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A Complete Derivation of Quantum Mechanics from Classical Field Theory - Part B: Emergence of Quantum Gravity

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A Complete Derivation of Quantum Mechanics from Classical Field Theory -Part B: Emergence of Quantum Gravity

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Abstract

Quantum gravity aims to reconcile general relativity, which governs the macroscopic dynamics of spacetime, with quantum mechanics, which describes matter and interactions on microscopic scales. The tension between the background-independent, deterministic framework of general relativity and the background-dependent, probabilistic nature of quantum mechanics underscores the need for a unified theoretical d escription. Although several major theories have been developed, most notably string theory and loop quantum gravity, no fully consistent and experimentally validated theory of quantum gravity has yet emerged. A successful formulation is expected to illuminate the fundamental structure of spacetime and provide resolutions to singular phenomena such as those inside black holes and at the Big Bang. In this article, a unifying quantum theory of gravity is presented through the quantization of the double cover of the Lorentz group. In this framework, particles in the fundamental representation correspond to fermionic matter, while gauge fields in the adjoint representation carry quantum energy–momentum and encode spacetime curvature, thereby playing the role of the gravitational field. Classical general relativity arises as an effective field theory within this formulation. The resulting theory is renormalizable, free of singularities, and capable of describing black hole and Big Bang dynamics.

Keywords: quantum gravity; string theory alternative; quantum mechanics foundation; general relativity; dark matter; Lorentz symmetry; cosmological constant problem; loop quantum gravity alternative

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1. Introduction

This paper is based on the previous paper [1] and is a direct continuation of it, so the reader is referred to read it before, but it is not mandatory, as this paper is written in a self-contained manner.

One of the most profound challenges in modern theoretical physics is the formulation of a quantum renormalizable theory of gravity, which is also known as *Quantum gravity*. Such a theory aims to unify general relativity of Einstein, which is a classical theory that describes the gravity at large-scale structure of spacetime, with quantum mechanics, which describes he behavior of matter and interactions at microscopic scales. While both theories are very successful in their respective realms, they rest on conceptual foundations that are in tension with each other and have not been solved yet.

Reconciling these two pictures requires a quantum description of the geometry of spacetime itself, a goal that has motivated decades of research, which has led to the development of multiple, largely independent approaches. The most prominent are the loop quantum gravity [16–18], which is based on the quantization of spacetime geometry using Ashtekar variables and background-independent methods, and the string theory [19–21], which is based on excitations of one-dimensional extended objects called strings living in high-dimensional space-time. Other promising directions, such as causal dynamical triangulations and emergent gravity models, emphasize different aspects of the

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problem. Each approach has produced important insights, including advances in understanding black hole entropy and possible resolutions of singularities. Nonetheless, no framework has yet provided definitive experimental predictions that can distinguish between competing scenarios.

The quest for quantum gravity is not only a matter of theoretical issues, but also of observational and practical relevance. Such as spacetime singularities in the center of black holes or the singularity of the beginning of space time, i.e., the Big Bang. Near these singularities, classical general relativity breaks down, and we cannot make predictions anymore. So, a complete understanding of high-energy phenomena in the early universe and black hole interiors is needed. The study of quantum gravity remains central to the effort of building a unified and self-consistent description of space-time and interactions at all scales.

Here, we present a theory that combines quantum mechanics and gravity within a single, consistent framework, using a simple model. This theory is renormalizable and free of singularities. In the Section 2 we gauge the space-time symmtery groups and then derive the corresponding gauge fields. Then find the equation of motion of a particle charged under the symmetry group of space-time and reproduce the quantum state of a particle in space-time. In the Section 3 we calculate the field strength tensor of the gauge fields of the local translational spacetime symmetry group and show how it is equal to the curvature of spacetime. Classical gravity is an effective field of this field strength tensor.

In the Section 4 we calculate the field strength tensor of the gauge fields of the local rotational symmetry of spacetime and show how it is also equal to the curvature of spacetime, and therefore produces a gravity-like force that can explain the dark matter and energy, as further discussed in another paper [15].

In the Appendix 5, we explore the structure of space-time as Hilbert space, the measure of space-time.

2. Gauging the Double Cover of the Lorentz Symmetry Group and Emergence of Quantum States

the $SU(2) \times SU(2)$ group is the double cover of the Lorentz Group $SO(1,3)$. $\{J_i, K_i\}$ are the generators of the Lorentz Group $SO(1,3)$.

Defining:

$$j_i^L \equiv J_i + iK_i, \;\; j_i^R \equiv J_i - iK_i, \tag{2.1}$$

where $i = 1,2,3$ [2] , $(\vec{j}^L, \vec{j}^R)$ are the generators of $SU(2)_{L/R}$, satisfiy [5,6] :

$$\begin{aligned} [j_i^L, j_j^L] &= i\epsilon_{ijk} j_k^L, \\ [j_i^R, j_j^R] &= i\epsilon_{ijk} j_k^R, \\ [j_i^L, j_j^R] &= 0, \end{aligned} \tag{2.2}$$

The Lagrangian for a massless particle charged under the local Lorentz symmetry is:

$$\mathcal{L} = (D_\mu \phi^{R/L}(x_\mu))^\dagger D_\mu \phi^{R/L}(x_\mu), \tag{2.3}$$

Where the covariant derivative is:

$$D_\mu \phi^{R/L}(x_\mu) = [\partial_\mu - igq^{R/L} \omega_\mu^{R/Li} j_i^{R/L}), \tag{2.4}$$

Where $i = 1,2,3$. The kinetic energy of the gauge fields is omitted and will be addressed in another paper (relevant to the curvature tensor of space-time). ϕ^R Lies in $(0,n)_{0,q_0^R}$ representation and ϕ^L lies in $(n,0)_{q_0^L,0}$. So they have spin by definition. $\omega_\mu^{R/L0}, \omega_\mu^{R/Li}$ are the gauge fields of $SU(2)_{R/L}$. Using the symmetry between left and right-handed groups seen in nature, we assign: $\omega_\mu^{R/Li} = \omega_\mu^i$. The $q^{R/L}$ are the conserved charges of $SU(2)_{R/L}$. g is the coupling constant. $j_\nu^{R/L}, \phi^{R/L}$ lie in any representation. For example, in the spinor representation, it has spin 1/2(the charge), which are just the left and right-handed Weyl fermions $\phi^{R/L} \equiv \psi^{R/L}$.

