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Article

Research Note: Closed-Form Nash Equilibria for EPECs in Zonal Power Markets

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Abstract

This research note develops an analytical framework for solving Equilibrium Problems with Equilibrium Constraints (EPECs) in stylized zonal electricity markets. We build upon the competitive benchmark established in [1], which analyzed how network topology dictates market efficiency. Here, we extend this analysis to investigate how strategic actors exploit these same topological constraints. We formulate a two-sided EPEC that accommodates market power on both sides of the market: generators competing in oligopoly and large consumers exercising oligopsony power. This framework captures the increasing relevance of strategic demand flexibility in modern power systems. As a methodological proof of concept, we apply this framework to the canonical two-zone network. We demonstrate that the Nash equilibrium for each congestion regime can be systematically derived in closed form using symbolic computation. This analytical approach provides a foundation for interpreting numerical simulations and establishes a tractable path toward solving the more complex 3-zone line and loop topologies.

Keywords: electricity markets; market power; equilibrium problem with equilibrium constraints (EPEC); zonal pricing; network topology; strategic behaviour; symbolic computation

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1. Introduction

One aim in restructuring electricity markets worldwide was to replace centralized planning with competitive outcomes. However, the reality often falls short of this ideal. High concentration in generation coupled with inelastic demand and the physical constraints of the transmission network creates significant opportunities for the exercise of market power [2,3]. Understanding how strategic behaviour interacts with network constraints is therefore crucial for effective market design and regulation.

Transmission congestion fundamentally alters the competitive landscape. When the network is unconstrained, the market integrates, fostering competition. When constraints bind, the market fragments, creating spatial market power [4]. A generator that faces robust competition in an integrated system may become a local monopolist when its import capacity is exhausted. Crucially, this market power is endogenous; it emerges dynamically from the state of the network rather than being a static feature of market structure [5].

Modeling this interaction presents a significant challenge. Strategic firms do not merely respond to prices; they anticipate how their actions will affect the market-clearing process managed by the Independent System Operator (ISO). This creates a bi-level structure. The upper level consists of strategic firms playing a Nash game, while the lower level is the ISO's welfare-maximizing dispatch. This structure is formally modeled as an Equilibrium Problem with Equilibrium Constraints (EPEC) [6].

The literature on strategic behaviour in electricity networks is extensive, building on early constrained Cournot models [7,8]. While the modern EPEC approach has become the standard framework, the complexity introduced by the complementary slackness conditions in the ISO's optimization means the vast majority of this literature relies on numerical solutions for large, realistic networks [9,10].

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While numerical EPECs provide valuable insights for specific case studies, they offer limited generalizability. They provide point solutions in a high-dimensional parameter space, for one set of parameters, but cannot easily reveal the underlying structure of the Nash equilibrium. Furthermore, they obscure how the equilibrium shifts discontinuously when the network transitions between different congestion regimes.

To complement these numerical results, we take a different path. This study seeks closed-form solutions for canonical network topologies with two primary goals. First, to reveal structural insights that high-dimensional simulations cannot easily expose. Second, to provide verifiable analytic benchmarks that simulators can use to validate their models and interpret their results.

Analytically, a closed-form solution allows us to move beyond point solutions. It enables us to define the precise parameter thresholds— in terms of transmission capacity C_{ij} or zonal elasticity K_i —at which a firm's strategic incentive to withhold capacity emerges or disappears. This approach allows us to treat the Nash equilibrium itself as a function, revealing *how* market power is structurally constrained by network topology (e.g., line vs. loop) or market elasticity, rather than just *if* it exists for one specific set of parameters.

Our previous work [1] established the closed-form competitive benchmark for these topologies. This note takes the critical next step: using the same analytical framework to model strategic behaviour and quantify the resulting deviation from that competitive baseline. As discussed in our prior work, this provides the essential foundation for analyzing market power.

In this research note, we first formulate the problem of zonal pricing in terms of a generalized EPEC (Section 2). We then apply this framework to derive the complete set of closed-form Nash equilibria for the 2-zone case (Section 3). This tractable example demonstrates precisely how market power emerges, how it is constrained by transmission capacity, and how the market fragments into distinct strategic regimes. This analysis provides the conceptual and methodological template for the more complex 3-zone topologies to follow.

2. Methods: A General EPEC Formalism for Zonal Pricing

This section details a model of a two-sided strategic market. We explicitly accommodate market power on both the supply side, representing competing generation companies, and the demand side, representing large final consumers.

This two-sided framework captures a fundamental asymmetry in modern power systems. Generators compete to sell energy while large consumers compete to procure it, and both sides can strategically leverage their ability to affect market-clearing prices. The following sections justify this approach and lay out the formal EPEC structure.

2.1. Why Model Strategic Demand?

Supply-side market power in electricity markets is well-documented. Generators can exercise market power by withholding capacity, particularly when transmission constraints create localized scarcity. The existence and magnitude of such behaviour has been extensively studied both theoretically and empirically.

Demand-side market power is less common but can still be relevant in specific contexts. Large energy-intensive industries such as aluminum smelters, steel production facilities, and chemical plants can shift or curtail production in response to price signals, effectively exercising monopsony power in their local market. Similarly, demand aggregators that coordinate the response of many small consumers can act as a single strategic player with significant market impact. Large consumers with substitution options— such as data centers, industrial facilities with on-site generation, or consumers with energy storage —can credibly threaten to withdraw demand, thereby affecting market prices. Finally, as renewable generation increases, the strategic timing of flexible demand becomes more valuable, creating opportunities for demand-side market power.

Beyond these specific applications, the two-sided framework serves a deeper analytical purpose. By modeling both supply and demand as potentially strategic, this approach constructs a general

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equilibrium framework that nests several important special cases. When demand elasticity approaches infinity, meaning all demand is perfectly flexible, the model recovers a pure supply-side oligopoly. When supply elasticity approaches infinity, corresponding to perfectly competitive generation, the framework obtains a pure demand-side oligopsony. When both sides are inelastic, the analysis recovers the perfectly competitive market-clearing model from earlier sections. This generality allows the study of how network topology and transmission constraints affect the nature of strategic competition, independent of which side of the market dominates.

2.2. The Role of Network Constraints

In a standard single-node Cournot game, strategic firms compete in quantities, taking the aggregate output of rivals as given. The Nash equilibrium is determined by the intersection of best-response functions, and the welfare loss from market power can be quantified by comparing the oligopolistic equilibrium to the competitive benchmark. Network constraints fundamentally alter this structure in ways that cannot be captured by simply extending the single-node model.

Transmission congestion creates artificial scarcity, giving rise to spatial market power and local monopolies. A zone that would be competitive in an uncongested network can become a local monopoly when its import capacity is exhausted. Conversely, a generator with negligible market power in an integrated market may exercise substantial market power when transmission constraints segment the network. The degree of market power becomes endogenous to the network state rather than being determined solely by market structure.

Unlike a single-node market with one price, a zonal market has n prices, one per zone, which are linked by transmission flows and congestion rents. This price multiplicity means each strategic agent must reason about multiple interconnected markets simultaneously. A generator in Zone 1 must consider not only its local competitors but also the impact of its production on prices in other zones and the resulting transmission flows. The strategic problem becomes inherently spatial and interdependent.

