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Article

Gravitational and Dark-Matter Concepts That Can Help Explain Cosmic Data

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Abstract

We discuss gravitational concepts and candidate specifications for dark matter that, together, can help explain known ratios of dark-matter effects to ordinary-matter effects and can help explain eras in the rate of expansion of the universe. The ratios pertain to galaxies and galaxy evolution, galaxy clusters, and densities of the universe. The candidate specifications for dark matter reuse, with variations, a set of known elementary particles. Regarding galaxy evolution and the rate of expansion of the universe, we deploy multipole-expansion methods that combine Newtonian gravity, aspects of motions of sub-objects of gravitationally interacting objects, and Lorentz invariance.

Keywords: dark matter; elementary particles; gravity; galaxy formation; rate of expansion of the universe; multipole expansions; particle properties

1. Introduction

This unit (of this paper) places our work in a context of ninety years of cosmology, gravitation, and elementary-particle research.

Our work addresses the following four physics opportunities.

1. Understand mechanisms that govern the extent to which, at various times in the evolution of the universe, large objects (such as galaxy clusters) accelerate away from each other or accelerate toward each other. (One notion of multiple large-scale objects dates back at least as far as to the discovery of galaxies other than the Milky Way galaxy during the 1920s [1–3]. At least as far back as the 1990s, popular modeling associated the six-word term rate of expansion of the universe with notions of the moving apart from each other of large cosmological objects [4].)
2. Understand phenomena, such as likely-gravitational effects, that seem not to associate with matter that emits light that people or equipment detect. (One notion of such likely gravitational effects dates back at least as far as to the 1930s [5,6]. Popular modeling associates the two-word term ordinary matter with matter that emits light that people or equipment detect. Popular modeling associates the two-word term dark matter with some notions of sources of gravitational effects that popular modeling suggests could satisfy the relevant notions of such likely-gravitational effects.)
3. Extend or complete the list of elementary particles. (One notion of elementary particles dates back at least as far as to the 1890s [7].)
4. Catalog properties that pertain to objects and understand model-related relationships between properties that pertain to objects and choices of dynamics models. (One notion regarding properties that pertain to objects features the possible equivalence between the property of inertial mass and the property of gravitational mass. That notion dates back at least as far as to the 1680s [8].)

During those ninety years, each one of many individual steps forward regarding those opportunities has tended to associate directly with no more than three of the four opportunities. (An appendix, in this paper, regarding some research that has tried to address rate-of-expansion, gravitational-repulsion,

dark-matter, and elementary-particle topics alludes to some aspects of such steps forward.) Our work stems from data that, collectively, associate with all four opportunities. Our work develops and uses methods that, collectively, associate with all four opportunities. Our work attempts to point to foundational concepts that, for example, can limit the ranges of and can help explain quantities that popular modeling treats as parameters.

We propose concepts that, when combined with each other, seem to offer steps forward regarding each of the four opportunities.

We try to base our concepts on data, widely familiar physics, and widely familiar mathematics. The following notions pertain regarding opportunity 1.

- We explore notions that two-body gravity can point to circumstances in which two objects (such as two galaxy clusters) repel each other.
- We propose that such repulsion can help explain the recent multibillion-year era of increasing rate of expansion of the universe. (We note, here as an aside, that we propose that similar repulsion can help explain some aspects regarding galaxy formation and evolution.)
- We indicate that gravitational repulsion can associate with the following bases: the notion that, within objects, sub-objects move; Newtonian gravity; Lorentz invariance; and multipole-expansion mathematics. (Two appendices, one regarding two-body gravitational repulsion and the other regarding gravitational multipole expansions, provide perspective.) We are not aware of other work that tries to combine those four bases. Using those four bases allows, for addressing natural circumstances that our work discusses, for de-emphasizing aspects regarding popular modeling gravity-related topics such as the cosmological constant (for which Λ is often a symbol), emerging dark energy, equations of state (for use with general relativity), and MOND (as in modified Newtonian dynamics).
- Using the four bases points to the following multipole-expansion notions. Monopole aspects of gravity associate with two-object mutual attraction. Dipole aspects of gravity associate with two-object mutual repulsion. Quadrupole aspects of gravity associate with two-object mutual attraction. The dipole aspects and quadrupole aspects increase with increasing kinetic energy of sub-objects.
- We anticipate proposing (in this paper) that our notions provide useful insight regarding opportunity 1.

The following notions pertain regarding opportunities 2 and 3.

- We explore notions that so-called dark matter associates with the likely-gravitational effects and that at least most dark matter has bases in elementary particles that are like the elementary particles that underlie ordinary matter.
- We propose that nature includes six sets of similar elementary particles. One set underlies ordinary-matter stuff. (We use the word stuff to denote objects. For ordinary matter stuff, examples of objects include electrons, protons, atomic nuclei, atoms, stars, and solar systems.) The other five sets underlie at least most dark matter stuff. The six sets of elementary particles can be sufficiently similar that we use the word isomer to refer to each set. (The appendix regarding uses of the word isomer provides perspective.) For example, across the six isomers, we propose that the masses of counterpart elementary particles can be the same. However, we leave some aspects, such as elementary-particle handedness and such as near-matches between elementary-fermion flavours (or generations) and elementary-fermion masses, as parameters. We associate the three-word term isomeric dark matter (and the acronym IDM) with a so-called class of such possible specifications.
- Using notions of IDM allows, for addressing natural circumstances that our work discusses, considering candidate dark-matter specifications that tend to be better-defined than some popular modeling candidate specifications for dark matter. Such better-defined candidate specifications allow for de-emphasizing reliance on popular modeling concepts such as cold dark matter (or, CDM), collisionless dark matter, and self-interacting dark matter (or, SIDM).

- We anticipate proposing (in this paper) that some members of the IDM class of candidate specifications for dark matter provide useful insight regarding opportunities 2 and 3.

The following notions pertain regarding opportunity 4.

- We explore notions that two-body gravity can point to the usefulness of multipole expansions that include the gravitational property of mass and gravitational properties other than mass.
- We propose that including gravitational properties other than mass can help explain some aspects regarding galaxy formation and evolution. (We note, here as an aside, that we propose that gravitational properties other than mass can help explain the recent multibillion-year era of increasing rate of expansion of the universe.)
- We indicate that gravitational-dipole-related properties and gravitational-quadrupole-related properties can associate with the following bases: the notion that, within objects, sub-objects move; Newtonian gravity; Lorentz invariance; and multipole-expansion mathematics. We are not aware of other work that tries to combine those four bases.
- Using the notions of gravitational-dipole-related properties and gravitational-quadrupole-related properties points to the following notions. Object-internal angular momentum might, for some combinations of physical circumstances and choices of modeling techniques, be a useful gravitational dipole property for objects. For large-scale objects, for some combinations of physical circumstances and choices of modeling techniques, object-internal sub-object kinetic energy might be a more useful object-internal gravitational dipole property than would be object-internal angular momentum.
- We anticipate proposing (in this paper) that our notions provide useful insight regarding opportunity 4.

We anticipate proposing (in this paper) that our work helps explain known cosmic data and can help anticipate and explain future cosmic data.

2. Methods

2.1. Data for Which We Seek Underlying Explanations

This unit discusses data that motivate our work, that inform our work, and for which our work seeks to help provide underlying explanations.

The following items discuss observed ratios of not-ordinary-matter effects to ordinary-matter effects. We seek to help provide underlying explanations.

- 1:0+ – Amounts of stuff in some individual galaxies [9–18].
- 0+:1 – Amounts of stuff in some individual galaxies. (Popular modeling associates the symbol z with redshift. Popular modeling associates redshifts of zero with the present universe. Popular modeling associates larger redshifts with earlier times in the history of the universe.)
 - Redshifts of more than approximately seven [19,20].
 - Redshifts of approximately six [21].
 - Redshifts of less than six through redshifts of nearly zero [22–29].
- \sim 4:1 – Amounts of stuff in some individual galaxies [30,31].
- 5+:1 – Amounts of stuff in many individual galaxies [9,32].
- 5+:1 – Amounts of stuff in many individual galaxy clusters [32–36].
- 5+:1 – Densities of the universe [37].
- \approx 0:1 – Amounts of stuff in observed or optically observable solar systems.
 - Observations about our solar system [38].
 - Papers about other solar systems tend to feature possible methods for future observations and seem to imply that, presently, \approx 0:1 pertains regarding known solar systems other than our own solar system [39].

The following items summarize some observations about large-scale aspects of the universe. We seek to help provide insight that can underlie explanations for these data.