Next, substituting $q = \hbar, g = 1$,$\hbar$ is the Planck constant, and $\partial_\mu \equiv I\partial_\mu$, we obtain,

$$D_\mu \phi^{R/L}(x_\mu) = [\partial_\mu \pm i\hbar {\omega^i}_\mu j_i^{R/L}]\phi^{R/L}(x_\mu), \tag{2.5}$$

Defining ${\omega^i}_\mu j_i := \omega_\mu$. The covariant derivative becomes:

$$D_\mu \phi^{R/L}(x_\mu) = [\partial_\mu \pm i\hbar \omega_\mu]\phi^{R/L}(x_\mu), \tag{2.6}$$

And the Lagrangian is again:

$$\mathcal{L} = (D_\mu \phi^{R/L}(x_\mu))^\dagger D_\mu \phi^{R/L}(x_\mu), \tag{2.7}$$

Notice that when $\hbar = 0$ it gives back the classical dynamics by the classical Lagrangian :

$$\mathcal{L} = \partial_\mu \phi^{R/L}(x_\mu)\partial^\mu \phi^{R/L}(x_\mu) \tag{2.8}$$

And the equation of motion is

$$\Box \phi^{R/L}(x_\mu) = 0, \tag{2.9}$$

The equations of motion of Equation (2.7)are :

$$D_\mu D^\mu \phi^{R/L} = 0\,, \tag{2.10}$$

explicitly,

$$\Big(\Box - 2i\hbar\, \omega^\mu \partial_\mu - \hbar^2 \omega_\mu \omega^\mu\Big)\phi^{R/L}(x_\mu) = 0, \tag{2.11}$$

For simplicity, assuming ω_μ is constant, i.e., $\partial_\mu \omega^\mu = 0$, the solution is as follows, We try a plane-wave solution:

$$\phi_c^{kR/L}(x_\mu) = e^{ik_\mu x^\mu}\phi_c^{kR/L}(x_\mu = 0) \tag{2.12}$$

which also satisfy $\Box\phi_c^{kR/L} = 0$, i.e., the classical solution in Equation (2.9). Next, calculating:

$$\Box \phi^{R/L} = -k^\mu k_\mu \phi^{R/L} = -p^2 \phi^{R/L}, \tag{2.13}$$

$$\omega^\mu \partial_\mu \phi^{R/L} = i(\omega^\mu k_\mu)\phi^{R/L}, \tag{2.14}$$

Then plugging into Equation (2.11), we obtain:

$$\Big(p^2 + 2\hbar(\omega^\mu k_\mu) + \hbar^2 \omega^2\Big)\phi^{R/L} = 0. \tag{2.15}$$

Dividing out $\phi^{R/L} \neq 0$, we get the extended dispersion relation:

$$p^2 - 2\hbar(\omega^\mu k_\mu) + \hbar^2 \omega^2 = 0. \tag{2.16}$$

We complete the square:

$$(k^\mu + \hbar\, \omega^\mu)^2 = 0, \tag{2.17}$$

Which implies that the shifted momentum $p^\mu := k^\mu + \hbar\, \omega^\mu$ is null:

$$p^\mu p_\mu = 0. \tag{2.18}$$

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So the general solution of Equation (2.10) is a superposition of plane waves satisfying this dispersion relation:

$$\phi^{R/L}(x_\mu) = \int d^4k\, g(k)\, e^{ik_\mu x^\mu}, \quad \text{with} \quad (k^\mu + \hbar\,\omega^\mu)^2 = 0. \tag{2.19}$$

assigning $p^\mu = k^\mu + \hbar\,\omega^\mu$, then

$$\phi^{R/L}(x_\mu) = e^{-i\hbar\,\omega_\mu x^\mu} \int d^4p\, f(p)\, e^{ip_\mu x^\mu}, \tag{2.20}$$

defining,

$$\phi_c^{R/L}(x_\mu) = \int d^4p\, f(p)\, e^{ip_\mu x^\mu}, \tag{2.21}$$

Which is a superposition of classical plane waves satisfying Equation (2.12). then Equation (2.20) becomes:

$$\phi^{R/L}(x_\mu) = e^{-i\hbar\,\omega_\mu x^\mu}\phi_c^{R/L}(x_\mu), \quad \text{where} \quad \Box\phi_c^{R/L}(x_\mu) = 0 \tag{2.22}$$

- $\phi_c^{R/L}(x_\mu)$ is a solution to the free massless classical particle ($\hbar = 0$ or $\omega_\mu = 0$).
- The exponential factor $e^{-i\hbar\,\omega_\mu x^\mu}$ accounts for the phase shift introduced by the constant field ω_μ.

Next, choosing the classical solution to be a constant instead of a wave:

$$\phi_c^{R/L}(x_\mu) = e^{-ik_\mu x^\mu}\phi_c^{R/L}(x_\mu = 0) = \text{constant} \tag{2.23}$$

Which is obtained by choosing $k_\mu \equiv 0$, meaning it is a static solution and has no classical energy-momentum (k_μ). Equation (2.22) becomes:

$$\phi^{R/L}(x_\mu) = ce^{-i\hbar\,\omega_\mu x^\mu} \tag{2.24}$$

assigning initial conditions $c \equiv \phi^{R/L}(x_\mu = 0)$, the particle becomes,

$$\begin{aligned}\phi^{R/L}(x_\mu) &= e^{-i\hbar\,\omega_\mu x^\mu}\phi^{R/L}(x_\mu = 0)\\ &= e^{-i\hbar\omega^\nu{}_\mu j_\nu^{R/L} x^\mu}\phi^{R/L}(x_\mu = 0)\end{aligned} \tag{2.25}$$

assigning $k^\mu \equiv 0$ for p^μ in Equation (2.17), we obtain,

$$p^\mu = \hbar\,\omega^\mu \tag{2.26}$$

Which is just *the de Broglie relation!* Hence, we obtain the *particle-wave duality* as the solution in Equation (2.25) is a wave in space-time together with this result. The choice $k_\mu \equiv 0$ means there is no more classical energy-momentum in the universe, only the quantized energy-momentum p_μ.

next, plugging $p^\mu = \hbar\,\omega^\mu$ into Equation (2.25), and changing coordinates $r_\mu := \hbar x_\mu$ gives,

$$\begin{aligned}\phi^{R/L}(r_\mu) &= e^{-ip^i{}_\mu j_i^{R/L} r^\mu/\hbar}\phi^{R/L}(r_\mu = 0)\\ &= e^{-ip_\mu r^\mu/\hbar}\phi^{R/L}(r_\mu = 0),\end{aligned} \tag{2.27}$$

Where $p_\mu^\nu = \hbar\omega_\mu^\nu$. This is the familiar evolution in space-time of a *quantum state / wave function*!