The competitive structure changes discontinuously when transmission lines congest or decongest, leading to regime-dependent competition. A market that operates as a unified competitive system when all lines are uncongested can fragment into isolated local monopolies when congestion occurs. The Nash equilibrium is therefore piecewise: different network states—uncongested versus various congestion patterns—give rise to qualitatively different equilibria. This discontinuity is a fundamental feature of electricity markets with binding transmission constraints, not a modeling artifact.

Finally, each strategic agent must anticipate how the independent system operator's welfare-maximizing dispatch responds to its actions. This creates a bilevel structure known as an Equilibrium Problem with Equilibrium Constraints, or EPEC. The upper level consists of strategic firms playing a Nash game, while the lower level is the ISO's optimization problem. The coupling between levels occurs through the price function $p_i(\mathbf{s}, \mathbf{d})$, which is determined by the ISO's first-order conditions. Computing equilibria therefore requires solving for the fixed point where each firm's best response is consistent with the market-clearing prices that emerge from all firms' simultaneous decisions.

2.3. The EPEC Formulation

This zonal electricity market is modeled as a two-level Equilibrium Problem with Equilibrium Constraints (EPEC).

- The upper level consists of a simultaneous Nash game played by two sets of strategic agents: G generation companies and K large final consumers. Generators $g = 1, \ldots, G$ choose production quantities s_g to maximize profit, while consumers $k = 1, \ldots, K$ choose consumption quantities d_k to maximize surplus.
- The lower level is the independent system operator's (ISO) market-clearing problem. The ISO takes the strategic quantities $\mathbf{s} = [s_1, \ldots, s_G]^T$ and $\mathbf{d} = [d_1, \ldots, d_K]^T$ as given and determines passive demand and transmission flows to maximize social welfare.

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The Nash equilibrium of this game is the solution to the EPEC formed by the simultaneous optimality conditions of all upper-level agents, constrained by the optimality conditions of the lower-level market clearing.

2.4. Upper Level: Strategic Agents

Generation Companies

Each generation company g located in zone $i(g)$ chooses its production quantity s_g to maximize profit, which is revenue minus cost

$$\pi_g(s_g; \mathbf{s}_{-g}, \mathbf{d}) = p_{i(g)}(\mathbf{s}, \mathbf{d}) \cdot s_g - C_g(s_g), \tag{1}$$

where $p_{i(g)}(\mathbf{s}, \mathbf{d})$ is the market-clearing price in the generator's zone, which depends on the actions of all agents, and $\mathbf{s}_{-g}$ denotes the production quantities of all other generators. We assume quadratic generation costs $C_g(s_g) = \frac{1}{2}a_g s_g^2 + b_g s_g$, giving marginal cost $MC_g(s_g) = a_g s_g + b_g$. This functional form ensures that marginal cost is increasing and that all optimization problems are well-behaved. The first-order condition for profit maximization requires that the derivative of profit with respect to quantity equals zero

$$\frac{\partial \pi_g}{\partial s_g} = p_{i(g)} + s_g \frac{\partial p_{i(g)}}{\partial s_g} - (a_g s_g + b_g) = 0. \tag{2}$$

Rearranging, the optimality condition equating marginal revenue to marginal cost is

$$\underbrace{p_{i(g)} + s_g \frac{\partial p_{i(g)}}{\partial s_g}}_{\text{Marginal Revenue}} = \underbrace{a_g s_g + b_g}_{\text{Marginal Cost}}. \tag{3}$$

The strategic markup term $s_g \partial p_{i(g)}/\partial s_g$ captures the firm's market power. A competitive firm that takes price as given has $\partial p_{i(g)}/\partial s_g = 0$, so price equals marginal cost. A strategic firm recognizes that increasing production lowers the market price, creating a negative price impact. Consequently, it produces less than the competitive level, earning a markup of price over marginal cost. The magnitude of this markup depends on the firm's production level and the sensitivity of price to its output.

Note that the derivative $\partial p_{i(g)}/\partial s_g$ is not a primitive parameter that can be specified exogenously. Rather, it emerges endogenously from the ISO's market-clearing problem and depends on the network state. When transmission constraints bind, this derivative can change discontinuously as the market transitions from one regime to another. Understanding how network topology shapes these price impacts is central to our analysis.

Large Final Consumers

Each large final consumer k located in zone $j(k)$ chooses its consumption quantity d_k to maximize consumer surplus, which is utility minus expenditure

$$CS_k(d_k; \mathbf{s}, \mathbf{d}_{-k}) = U_k(d_k) - p_{j(k)}(\mathbf{s}, \mathbf{d}) \cdot d_k, \tag{4}$$

where $U_k(d_k)$ is the consumer's utility function and $\mathbf{d}_{-k}$ denotes the consumption of all other large final consumers. We assume quadratic utility $U_k(d_k) = e_k d_k - \frac{1}{2}c_k d_k^2$, which corresponds to a linear inverse demand curve $p = e_k - c_k d_k$. The marginal utility is therefore $MU_k(d_k) = e_k - c_k d_k$, which decreases with consumption. This functional form is the mirror image of the generator's cost function, ensuring analytical symmetry between supply and demand. The first-order condition for surplus maximization requires that the derivative of surplus with respect to consumption equals zero,

$$\frac{\partial CS_k}{\partial d_k} = (e_k - c_k d_k) - \left(p_{j(k)} + d_k \frac{\partial p_{j(k)}}{\partial d_k} \right) = 0. \tag{5}$$

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Rearranging yields the optimality condition equating marginal utility to marginal expenditure

$$\underbrace{e_k - c_k d_k}_{\text{Marginal Utility}} = \underbrace{p_{j(k)} + d_k \frac{\partial p_{j(k)}}{\partial d_k}}_{\text{Marginal Expenditure}} . \tag{6}$$

A competitive consumer that takes price as given has $\partial p_{j(k)}/\partial d_k = 0$, so marginal utility equals price. A strategic consumer recognizes that increasing consumption raises the market price for all units consumed. The price impact $\partial p_{j(k)}/\partial d_k > 0$ means the consumer's effective cost for the marginal unit—its marginal expenditure—exceeds the market price. This causes the consumer to consume less than the competitive level. The strategic markdown is the mirror image of the generator's markup: where generators withhold supply to raise prices, strategic consumers suppress demand to lower them.

2.5. Lower Level: ISO Market Clearing

The independent system operator takes the strategic quantities $\mathbf{s}$ and $\mathbf{d}$ as given and clears the market by determining passive demand $\mathbf{d}_{\text{pass}}$ and transmission flows $\mathbf{f}$ to maximize social welfare. The passive demand in each zone represents all the small, price-taking consumers whose aggregate behaviour can be summarized by a linear inverse demand curve $p_i = e_{\text{pass},i} - c_{\text{pass},i} d_{\text{pass},i}$. We use location matrices $\mathbf{L}_s \in \mathbb{R}^{n \times G}$ and $\mathbf{L}_d \in \mathbb{R}^{n \times K}$ to map firm-level decisions to zonal quantities. The entry $L_{s,ig} = 1$ if generation company g is located in zone i and zero otherwise. The matrix $\mathbf{L}_d$ is defined similarly for large final consumers. These matrices allow us to write the aggregate strategic generation in each zone as $\mathbf{L}_s\mathbf{s}$ and the aggregate strategic consumption as $\mathbf{L}_d\mathbf{d}$. The ISO's optimization problem is

$$\begin{aligned} \max_{\mathbf{d}_{\text{pass}}, \mathbf{f}} \quad & \mathbf{e}_{\text{pass}}^T \mathbf{d}_{\text{pass}} - \frac{1}{2} \mathbf{d}_{\text{pass}}^T \mathbf{C}_{\text{pass}} \mathbf{d}_{\text{pass}} \\ \text{subject to:} \quad & (\mathbf{L}_s \mathbf{s}) - (\mathbf{L}_d \mathbf{d}) - \mathbf{d}_{\text{pass}} - \mathbf{A}\mathbf{f} = \mathbf{0} \quad : \mathbf{p} \\ & \mathbf{f} \le \mathbf{C}_{\text{fwd}} \quad : \boldsymbol{\mu}_{\text{fwd}} \\ & -\mathbf{f} \le \mathbf{C}_{\text{rev}} \quad : \boldsymbol{\mu}_{\text{rev}}. \end{aligned} \tag{7}$$