- Popular modeling suggests two observed multibillion-year eras regarding the rate of expansion of the universe [40–43]. Chronologically, the first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases.
- Data and popular modeling might provide hints that the second multibillion-year era might be ending [44,45] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

The following items summarize some other observations. We seek to provide a framework that can accommodate results of future such (and other) observations and thereby help people evolve our work.

- 1:1 or 0:1 – Amounts of some depletion of cosmic microwave background radiation. (Ordinary-matter effects that associate with the depletion of cosmic microwave background radiation via hyperfine transitions in ordinary-matter hydrogen atoms might account for half of the observed depletion or for all the observed depletion. The case of half associates with 1:1. The case of all associates with 0:1.)
 - An observation [46–48] suggests 1:1.
 - Other pieces of research [49–52] suggest 0:1.
- The extent to which dark-matter stuff includes IGM (as in intergalactic medium).
 - Data about collisions of galaxy clusters [53,54] suggest that at least much dark matter stuff does not include IGM stuff.

2.2. Perspective About Our Approach and Methods

This unit provides perspective about the approach and methods we develop and use. Our approach emphasizes the following five notions and five-step process.

1. Use patterns that data exhibit.
2. Reuse familiar physics.
3. Use simple mathematics.
4. State or label key concepts.
5. Anticipate uses of key concepts.

The appendix about usages of a five-step from-data-to-uses approach discusses aspects of using that approach regarding the following.

- Large-scale presences of dark matter.
- Presences, in galaxies, of dark matter and of ordinary matter.
- The formation of objects such as stars, solar systems, and low-mass galaxies.
- The rate of expansion of the universe.
- Ordinary-matter fermion elementary particles.

2.3. Objects, Interactions Between Objects, and Isomeric Reaches of Interactions

This unit posits values for integers that can help describe the extent to which dark matter and ordinary matter interact with each other.

People and ordinary-matter equipment do not sense light that dark matter might emit. We posit that, at least to a first approximation, each one of the six isomers associates with its own instance of electromagnetism. We say that, for each one of the six instances of electromagnetism, the interaction reach per instance is one isomer.

Popular modeling suggests that each nonzero-mass object can interact gravitationally with all other nonzero-mass objects. We posit that, at least to a first approximation, nature includes one instance of gravitation. We say that the reach per instance for some, but not all, aspects of gravitation is six isomers.

We use the symbol n_{in} to denote the number of instances of a type of interaction. (One type of interaction is so-called two-body gravitational monopole pull interactions.) We use the symbol R_{in} to

denote the reach of an instance of the type of interaction. The reach is a number of isomers. Each one of n_{in} and $R_{/in}$ is a positive integer.

We also use the symbol n_{in} to denote the number of instances of a property that objects exhibit. (Examples of properties include charge and blackbody temperature.) For example, blackbody temperature is a property that people and equipment observe regarding stars. We use the symbol $R_{/in}$ to denote the reach of an instance of the property. The reach is a number of isomers. Each one of n_{in} and $R_{/in}$ is a positive integer.

We posit that, for each relevant aspect of electromagnetic interactions, for each relevant aspect of gravitational interactions, for each electromagnetic property, and for each gravitational property, Eq. (1) pertains.

$$n_{in} \cdot R_{/in} = 6 \quad (1)$$

Eq. (1) associates with four potentially relevant solutions. Each solution associates with one of $R_{/in} = 1$, $R_{/in} = 2$, $R_{/in} = 3$, or $R_{/in} = 6$.

For a solution for which $n_{in} = 3$ and $R_{/in} = 2$, we posit that each one of the three instances associates with a so-called isomer-pair and that, for each instance, the reach of two isomers associates with the two isomers that associate with the isomer-pair.

We posit that listing an adequately robust set of interaction reaches per instance can associate with candidate explanations for some cosmic data.

Our work regarding gravitational multipole expansions indicates that, for modeling that treats two objects as pointlike, the following notions pertain. (Here, we summarize key concepts. Two appendices, one regarding two-body gravitational repulsion and the other regarding gravitational multipole expansions, provide perspective.) For discussion purposes, we assume that the two objects are not (instantaneously) moving relative to each other. Our work indicates that a monopole component of the gravitational interaction associates with force components that attract (or pull) the two objects toward each other. Our work indicates that a dipole component of the gravitational interaction associates with force components that repel (or push) the two objects away from each other. Our work indicates that a quadrupole component of the gravitational interaction associates with force components that attract (or pull) the two objects toward each other.

Table 1 posits instances and reaches per instance for some components of some interactions. (We note, as an aside, that one might consider that some of the instances and reaches are guesses. However, below in this paper, we indicate how the Table 1 numeric values of instances and reaches per instance can underlie steps forward regarding explaining observed ratios of presumed-dark-matter effects to ordinary-matter effects and regarding explaining eras in the rate of expansion of the universe. One might consider that the consistency, across the explanations, of the instances and reaches helps validate the might-be-considered guesses. The discussion unit, in this paper, regarding the extent of the assumptions that our work makes and the extent to which our work helps explain data discusses the above notions about consistency and discusses other notions about possible consistency.)

Table 1. Posited instances and reaches per instance for some components of some interactions. Electromagnetic dynamics-properties interactions feature electromagnetic properties such as charge and magnetic moment. Pull associates with attraction between two objects. Push associates with repulsion between two objects. n_{in} denotes the number of instances of the interaction component. $R_{/in}$ denotes the interaction reach, in number of isomers, per instance.

Interaction component	n_{in}	$R_{/in}$
Two-body gravitational monopole pull interactions	1	6
Two-body gravitational dipole push interactions	3	2
Two-body gravitational quadrupole pull interactions	6	1
Two-body electromagnetic dynamics-properties interactions	6	1
Sensing of one-body blackbody temperature	6	1

We note, as an aside, that, for each row in Table 1, the following notions, which pertain to at least the properties of interacting non-colliding cosmological bodies, pertain. For each relevant body, the sum of the rest masses of the sub-objects of the body does not change. For each relevant body, the sum of the rest charges of the sub-objects of the body does not change. Popular modeling can consider that aspects that associate with the first three (or, gravitational) rows and aspects that associate with the last two (or, electromagnetic) rows can pertain independently of each other.

The appendix regarding gravitational multipole expansions indicates the following notions regarding gravitational interactions. Non-monopole effects that associate with positions (and not with velocities) of sub-objects of objects associate gravitationally with one instance ($n_{in} = 1$) and a reach per instance of six isomers ($R_{/in} = 6$). For objects such as hadrons, modeling regarding gravitation can feature the masses and other aspects of the objects and de-emphasize the masses and other aspects of sub-objects such as quarks.

Possibly, physics will evolve Table 1, for example by changing terminology that the table uses or by adding rows to the table. (We note, as an aside, that Table 4 indicates some possibilities for additional rows.)

2.4. Two Members of the IDM Class of Candidate Specifications for Dark Matter

This unit discusses two members of the IDM class of candidate specifications for dark matter.

We note that 5+:1 observed ratios of not-ordinary-matter effects to ordinary-matter effects associate with large-scale phenomena.

We posit that dark matter underlies (at least much of) the not-ordinary-matter effects.

We posit that nature includes six near-copies of a set of elementary particles and that one copy includes the set of all known fermion elementary particles. We posit that, across the six near-copies, counterpart particles have identical masses. These posits could underlie explanations for $\sim 5:1$ ratios that might associate with the known 5+:1 ratios.

We use the word isomer to associate with the notion of near-copy. Our use of the word isomer comports with typical use, in science, of the word isomer in situations in which science discusses the relevance of exact symmetries or approximate symmetries. We use the word isomer to denote each one of the six sets of elementary particles.

We use the word stuff to associate with occurrences of elementary particles, atomic nuclei, atoms, and so forth that exist in nature.

As an aside, we note that, because stuff that associates with one isomer interacts gravitationally with stuff that associates with other isomers, the possible graviton elementary particle would not be a member of any one of the six relevant sets of elementary particles. At this point in our discussion, we leave open the question of the extent to which one might include elementary bosons in the relevant set of known elementary particles.

The number six, as in six sets, factors into a factor of two and a factor of three. We propose that each one of the two and the three might associate with a symmetry or approximate symmetry.

We propose that the factor of two might associate with at least one of the following notions.

- Popular modeling notions that associate the one-element term left-handed with the set of known elementary particles [55,56]. (For this notion, at least one dark-matter isomer associates with right-handed counterpart-to-ordinary-matter elementary particles.)
- The popular modeling notion of matter-antimatter asymmetry (which is also known as baryon asymmetry) [57]. (For this notion, one dark-matter isomer underlies stuff that enables popular modeling to consider, in an adequately broad context, notions of matter-antimatter symmetry.)
- The 1:1 ratio that possibly pertains regarding some depletion of cosmic microwave background radiation. (For this notion, the stuff that associates with one dark-matter isomer includes hydrogen-like atoms that account for one-half of the relevant depletion of cosmic microwave background radiation.)