3. The Translational Invariance and Gravity

The Poincaré group is given by $SO(1,3) \times \mathbf{R}^{1,3}$ using the double cover instead: $SU(2)_L \times SU(2)_R \times \mathbf{R}^{1,3}$, where $SU(2)_L \times SU(2)_R$ is the double cover of $SO(1,3)$. The quantization of the

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translation symmetry group $\mathbf{R}^{1,3}$ is as follows: The evolution of a quantum particle according to Equation (2.27) is :

$$\psi(r_\mu + r^0_\mu) = e^{\frac{-ij_i^{L/R} p^i_\mu r^\mu}{\hbar}} \psi(r^0_\mu) \tag{3.1}$$

$i = 1, 2, 3$.This, in fact, is a translation of the particle over the space-time:

$$\psi(r_\mu) \to e^{\frac{-ij_i^{L/R} p^i_\mu r^\mu}{\hbar}} \psi(r^0_\mu) = \psi(r_\mu + r^0_\mu) \tag{3.2}$$

defining: $j_i^{L/R} p^i_\mu := p^{L/R}_\mu := p_\mu$ (we omit the L/R for simplicity), Equation (3.2) becomes :

$$\psi(r_\mu) \to e^{\frac{-ip_\mu r^\mu}{\hbar}} \psi(r^0_\mu) = \psi(r_\mu + r^0_\mu) \tag{3.3}$$

Making infinitesimal successive j translations,

$$\psi(r_\mu) = \prod_j e^{\frac{-ip^j_\mu dr^\mu}{\hbar}} \psi(r^0_\mu) \tag{3.4}$$

adding the exponents,

$$\psi(r_\mu) = e^{\frac{-i\sum_j p^j_\mu dr^\mu}{\hbar}} \psi(r^0_\mu) = e^{\frac{-i\int p_\mu dr^\mu}{\hbar}} \psi(r^0_\mu) \tag{3.5}$$

Demanding invariance under translation, we obtain,

$$\frac{\partial \psi(r^\mu)}{\partial r^\mu} = \frac{\partial}{\partial r^\mu}\left(\frac{-i\int_{r_0^\mu}^{r^\mu} p dr^\mu}{\hbar}\right)\psi(r^\mu) = 0 \tag{3.6}$$

using the Leibniz integral rule,

$$p(r^\mu) - p(r^\mu_0) = 0 \tag{3.7}$$

We obtain the *conservation of energy-momentum*. Hence, the conserved charge of the translational symmetry is the p_μ.

For massless particles, the energy-momentum is given by $p_\mu = \hbar(\omega_0, \vec{k})$. Using the dispersion relation for massless particles, $\omega_0 = c|\vec{k}|$, we obtain, $p_\mu = \hbar|\vec{k}|(c, \vec{1})$, where $(c, \vec{1})$ is a vector in four spacetime dimensions, we denote by 1_μ. Therefore, $p_\mu = p_0 1_\mu$. The same we have $\omega_\mu = \omega_0 1_\mu$, plugging into Equation (3.3) , we obtain:

$$\psi(r_\mu) = e^{\frac{-ip_0 1_\mu r^\mu}{\hbar}} \psi(r^0_\mu) \tag{3.8}$$

Defining the four vector $r^{L/R} := 1_\nu r^\nu$, choosing $1_\mu = \sigma_\mu$ (The Pauli four-vector) we have $r := \sigma_\nu r^\nu$ which is the $(1/2, 1/2)$ representation of Lorentz symmetry. Rewriting Eq(3.2) gives :

$$\psi(r_0) \to e^{-i\theta} \psi(r_0) := \psi(r + r_0) \tag{3.9}$$

Where $\theta = r/\hbar$ is the "angle of rotation" and has mass dimension (-1). This symmetry is abelian in this form; the conserved charge is p_0, which has mass dimension 1. The gauge fields denoted by ω'_μ of the local symmetry transform as,

$$\omega'^{L/R}_\mu \to \omega'^{L/R}_\mu + \frac{1}{G_0\hbar}\partial_\mu r^{L/R} \tag{3.10}$$

G_0 is the coupling constant. We omit the superscript L/R from ω', r for simplicity. This symmetry is abelian. On the other hand, $\omega'_\mu = j_i^{L/R}\omega'^i_\mu$ means they are charged under the Lorentz symmetry (adjoint representation), substituting in Equation (3.10) above, we obtain,

$$j_i^{L/R}\omega'^i_\mu \to j_i^{L/R}\omega'^i_\mu + \frac{1}{G_0\hbar}\partial_\mu r \tag{3.11}$$

$i = 1, 2, 3$. multiply by $\hbar$,

$$\hbar j_i^{L/R}\omega'^i_\mu \to \hbar j_i^{L/R}\omega'^i_\mu + \frac{1}{G_0}\partial_\mu r \tag{3.12}$$

Which also equals,

$$\hbar\omega'_\mu \to \hbar\omega'_\mu + \frac{1}{G_0}\partial_\mu r \tag{3.13}$$

The covariant derivative is given by :

$$D_\mu = \partial_\mu - iG_0 p_0 \omega'^i_\mu j_i^{L/R} \tag{3.14}$$

The $\omega'^{L/R}_\mu \equiv j_i^{L/R}\omega'^i_\mu$ is just the connection of space-time. $\omega'^{L/R}_\mu$ lies in the adjoint representation of $SU(2)_{L/R}$. $\omega'^{L/R}_\mu$ is a spin-2 field because the gauge field bosons $\omega'^{L/R\nu}_\mu$ have spin 1 by definition. But they also lie in the adjoint representation of $SU(2)_{L/R}$, which gives them an extra charge $q = 1\hbar$. Thus, in total, they have spin 2, i.e., a tensor field with rank 2 (two indices).