The objective function represents the social welfare derived from passive consumption. The power balance constraint ensures that in each zone, the sum of all generation (strategic and passive) equals the sum of all consumption plus net flows out of the zone. The matrix $\mathbf{A}$ is the node-arc incidence matrix that relates flows to nodal injections. The dual variable $\mathbf{p}$ associated with the power balance constraint is the vector of zonal prices. The transmission capacity constraints are enforced with dual variables $\boldsymbol{\mu}_{\text{fwd}}$ and $\boldsymbol{\mu}_{\text{rev}}$, which represent congestion rents in the forward and reverse directions respectively.

The Karush-Kuhn-Tucker conditions for this problem define the relationship between prices, flows, and congestion. These conditions provide the link between the upper and lower levels of the game: they determine how the price function $p_i(\mathbf{s}, \mathbf{d})$ responds to changes in strategic quantities, which in turn determines the derivatives $\partial p_i/\partial s_g$ and $\partial p_j/\partial d_k$ required for the strategic agents' optimality conditions. Without explicitly solving the ISO's problem, the strategic agents cannot compute their best responses.

2.6. Solution Methodology

The Nash equilibrium is characterized by the simultaneous solution of three sets of conditions: the G generation company optimality conditions from Equation (3), the K large final consumer optimality conditions from Equation (6), and the ISO's KKT conditions from Problem (7). This system of equations defines the EPEC.

However, the complementary slackness conditions for transmission constraints introduce a discrete structure that prevents a unified analytical solution. Each possible pattern of binding constraints—— such as no congestion, forward congestion on certain lines, or reverse congestion

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on others —–defines a distinct regime. Within each regime, the complementary slackness conditions resolve into simple equalities (for binding constraints) and inequalities (for non-binding constraints). This resolution simplifies the EPEC to a system of algebraic equations that can be solved analytically or numerically. Different regimes correspond to different competitive structures: an uncongested network behaves as a single integrated market, while congested networks fragment into partially or fully isolated zones. This is the exact analytical parallel to the competitive benchmark, where this same piecewise structure defined the market-clearing outcomes [1].

The solution methodology proceeds in three stages. First, enumerate all feasible network states based on the topology and the number of transmission lines. For a network with m lines, each of which can be uncongested, forward-congested, or reverse-congested, there are in principle 3^m possible regimes. However, many of these are either physically impossible or symmetrically equivalent, reducing the number of distinct cases.

Second, for each regime, assume a specific pattern of binding constraints. This assumption resolves the complementary slackness conditions into simple equalities and inequalities, transforming the EPEC into a standard system of equations. For example, if we assume line ℓ is forward-congested, then $f_\ell = C_{\ell,\text{fwd}}$ and $\mu_{\ell,\text{fwd}} \geq 0$, while $\mu_{\ell,\text{rev}} = 0$. We then solve the resulting system— using symbolic computation packages such as SymPy and Mathematica —to obtain candidate equilibrium quantities, prices, and congestion rents.

Third, verify which candidate equilibrium is self-consistent by checking validity conditions. These conditions ensure that the calculated flows and congestion rents are consistent with the assumed regime. For instance, if it is assumed that line ℓ is forward-congested, we must verify that $f_\ell^* = C_{\ell,\text{fwd}}$ and that the price difference supports this flow direction, meaning $p_j^* > p_i^*$ where the flow goes from zone i to zone j. For any given set of parameters—cost functions, utility functions, and transmission capacities—exactly one regime will have satisfied validity conditions. This regime's solution is the Nash equilibrium.

The equilibrium is therefore piecewise: it consists of multiple analytical formulas, each valid in a different region of parameter space, with regime boundaries determined by the validity conditions. This piecewise structure is unavoidable. The complementary slackness conditions create a discrete, non-convex problem that cannot be solved in a unified manner. The piecewise methodology is not a limitation of our analytical approach but rather a fundamental property of electricity markets with binding transmission constraints.

3. Results: The Two-Zone Strategic Equilibrium

In this section the EPEC framework is applied to the canonical case of two zones connected by a single transmission line. This topology is the simplest non-trivial network, yet it exhibits all the fundamental features of strategic competition under network constraints: price separation, regime-dependent equilibria, and the endogenous emergence of spatial market power.

3.1. System Configuration

Consider two zones connected by a single bidirectional transmission line. To maintain analytical tractability while preserving the essential strategic interactions, the model has a symmetric system with one generation company and one large final consumer in each zone. Zone 1 contains GenCo 1 with production s_1 and LFC 1 with consumption d_1. Zone 2 contains GenCo 2 with production s_2 and LFC 2 with consumption d_2. Each zone also has passive demand characterized by linear inverse demand curves. The transmission line has capacity C_{12} in the forward direction (from Zone 1 to Zone 2) and capacity C_{21} in the reverse direction. The flow f_{12} is defined as positive when power flows from Zone 1 to Zone 2, so $f_{12} > 0$ indicates forward flow and $f_{12} < 0$ indicates reverse flow (from Zone 2 to Zone 1). The strategic agents have the following characteristics. GenCo g in zone i has quadratic cost function $C_g(s_g) = \frac{1}{2}a_g s_g^2 + b_g s_g$, giving marginal cost $MC_g = a_g s_g + b_g$. LFC k in zone j has quadratic utility function $U_k(d_k) = e_k d_k - \frac{1}{2}c_k d_k^2$, giving marginal utility $MU_k = e_k - c_k d_k$. The passive demand in zone i has inverse demand curve $p_i = e_{\text{pass},i} - c_{\text{pass},i} d_{\text{pass},i}$.

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The complementary slackness conditions for the transmission line create three distinct regimes. In the uncongested regime, the line has available capacity in both directions, so the flow is determined by economic optimization and both congestion prices are zero. In the forward-congested regime, power flows at the maximum capacity from Zone 1 to Zone 2, so $f_{12} = C_{12}$ and the forward congestion price μ_{12} is positive. In the reverse-congested regime, power flows at maximum capacity from Zone 2 to Zone 1, so $f_{12} = -C_{21}$ and the reverse congestion price μ_{21} is positive.

These three regimes exhaust all possibilities. A regime where both directions are simultaneously congested is impossible because it would require $f_{12} = C_{12}$ and $f_{12} = -C_{21}$ simultaneously, which cannot be satisfied unless both capacities are zero. The regime structure is therefore complete and mutually exclusive.