The appendix that discusses a possible symmetry regarding the factor of two discusses notions and details regarding this possible symmetry. We do not try to fully characterize the would-be symmetry or approximate symmetry. Regardless of the details, we use the term left-handed to describe the ordinary-matter isomer and we use the term right-handed to describe the Table 2 (below) dark-matter isomer that would be most like the ordinary-matter isomer.

We propose that the factor of three might associate with at least one (and possibly all) of the following notions.

- Popular modeling notions of neutrino oscillations [58,59]. (Regarding this notion, popular modeling states that neutrino flavour-eigenstates do not fully align with neutrino mass-eigenstates.)
- Aspects of a formula that interrelates the masses of the three charged leptons (the electron, the muon, and the tau) and the geometric-mean masses for each of the three generations of quarks (Table 3.9.10 in [60] or Table 14 in [61]).
- A might-be approximate symmetry that would associate with near-matches between charged-lepton flavours and charged-lepton masses.

The appendix that discusses the possible approximate symmetry regarding the factor of three discusses notions and details regarding this possible approximate symmetry. We do not try to fully characterize the possible approximate symmetry.

We discuss two members of the class of IDM candidate specifications for dark matter. We use the one-element term IDM-1 to denote one member. We use the one-element term IDM-2 to denote one member.

The IDM-1 member of the class of IDM candidate specifications for dark matter associates with the notions regarding a factor of two and a would-be symmetry and with the notions regarding a factor of three and a would-be symmetry.

We use the acronym SEA (as in significantly-electromagnetically-active) to describe aspects of the stuff that associates with the ordinary-matter isomer and aspects of the stuff that associates with the dark-matter isomer that associates with the isomer-pair that includes the isomer that underlies ordinary-matter stuff.

We use the acronym MEA (as in marginally-electromagnetically-active) to describe aspects of the stuff that associates with the other four dark-matter isomers. (The appendix regarding the evolution of MEA dark-matter stuff proposes means by which the stuff that associates with each one of the four MEA isomers evolves to feature neutron-like analogs and proposes that the neutron-like analogs do not decay significantly into proton-analogs and electron-analogs.)

Table 2 discusses a numbering scheme for the posited IDM-1 six isomers of all known elementary fermions, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer.

Table 2. A numbering scheme for the posited IDM-1 six isomers of all known elementary fermions, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer. The symbol $l_{is,pr}$ denotes the isomer-pair number. The symbol l_{is} denotes the isomer number. The masses of counterpart elementary particles are, across the isomers, the same. Handedness associates with the factor of two that associates with the number, six, of isomers. For each row, the quark generations column assigns the three generation numbers in order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the generation. (The following pertain for the ordinary-matter isomer. Generation-1 associates with the up quark and the down quark. Generation-2 associates with the charm quark and the strange quark. Generation-3 associates with the top quark and the bottom quark.) For each row, the lepton flavours column assigns the three flavor numbers in order of increasing mass for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.) The notion that, for four isomers, the lepton-flavours order does not match the quark-generations order associates with our notions of a possible symmetry or approximate symmetry regarding charged-lepton flavours and charge-lepton masses. The stuff column identifies stuff made from the isomer that associates with the table row as OM, as in ordinary matter, or DM, as in dark matter. The acronym SEA abbreviates our term significantly-electromagnetically-active. The stuff that associates with DM (SEA) interacts electromagnetically with itself on a par with OM stuff interacting electromagnetically with OM stuff. The acronym MEA abbreviates our term marginally-electromagnetically-active. The stuff that associates with MEA interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-net-charge objects.

$l_{is,pr}$	l_{is}	Handedness	Quark generations	Lepton flavours	Stuff
0	0	Left	1, 2, 3	1, 2, 3	OM (SEA)
0	3	Right	1, 2, 3	1, 2, 3	DM (SEA)
1	1	Left	1, 2, 3	3, 1, 2	DM (MEA)
1	4	Right	1, 2, 3	3, 1, 2	DM (MEA)
2	2	Left	1, 2, 3	2, 3, 1	DM (MEA)
2	5	Right	1, 2, 3	2, 3, 1	DM (MEA)

The stuff that associates with each dark matter isomer comports with the popular modeling acronym CDM (as in cold dark matter). The stuff that associates with the SEA dark-isomer comports with the popular modeling acronym SIDM (as in self-interacting dark matter).

We note, as an aside, the following. Some observational results [62–64] suggest that some dark matter might comport with popular modeling notions of self-interacting dark matter [65,66]. Some popular modeling results [67–70] point to possible benefits of considering that some dark matter is self-interacting dark matter.

The IDM-2 member of the class of IDM candidate specifications for dark matter de-emphasizes the would-be notions regarding a factor of two and extends would-be notions regarding a factor of three.

Table 3 discusses a numbering scheme for the posited IDM-2 six isomers of all known elementary fermions, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer.

Table 3. A numbering scheme for the posited IDM-2 six isomers of all known elementary fermions, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer. The symbol $l_{is,pr}$ denotes the isomer-pair number. The symbol l_{is} denotes the isomer number. The masses of counterpart elementary particles are, across the isomers, the same. For each row, the quark generations column assigns the three generation numbers in order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the generation. (The following pertain for the ordinary-matter isomer. Generation-1 associates with the up quark and the down quark. Generation-2 associates with the charm quark and the strange quark. Generation-3 associates with the top quark and the bottom quark.) For each row, the lepton flavours column assigns the three flavor numbers in order of increasing mass for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.) The stuff column identifies stuff made from the isomer that associates with the table row as OM, as in ordinary matter, or DM, as in dark matter. The acronym SEA abbreviates our term significantly-electromagnetically-active. The acronym MEA abbreviates our term marginally-electromagnetically-active. The stuff that associates with MEA interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-net-charge objects. TBD denotes the three-word phrase to be determined.

$l_{is,pr}$	l_{is}	Handedness	Quark generations	Lepton flavours	Stuff
0	0	Left	1, 2, 3	1, 2, 3	OM (SEA)
0	3	TBD	1, 2, 3	1, 3, 2	DM (TBD)
1	1	TBD	1, 2, 3	3, 1, 2	DM (MEA)
1	4	TBD	1, 2, 3	3, 2, 1	DM (MEA)
2	2	TBD	1, 2, 3	2, 3, 1	DM (MEA)
2	5	TBD	1, 2, 3	2, 1, 3	DM (MEA)

Table 3 differs from Table 2. For example, Table 3 includes all six possible permutations of lepton flavours. This paper does not address the extent to which IDM-2 isomer-3 stuff would evolve similarly to either one of IDM-1 DM (SEA) and IDM-2 DM (MEA).

Each one of Table 2 and Table 3 associates with a rather-well specified IDM candidate description of dark-matter elementary fermions. Considering the notion of the IDM class of candidate specifications for dark matter, we suggest that people might consider other similar, but different, candidate notions regarding IDM dark matter.

3. Results

3.1. Galaxy Formation and Galaxy Evolution

This unit indicates that our work can help provide explanations for some observations regarding galaxy formation and evolution and that our work adds insight regarding galaxy formation and galaxy evolution.

Our work proposes that, early in the history of the universe, single-isomer objects clumped based on reach-1 effects of at least one of gravitational quadrupole pull and chromodynamics pull. Some smaller single-object clumps evolved into solar systems. Some larger clumps (that could have included multiple solar systems) evolved into early galaxies.

Our work proposes that the discussion above explains 1:0+ ratios that pertain to some early galaxies and 0+:1 ratios that pertain to some early galaxies.

Our work proposes that some later 1:0+ galaxies and some later 0+:1 galaxies retain their ratios from early in the evolution of the universe.

For each of some $\sim 4:1$ galaxies, our work proposes the following scenario. The galaxy started as a 0+:1 galaxy. Reach-2 dipole push contributions to gravity drove away some ordinary-matter stuff and the stuff that associated with one dark-matter isomer. Then, reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim 4:1$.

For each of some $\sim 4:1$ galaxies, our work proposes the following scenario. The galaxy started as an MEA-isomer 1:0+ galaxy. Reach-2 dipole push contributions to gravity drove away some dark-

matter stuff but essentially no ordinary-matter stuff. Then, reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim 4:1$.

We note that observational research that found $\sim 4:1$ galaxies found that the $\sim 4:1$ galaxies were at the low end, regarding ratios of dark-matter presence to ordinary-matter presence, of the range of ratios for galaxies studied during the research [30,31].