Next, ω'^i_μ, $i = 1, 2, 3$, doesn't commute because it is charged under the Lorentz symmetry group, so the field strength tensor behaves like a non-Abelian, then the field strength tensor is :

$$\mathcal{F}'^i_{\mu\nu} = \partial_\nu\omega'^i_\mu - \partial_\mu\omega'^i_\nu + g_0\epsilon^{ijk}\omega'^j_\mu\omega'^k_\nu \tag{3.15}$$

$$= \partial_\nu\omega'^i_\mu - \partial_\mu\omega'^i_\nu + g_0\omega'_\mu \times \omega'_\nu \tag{3.16}$$

ϵ^{ijk} is the structure constant of $SU(2)$, expanding further,

$$\mathcal{F}'^i_{\mu\nu} = \partial_\nu\omega'^i_\mu - \partial_\mu\omega'^i_\nu + [\omega'_\mu\omega'_\nu - \omega'_\nu\omega'_\mu]^i \tag{3.17}$$

$[\omega'_\mu\omega'_\nu - \omega'_\nu\omega'_\mu]^i$ Is the i element of the cross product. $F^{L/Ri}_{\mu\nu}$ is just the curvature tensor of spacetime. In the next sections, we compare it to the Riemann curvature tensor $R^\rho{}_{\sigma\mu\nu}$.

3.1. Lorentz Generators in Weyl Representation

Conventions:

$$\sigma^\mu = (\mathbf{1}, \sigma^i), \qquad \bar{\sigma}^\mu = (\mathbf{1}, -\sigma^i), \qquad \sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu). \tag{3.18}$$

Let $\omega_{\mu\nu}$ be antisymmetric and define rotation and boost parameters by $\omega_{ij} = \epsilon_{ij\ell}\theta^\ell$ and $\omega_{0i} = \xi_i$.

Compute components for left-handed generators:

$$\sigma^{0i} = -\tfrac{i}{2}\sigma^i, \qquad \sigma^{ij} = \tfrac{1}{2}\epsilon^{ijk}\sigma^k. \tag{3.19}$$

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Then

$$\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}=\frac{i}{2}\left(2\omega_{0i}\sigma^{0i}+\omega_{ij}\sigma^{ij}\right)=\frac{1}{2}\omega_{0i}\sigma^{i}+\frac{i}{2}\theta^{k}\sigma^{k}=\frac{1}{2}(\vec{\xi}+i\vec{\theta})\cdot\vec{\sigma}. \tag{3.20}$$

Thus, the infinitesimal left-handed Lorentz transformation is

$$\psi_L'=\left(1-\tfrac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\right)\psi_L=\left(1-\tfrac{1}{2}(\vec{\xi}+i\vec{\theta})\cdot\vec{\sigma}\right)\psi_L, \tag{3.21}$$

and the finite transformation is

$$\psi_L'=\exp\left[-\tfrac{1}{2}(\vec{\xi}+i\vec{\theta})\cdot\vec{\sigma}\right]\psi_L, \tag{3.22}$$

For right-handed spinors use $\bar{\sigma}^{\mu\nu}=\frac{i}{4}(\bar{\sigma}^{\mu}\sigma^{\nu}-\bar{\sigma}^{\nu}\sigma^{\mu})$. One finds

$$\frac{i}{2}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}=\frac{1}{2}(-\vec{\xi}+i\vec{\theta})\cdot\vec{\sigma}, \tag{3.23}$$

so

$$\psi_R'=\exp\left[-\tfrac{1}{2}(-\vec{\xi}+i\vec{\theta})\cdot\vec{\sigma}\right]\psi_R. \tag{3.24}$$

Remark. Different texts flip the sign of ξ_i (defining $\omega_{0i}=-\xi_i$). That changes the displayed combination between the equivalent forms $(\vec{\xi}+i\vec{\theta})$ and $(\vec{\theta}-i\vec{\xi})$; the difference is purely a convention.

Next, choosing $j^L_\mu=\overline{\sigma}_\mu, j^R_\mu=\sigma_\mu$. We define the following gauge fields:

$$\begin{aligned}\omega'^{Li}_{\mu}\sigma_i&\equiv(\Omega^i_\mu+iN^i_\mu)\sigma_i=\Omega^i_\mu\sigma_i+N^i_\mu(i\sigma_i)\\ \omega'^{Ri}_{\mu}\sigma_i&\equiv(\Omega^i_\mu-iN^i_\mu)\sigma_i=\Omega^i_\mu\sigma_i-N^i_\mu(i\sigma_i),\end{aligned} \tag{3.25}$$

But $i\sigma_i\equiv K_i$. the K_i are the boost generators of the Lorentz group $SO(1,3)$. the J_i are rotation generators of the Lorentz group $SO(1,3)$ according to Equation (2.1) , in short

$$\omega'^{L/Ri}_{\mu}\sigma_i=\Omega^i_\mu J_i\pm N^i_\mu K_i, \tag{3.26}$$

where Ω^i_μ, N^i_μ are the gauge fields, coupled to the rotation/boost generators J_i, K_i, respectively. Next, Defining:

$$\begin{aligned}\mathcal{J}^L_\mu&:=\mathcal{J}^{L\rho\sigma}_\mu\sigma_{\rho\sigma}\\ \mathcal{J}^R_\mu&:=\mathcal{J}^{R\rho\sigma}_\mu\overline{\sigma}_{\rho\sigma},\end{aligned} \tag{3.27}$$

$\mathcal{J}^{L/R\rho\sigma}_\mu$ are the gauge field coupled to the Lorentz generators of $SO(1,3)$. first define $\overline{\sigma}^{\mu\nu}:=(\sigma^{\mu\nu})^R, \sigma^{\mu\nu}:=(\sigma^{\mu\nu})^L$ then define:

$$\begin{aligned}\omega'^{L/R}_\mu&:=\omega'^{L/Ri}_\mu\sigma_i\\ \mathcal{J}^{L/R}_\mu&:=\mathcal{J}^{L/R\rho\sigma}_\mu\sigma^{L/R}_{\rho\sigma},\end{aligned} \tag{3.28}$$

Then the covariant derivative becomes:

$$\begin{aligned}D_\mu&=\partial_\mu-iG_0p_0\mathcal{J}^{L/R\rho\sigma}_\mu\sigma^{R/L}_{\rho\sigma}=\partial_\mu-iG_0p_0\mathcal{J}^{L/R}_\mu\\ D_\mu&=\partial_\mu-iG_0p_0\omega'^{L/Ri}_\mu j_i=\partial_\mu-iG_0p_0\omega'^{L/R}_\mu\end{aligned} \tag{3.29}$$

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The covariant derivatives must be equal. therefore,

$$\omega'^{L/R}_{\mu} \equiv \mathcal{J}^{L/R}_{\mu}, \tag{3.30}$$

the representation of $\mathcal{J}^{L/R\rho\sigma}_{\mu}$ in the antisymmetric indices, where it is charged under the Lorenz symmetry

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right)\Big|_{\text{antisym.}} = (1,0)\oplus(0,1) \tag{3.31}$$

This corresponds exactly to the Lorentz generators, i.e., the adjoint representation of $SO(1,3)$. Hence, spin 1 . Adding the spin-1 also for the lower index μ, as it behaves as a vector, the total spin is 2, hence it is a spin-2 particle, as same as ω'_{μ}.