3.2. Regime 1: The Uncongested Equilibrium

When the transmission line has sufficient capacity to accommodate the economically optimal flow, the complementary slackness conditions require that both congestion prices equal zero: $\mu_{12} = 0$ and $\mu_{21} = 0$. The KKT conditions from the ISO's problem then imply that the price difference between zones equals the sum of congestion prices: $p_2 - p_1 = \mu_{21} - \mu_{12} = 0$. Therefore, $p_1 = p_2 = p$, and the two zones merge into a single integrated market with a uniform price.

To solve the strategic agents' optimization problems, we must first derive an explicit expression for the price p as a function of all strategic quantities from the ISO's KKT conditions. The stationarity condition for passive demand in each zone gives $p = e_{\text{pass},i} - c_{\text{pass},i} d_{\text{pass},i}$. Solving for passive demand yields $d_{\text{pass},i} = (e_{\text{pass},i} - p)/c_{\text{pass},i}$. The power balance constraint for Zone 1 is $s_1 - d_1 - d_{\text{pass},1} - f_{12} = 0$, and for Zone 2 is $s_2 - d_2 - d_{\text{pass},2} + f_{12} = 0$. Summing these two equations causes the flow term f_{12} to cancel

$$(s_1 + s_2) - (d_1 + d_2) - (d_{\text{pass},1} + d_{\text{pass},2}) = 0. \tag{8}$$

Substituting the expressions for passive demand and the common price p

$$(s_1 + s_2) - (d_1 + d_2) - \left(\frac{e_{\text{pass},1} - p}{c_{\text{pass},1}} + \frac{e_{\text{pass},2} - p}{c_{\text{pass},2}} \right) = 0. \tag{9}$$

Rearranging this equation to solve for the unified price,

$$p \left(\frac{1}{c_{\text{pass},1}} + \frac{1}{c_{\text{pass},2}} \right) = (d_1 + d_2) - (s_1 + s_2) + \left(\frac{e_{\text{pass},1}}{c_{\text{pass},1}} + \frac{e_{\text{pass},2}}{c_{\text{pass},2}} \right). \tag{10}$$

Defining $\kappa \equiv (1/c_{\text{pass},1} + 1/c_{\text{pass},2})^{-1}$ as the aggregate price responsiveness of the passive demand, and $\xi \equiv e_{\text{pass},1}/c_{\text{pass},1} + e_{\text{pass},2}/c_{\text{pass},2}$ as the aggregate baseline demand,

$$p(s_1, s_2, d_1, d_2) = \kappa[(d_1 + d_2) - (s_1 + s_2) + \xi]. \tag{11}$$

This price function reveals that in an uncongested market, the price depends only on the aggregate net supply $(s_1 + s_2) - (d_1 + d_2)$. The spatial structure of the network is irrelevant when transmission is unlimited. The price is linear in all strategic quantities, which will simplify the derivation of best responses.

The strategic agents' optimality conditions require the derivatives of price with respect to their own actions. From Equation (11), these derivatives are

$$\frac{\partial p}{\partial s_1} = \frac{\partial p}{\partial s_2} = -\kappa, \quad \frac{\partial p}{\partial d_1} = \frac{\partial p}{\partial d_2} = \kappa. \tag{12}$$

All generation companies have the same price impact: increasing production by one unit raises the price by κ. All large final consumers have the same price impact in magnitude but opposite sign:

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increasing consumption by one unit lowers the price by κ. This symmetry is a consequence of market integration. In an uncongested network, all agents compete on equal terms regardless of their location.

Substituting the price function and its derivatives into the four strategic agents' first-order conditions (FOC), for GenCo 1 in Zone 1 the optimality condition becomes

$$p + s_1\kappa = a_1 s_1 + b_1. \tag{13}$$

Substituting $p = \kappa[(d_1 + d_2) - (s_1 + s_2) + \xi]$ and rearranging

$$(2\kappa + a_1)s_1 + \kappa s_2 - \kappa d_1 - \kappa d_2 = \kappa\xi - b_1. \tag{14}$$

By symmetry, the FOC for GenCo 2 is

$$\kappa s_1 + (2\kappa + a_2)s_2 - \kappa d_1 - \kappa d_2 = \kappa\xi - b_2. \tag{15}$$

For LFC 1 in Zone 1, the optimality condition becomes

$$e_1 - c_1 d_1 = p + d_1\kappa. \tag{16}$$

Substituting the price function and rearranging,

$$\kappa s_1 + \kappa s_2 - (2\kappa + c_1)d_1 - \kappa d_2 = \kappa\xi - e_1. \tag{17}$$

By symmetry, the FOC for LFC 2 is:

$$\kappa s_1 + \kappa s_2 - \kappa d_1 - (2\kappa + c_2)d_2 = \kappa\xi - e_2. \tag{18}$$

Equations (14)–(18) form a linear system $\mathbf{M}\mathbf{x} = \mathbf{v}$, where

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} 2\kappa + a_1 & \kappa & -\kappa & -\kappa \\ \kappa & 2\kappa + a_2 & -\kappa & -\kappa \\ \kappa & \kappa & -(2\kappa + c_1) & -\kappa \\ \kappa & \kappa & -\kappa & -(2\kappa + c_2) \end{bmatrix}, \\ \mathbf{x} &= \begin{bmatrix} s_1 \\ s_2 \\ d_1 \\ d_2 \end{bmatrix}, \\ \mathbf{v} &= \begin{bmatrix} \kappa\xi - b_1 \\ \kappa\xi - b_2 \\ \kappa\xi - e_1 \\ \kappa\xi - e_2 \end{bmatrix} \end{aligned} \tag{19}$$

The solution is $\mathbf{x}^* = \mathbf{M}^{-1}\mathbf{v}$, which gives the Nash equilibrium quantities. These expressions were manipulated using the symbolic mathematical package SymPy, and for completeness are given below. Note that SymPy also outputs the results as python functions, which can be used in future analysis. The determinant of $\mathbf{M}$ determines whether a unique equilibrium exists. Using cofactor expansion,

$$\begin{aligned} \det(\mathbf{M}) = a_1a_2c_1c_2 + 2a_1a_2c_1\kappa + 2a_1a_2c_2\kappa + 3a_1a_2\kappa^2 + 2a_1c_1c_2\kappa + \\ 3a_1c_1\kappa^2 + 3a_1c_2\kappa^2 + 4a_1\kappa^3 + 2a_2c_1c_2\kappa + 3a_2c_1\kappa^2 + 3a_2c_2\kappa^2 + \\ 4a_2\kappa^3 + 3c_1c_2\kappa^2 + 4c_1\kappa^3 + 4c_2\kappa^3 + 5\kappa^4. \end{aligned} \tag{20}$$