Many later galaxies are 5+:1 galaxies. Our work proposes that many 5+:1 galaxies resulted from mergers of smaller, previous galaxies. Our work proposes that such mergers associate with reach-6 monopole gravitational pull. Our work proposes that the earliest mergers that led to a 5+:1 galaxy could have been mergers that involved 1:0+ galaxies and 0+:1 galaxies.

3.2. *The Fives in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects*

This unit indicates that our work can help provide explanations for the fives in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling proposes that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

Our work proposes that the notion of five dark-matter isomers explains the fives in such 5+:1 ratios of dark-matter effects to ordinary-matter effects.

3.3. *Our Solar System and Other Optically Observable Solar Systems*

This unit indicates that our work can help provide explanations for some observations regarding solar systems.

Our discussion above regarding early galaxies proposes that components, including solar systems, of galaxies feature stuff that associates with just one isomer. Our work proposes that presently optically observable solar systems stem from early 0+:1 clumps and do or would usually measure as $\approx 0:1$.

3.4. *Eras in the Rate of Expansion of the Universe*

This unit indicates that our work can help provide explanations for some observations regarding the rate of expansion of the universe.

Our work proposes that the relevant two or three eras in the rate of expansion of the universe associate with the moving apart from each other of neighboring, but not colliding, large objects.

Our work proposes that the rate of expansion of the universe associates with interactions between neighboring non-colliding large objects.

Our work proposes that the start of the first multibillion-year era associates with a transition to dominance, regarding interactions between many neighboring non-colliding large objects, including galaxy clusters, by gravitational quadrupole pull.

Our work proposes that the start of the second multibillion-year era associates with a transition to dominance, regarding interactions between many neighboring non-colliding large objects, including galaxy clusters, by gravitational dipole push.

Our work proposes that the possible start of a new era associates with a transition to dominance, regarding interactions between many neighboring non-colliding large objects, including galaxy clusters, by gravitational monopole pull.

4. Discussion

4.1. *The Extent of the Assumptions That Our Work Makes and the Extent to Which Our Work Helps Explain Data*

This unit compares the extent of the assumptions that underlie our work and the extent to which our work can help explain data.

Assumptions that our work makes associate with some notions about IDM elementary particles and with the five rows in Table 1.

Data that our work can help explain associate with four ratios (1:0+, 0+:1, $\sim 4:1$, and 5+:1) of dark-matter presence to ordinary matter presence regarding galaxies, with aspects of (especially the

early) formation and evolution of galaxies, with the lack of dark matter in some solar systems, with two 5+:1 ratios of dark-matter presence to ordinary matter presence regarding aspects that are larger than galaxies, and with aspects regarding eras in the rate of expansion of the universe.

One might say that comparing the extent of the data that our work can help explain with the extent of the assumptions that our work makes provides credibility for our work.

4.2. The Pluses in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit indicates that our work might help provide explanations for at least some portions of the pluses in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, for many galaxy clusters, and for densities of the universe.

Our work does not necessarily rule out the possibility that some portions of (or the entireties of) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence associate with axions, with other unfound elementary particles, or with other popular modeling suggestions regarding the nature of dark matter.

The following notions indicate that, for IDM-1 dark matter, our work might explain all or some of the amounts that underlie the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence. Our work proposes that the stuff that associates with either one of the two SEA isomers (one of which associates with ordinary matter and one of which associates with dark matter) associates with more electromagnetic energy than does the stuff that associates with any one of the four MEA isomers (each of which associates with dark matter). Table 1 provides an example of a reach-6 interaction component. Our work proposes that, at least early in the history of the universe, reach-6 or reach-3 interaction components might have enabled (via electromagnetic or other means) flows of electromagnetic energy between isomer-pairs. The net flows could have resulted in each MEA isomer having more stuff than each SEA isomer has. This notion of more stuff might explain all or some of the amounts that underlie the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence.

This paper does not discuss the extent to which similar (to IDM-1) notions might pertain for other notions, including IDM-2 dark matter, of IDM dark matter.

4.3. Hyperfine Depletion of Cosmic Microwave Background Radiation

This unit indicates that our work can help provide explanations for each of two possibilities regarding some depletion of cosmic microwave background radiation.

Regarding the depletion of cosmic microwave background radiation, popular modeling suggests that the second 1 in the possible 1:1 ratio or the only 1 in the possible 0:1 ratio associates with hyperfine effects of ordinary-matter hydrogen atoms.

Should popular modeling eventually settle on the 1:1 ratio, our work proposes that a reach per instance of at least two isomers pertains regarding hyperfine interactivity. For IDM-1 dark matter, our work proposes the following notions. MEA-isomers do not underlie significant numbers of hydrogen-like atoms. The first 1 in the 1:1 ratio associates with hyperfine effects of hydrogen-like atoms that associate with the SEA dark-matter isomer.

Should popular modeling eventually settle on the 0:1 ratio, our work proposes that a reach per instance of one isomer pertains regarding hyperfine interactivity.

4.4. The So-Called Hubble Tension and Some Other Possible Gaps Between Data and Popular Modeling

This unit indicates that our work might help close some possible gaps between data and popular modeling.

Popular modeling discusses some possible so-called tensions or gaps between data and popular modeling. One such possible tension is the so-called Hubble tension [71–73]. Other such possible tensions associate with large-scale lumpiness [74–82] and include the so-called S8 tension.

We propose that such tensions might associate with trying to extrapolate from popular modeling that works adequately well regarding phenomena that our work would associate with $R_{/in} = 1$ gravitational quadrupole pull to estimate later phenomena that our work would associate with $R_{/in} = 2$ gravitational dipole push. Such popular modeling extrapolations might, in effect, assume that gravitational dipole push associates with $R_{/in} = 1$ and, thereby, underestimate gravitational push. The underestimates might associate with overestimating, compared to data, some clumping of stuff. These notions associate with two of the three rows in Table 1 that discuss two-body gravitational interactions.

To the extent that popular modeling and further data fail to resolve some of the so-called tensions between popular modeling and data regarding some large-scale phenomena, our work regarding instances and reaches per instance might help resolve remaining tensions.

4.5. Reaches for Interaction Components Not Discussed Above

This unit discusses interaction components that Table 1 does not discuss.

Table 4 posits instances and reaches per instance for some components of some interactions.

Table 4. Posited instances and reaches per instance for some components of some interactions. Pull associates with attraction between two objects. Push associates with repulsion between two objects. n_{in} denotes the number of instances of the interaction component. $R_{/in}$ denotes the interaction reach, in number of isomers, per instance. For one-atom hyperfine absorption of light, a choice between reach-1 and a reach of more than one depends on further analysis of data. Assuming that a 1:1 data ratio pertains and that IDM-1 dark matter pertains, the lack of hydrogen-like atoms in the stuff that associates with the four dark-matter isomers that do not associate with SIDM (as in self-interacting dark matter) stuff points to a practical reach of two and could be compatible with a theoretical reach of six. TBD abbreviates the three-word phrase to be determined. Our work might not necessarily depend significantly on TBD numbers.

Interaction component	n_{in}	$R_{/in}$
One-atom hyperfine absorption of light (1:1 data)	3 or 1	2 or 6, respectively
One-atom hyperfine absorption of light (0:1 data)	6	1
Strong-force pull and push interactions	TBD (possibly 6)	TBD (possibly 1)
Weak interactions	TBD	TBD
Higgs mechanism interactions that enable non-zero mass	TBD	TBD

We note, as an aside, that, for one-atom hyperfine absorption of light, each one of the following two sentences, might have significance. The absorption changes electromagnetic properties of the atom. The absorption changes gravitational properties of the atom. For the 1:1 data case, notions (including reaches) that associate with aspects regarding monopole gravity (for example, the first row in Table 1) or dipole gravity (for example, the second row in Table 1) might pertain. Such concepts might not disturb notions (which might parallel notions that might associate with Table 1) that interactions that involve electromagnetism and do not significantly involve gravitation associate with reaches of one.

Table 4 is not necessarily complete. For example, popular modeling suggests the notion of an inflationary epoch (which might have preceded the known multi-billion year era of decreasing rate of expansion of the universe) and an inflaton elementary particle that might have played a role in driving increases in the rate of expansion of the universe during the inflationary epoch [83]. Popular modeling discusses notions of spatial uniformity regarding the distribution of stuff and notions of variations from uniformity that might associate with the inflationary epoch. One might suppose that notions of high uniformity might associate with an inflation-epoch interaction component for which a reach of six pertains.

4.6. Opportunities to Interrelate Physics Constants and to Reduce the Number of So-Called Fundamental Physics Constants

This unit discusses relationships, among data, that people might find useful for extending our work and possibly for reducing the number of physics constants that popular modeling assumes to be independent of each other.