For simplicity of notation, we will drop the superscript L/R. The covariant derivative under the local group $SO(1,3)$ is then :

$$D_\mu = \partial_\mu - iG_0 p_0 \mathcal{J}^{\rho\sigma}_{\mu}\sigma_{\rho\sigma} = \partial_\mu - iG_0 p_0 \mathcal{J}_\mu \tag{3.32}$$

The field strength tensor is :

$$\mathcal{F}'^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \mathcal{J}^{\rho}_{\nu\sigma} - \partial_\nu \mathcal{J}^{\rho}_{\mu\sigma} + g_0 \epsilon^{(\rho\sigma)}_{(\rho'\sigma'),(\rho''\sigma'')} \mathcal{J}^{(\rho'\sigma')}_{\mu} \mathcal{J}^{(\rho''\sigma'')}_{\nu} \tag{3.33}$$

$$= \partial_\mu \mathcal{J}^{\rho}_{\nu\sigma} - \partial_\nu \mathcal{J}^{\rho}_{\mu\sigma} + g_0 [\mathcal{J}_\mu \times \mathcal{J}_\nu]^{\rho}_{\sigma} \tag{3.34}$$

The indices $(\rho\sigma)$ run as one index from $0,1,...,5$. $[\mathcal{J}_\mu \times \mathcal{J}_\nu]^{\rho}_{\sigma}$ is the $(\rho\sigma)$-element of the cross product. The cross product in 6 dimensions is calculated by using the wedge product instead (the Lie algebra structure constant ϵ^{ijk} doesn't change):

$$\mathcal{F}'^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \mathcal{J}^{\rho}_{\nu\sigma} - \partial_\nu \mathcal{J}^{\rho}_{\mu\sigma} + g_0 [\mathcal{J}_\mu \wedge \mathcal{J}_\nu]^{\rho}_{\sigma} \tag{3.35}$$

expanding further,

$$\begin{aligned} \mathcal{F}'^{\rho}{}_{\sigma\mu\nu} &= \partial_\mu \mathcal{J}^{\rho}_{\nu\sigma} - \partial_\nu \mathcal{J}^{\rho}_{\mu\sigma} + \mathcal{J}^{\rho}_{\mu} \cdot \mathcal{J}_{\nu\sigma} - \mathcal{J}^{\rho}_{\nu} \cdot \mathcal{J}_{\mu\sigma} \\ &= \partial_\mu \mathcal{J}^{\rho}_{\nu\sigma} - \partial_\nu \mathcal{J}^{\rho}_{\mu\sigma} + \mathcal{J}^{\rho}_{\mu\lambda} \mathcal{J}^{\lambda}_{\nu\sigma} - \mathcal{J}^{\rho}_{\nu\lambda} \mathcal{J}^{\lambda}_{\mu\sigma} \end{aligned} \tag{3.36}$$

On the other hand, the classical (Riemann) curvature tensor $R^{\rho}{}_{\sigma\mu\nu}$ is given by :

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}, \tag{3.37}$$

where $\Gamma^{\rho}{}_{\mu\sigma}$ is the Christoffel symbol, then by equating with Equation (3.36) we have the equivalences,

$$\begin{aligned} (\mathcal{F}'^{\rho}{}_{\sigma\mu\nu})^{L/R} &\equiv (R^{\rho}{}_{\sigma\mu\nu})^{L/R} \\ (\mathcal{J}^{\rho}_{\mu\sigma})^{L/R} &\equiv (\Gamma^{\rho}{}_{\mu\sigma})^{L/R} \end{aligned} \tag{3.38}$$

This means the *strength field tensor $\mathcal{F}$ is equal to the curvature of spacetime!* then Equation (3.36) becomes:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma} \tag{3.39}$$

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The Lagrangian for a particle ψ moving in a gravitational field.

$$\begin{aligned}\mathcal{L} &= D_\mu\psi D^\mu\psi + \mathcal{F}'^{R}_{\mu\nu}\mathcal{F}'^{R\mu\nu} + \mathcal{F}'^{L}_{\mu\nu}\mathcal{F}'^{L\mu\nu} \\ &= D_\mu\psi D^\mu\psi + R^{R}_{\mu\nu}R^{R\mu\nu} + R^{L}_{\mu\nu}R^{L\mu\nu}\end{aligned} \tag{3.40}$$

where $(\mathcal{F}'^{\rho}{}_{\sigma\mu\nu})^{L/R} \equiv \mathcal{F}'^{L/R}_{\mu\nu}, (\mathcal{R}'^{\rho}{}_{\sigma\mu\nu})^{L/R} \equiv \mathcal{R}'^{L/R}_{\mu\nu}$. This is a linear renormalizable Lagrangian. Therefore, a *quantized gravity!*

In the case ω_μ is constant, as in Equation (2.12), we have $R^{L/R}_{\mu\nu} \equiv 0$, according to Equation (3.15), which gives a flat space-time and background-dependent quantum theory in a flat universe as we used to.

3.2. Background Dependence

The condition for a constant ω^i_μ gives a constant field strength tensor $F_{\mu\nu}F^{\mu\nu}$ = constant, which is also gauge invariant, hence a constant gauge invariant space-time curvature. This means that we have picked a background-dependent quantum field theory in the previous paper. Generalizing to any ω^i_μ gives background-independent quantum field theory combined with quantum gravity.