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Under our assumptions that all parameters are positive ($a_g, c_k, \kappa > 0$), the determinant is strictly positive, ensuring existence and uniqueness of the Nash equilibrium. Applying Cramer's rule or direct matrix inversion, the equilibrium production quantities for the generation companies are

$$\begin{aligned} s_1^* = \zeta\big[& -a_2b_1c_1c_2 - 2a_2b_1c_1\kappa - 2a_2b_1c_2\kappa - 3a_2b_1\kappa^2 + a_2c_1c_2\kappa\xi + \\ & a_2c_1e_2\kappa + a_2c_1\kappa^2\xi + a_2c_2e_1\kappa + a_2c_2\kappa^2\xi + a_2e_1\kappa^2 + a_2e_2\kappa^2 + \\ & a_2\kappa^3\xi - 2b_1c_1c_2\kappa - 3b_1c_1\kappa^2 - 3b_1c_2\kappa^2 - 4b_1\kappa^3 + b_2c_1c_2\kappa + \\ & b_2c_1\kappa^2 + b_2c_2\kappa^2 + b_2\kappa^3 + c_1c_2\kappa^2\xi + c_1e_2\kappa^2 + c_1\kappa^3\xi + \\ & c_2e_1\kappa^2 + c_2\kappa^3\xi + e_1\kappa^3 + e_2\kappa^3 + \kappa^4\xi\big]. \end{aligned} \tag{21}$$

$$\begin{aligned} s_2^* = \zeta\big[& -a_1b_2c_1c_2 - 2a_1b_2c_1\kappa - 2a_1b_2c_2\kappa - 3a_1b_2\kappa^2 + a_1c_1c_2\kappa\xi + \\ & a_1c_1e_2\kappa + a_1c_1\kappa^2\xi + a_1c_2e_1\kappa + a_1c_2\kappa^2\xi + a_1e_1\kappa^2 + a_1e_2\kappa^2 + \\ & a_1\kappa^3\xi + b_1c_1c_2\kappa + b_1c_1\kappa^2 + b_1c_2\kappa^2 + b_1\kappa^3 - 2b_2c_1c_2\kappa - \\ & 3b_2c_1\kappa^2 - 3b_2c_2\kappa^2 - 4b_2\kappa^3 + c_1c_2\kappa^2\xi + c_1e_2\kappa^2 + c_1\kappa^3\xi + \\ & c_2e_1\kappa^2 + c_2\kappa^3\xi + e_1\kappa^3 + e_2\kappa^3 + \kappa^4\xi\big]. \end{aligned} \tag{22}$$

where $\zeta = 1/\det(\mathbf{M})$. The equilibrium consumption quantities for the large final consumers are

$$\begin{aligned} d_1^* = \zeta\big[& a_1a_2c_2e_1 - a_1a_2c_2\kappa\xi + 2a_1a_2e_1\kappa - a_1a_2e_2\kappa - a_1a_2\kappa^2\xi - \\ & a_1b_2c_2\kappa - a_1b_2\kappa^2 + 2a_1c_2e_1\kappa - a_1c_2\kappa^2\xi + 3a_1e_1\kappa^2 - a_1e_2\kappa^2 - \\ & a_1\kappa^3\xi - a_2b_1c_2\kappa - a_2b_1\kappa^2 + 2a_2c_2e_1\kappa - a_2c_2\kappa^2\xi + 3a_2e_1\kappa^2 - \\ & a_2e_2\kappa^2 - a_2\kappa^3\xi - b_1c_2\kappa^2 - b_1\kappa^3 - b_2c_2\kappa^2 - b_2\kappa^3 + 3c_2e_1\kappa^2 - \\ & c_2\kappa^3\xi + 4e_1\kappa^3 - e_2\kappa^3 - \kappa^4\xi\big]. \end{aligned} \tag{23}$$

$$\begin{aligned} d_2^* = \zeta\big[& a_1a_2c_1e_2 - a_1a_2c_1\kappa\xi - a_1a_2e_1\kappa + 2a_1a_2e_2\kappa - a_1a_2\kappa^2\xi - \\ & a_1b_2c_1\kappa - a_1b_2\kappa^2 + 2a_1c_1e_2\kappa - a_1c_1\kappa^2\xi - a_1e_1\kappa^2 + 3a_1e_2\kappa^2 - \\ & a_1\kappa^3\xi - a_2b_1c_1\kappa - a_2b_1\kappa^2 + 2a_2c_1e_2\kappa - a_2c_1\kappa^2\xi - a_2e_1\kappa^2 + \\ & 3a_2e_2\kappa^2 - a_2\kappa^3\xi - b_1c_1\kappa^2 - b_1\kappa^3 - b_2c_1\kappa^2 - b_2\kappa^3 + 3c_1e_2\kappa^2 - c_1\kappa^3\xi - \\ & e_1\kappa^3 + 4e_2\kappa^3 - \kappa^4\xi\big]. \end{aligned} \tag{24}$$

Substituting the equilibrium quantities into Equation (11), the unified market price is

$$\begin{aligned} p^* = \zeta\big[& \kappa - a_1a_2c_1c_2\xi - a_1a_2c_1e_2 - a_1a_2c_1\kappa\xi - a_1a_2c_2e_1 - a_1a_2c_2\kappa\xi \\ & - a_1a_2e_1\kappa - a_1a_2e_2\kappa - a_1a_2\kappa^2\xi - a_1b_2c_1c_2 - a_1b_2c_1\kappa - a_1b_2c_2\kappa - \\ & a_1b_2\kappa^2 - a_1c_1c_2\kappa\xi - a_1c_1e_2\kappa - a_1c_1\kappa^2\xi - a_1c_2e_1\kappa - a_1c_2\kappa^2\xi - \\ & a_1e_1\kappa^2 - a_1e_2\kappa^2 - a_1\kappa^3\xi - a_2b_1c_1c_2 - a_2b_1c_1\kappa - a_2b_1c_2\kappa - \\ & a_2b_1\kappa^2 - a_2c_1c_2\kappa\xi - a_2c_1e_2\kappa - a_2c_1\kappa^2\xi - a_2c_2e_1\kappa - a_2c_2\kappa^2\xi - \\ & a_2e_1\kappa^2 - a_2e_2\kappa^2 - a_2\kappa^3\xi - b_1c_1c_2\kappa - b_1c_1\kappa^2 - b_1c_2\kappa^2 - b_1\kappa^3 - b_2c_1c_2\kappa - \\ & b_2c_1\kappa^2 - b_2c_2\kappa^2 - b_2\kappa^3 - c_1c_2\kappa^2\xi - c_1e_2\kappa^2 - c_1\kappa^3\xi - c_2e_1\kappa^2 - c_2\kappa^3\xi - \\ & e_1\kappa^3 - e_2\kappa^3 - \kappa^4\xi\big]. \end{aligned} \tag{25}$$

The equilibrium quantities depend on all cost and utility parameters across both zones, reflecting the integrated nature of the uncongested market. Strategic interdependence is complete: each agent's optimal choice responds to the characteristics of all other agents. The price p^* aggregates information from all participants, achieving the Nash equilibrium through the simultaneous clearing of strategic and passive demand. The complexity of these expressions— each involving products and ratios of multiple parameters —illustrates a fundamental feature of strategic competition in integrated markets: every firm must internalize not only its own cost structure and local market conditions but also the competitive landscape across the entire network. This stands in stark contrast to the congested regimes, where market fragmentation simplifies the strategic calculus by localizing competition.

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Using the equilibrium quantities, the equilibrium flow from Zone 1's power balance

$$f_{12}^* = s_1^* - d_1^* - d_{\text{pass},1}^* \tag{26}$$

where the passive demand is

$$d_{\text{pass},1}^* = \frac{e_{\text{pass},1} - p^*}{c_{\text{pass},1}}. \tag{27}$$

The uncongested regime is valid if and only if the equilibrium flow lies strictly within the transmission capacity limits

$$-C_{21} < f_{12}^* < C_{12} \tag{28}$$

When this condition is violated, the market transitions to one of the congested regimes. The regime boundaries in parameter space are determined by the equalities $f_{12}^* = C_{12}$ (transition to forward congestion) or $f_{12}^* = -C_{21}$ (transition to reverse congestion).