Eq. (2) might associate with a relationship that links a strength of the electromagnetic interaction, a strength of the gravitational interaction, the masses of two elementary fermions, and the number of isomers [60,84]. m_τ denotes the mass of the tau. m_e denotes the mass of the electron. The exponent 6 might associate with the number, six, of isomers. The right-hand side of the equation is the ratio of the electromagnetic repulsion between two electrons to the gravitational attraction between the same two electrons.

$$(4/3)(m_\tau^2/m_e^2)^6 = ((1/(4\pi\epsilon_0))(q_e)^2)/(G(m_e)^2) \quad (2)$$

An equation might interrelate the masses of all known non-neutrino fermion elementary particles. (Eqs. (12), (13), (14), and (15) in this paper associate with the equation. Table 3.9.10 in [60] and Table 14 in [61] discuss the equation.)

The following paragraphs discuss relationships regarding properties of boson elementary particles.

Regarding boson elementary particles, we define $(N')^2$ via Eqs. (3) and (4). M' denotes $m/(m_Z/3)$, in which m denotes the mass of an elementary boson and m_Z denotes the mass of the Z boson. S' denotes S (as in the spin, in units of \hbar). Q' denotes the magnitude of the charge, in units of the magnitude of the charge of the W boson. (Popular modeling equates the magnitude of the charge of the W boson to the magnitude of the charge of the electron.) μ' denotes the magnitude of the magnetic moment, in units of the magnitude of the magnetic moment of the W boson.

$$(N')^2 \equiv (M')^2 + (S')^2 + (Q')^2 + (\mu')^2 - (T')^2 \quad (3)$$

$$(T')^2 = 1 \Leftrightarrow M' > 0; \quad (T')^2 = 0 \Leftrightarrow M' = 0 \quad (4)$$

Based on data [84], we propose that Eqs. (5) and (6) might pertain regarding all known boson elementary particles.

$$N' \in \{0, 1, 2, 3, 4\} \quad (5)$$

$$N' = 4 - S' \geq 3 \Leftrightarrow M' > 0; \quad N' = S' \Leftrightarrow M' = 0 \quad (6)$$

Eq. (7) comports with data and with Eq. (3).

$$(m_W)^2 : (m_Z)^2 : (m_{\text{Higgs}})^2 :: 7 : 9 : 17 \quad (7)$$

4.7. Modeling, Interactions, Properties, and Objects

This unit provides perspective regarding relationships among choices of modeling techniques, relevant interactions, relevant properties, and relevant notions regarding objects.

Popular modeling includes notions that choices of relevant interactions, properties, and objects associate with choices of modeling techniques. The modeling technique general relativity provides an example. Relevant properties can associate with densities, such as energy density and momentum density, and not necessarily with objects or properties, such as masses, charges, and magnetic moments, of individual objects.

Our work points to notions of modeling-choice dependence regarding interactions, properties, and objects.

We discuss modeling that associates with the rest frame of an object and that pertains regarding electromagnetism.

One popular modeling basis for discussing electromagnetism features notions of pointlike as a modeling choice regarding objects, charge as a property of objects, and monopole potentials as a characterization of electromagnetic effects. The following are popular modeling paths that start from that basis and potential paths that start from that basis.

- One popular modeling path removes the constraint of pointlike, constrains itself to the property of charge, and deploys multipole expansion techniques that associate with spatial distributions of charge.
- One popular modeling path that retains pointlike, adds the property of magnetic moment, and associates dipole potentials with magnetic moments.
- Our work suggests a path that features notions of approximately pointlike, notions of charge and internally moving charge, and notions of a multipole expansion that features monopole potentials that associate with rest charges and dipole potentials that associate with the internal nonzero velocities of nonzero-charge sub-objects of the object.

We discuss modeling that associates with the rest frame of an object and that pertains regarding gravitation. This discussion has parallels to the discussion above regarding electromagnetism.

One popular modeling basis for discussing gravitation features the notions of pointlike as a modeling choice regarding objects, mass as a property of objects, and monopole potentials as a characterization of gravitational effects. The following are popular modeling paths that start from that basis and potential paths that start from that basis.

- One popular modeling path removes the constraint of pointlike, constrains itself to the property of mass, and deploys multipole expansion techniques that associate with spatial distributions of mass.
- We suggest a path that retains pointlike, adds the property of spin (as in object-internal angular momentum), and associates dipole potentials with spin.
- Our work suggests a path that features notions of approximately pointlike, notions of mass and internally moving mass, and notions of a multipole expansion that features monopole potentials that associate with rest masses and dipole potentials that associate with the internal nonzero velocities of nonzero-mass sub-objects of the object.
 - We suggest the possibility of considering the notion of modeling the monopole-potential-generating phenomena as associating with the rest masses of objects and sub-objects.
 - We suggest the possibility of considering the notion of modeling the dipole-potential-generating phenomena as having two components. One component associates with the spin of the object. One component associates with thermal-motion-like notions regarding sub-objects.

We discuss notions that pertain to properties that associate with components of interactions or with objects.

Regarding electromagnetism, our work suggests modeling nature as having six instances (with a one-to-one association with the six isomers) of charge.

Regarding gravity, our work suggests considering that instances, reaches per instance, and relevant isomer numbers are properties of components of interactions. The proportions of the rest masses that associate with each one of the six isomers can associate with properties of objects.

Regarding mass, one might want to be cautious regarding the modeling extents to which various terms pertain. For example, notions of gravitational mass might not necessarily meaningfully pertain to quarks.

4.8. Potential Future Endeavors and Directions for Observational, Experimental, and Theoretical Physics

This unit points to potential future directions for some aspects of cosmology, gravitation, and elementary-particle physics.

Future endeavors and directions for physics might include the following.

Determine, via data, which of the two Table 4 rows regarding one-atom hyperfine absorption of light pertains to nature. To the extent that the relevant reach is not one, reconcile theoretically the reach of more than one with the two electromagnetism reaches of one that Table 1 indicates.

Determine the extent to which self-interacting-dark-matter stuff includes IGM (as in intergalactic medium), perhaps including by analyzing data about collisions of galaxy clusters [53,54]. Thereby, narrow the class of IDM candidate descriptions for dark matter.

Determine the extent to which IDM candidate specifications for dark matter are compatible with data that associate with various types of studies that involve ratios of dark-matter effects to ordinary-matter effects that our work does not directly address [85–88]. The following notions potentially pertain. Narrow the class of IDM candidate descriptions for dark matter and better determine interaction-related information that belongs in Table 1 and Table 4. Add insight regarding various types of ongoing studies. Refute aspects of our work.

Determine the extent to which adding, to simulations, IDM candidate specifications for dark matter can be useful.

Determine the extent to which gravitational wave signatures differ between collisions that involve same-isomer small-mass black holes and neutron (or dark-matter neutron-counterpart) stars and collisions that involve different-isomer small-mass black holes and neutron (or dark-matter neutron-counterpart) stars.

Associating with perspective that general relativity has passed so-called precision tests [89] and that those tests seem not to involve dark matter, explore the extent to which general relativity might not be adequately accurate for circumstances in which the isomeric composition of stuff varies significantly between regions of the universe or for circumstances in which significant (or dominant) effective gravitational reaches per instance vary with time. (We note, as an aside, that popular modeling recognizes circumstances for which tests of general relativity have yet to be very precise and for which alternative theories of gravity might be appropriate [90].)

Explore notions of a possible symmetry (or approximate symmetry) regarding quark masses, quark generations, lepton masses, and lepton flavours.

Explore the relevance of knowing the reaches that Table 4 lists as TBD (as in to be determined). For example, a weak interaction reach of more than one isomer might associate with a possible symmetry (or approximate symmetry) regarding quark masses, quark generations, lepton masses, and lepton flavours. At least to the extent that knowledge of such reaches might be relevant, determine the reaches.

Explore relationships among physics constants and possibly reduce the number of independent constants.

Explore relationships between choices regarding properties (of objects and interaction fields) that people attribute to nature and choices among models that people use.

Explore the extent to which our work can help focus and accelerate cosmology and elementary-particle research.

5. Conclusion

This unit summarizes results that our work achieves.

We propose seemingly new notions regarding gravitational multipole expansions and gravitational properties of objects. A key aspect features the motions, within gravitationally interacting objects, of sub-objects of the interacting objects. Within popular modeling that combines Newtonian modeling and multipole methods, monopole components of gravity associate with the attraction of objects toward each other. Within our modeling framework that combines Newtonian gravitation and Lorentz invariance, the motions of the sub-objects contribute to a dipole component of gravity that repels objects from each other.