3.3. Vacuum Energy

in the case of a constant isotropic $\omega'^{L/R}_\mu = |\omega_{\text{Planck}}|$, which is the energy of the vacuum. The curvature by substitution in Equation (3.17) is:

$$\mathcal{F}'^{i}_{\mu\nu} = 0 - 0 + [\omega_{\text{Planck}}\omega_{\text{Planck}} - \omega_{\text{Planck}}\omega_{\text{Planck}}]^i = 0 \tag{3.41}$$

Therefore, the vacuum energy does not contribute to curvature of space-time in total!. This explains the vacuum energy catastrophe [25,26]as we explain why does gravity “sees” almost none of the huge zero-point energy that QFT predicts.

3.4. Einstein field equation

The relation between the Christofel symbol and the metric is given by,

$$\mathcal{J}^i_{kl} \equiv \Gamma^i_{kl} = \frac{1}{2}g^{im}(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}) \tag{3.42}$$

$i, k, l, m = 0, 1, 2, 3$. Einstein’s gravitational field is formulated by the metric $g_{\mu\nu}$ as a field, by the Einstein-Hilbert action,

$$S = \int (\frac{1}{2\kappa}R + \mathcal{L}_M)\sqrt{-g}d^4r \tag{3.43}$$

where $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor matrix. R is the Ricci scalar, $R = g^{ab}R_{ab} = g^{ab}R^c_{acb}$ and $\kappa = 8\pi Gc^{-4}$, G is the Einstein gravitational constant. $a, b, c = 0, 1, 2, 3$. $\mathcal{L}_M$ is the matter’s Lagrangian. It is a non-linear, non-renormalizable Lagrangian. This is clear why, because they used the metric as a fundamental field, where it is not. It is just an effective approximation. When dealing instead with the gauge fields (ω'^{ν}_μ) as the fundamental fields, the Lagrangian becomes renormalizable like the other Yang-Mills fields. The action of this Lagrangian is again:

$$S = \int (D_\mu\psi D^\mu\psi + R^{R}_{\mu\nu}R^{R\mu\nu} + R^{L}_{\mu\nu}R^{L\mu\nu})\sqrt{-g}d^4r \tag{3.44}$$

From this, one can derive the effective metric. Adding$\sqrt{-g}$ doesn’t change the convergence of the integral, because it is just a coordinate change, which shouldn’t change the convergence status of the integral; besides, the metric is a dimensionless quantity. Therefore, the gravity is renormalized.

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3.5. The Torsion Tensor

The torsion tensor T is given by:

$$T^{\rho}_{\mu\sigma} = \Gamma^{\rho}{}_{\mu\sigma} - \Gamma^{\rho}{}_{\sigma\mu} \tag{3.45}$$

which calculates the anti-symmetry of the connection. Replacing $\mathcal{J}^{\rho}_{\sigma\mu} \leftrightarrow \Gamma^{\rho}{}_{\sigma\mu}$ by Equation (3.38)and multiplying by S_{ρ}^{σ} gives:

$$S_{\rho}^{\sigma} T^{\rho}_{\mu\sigma} = S_{\rho}^{\sigma} \mathcal{J}^{\rho}_{\mu\sigma} - S_{\rho}^{\sigma} \mathcal{J}^{\rho}_{\sigma\mu} \tag{3.46}$$

Using relation in Eqs. (3.28),(3.30) We obtain:

$$S_{\rho}^{\sigma} T^{\rho}_{\mu\sigma} = j_i \omega^i_{\mu} - j_i \omega^i_{\mu} \equiv 0 \tag{3.47}$$

hence,

$$T^{\rho}_{\mu\sigma} \equiv 0 \tag{3.48}$$

This is true for L/R:

$$S_{\rho}^{\sigma} T^{L/R\rho}_{\mu\sigma} = j_i \omega^{L/Ri}_{\mu} - j_i \omega^{L/Ri}_{\mu} \equiv 0 \tag{3.49}$$

i.e.

$$T^{L/R\rho}_{\mu\sigma} = 0 \tag{3.50}$$

Hence, our universe is torsionless as known [22–24].

4. The Rotational Invariance and the Gravity-Like Force

4.1. The Spin Operator, Angular Momentum Operator, and Quantization Conditions

The rotation of the particle according to the group $SU(2)_{L/R}$ is given by:

$$\phi^{R/L}(r_{\mu}) = e^{-i\hbar j_i^{R/L} \gamma^i} \phi^{R/L}(r_{\mu} = 0), \tag{4.1}$$

Equating to the solution in Equation (2.25) in the linear coordinates, we obtain the relation,

$$\omega^i_{\mu} r^{\mu} := \gamma^i \tag{4.2}$$

which is a generalization of rotation in 2d dimensions $\omega t = \gamma$. This explains why the gauge fields ω^i_{μ} have angular velocity units. The *angular momentum operator* is easily derived from the exponent of the solution in Equation (4.1), which is the part coupled to angles γ^i:

$$\hat{S}_i = -i\hbar j_i^{R/L} \tag{4.3}$$

For example, for a spinor representation, $j_i^{R/L} = \pm\vec{\sigma}/2$ the operator becomes:

$$\vec{\hat{S}} = \pm i \frac{\hbar}{2} \vec{\sigma} \tag{4.4}$$

$\vec{L}$ is the familiar operator for the angular momentum of spin 1/2 particles in quantum mechanics the Equation (4.1) is re-written as:

$$\phi^{R/L}(r_\mu) = e^{-iS_i\gamma^i}\phi^{R/L}(r_\mu = 0), \tag{4.5}$$

Next, moving to cyclic coordinate $(t, \vec{r})$ to $(t, \vec{\theta})$ and $(E, \vec{P})$ to $(E, \vec{L})$, Substituting in Equation (2.27) and scaling $\theta^\mu \to \theta^\mu/\hbar$, we obtain:

$$\phi^{R/L}(\vec{\gamma}) = e^{-i(L_i\theta^i/\hbar + Et)}\phi^{R/L}(\vec{\gamma}_\mu = 0), \tag{4.6}$$

where $L_0 = E$ the energy and $\gamma_0 = t$ the time, which is not periodic. Applying periodic conditions on the rotations of the spatial part $i = 1, 2, 3$, for bosonic representation, we have,

$$\phi^{R/L}(2\pi) = e^{-i\hat{L}_i 2\pi/\hbar}\phi^{R/L}(0) \equiv e^{-i2\pi n_i}\phi^{R/L}(0), \tag{4.7}$$

then,

$$\hat{L}_i = n_i\hbar \tag{4.8}$$

in the same manner for fermionic representation $\phi^{R/L}(2\pi) = -\phi^{R/L}(0)$:

$$\hat{L}_i = (n_i + 1/2)\hbar \tag{4.9}$$

Which are the quantization of angular momentum.