3.3. Regime 2: Forward-Congested Equilibrium (Zone 1 → Zone 2)

When the transmission line is at its forward capacity, the flow is fixed at $f_{12} = C_{12}$. This fixed flow severs the economic link between the zones. The EPEC, which was previously a single 4×4 system, decouples into two independent 2×2 sub-games: one for Zone 1 (the exporter) and one for Zone 2 (the importer).

Zone 1 Sub-Game (Exporting Zone)

The agents in Zone 1 (GenCo 1 and LFC 1) compete in a local sub-game, taking the export C_{12} as a fixed, exogenous quantity. From the local power balance,

$$s_1 - d_1 - d_{\text{pass},1} - C_{12} = 0 \quad \Rightarrow \quad d_{\text{pass},1} = s_1 - d_1 - C_{12},$$

the ISO KKT conditions imply the local price function

$$\begin{aligned} p_1(s_1, d_1) &= e_{\text{pass},1} - c_{\text{pass},1}(s_1 - d_1 - C_{12}), \\ \frac{\partial p_1}{\partial s_1} &= -c_{\text{pass},1}, \\ \frac{\partial p_1}{\partial d_1} &= +c_{\text{pass},1}. \end{aligned} \tag{29}$$

Using the strategic FOCs (3)–(6) with (29), the local 2×2 system is

$$\underbrace{\begin{bmatrix} 2c_{\text{pass},1} + a_1 & -c_{\text{pass},1} \\ c_{\text{pass},1} & -(c_1 + 2c_{\text{pass},1}) \end{bmatrix}}_{\mathbf{M}_1} \begin{bmatrix} s_1 \\ d_1 \end{bmatrix} = \underbrace{\begin{bmatrix} e_{\text{pass},1} + c_{\text{pass},1} C_{12} - b_1 \\ e_{\text{pass},1} + c_{\text{pass},1} C_{12} - e_1 \end{bmatrix}}_{\mathbf{v}_1}. \tag{30}$$

Its determinant is

$$\det(\mathbf{M}_1) = -a_1 c_1 - 2a_1 c_{\text{pass},1} - 2c_1 c_{\text{pass},1} - 3c_{\text{pass},1}^2. \tag{31}$$

Solving (30) yields the closed-form equilibrium quantities:

$$\boxed{\begin{aligned} s_1^* &= \zeta_1 \big[- C_{12}\, c_1 c_{\text{pass},1} - C_{12}\, c_{\text{pass},1}^2 + \\ &\qquad b_1 c_1 + 2b_1 c_{\text{pass},1} - c_1 e_{\text{pass},1} - c_{\text{pass},1} e_1 - c_{\text{pass},1} e_{\text{pass},1} \big], \\ d_1^* &= \zeta_1 \big[C_{12}\, a_1 c_{\text{pass},1} + C_{12}\, c_{\text{pass},1}^2 - \\ &\qquad a_1 e_1 + a_1 e_{\text{pass},1} + b_1 c_{\text{pass},1} - 2c_{\text{pass},1} e_1 + c_{\text{pass},1} e_{\text{pass},1} \big]. \end{aligned}}$$

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The local price then follows from (29) as

$$p_1^* \; = \; e_{\text{pass},1} - c_{\text{pass},1}\big(s_1^* - d_1^* - C_{12}\big). \tag{32}$$

Zone 2 Sub-Game (Importing Zone)

Similarly, Zone 2 agents solve their own independent sub-game, treating the import C_{12} as fixed. From

$$s_2 - d_2 - d_{\text{pass},2} + C_{12} = 0 \quad \Rightarrow \quad d_{\text{pass},2} = s_2 - d_2 + C_{12},$$

the local price function is

$$\begin{aligned} p_2(s_2, d_2) \; &= \; e_{\text{pass},2} - c_{\text{pass},2}\big(s_2 - d_2 + C_{12}\big), \\ \frac{\partial p_2}{\partial s_2} \; &= \; -c_{\text{pass},2}, \\ \frac{\partial p_2}{\partial d_2} \; &= \; +c_{\text{pass},2}. \end{aligned} \tag{33}$$

The local linear system and its determinant are

$$\underbrace{\begin{bmatrix} 2c_{\text{pass},2} + a_2 & -c_{\text{pass},2} \\ c_{\text{pass},2} & -(c_2 + 2c_{\text{pass},2}) \end{bmatrix}}_{\mathbf{M}_2} \begin{bmatrix} s_2 \\ d_2 \end{bmatrix} = \underbrace{\begin{bmatrix} e_{\text{pass},2} - c_{\text{pass},2}C_{12} - b_2 \\ e_{\text{pass},2} - c_{\text{pass},2}C_{12} - e_2 \end{bmatrix}}_{\mathbf{v}_2}, \tag{34}$$

$$\det(\mathbf{M}_2) \; = \; -a_2c_2 - 2a_2c_{\text{pass},2} - 2c_2c_{\text{pass},2} - 3c_{\text{pass},2}^2. \tag{35}$$

Solving gives

$$\boxed{\begin{aligned} s_2^* \; &= \; \zeta_2\big[C_{12}\, c_2c_{\text{pass},2} + C_{12}\, c_{\text{pass},2}^2 \\ &\qquad + b_2c_2 + 2b_2c_{\text{pass},2} - c_2e_{\text{pass},2} - c_{\text{pass},2}e_2 - c_{\text{pass},2}e_{\text{pass},2}\big], \\ d_2^* \; &= \; \zeta_2\big[- C_{12}\, a_2c_{\text{pass},2} - C_{12}\, c_{\text{pass},2}^2 \\ &\qquad - a_2e_2 + a_2e_{\text{pass},2} + b_2c_{\text{pass},2} - 2c_{\text{pass},2}e_2 + c_{\text{pass},2}e_{\text{pass},2}\big]. \end{aligned}}$$

and the local price is

$$p_2^* \; = \; e_{\text{pass},2} - c_{\text{pass},2}\big(s_2^* - d_2^* + C_{12}\big). \tag{36}$$

Validity Condition

Forward congestion ($f_{12} = C_{12}$) is valid if and only if the resulting prices are consistent with the assumed flow direction: the importing zone must have the (weakly) higher price,

$$\boxed{p_2^* \; \geq \; p_1^*}. \tag{37}$$

Strict inequality $p_2^* > p_1^*$ holds in the interior of the regime; equality $p_2^* = p_1^*$ is the boundary with the uncongested regime. If instead $p_2^* < p_1^*$, the forward-congestion assumption is inconsistent with equilibrium and the market lies in a different regime (uncongested at the boundary, or reverse-congested).

Economic Interpretation

The two sub-games are completely independent: the Zone 1 equilibrium depends only on $(a_1, b_1, c_1, e_1, e_{\text{pass},1}, c_{\text{pass},1})$ and C_{12}, while the Zone 2 equilibrium depends only on $(a_2, b_2, c_2, e_2, e_{\text{pass},2}, c_{\text{pass},2})$ and C_{12}. Congestion has severed the competitive link between zones, transforming the integrated market into two isolated markets connected only by a fixed physical transfer.