Thus, our work proposes new steps forward regarding the topics of gravitational phenomena and of multipole properties of objects.

We propose a seemingly new class of candidate specifications for dark-matter elementary particles. A key aspect features five dark-matter counterparts for each ordinary-matter fermion elementary particle and has bases in the notion that ordinary-matter elementary fermions associate with near-matches, but not exact matches, between flavour states and mass states.

Thus, our work proposes new steps forward regarding the topics of dark matter and elementary particles.

We combine our work regarding gravitation, properties of objects, dark matter, and elementary particles. We propose (via Eq. (1) and Table 1) a way to characterize components of gravitational and electromagnetic interactions between objects. This way associates with using data to choose, for each component of interactions, one of four mathematically possible solutions to an equation that involves only integers.

We indicate that choices (that Table 1 lists) of solutions can help explain known ratios of dark-matter effects to ordinary-matter effects and can help explain eras in the rate of expansion of the universe.

Comparing the extent of the data that our work can help explain with the extent of the assumptions that our work makes seems to provide credibility for our work.

We indicate that data seem to point to interrelationships among some physics constants.

We suggest potential future specific endeavors and general directions for cosmology, gravitation, and elementary-particle physics.

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Appendices

A.1. Some Research That Has Tried to Address Rate-of-Expansion, Gravitational-Repulsion, Dark-Matter, and Elementary-Particle Topics

This unit provides or points to perspective about research regarding the rate of expansion of the universe, gravitational repulsion, dark matter, and dark-matter elementary particles.

Measurements that point to increases in the rate of expansion of the universe point to the possibility of gravitational repulsion. Popular physics modeling includes theories of gravity that include gravitational repulsion [91,92]. Regarding explaining repulsion via so-called dark energy, popular modeling discusses reasons to find alternatives to invoking the cosmological constant [93].

Generally, the following notions pertain regarding research related to the rate of expansion of the universe [94–98]. Research focuses on data that popular modeling associates with supernovae, cosmic microwave background radiation, baryon acoustic oscillations, galaxy clustering and large-scale structure, and gravitational lensing. Parameters that people use in theories include the Hubble constant, baryon density, cold dark matter density, dark energy density, equations of state for dark energy, and measures of the clumpiness of matter.

People have yet to settle on a preferred description of dark matter [65,99]. Some proposed descriptions base dark matter on yet-to-be-found elementary particles. Some proposed descriptions feature copies or near-copies of standard model matter [92,100].

Generally, the following notions pertain regarding research related to dark matter [84,101–104]. Research focuses on data that popular modeling associates with cosmic microwave background radiation, large-scale structure, baryon acoustic oscillations, supernovae, galaxy rotation curves and gravitational lensing, redshift space distortions, and the Lyman- α forest. Parameters that people use in

theories include the dark matter density parameter, the equation of state parameter, particle properties such as masses and cross-sections, temperature (as in cold, warm, or hot), and parameters regarding interactions between dark matter and baryons or dark energy.

As far as we know, the following notions pertain.

- Popular modeling does not try to use gravitational multipole expansions regarding two-body gravitational interactions to help explain the rate of expansion of the universe or to help explain aspects of gravity that repel objects from each other.
- Popular modeling does not deploy multipole expansions for which the motions of sub-objects of a gravitationally interacting object move within the object and, via such motions, associate with a multipole property of the object.
- Popular modeling treats the 5+:1 ratio of dark-matter density of the universe to ordinary-matter density of the universe as a ratio of two parameters (the two relevant densities of the universe) and does not provide an explanation, based on physics principles, for the ratio.
- Popular modeling does not consider that most dark matter might have bases in fermion elementary particles that have masses that are similar to the masses of ordinary matter fermion elementary particles.

A.2. Two-Body Gravitational Repulsion

This unit provides a thought experiment that indicates that notions of Newtonian gravity, notions of Lorentz invariance, and an analog to one Maxwell equation point to circumstances in which two cosmological objects repel each other gravitationally.

For an object and a body that do not move relative to each other, consider the gravitational effects of the object on the body. Assume that the object models as having sub-objects that move relative to the center-of-mass of the object.

Deploy Newtonian gravitational modeling, including ...

- The notion that masses and forces do not depend on velocities.
- The notion that the rest masses of the sub-objects sum to the rest mass of the object.

Assume that the object and the body are sufficiently far apart that modeling can treat the object as spatially pointlike.

Note that, for each moving sub-object, Lorentz invariance suggests that the body senses a mass that exceeds the rest mass of the sub-object.

Consider a gravitational analog to the electromagnetic Maxwell equation that associates the electric field with a sum of two terms, with one term associating with a spatial gradient of a scalar-potential field and with the other term associating with a temporal partial derivative of a 3-vector-potential field.

For each moving sub-object, ...

- Associate the rest mass of the sub-object with a scalar-potential field.
- Associate the body-perceived mass minus rest mass with a 3-vector-potential field (and not with a scalar-potential field).
- Note that the 3-vector-potential associates with the notion of spatially dipole (and not with the notion of spatially monopole that associates with the scalar-potential that associates with the rest mass of the sub-object).
- Conclude that, from the perspective of the body, the dipole contribution to the gravitational field detracts from the monopole contribution to the gravitational field.
- Conclude that the monopole contribution associates with a gravitational pull of the body toward the object and that the dipole contribution associates with a gravitational push of the body away from the object.

Associate the overall gravitational effect of the object on the body with the sum of the sub-object effects.

Note that, for the object being sufficiently far from the body, gravity attracts the body toward the object.

Note that, for the object being sufficiently close to the body, gravity repels the body away from the object.

A.3. Gravitational Multipole Expansions

This unit develops the notions of gravitational monopole pull, gravitational dipole push, and gravitational quadrupole pull.

We discuss interactions between an object-A and an object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as electromagnetic fields and gravitational fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

We review aspects of seventeenth century Newtonian gravity.

Eq. (8), Eq. (9), and Eq. (10) describe aspects regarding the motion of object-P [8]. G is the gravitational constant. m_{oA} is the mass of object-A. Mass is a scalar property. m_{oP} is the mass of object-P. r is the 3-vector distance that object-P is away from object-A. ∇ is the gradient operator. ∇ produces a 3-vector field from a scalar field. F_{oP} is the force that object-P feels. Object-P might sense effects of that force via an accelerometer that associates with object-P. In equations such as Eq. (8), V is a scalar field. Popular modeling associates with V the word potential. In equations such as Eq. (9), r^{n_r} denotes the n_r -th power of the magnitude of the 3-vector r .

$$Gm_{oA}m_{oP}(-\nabla V) = F_{oP} \quad (8)$$

$$V = -1/r^{n_r} \quad (9)$$

$$n_r = 1 \quad (10)$$

The left-hand side of Eq. (8) does not involve a velocity of object-P, a velocity of object-A, a velocity of object-A relative to object-P, or a velocity of object-P relative to object-A. With respect to recent popular modeling, one can consider that m_{oA} associates with the rest mass of object-A. With respect to recent popular modeling, one can consider that m_{oP} associates with the rest mass of object-P.

We review aspects of eighteenth century two-body electromagnetism.

Eq. (11) is an aspect of eighteenth century two-body electromagnetism [105,106]. ϵ_0 denotes the vacuum electric permittivity. q_{oA} is the charge of object-A. Charge is a scalar property. q_{oP} is the charge of object-P. Eq. (9), and Eq. (10) pertain.

$$-(1/(4\pi\epsilon_0))q_{oA}q_{oP}(-\nabla(V)) = F_{oP} \quad (11)$$

Eq. (11) has similarities to Eq. (8).

The left-hand side of Eq. (11) does not involve a velocity of object-P, a velocity of object-A, a velocity of object-A relative to object-P, or a velocity of object-P relative to object-A. With respect to recent popular modeling, one can consider that q_{oA} associates with the rest charge of object-A. With respect to recent popular modeling, one can consider that q_{oP} associates with the rest charge of object-P.

For each one of gravity and electromagnetism, popular modeling includes notions of multipole expansions that have bases in spatial distributions of a scalar property that can associate with sub-objects of an object. For gravity, the scalar property is mass. For electromagnetism, the scalar property is charge. For popular modeling multipole expansions regarding each one of charge and mass, the following notions pertain. The sum of the values of the scalar property for the sub-objects equals the value of the scalar property for the object. Popular modeling de-emphasizes the notion that the sub-objects might move within the object. Popular modeling de-emphasizes forces by which the

sub-objects might attract or repel each other. Popular modeling de-emphasizes energies that might associate with keeping the sub-objects in their respective places.