4.2. The Field Strength Tensor and a New Force of Nature

The covariant derivative of $D_\mu^{L/R} = \partial_\mu - ig_0\hbar\omega_\mu^{L/Ri} j_i^{L/R}$ is similar to The covariant derivative $D_\mu^{L/R} = \partial_\mu - iG_0 p^0_{(\mu)}\omega_\mu'^{L/Ri} j_i^{L/R}$ from Equation (3.14) by swapping $\omega'^{L/R\mu}$ with $\omega^{L/R\mu}$ Therefore, it produces the same force as gravity but instead the energy-momentum p_μ as its charge, we have quantum angular momentum in units of $n\hbar$ as the charge. This new force produces curvature of the space-time R, also according to the Eqs. (3.39) (3.36)

$$R_{\mu\nu}^{L/R} \equiv \mathcal{F}_{\mu\nu}^{L/R}, \tag{4.10}$$

where

$$\mathcal{F}_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu + \omega_\mu\omega_\nu - \omega_\nu\omega_\mu \tag{4.11}$$

For the non-scalar representation $(0, n)_{0,0} \oplus (n, 0)_{0,0}$ e.g. $(0, 1/2)_{0,0} \oplus (1/2, 0)_{0,0}$ this state is charged under the spatial part, $\omega^i, i = 1, 2, 3$, and the temporal part $\omega_\nu^0, i = 0$ is zero, the field strength tensor then is,

$$\mathcal{F}_{\mu\nu}^i = \partial_\nu\omega_\mu^i - \partial_\mu\omega_\nu^i + G_0\hbar\epsilon^{ijk}\omega_\mu^j\omega_\nu^k \neq 0 \tag{4.12}$$

The angular momentum operators from Equation (4.8) :

$$\hat{L}_i^L = j_i^L\hbar$$
$$\hat{L}_i^R = j_i^R\hbar \tag{4.13}$$

And the quantized angular momentum is:

$$L_i^L = n^L \hbar$$
$$L_i^R = n^R \hbar \tag{4.14}$$

$n^{L/R}$ are natural numbers. According to the Equation (2.1) we have,

$$J_i = \frac{j_i^L + j_i^R}{2}$$
$$K_i = \frac{j_i^L - j_i^R}{2i}, \tag{4.15}$$

defining the following operators accordingly:

$$\hat{\Omega}_i = \hbar J_i = \hbar(\frac{j_i^L + j_i^R}{2})$$
$$c\hat{N}_i = \hbar K_i = \hbar(\frac{j_i^L - j_i^R}{2i}), \tag{4.16}$$

c is the speed of light, their quantized value is:

$$\Omega_i = \hbar(\frac{n_i^L + n_i^R}{2})$$
$$cN_i = \hbar(\frac{n_i^L - n_i^R}{2i}), \tag{4.17}$$

Then we have,

$$(L_i^L)^2 + (L_i^R)^2 = [(n_i^L)^2 + (n_i^R)^2]\hbar^2 = 2[(\Omega_i)^2 - (cN_i)^2] \tag{4.18}$$

On the other hand:

$$2[(\Omega_i)^2 - (cN_i)^2] := L^{\mu\nu} L_{\mu\nu} \tag{4.19}$$

$L^{\mu\nu}$ is the angular momentum tensor. $L^{\mu\nu} = r^\mu p^\mu - r^\mu p^\mu = r^\mu \wedge p^\nu$. Ω_i is spatial angular momentum, and N_i is the dynamic mass moment [3,4]. Hence,

$$(L_i^L)^2 + (L_i^R)^2 = L^{\mu\nu} L_{\mu\nu} \tag{4.20}$$

Then, the resulting potential is:

$$V_2(r) = -g_0 \frac{L_{1\mu\nu} L_2^{\mu\nu}}{r} = \tag{4.21}$$
$$= -2g_0 \frac{\vec{L}_1 \cdot \vec{L}_2 - \vec{N}_1 \cdot \vec{N}_2}{r} \tag{4.22}$$

This new potential can explain the Dark matter and Dark Energy, which is dealt with in another paper, Cf. [15].

4.3. The Torsion Tensor

The torsion tensor here is also zero, similar to what was done in Equation (3.50),

$$T^{L/R\rho}_{\mu\sigma} = 0 \tag{4.23}$$

again due to the similarity between ω'_μ and ω_μ.

5. Appendix-The Quantum Structure of Space-Time

The distances in space-time we measure are the eigenvalues of the position quantum state in bra-ket notation:

$$\hat{r}_{\mu}^{L/R}|r_{\mu}^{L/R}\rangle = r_{\mu}^{L/R}|r_{\mu}^{L/R}\rangle, \tag{5.1}$$

The state is $|r_{\mu}^{L/R}\rangle$ dimensionless (as every unitary representation state), and its conjugate state $|p_{\mu}^{L/R} = \hbar\omega_{L/R\mu}\rangle$, but their eigenvalues have. The coordinate (mass dimension 1) and its conjugate momentum (mass dimension -1), respectively. Together, give back $p^{L/R\mu}r_{\mu}^{L/R} = \gamma^{L/R} = \gamma^{L/R\mu}j_{\mu}^{L/R}$, which is dimensionless. $p_{\mu}^{L/R}, r_{\mu}^{L/R}$ are conjugate coordiantes in the phase space and are related by the Fourier transformation.