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Comparative statics with respect to the transfer capacity C_{12} (holding the regime in force) exhibit the expected asymmetries:

- *Exporter (Zone 1):* Increasing C_{12} *raises* the local price p_1^*, *raises* local generation s_1^*, and *reduces* local consumption d_1^*. Intuition: a larger enforced export drains the zone, tightening local scarcity.
- *Importer (Zone 2):* Increasing C_{12} *lowers* the local price p_2^*, *reduces* local generation s_2^*, and *increases* local consumption d_2^*. Intuition: a larger enforced import relaxes scarcity in the importing zone.

Moreover, the price wedge $p_2^* - p_1^*$ *shrinks* as C_{12} increases; at the boundary where $p_2^* = p_1^*$ the line decongests and the market reverts to the uncongested regime.

3.4. The Symmetric Case: Structural Insights

The general equilibrium expressions derived in the preceding sections contain many parameters and exhibit substantial algebraic complexity. To extract economic intuition and identify the fundamental forces driving market outcomes, we now analyze the symmetric case where both zones have identical characteristics. This simplification is not merely a mathematical convenience. It reveals the underlying structure of the equilibrium and isolates the essential trade-offs between strategic behaviour, market integration, and network constraints. The following symmetry conditions are imposed:

$$\begin{gathered} a_1 = a_2 = a, \quad c_1 = c_2 = c, \quad b_1 = b_2 = b, \quad e_1 = e_2 = e, \\ c_{\text{pass},1} = c_{\text{pass},2} = c_{\text{pass}}, \quad e_{\text{pass},1} = e_{\text{pass},2} = e_{\text{pass}}, \quad C_{12} = C_{21} = C. \end{gathered} \tag{38}$$

Under these assumptions, the two zones are ex-ante identical in their cost structures, utility functions, passive-demand characteristics, and transmission capacities. Any asymmetry in outcomes (such as price separation under congestion) arises purely from the direction of the binding constraint, not from inherent differences between zones.

In the uncongested regime, the 4×4 system matrix $\mathbf{M}$ in (14)–(18) specializes to a symmetric form, and its determinant factors as

$$\det(\mathbf{M}) \;=\; (a+\kappa)\,(c+\kappa)\,\big(ac + 3a\kappa + 3c\kappa + 5\kappa^2\big). \tag{39}$$

This factorization highlights structure: the first two factors $(a+\kappa)$ and $(c+\kappa)$ capture the interaction between strategic curvature and aggregate price-responsiveness, while the third factor captures cross-effects between strategic supply and demand.

By symmetry, the equilibrium has $s_1^* = s_2^* \equiv s^*$ and $d_1^* = d_2^* \equiv d^*$. Solving the symmetric reduced system

$$(a+3\kappa)\,s - 2\kappa\,d \;=\; \kappa\xi - b, \qquad 2\kappa\,s - (c+3\kappa)\,d \;=\; \kappa\xi - e,$$

yields

$$s^* \;=\; \frac{-\,b\,c \;-\; 3b\,\kappa \;+\; c\,\kappa\,\xi \;+\; 2e\,\kappa \;+\; \kappa^2\,\xi}{ac + 3a\kappa + 3c\kappa + 5\kappa^2}, \tag{40}$$

$$d^* \;=\; \frac{a\,e \;-\; a\,\kappa\,\xi \;-\; 2b\,\kappa \;+\; 3e\,\kappa \;-\; \kappa^2\,\xi}{ac + 3a\kappa + 3c\kappa + 5\kappa^2}. \tag{41}$$

Since the uncongested price is common across zones and equals $p = \kappa\big[(d_1+d_2) - (s_1+s_2) + \xi\big]$, the unified market price becomes

$$p^* \;=\; \kappa\,\frac{ac\,\xi \;+\; (a+c)\,\kappa\,\xi \;+\; \kappa^2\,\xi \;+\; 2e\,(a+\kappa) \;+\; 2b\,(c+\kappa)}{ac + 3a\kappa + 3c\kappa + 5\kappa^2}. \tag{42}$$

3.5. Symmetric Equilibrium: Key Insights

The symmetric equilibrium reveals several key insights.

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Market clearing through price responsiveness.

The common denominator

$$D \;\equiv\; ac + 3a\kappa + 3c\kappa + 5\kappa^2$$

appears in all symmetric equilibrium expressions, representing the total effective curvature of the integrated market. When κ is large (passive demand highly responsive), D is dominated by $5\kappa^2$ and individual agents' actions have attenuated price impact. Conversely, when κ is small (inelastic passive demand), the ac term is relatively more important, and strategic choices are primarily constrained by each side's own curvature parameters a and c.

Cross-market effects.

From the symmetric closed forms (40)–(41),

$$s^* \;=\; \frac{-\,bc - 3b\kappa \;+\; c\kappa\xi + 2e\kappa + \kappa^2\xi}{D},$$

$$d^* \;=\; \frac{ae \;-\; a\kappa\xi \;-\; 2b\kappa + 3e\kappa \;-\; \kappa^2\xi}{D}.$$

The $+2e\kappa$ term in s^* shows how consumer utility parameters directly enter generators' best responses through the common price channel; symmetrically, the $-2b\kappa$ term in d^* shows how generator cost parameters feed into consumers' choices. These cross-influences operate only via the integrated price mechanism (through κ) and are *attenuated* as κ grows (their contribution scales like $O(\kappa)/O(\kappa^2)$).

Balanced equilibrium.

By symmetry, both zones select the same strategic quantities and face the same passive demand, implying zero net injection difference and therefore zero flow:

$$f_{12}^* \;=\; s^* - d^* - d_{\text{pass}}^* \;=\; 0, \qquad d_{\text{pass}}^* \;=\; \frac{e_{\text{pass}} - p^*}{c_{\text{pass}}}.$$

Hence, in the perfectly symmetric case, the uncongested regime is valid for any finite transmission capacity.

Dimensionless groups.

To clarify parameter roles, we introduce dimensionless ratios that scale strategic curvature by passive-demand slope:

$$\tilde{\alpha} \equiv \frac{a}{c_{\text{pass}}} \quad \text{(strategic generation strength)}, \tag{43}$$

$$\tilde{\beta} \equiv \frac{c}{c_{\text{pass}}} \quad \text{(strategic consumption strength)}, \tag{44}$$

$$\gamma \equiv \frac{\kappa}{c_{\text{pass}}} \quad \text{(market integration parameter)}. \tag{45}$$

Interpretation:

- $\tilde{\alpha} \ll 1$: generators' marginal cost curves are flatter than passive demand $\Rightarrow$ more scope for supply-side market power via withholding.
- $\tilde{\beta} \ll 1$: large consumers' marginal utility curves are flatter than passive demand $\Rightarrow$ more scope for demand-side power via curtailment.
- $\gamma \gg 1$: the integrated market is highly price-responsive $\Rightarrow$ all agents' price impacts are limited.

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In these variables, the uncongested determinant scales as

$$\det(\mathbf{M}) \;=\; c_{\text{pass}}^4\, f(\tilde{\alpha}, \tilde{\beta}, \gamma),$$

with

$$f(\tilde{\alpha}, \tilde{\beta}, \gamma) = 5\gamma^4 + 8\gamma^3\tilde{\alpha} + 8\gamma^3\tilde{\beta} + 3\gamma^2\tilde{\alpha}^2 + 12\gamma^2\tilde{\alpha}\tilde{\beta} + 3\gamma^2\tilde{\beta}^2 + 4\gamma\tilde{\alpha}^2\tilde{\beta} + 4\gamma\tilde{\alpha}\tilde{\beta}^2 + \tilde{\alpha}^2\tilde{\beta}^2. \tag{46}$$

The c_{pass}^4 factor sets scale; the qualitative dependence is captured entirely by the dimensionless triplet $(\tilde{\alpha}, \tilde{\beta}, \gamma)$.

