hubble Constant

A central prediction of Timeflow Gravity is that the Hubble constant is not a fundamental constant of nature, but an environmental variable that depends on the local energy density of the region being measured.

As derived in the Holographic Partition, the active vacuum energy density (ρ_{chaos}) is complementary to the local matter density (ρ_{order}). The vacuum does not have a fixed value; it fills the capacity left unused by matter.

$$\Omega_{vac}^{local} = 1 - \Omega_m^{local} \quad (137)$$

Since the expansion rate is driven by the square root of the vacuum density Equation (93), it follows that the measured expansion rate must correlate inversely with the large-scale matter density of the environment.

$$H_{local} = H_{vac} \sqrt{1 - \Omega_m^{local}} \quad (138)$$

In voids ($\Omega_m \rightarrow 0$) the vacuum potential is nearly maximal. The expansion rate should approach the theoretical vacuum maximum ($H \approx H_{vac} \approx 80$ km/s/Mpc). In superclusters ($\Omega_m \rightarrow 1$): the vacuum potential is suppressed by the high matter density. The local expansion rate should be significantly lower than the global average.

This environmental dependence offers a natural resolution to the "Hubble Tension"—the persistent $4 - 6\sigma$ discrepancy between local measurements of H_0 (using Cepheids/Supernovae) and global measurements (using the CMB).

Consequently, Timeflow Gravity asserts that there is no "true" single value for H_0 . The discrepancy between the $73 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (local Cepheid/SN ladder) and the $67 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (global CMB) is a physical manifestation of the vacuum filling the capacity left unused by matter. The local measurement is a high-vacuum/low-load observation, while the CMB measurement is a global-average-load observation. Both are mathematically consistent within the TG framework, effectively moving the problem from a measurement crisis to a predicted feature of vacuum thermodynamics.

19.2. The Active Repulsion of Cosmic Voids

In Timeflow Gravity, the faster rate of time in underdense regions creates a dynamically enhanced repulsive gravitational effect. This "active repulsion" is a direct consequence of the entropy-equilibrium law, which leads to steeper curvature gradients across void boundaries. This predicts a measurable

increase in the weak gravitational lensing signal produced by cosmic voids compared to the standard Λ CDM model.

The enhancement arises from the theory's effective gravitational coupling Equation (53). Since the gravitational lensing potential is proportional to the gravitational constant, the lensing shear (γ) predicted by Timeflow Gravity (TG) is enhanced by a factor of $1/\mu$ relative to General Relativity (GR):

$$\gamma_{\text{TG}} \approx \frac{1}{\mu} \gamma_{\text{GR}} \quad (139)$$

The value of μ is determined by the total gravitational acceleration, which, due to the External Field Effect (EFE), is dominated by the background field from the cosmic web, g_{ext} , rather than the void's negligible internal field. For the typical range of external fields found in the environments of large voids, $g_{\text{ext}} \in [1a_0, 2a_0]$, we can calculate the predicted enhancement using the function $\mu(x) = x/(1+x)$, where $x = g_{\text{ext}}/a_0$.

For a stronger external field of $g_{\text{ext}} = 2a_0$, the enhancement factor is:

$$\frac{1}{\mu(2)} = \frac{1}{2/(1+2)} = 1.5 \quad (140)$$

For a weaker external field of $g_{\text{ext}} = 1a_0$, the enhancement factor is:

$$\frac{1}{\mu(1)} = \frac{1}{1/(1+1)} = 2.0 \quad (141)$$

Therefore, the theory makes a firm prediction of a 1.5x to 2.0x enhancement in the tangential shear amplitude produced by cosmic voids. This provides a clean observational test to distinguish Timeflow Gravity from Λ CDM, where no such environmental enhancement is expected.

19.3. State-Dependent Gravity

A defining feature of General Relativity is that gravity is state-independent; the internal arrangement of matter is irrelevant to its far-field flux (Gauss's Law). Timeflow Gravity challenges this paradigm by identifying gravitational dynamics as an entropic response governed by the phase coherence of the Timeflow field.

As shown in wave-mechanics approach Equation (32), we may associate gravitational amplification with the interference term between the matter wavefunction Ψ_{order} and the vacuum background Ψ_{chaos} . The strength of this interference is dictated by the kinematic entropy of the source.

In rotationally supported stellar disks, the ordered motion of baryons maintains a stable phase relationship with the vacuum. This constructive interference maximizes the thermodynamic efficiency (μ), leading to the characteristic MONDian gravitational boost. In diffuse gas or pressure-supported systems, the chaotic, random motion of baryons leads to phase decoherence. The interference term averages toward zero, increasing the chaotic entropy contribution (Ψ_{chaos}) and suppressing the field's response.