Popular modeling regarding gravitation points to situations in which dipole contributions dilute monopole contributions and to situations in which dipole contributions augment monopole contributions. For example, consider an object that models as being two separated, equal-mass pointlike sub-objects. From the perspective of another body that lies along a line that runs through the center-of-mass of the object and is perpendicular to the line that runs through the two sub-objects, each sub-object is farther away than is the center-of-mass of the object; the body senses less gravitational pull than the body would sense if the two sub-objects existed at the center-of-mass point. However, if the other body lies along a line that runs through the two sub-objects and is farther away from the center-of-mass than is each sub-object, the body senses more gravitational pull than the body would sense if the two sub-objects existed at the center-of-mass point. From such notions, one might conclude that whether there is net dipole dilution of gravitational forces or net dipole augmentation of gravitational forces depends (for two objects) on details and (across several two-body interactions) on statistics.

For now (in this discussion), we de-emphasize the notion of spatial distributions of sub-objects. We focus instead on the motions of sub-objects and assume that modeling can treat an overall object as spatially pointlike. Also, for now (in this discussion), we treat an overall object as not moving.

Within cosmological (and other) objects, sub-objects can move. For example, galaxies move within galaxy clusters. Also, solar systems move within galaxies. Popular modeling Lorentz invariance [107] suggests that a body that is remote from the object and that does not move relative to the object senses, for each sub-object that moves within the object and has a nonzero value of a scalar property, a larger (than if the sub-object did not move within the object) magnitude of the value of the scalar property.

We develop multipole expansion techniques that address the notion that, if sub-objects model as moving, a remote body likely (for electromagnetism) or always (for gravity) would sense a sum, across sub-objects, of the values of the scalar property (charge for electromagnetism or mass for gravity) that differs from the value of the scalar property that the body associates with the object.

Our development considers quantities that pertain in the rest frame of the body. Our development considers that the object models as approximately pointlike and that all sub-objects model as being (instantaneously) at the same point as the object. We de-emphasize notions that popular modeling associates with the two-word term retarded time.

We consider a case for which the object does not move relative to the body, the only significant interactions between the object and the body are electromagnetic, and the only significant interactions between the sub-objects and the body are electromagnetic. (Our discussions regarding electromagnetism echo popular modeling [106].)

We consider a sub-object that has a rest charge q_0 and that moves (relative to each of the object and the body) with a 3-vector velocity v . We define γ^* by $\gamma^* \equiv 1 + (\gamma - 1)$, in which $\gamma = (1 - (|v|/c)^2)^{-1/2}$ is the so-called Lorentz factor. The first term (as in the first 1) in γ^* associates with $v = 0$. The second term (as in $\gamma - 1$) in γ^* associates with $|v| > 0$. Popular modeling associates the scalar potential (perceived by the body) with a constant multiplied by $q_0\gamma^*/|r|$, in which r denotes a 3-vector distance from the sub-object. Popular modeling associates the vector potential (perceived by the body) with a constant multiplied by $q_0\gamma^*(v/c)/|r|$. Based on aspects that are compatible with popular modeling, we replace the motion-related portion ($q_0(\gamma - 1)/|r|$) of the scalar potential with a vector-potential term that is proportional to $q_0(\gamma - 1)vt/|r|^2$, in which t denotes the temporal coordinate. (The relevant popular modeling equation is $E = -\nabla\phi - \partial A/\partial t$, in which E is the 3-vector electric field, ∇ is the spatial-gradient operator, ϕ is the scalar electromagnetic scalar potential, A is the 3-vector electromagnetic vector potential, and t is the temporal coordinate. Our work, in effect, replaces the motion-related, as perceived by the body, portion of ϕ with a motion-related, as perceived by the body, component of A that does not contribute to the magnetic field perceived by the body. For $|v|/c \ll 1$, $q_0(\gamma - 1)vt/|r|^2 \approx q_0(v|v|/2)(t/(c|r|^2))$. The motion-related portion of ϕ would have associated with

a push or pull component of force on the body. Thus, the replacement motion-related component of A should associate with a push or pull component of force on the body. A vector representation that comports with $q_0(v|v|/2)(t/(c|r|^2))$ can comport with a notion of $v \times (v \times r)$, which is parallel or antiparallel to r . \times denotes the cross product.)

Across all sub-objects, there are now the following three types of terms.

1. Scalar potential terms that associate with rest charges and with monopole (as in $1/|r|$) spatial potentials.
2. Vector potential terms that associate with $v \neq 0$ and with monopole (as in $1/|r|$) spatial potentials.
3. Vector potential terms that associate with $v \neq 0$ and that popular modeling can associate with dipole (as in $1/|r|^2$) spatial potentials.

The type-1 terms comport with popular modeling notions of multipole expansions. The type-2 terms comport with popular modeling notions of magnetic fields and are not necessarily relevant to our discussion (because our discussion pertains regarding the rest frame of the affected body). (Popular modeling states that the relevant effects, on the body, of the magnetic field, symbolized by the 3-vector B , that associates with the object scale as $v_b \times B$, in which v_b is the 3-vector velocity of the body. In the rest frame of the body, $v_b = 0$.) The type-3 terms comport with popular modeling notions of zero contributions to magnetic fields and do not necessarily comport with popular modeling notions of multipole expansions that involve only scalar properties of objects.

From the standpoint of the affected body, type-3 terms scale, for $|v|/c \ll 1$, proportionately to $|v|^2/c^2$. (We note that $q_0(v|v|/2)(t/(c|r|^2)) = q_0((v/c)((|v|/c)/2))(ct/(|r|^2))$.)

From the standpoint of the affected body, type-2 terms scale, for $|v|/c \ll 1$, proportionately to $|v|/c$. From the standpoint of the affected body, type-2 terms associate with notions of charge currents.

From the standpoint of the affected body, type-3 (or $(\gamma - 1)$) terms dilute the counterpart overall (or γ^*) scalar potential contributions.

Our work above features notions related to Lorentz invariance, to classical physics, and not necessarily to quantum physics. Our work above does not necessarily depend on popular modeling notions that quantum interactions with electromagnetic fields feature transfers (from or to electromagnetic fields) of angular momenta for which the magnitudes are one times \hbar . Our work above does not necessarily rely on popular modeling notions that electromagnetism associates with a spin-1 field.

We change our focus from electromagnetism to gravitation.

We anticipate that our work below does not necessarily need to associate with quantum physics and does not necessarily need to consider popular modeling notions that gravity associates with a spin-2 field.

We consider a case for which the object does not move relative to the body, the only significant interactions between the object and the body are gravitational, and the only significant interactions between the sub-objects and the body are gravitational. Compared to the above electromagnetic case, the following changes pertain. The word mass substitutes for the word charge. Rest masses are nonnegative. From the standpoint of the affected body, type-1 terms associate with notions of masses and with notions of gravitational attraction. From the standpoint of the affected body, type-2 terms associate with notions of momenta. From the standpoint of the affected body, the magnitudes of type-3 terms associate with notions of kinetic energies. (For a sub-object with a rest mass of m_0 and for $|v|/c \ll 1$, $m_0(\gamma - 1)vt/|r|^2 \approx m_0(v|v|/2)(t/(c|r|^2))$ and $|m_0(v|v|/2)| = (1/2)m_0|v|^2$. For $|v|/c \ll 1$, $(1/2)m_0|v|^2$ associates with a popular modeling notion of kinetic energy.)

For gravitation, monopole components of interactions associate with pull (as in gravitational attraction of the body toward the object) and type-3 dipole components of interactions associate with push (as in gravitational repulsion of the body away from the object). Quadrupole components of interactions can associate with considering that each one of the object and the body has moving sub-objects. Quadrupole effects correct for otherwise miscounting some dipole effects. Such so-called miscounting associates with the notion that a moving sub-object of the body senses effects

of the motions of sub-objects of the object. Quadrupole components of interactions associate with gravitational pull.

We note, as an aside, that our notions of pull and push do not necessarily encompass notions for which popular modeling use of the word torque would pertain.

We note, as an aside, the following possible associations between type-3 gravitational components and applications of general relativity in situations for which the energy flux is zero. Monopole might associate with energy density. Dipole might associate with pressure.

We note, as an aside, that we do not try to explore similarities and differences between gravito-electromagnetism [108–110] and our work regarding gravitational multipole expansions.

We return (in this discussion) to the notion that gravitational modeling does not necessarily need to treat objects as pointlike.

One use above of pointlike associates with the notion that, for discussing aspects related to the motions of sub-objects, one can assume that, across the sub-objects, just one origin related to vectors r pertains. This notion is useful for simplifying our discussion. We posit that the one-origin simplification is not necessarily an oversimplification regarding aspects related to motions of sub-objects.