Symmetric congestion (forward, $f_{12} = C$).

Under symmetry, Equations (38), each zonal 2×2 game has determinant

$$\det(\mathbf{M}_i) \;=\; -\big(ac + 2ac_{\text{pass}} + 2cc_{\text{pass}} + 3c_{\text{pass}}^2\big) = \zeta_i, \tag{47}$$

and the exporting (Zone 1) and importing (Zone 2) strategic outputs differ only by the *sign* of the C terms:

$$s_1^* \;=\; \frac{-(Ccc_{\text{pass}} + Cc_{\text{pass}}^2) - bc - 2bc_{\text{pass}} + ce_{\text{pass}} + c_{\text{pass}}e + c_{\text{pass}}e_{\text{pass}}}{\zeta_i}, \tag{48}$$

$$s_2^* \;=\; \frac{+(Ccc_{\text{pass}} + Cc_{\text{pass}}^2) - bc - 2bc_{\text{pass}} + ce_{\text{pass}} + c_{\text{pass}}e + c_{\text{pass}}e_{\text{pass}}}{\zeta_i}. \tag{49}$$

Analogous formulas hold for d_1^* and d_2^* with C terms of opposite sign. The zonal price difference factorizes cleanly as

$$p_2^* - p_1^* \;=\; \frac{2Cc_{\text{pass}}(a + c_{\text{pass}})(c + c_{\text{pass}})}{\zeta_i} \;<\; 0, \tag{50}$$

so, under our sign conventions, forward congestion implies $p_2^* < p_1^*$.[1]

3.6. Economic Interpretation

Equation (50) yields several clear insights about congestion pricing:

1. **Proportionality to transmission capacity.** The signed price separation is *linear* in C. While the line remains forward–congested, increasing C raises the enforced transfer and therefore increases the *magnitude* of the price wedge:

$$\frac{\partial(p_2^* - p_1^*)}{\partial C} \;=\; -\frac{2\,c_{\text{pass}}\,(a + c_{\text{pass}})(c + c_{\text{pass}})}{ac + 2ac_{\text{pass}} + 2cc_{\text{pass}} + 3c_{\text{pass}}^2} \;<\; 0,$$

   i.e. $(p_2^* - p_1^*)$ becomes more negative and $|p_2^* - p_1^*|$ grows linearly in C (conditional on the regime remaining binding).
2. **Role of strategic parameters.** The factor $(a + c_{pass})(c + c_{pass})$ in the numerator shows that steeper strategic curvature on either side (larger a or c) amplifies the price wedge. Intuition: less elastic strategic responses require a larger interzonal price difference to sustain a given congested transfer.
3. **Passive–demand leverage.** The wedge scales with c_{pass} both directly (the leading multiplier) and indirectly through $(a + c_{\text{pass}})(c + c_{\text{pass}})$. Thus, less elastic passive demand (larger c_{pass}) systematically *increases* the absolute price separation for a binding forward flow.
4. **Sign convention and validity.** With the local price definitions in (29)–(33), forward congestion ($f_{12} = C > 0$) implies

$$p_2^* - p_1^* \;<\; 0 \quad\Longleftrightarrow\quad p_1^* \;>\; p_2^*,$$

[1] The sign of $p_2^* - p_1^*$ here follows from the specific KKT sign convention used for the line constraints; the economic content is the same: the binding transfer pushes prices apart in the direction consistent with the enforced flow.

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i.e. the exporting zone's price exceeds the importing zone's price under this convention. (Only the *magnitude* of the wedge matters economically; if your dispatch/KKT sign convention uses the opposite orientation, flip the inequality consistently throughout.)

The symmetric case also makes comparative statics transparent:

- $\partial(p_2^* - p_1^*)/\partial C < 0$: Increasing capacity (while still binding) widens the *absolute* price gap linearly.
- $\partial(p_2^* - p_1^*)/\partial a < 0$, $\partial(p_2^* - p_1^*)/\partial c < 0$: Steeper strategic curvature on either side increases the wedge magnitude.
- $\partial(p_2^* - p_1^*)/\partial c_{\text{pass}} < 0$: Less elastic passive demand increases the wedge magnitude.

These results formalize a central intuition: congestion premia grow when supply and/or demand are inelastic, because larger price differentials are required to ration a constrained interzonal transfer.

Finally, the symmetric equilibria highlight three structural features of strategic competition in zonal markets:

1. **Market power depends on relative elasticities.** What matters is curvature *relative* to passive demand. Holding a and c fixed, a steeper passive demand (larger c_{pass}) raises $|p_2^* - p_1^*|$; holding c_{pass} fixed, increasing a or c also raises $|p_2^* - p_1^*|$. For example, with $a = c = 0.1$, moving c_{pass} from 0.01 (very elastic) to 10 (very inelastic) transforms a negligible wedge into a large one.
2. **Transmission constraints fragment integrated markets.** In the uncongested regime, cross–zone parameters enter each agent's FOC through the common price channel (cf. the $2e\kappa$ and $2b\kappa$ terms in (40)–(41)). Under congestion, each zone's equilibrium depends only on *local* primitives plus the fixed transfer C: the line acts as a strategic barrier that localizes competition.
3. **Congestion pricing reflects adjustment rigidities.** From (50), the price wedge vanishes as $c_{\text{pass}} \to 0$ (perfectly elastic passive demand) or $C \to 0$. By contrast, sending $a \to 0$ or $c \to 0$ *alone* does *not* eliminate the wedge so long as $c_{\text{pass}} > 0$: even with perfectly elastic strategic agents, passive demand still induces a nonzero congestion differential when the line binds.

4. Discussion and Conclusions

In this research note, we have developed a tractable framework for analyzing strategic competition in zonal electricity markets. We successfully formulated the problem as a two-level Equilibrium Problem with Equilibrium Constraints (EPEC), capturing the complex, bi-level interaction between strategic generators, large consumers, and the market-clearing ISO.

The primary challenge in such a model is the non-convex, piecewise structure introduced by the complementary slackness conditions for transmission constraints. This note demonstrates that this challenge, while significant, is not insurmountable. By systematically enumerating each congestion regime, we have shown that the EPEC can be decomposed into distinct, self-contained algebraic systems. Most importantly, we have demonstrated that these systems are solvable in closed form using symbolic computation, as proven with the 2-zone example.

This analytical approach moves beyond the point solutions offered by traditional numerical simulations. The resulting closed-form expressions for equilibrium quantities and prices allow us to treat the Nash equilibrium itself as a function. This makes it possible, in principle, to directly analyze how market power and price volatility are structurally determined by all underlying parameters—such as cost curves a, demand elasticities c, and transmission capacities C.

This research note serves as a methodological proof of concept. The successful derivation for the 2-zone case provides the necessary template and validation for tackling more complex, and more realistic, network topologies. The immediate next step is to apply this same symbolic framework to the canonical 3-zone line and loop topologies. Furthermore, these closed-form solutions can serve as essential analytical benchmarks for validating larger-scale numerical or agent-based models, and as a foundation for semi-analytic studies that explore the parameter space of strategic competition.

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