The 4D volume V is the multi-particle state of $|r_{\mu}\rangle$ using the anti-symmetric tensor product.

$$|V\rangle = \text{anti-sym}(|r_{\mu}^{1}\rangle \otimes |r_{\mu}^{2}\rangle \otimes |r_{\mu}^{3}\rangle \otimes |r_{\mu}^{4}\rangle), \tag{5.2}$$

Defining the operator $\hat{\mathbf{O}}$ works on the direct product of Hilbert spaces $\otimes_i H^i$:

$$\hat{\mathbf{O}}|\mathbf{r}\rangle = (\bigotimes_{i=1}^{N}\hat{O}_i)(\bigotimes_{i=1}^{N}|r_i\rangle) = \bigotimes_{i=1}^{N}(\hat{O}_i|r_i\rangle), \tag{5.3}$$

where $\hat{\mathbf{O}} = \bigotimes_{i=1}^{N}\hat{O}_i$, Then the volume operator can be defined as:

$$\hat{V} = \hat{r}_{\mu}^{0} \otimes \hat{r}_{\mu}^{1} \otimes \hat{r}_{\mu}^{2} \otimes \hat{r}_{\mu}^{3} \tag{5.4}$$

Where $\hat{r}_{\mu}|r_{\mu}^{L/R}\rangle = r_{\mu}|r_{\mu}^{L/R}\rangle$ is the position operator. Calculating the eigenvalue of the volume operator working on the eigenvector in Equation (5.2),

$$\begin{aligned}\hat{V}|V\rangle &= \text{anti-sym}(\hat{r}_{\mu}^{0}|r_{\mu}^{0}\rangle \otimes \hat{r}_{\mu}^{1}|r_{\mu}^{1}\rangle \otimes \hat{r}_{\mu}^{2}|r_{\mu}^{2}\rangle \otimes \hat{r}_{\mu}^{3}|r_{\mu}^{3}\rangle)\\ &= \text{anti-sym}(r_{\mu}^{0}|r_{\mu}^{0}\rangle \otimes r_{\mu}^{1}|r_{\mu}^{1}\rangle \otimes r_{\mu}^{2}|r_{\mu}^{2}\rangle \otimes r_{\mu}^{3}|r_{\mu}^{3}\rangle),\end{aligned} \tag{5.5}$$

Using the tensor product multiplication rule $(A \otimes B)(C \otimes D) = (AC \otimes BD)$gives,

$$\hat{V}|V\rangle = \text{anti-sym}(r_{\mu}^{0} \otimes r_{\mu}^{1} \otimes r_{\mu}^{2} \otimes r_{\mu}^{3}))|V\rangle, \tag{5.6}$$

Thus, the eigenvalue is,

$$\lambda_V = \text{anti-sym}(r_{\mu}^{0} \otimes r_{\mu}^{1} \otimes r_{\mu}^{2} \otimes r_{\mu}^{3}), \tag{5.7}$$

which is equivalent to the exterior product (wedge product)

$$\lambda_V = r_{\mu}^{0} \wedge r_{\mu}^{1} \wedge r_{\mu}^{2} \wedge r_{\mu}^{2} = \det(r_{\mu}^{0}, r_{\mu}^{1}, r_{\mu}^{2}, r_{\mu}^{3}), \tag{5.8}$$

Which is the 4D volume element.

In case the multi-particles are entangled, the states of the volume become

$$|V\rangle = \sum_i \beta_i |r_{\mu}^{0i}, r_{\mu}^{1i}, r_{\mu}^{2i}, r_{\mu}^{3i}\rangle, \tag{5.9}$$

With the eigenvalues

$$\lambda_i = r_{\mu}^{0i} \wedge r_{\mu}^{1i} \wedge r_{\mu}^{2i} \wedge r_{\mu}^{3i}, \tag{5.10}$$

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In total, the space-time is a Hilbert space with the states $|r_\mu^{L/R}\rangle, |\omega_\mu'^{L/R}\rangle$ that lie in the adjoint representation, the $|r_\mu^{L/R}\rangle$ make the volume while $|\omega_\mu'^{L/R}\rangle$ endowing it with a curvature (gravity).

6. Discussion

The search for a consistent theory of quantum gravity remains one of the most important open problems in modern physics. The central difficulty lies in reconciling the dynamical nature of spacetime in general relativity with the principles of quantum mechanics.

In this paper, we have shown that by gauging he double cover Lorentz symmetry of space-time, we can reproduce quantum mechanics and quantum gravity in one piece by a Lagrangian of a particle charged under this local Lorentz symmetry group. In the case that the gauge fields are constant in space-time, we reproduced the quantum states and their evolution in space-time, the quantization of the four-energy momentum $p_\mu = \hbar\omega_\mu$. Moving to a non-constant gauge field and calculating their kinetic energy, i.e, the field strength tensor, we retrieved the curvature of the space-time in a quantized manner, hence the quantum gravity. This curvature is coupled to the energy-momentum, which is the conserved Noether charge of invariance under space-time translations. The case with constant gauge fields becomes equal to a flat universe, i.e., Minkowski space. General relativity turns out to be an effective field theory of these fields. This model is renormalizable as it is a Yang-Mills theory of the Lorentz symmetry group double cover, so it has no singularities inside a black hole or at the Big Bang.

This model, compared to *string theory,* is simpler, living only in four dimensions, has no extra unobserved particle, and is formulated according to the well-known verified Yang-Mills framework. The reliance of string theory on higher dimensions, supersymmetry, and a specific compactification scheme and background dependence raises questions about its uniqueness and direct physical applicability. *Loop quantum gravity*, on the contrary, is background-independent and aims to quantize the very geometry of spacetime using Ashtekar variables. It provides a mathematically rigorous foundation for discrete spectra of geometric operators, suggesting that spacetime itself has a quantum granular structure. Yet, challenges remain in recovering the classical continuum limit, connecting with low-energy phenomenology, and unifying the framework with the standard model physics. In comparison to Quantum loop gravity (QLG), the space-time here is the representation space of the Lorentz symmetry group, hence it is a Hilbert space as it is postulated by quantum mechanics. There is no need to be dealt as an independent quantity; the position four-vector and the gauge fields live in the adjoint representation, hence real, and the gauge fields endow the Hilbert space with a curvature which plays the role of gravity. Therefore, the classical existence of space-time is no longer valid and should be replaced according to understanding. This universe is free from singularity inside the black hole or at the Big Bang, and now can be re-formulated according to this framework.

The invariance under rotation in spacetime gives a similar force to gravity, but it is coupled to the angular momentum tensor instead. It can explain the dark matter and dark energy, which are discussed in another paper.

7. Conclusions

Gravity and quantum mechanics can be unified in one frame, which derives both of them consistently. This frame is built by expanding the Lorentz symmetry by adding a new abelian group and making it local. The equation of motion of a particle charged under this symmetry reproduces quantum mechanics, while the field strength tensor reproduces the curvature of space-time, hence a renormalizable quantum gravity. The framework here is the Yang-Mills in four dimensions, which is well-established, no extra space-time dimension as in string theory, and no need for special treatment of space-time as in the loop quantum gravity. Space-time itself is just the representation space of this group of symmetry, i.e, Hilbert space, and the classical image of independent space-time is superfluous. There is no extra particle predicted, which makes this model economical and natural.

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