This hypothesis naturally explains the puzzling existence of Ultra-Diffuse Galaxies (UDGs) that appear to lack Dark Matter, such as NGC 1052-DF2 and DF4.

This leads to a specific, testable prediction: the residuals of the Radial Acceleration Relation (RAR) are not random noise but correlate with the baryonic state of the source. We define the entropic residual δ as the deviation from the universal MOND-like response:

$$\delta = \log(g_{\text{obs}}) - \log(g_{\text{RAR}}) \quad (142)$$

where g_{RAR} is the acceleration predicted by the universal response function $\mu(x) = x/(1+x)$.

For two systems with identical baryonic mass M_b , the theory distinguishes between ordered and disordered states based on their phase coherence. In ordered systems, such as stellar-dominated disks,

high phase coherence results in a positive residual ($\delta > 0$), placing the system above the mean RAR curve. In disordered systems dominated by diffuse gas, high kinematic entropy results in a negative residual ($\delta < 0$), placing the system below the mean RAR curve. This morphological shift represents the transition between the field's constructive interference and its decoherent thermal phase. We can use the gas fraction ($f_{\text{gas}} \equiv g_{\text{gas}}/g_{\text{bar}}$) as a direct proxy for the thermodynamic disorder of the source.

If gravity is observed to be strictly a function of mass regardless of state, the hypothesis is falsified. If the rotation velocity correlates with the gas fraction ($f_{\text{gas}} \equiv g_{\text{gas}}/g_{\text{bar}}$) independent of mass, the state-dependent gravity prediction is confirmed.

20. Conclusions

General Relativity and Λ CDM have been remarkably successful, providing a powerful mathematical description of gravity and the universe's large-scale evolution. However, their success relies on the introduction of two major, unexplained components: dark matter and dark energy. These are essentially placeholders for observed gravitational effects that cannot be accounted for by the visible matter in the universe. While Λ CDM describes what happens with incredible precision, it does not explain why these phenomena occur.

Timeflow Gravity offers a different perspective. Instead of adding new, unseen substances to the cosmos, it proposes a new physical principle: that gravity is an emergent thermodynamic phenomenon arising from a single quantum scalar field—the Timeflow field. This shift from a descriptive to an explanatory framework provides a physical origin for the universe's most profound puzzles.

Where Λ CDM requires dark matter to explain galactic rotation, TG derives the MOND acceleration scale (a_0) and the associated galactic scaling laws (Tully-Fisher, RAR) directly from the thermodynamics of causal horizons. Where Λ CDM requires dark energy to explain cosmic acceleration, TG predicts the Hubble constant from the fundamental transition rate of the quantum vacuum. Timeflow Gravity offers a compelling solution to the emerging puzzles from recent JWST observations, as an enhanced effective gravitational constant ($G_{\text{eff}} = G/\mu$) in the early universe naturally accelerates the formation of massive galaxies. TG respects the law of energy conservation at global scale—an important theoretical advantage over General Relativity. Even the puzzling Hubble tension and evolving dark energy find a natural explanation as a measurement of the universe in two different thermodynamic states. Finally, the theory's thermodynamic foundation presents a new avenue for explaining the universe's initial conditions, providing a mechanism for an inflationary-like epoch without requiring a separate inflaton field.

With just a single, minimal modification of General Relativity, Timeflow Gravity accounts for the phenomena attributed to dark matter, dark energy, and inflation within one unified framework. In this sense, TG could be not only more economical than the standard model, but also more predictive, offering testable explanations where Λ CDM introduces additional entities. This suggests that treating gravity as an emergent property of a Timeflow field may represent a significant step towards a more fundamental and complete theory of the cosmos.

A particularly interesting feature of this approach is the environment-dependent drive of energy to preserve ordered states. This principle may extend far beyond gravitation alone, potentially offering a universal mechanism for the emergence of complex structures. From the stability of particles, to the formation of galaxies, and even to the rise of biological life, the same thermodynamic law could be at work: the tendency of spacetime to sustain order against the backdrop of chaos.

This unification opens a path toward a deeper theory of reality, where matter, gravity, space and time are not isolated phenomena but representations of one continuous thermodynamic process. Timeflow Gravity therefore holds the potential not only to resolve the puzzles of dark matter and dark energy, but also to illuminate the fundamental principles that shape the evolution of the Universe.

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