Regarding the instantaneous positions of sub-objects, the notion of just one origin related to vectors r (or the notion of pointlike modeling for the object) is, in general, not necessarily adequately accurate. Per discussion above, stationary multipole aspects can detract from or add to monopole aspects. For our work, we posit that, at least statistically across similar objects, one can de-emphasize net non-monopole effects that associate with positions of sub-objects compared to non-monopole effects that associate with velocities of sub-objects. We note, as an aside, that, for non-monopole effects that associate with velocities of sub-objects, net effects associate with gross effects.

We note, as an aside, that net non-monopole effects that associate with positions of sub-objects or objects do not necessarily associate with needing modeling based on Lorentz invariance and would associate gravitationally with one instance ($n_{in} = 1$) and a reach per instance of six isomers ($R_{/in} = 6$).

We note, as an aside, that, for objects such as hadrons, modeling regarding gravitation can feature the masses and momenta of the objects and de-emphasize the masses and momenta of sub-objects such as quarks.

We note, as an aside, that the gravitational rows in Table 1 are compatible with discussion above.

A.4. Uses of the Word Isomer

This unit compares our use of the word isomer with other uses of the word isomer.

Our use of the word isomer can associate with notions of symmetries, including chirality (or mirror image) symmetry, and with notions of approximate symmetries. Chemistry uses the word isomer regarding the notion that molecules can be mirror images of each other.

Our use of the word isomer does not directly associate with some other uses of the word isomer, for example regarding alternative geometric arrangements (other than uses related to chiral symmetry) of atoms within molecules or regarding long-lived excited states of atomic nuclei.

We are not aware of attempts (to parallel, in elementary-particle physics, nuclear-physics notions of isomers and thereby ...) to use the word isomer in conjunction with the three flavour states of ordinary-matter charged leptons or the three generation-states of similarly-charged ordinary-matter quarks. Our work might be able to embrace such uses of the word isomer; however, our work does not presently use the word isomer for such purposes.

A.5. Usages of a Five-Step from-Data-to-Uses Approach

This unit provides, regarding five usages, information regarding our usages of an approach that starts from data patterns and progresses to key concepts and anticipated uses.

The following notions pertain regarding large-scale presences of dark matter.

1. For galaxy clusters and densities of the universe, a pattern is that 5+:1 ratios of presence of dark matter to presence of ordinary matter often pertain.

2. Based on the possibly relevant notion of reusing familiar physics, we propose that five near-copies of a set of elementary particles that underlie ordinary matter underlie dark matter.
3. Based on the possibly relevant notion of using simple mathematics, we explore the notion that similarities and differences between the total-of-six sets associate with symmetries or broken symmetries. Based on usage in other areas of science of the word isomer, we suggest using the concept of isomers to associate with the six sets. (The appendix regarding uses of the word isomer provides perspective.)
4. We use the three-word term isomeric dark matter (and the acronym IDM) to refer to a class of candidate specifications for dark matter, for which each member of the class associates with our notion of five sets out of six sets of isomers.
5. We anticipate discussing (in this paper) how two members of the IDM class of candidate specifications for dark matter can help explain cosmic data.

The following notions pertain regarding presences, in galaxies, of dark matter and of ordinary matter.

1. Galaxies that feature low proportions (such as 0+:1) of dark matter and galaxies that feature low proportions (such as 1:0+) of ordinary matter tend to have masses that are small compared to the masses of galaxies that feature ratios of dark-matter-presence to ordinary-matter-presence of around 5+:1.
2. Based on the familiar physics notion that galaxy formation and evolution associate with gravitational phenomena that attract stuff toward other stuff, we propose that some gravitational effects can clump ordinary matter without attracting much dark matter and that some gravitational effects can clump dark matter without attracting much ordinary matter.
3. Regarding applying mathematics, we have opportunities to deploy techniques (that stem from Newtonian gravity, the notion that large objects can have sub-objects that move within the large objects, and Lorentz invariance and) that feature one or more than one of using traditional gravitational multipole expansions based on spatial distributions of mass, considering new gravitational properties of objects, and developing and deploying new types of gravitational multipole expansions.
4. We use the two-element term Newton-Lorentz gravity to refer to a new type of gravitational multipole expansion for which multipole-expansion-terms (other than monopole terms) associate with motions of sub-objects of gravitationally interacting objects.
5. We anticipate discussing (in this paper) how gravitational multipole properties of objects and Newton-Lorentz multipole expansions can help explain cosmic data about galaxy formation and evolution.

The following notions pertain regarding the formation of objects such as stars, solar systems, and low-mass galaxies.

1. Galaxies that feature low proportions (such as 0+:1) of dark matter and galaxies that feature low proportions (such as 1:0+) of ordinary matter tend to have masses that are small compared to the masses of galaxies that feature dark-matter-presence to ordinary-matter-presence ratios of around 5+:1.
2. Based on the familiar physics notions that associate with the Lambda cold dark matter (also known as Λ CDM) concept of hierarchical structure formation and on our notions of Newton-Lorentz gravity, we suggest that Newton-Lorentz gravity can help explain the clumping of single-isomer objects such as stars or solar systems.
3. Regarding applying mathematics, we use the same techniques as we use regarding the presences, in galaxies, of dark matter and ordinary matter.
4. We use notions of Newton-Lorentz gravity.
5. We anticipate discussing (in this paper) how gravitational multipole properties of objects and Newton-Lorentz multipole expansions can help explain cosmic data about the formation of stars, solar systems, and low-mass galaxies.

The following notions pertain regarding the rate of expansion of the universe.

1. Data associate with an orderly progression, over billions of years, from decreasing rates of separation between large objects (such as galaxy clusters), to increasing rates of separation between large objects, to possible decreasing rates of separation between large objects.
2. Based on the possibly relevant notion of reusing familiar physics, we propose that notions of two-body gravitational interactions might help explain the relevant two or three eras. Here, the two-body interactions pertain to objects such as galaxy clusters.
3. Regarding applying mathematics, we propose using Newton-Lorentz multipole expansions.
4. Two-body interactions between large objects can provide insight regarding eras in the rate of expansion of the universe.
5. We anticipate discussing (in this paper) how gravitational multipole properties of objects and Newton-Lorentz multipole expansions can help explain cosmic data regarding the rate of expansion of the universe.

The following notions pertain regarding ordinary-matter fermion elementary particles.

1. For each one of four magnitudes of charge (namely, letting $|q_e|$ denote the absolute value of the charge of an electron, for $(2/3)|q_e|$ quarks, for $(1/3)|q_e|$ quarks, for $1|q_e|$ charged leptons, and for $0|q_e|$ neutrinos), there are three elementary fermions.
2. Popular modeling uses the two properties of mass and flavour (or generation) to help describe ordinary-matter elementary fermions. For each one of the four magnitudes of charge, the weak interaction underlies transitions between the three relevant elementary fermions. For each one of the four magnitudes of charge, popular modeling considers the notions that there are three mass eigenstates, that there are three flavour eigenstates, and that the three mass eigenstates might somewhat match and do not necessarily exactly match the three flavour eigenstates.
3. Based on the possibly relevant notion of using simple mathematics, we propose the notion that, across isomers, near-matches between lepton eigenstates can differ from near-matches between quark eigenstates. We propose notions that the differences across isomers can associate with the breaking of a six-fold symmetry or of a three-fold symmetry.
4. One might suggest the three-element term weak-interaction multi-isomer symmetry to refer to the possible six-fold or three-fold symmetry.
5. We anticipate discussing (in this paper) notions regarding such a six-fold symmetry or such a three-fold symmetry.

A.6. Possible Symmetry or Approximate Symmetry Regarding the Factor of Two (That Associates with the Number, Six, of Isomers)

This unit discusses aspects related to the factor of two (that associates with the number, six, of isomers), to possible right-handed counterpart-to-ordinary-matter dark-matter elementary particles, to matter-antimatter asymmetry, and to possible dark-matter hydrogen-like atoms.

This discussion associates with our notions of IDM-1 dark-matter elementary particles. Some aspects of this discussion do not necessarily associate with our notions of IDM-2 dark-matter elementary particles. Parts of or the entirety of this discussion might associate with other notions of IDM dark-matter elementary particles.

The following notions might cover all three of right-handed counterpart-to-ordinary-matter dark-matter elementary particles, matter-antimatter asymmetry, and dark-matter hydrogen-like atoms.

- People, informally, use the one-element term left-handed to describe the ordinary matter isomer.
- In more technical terms, the following popular modeling notions pertain for the ordinary-matter isomer. Left-chiral components of matter elementary fermions and right-chiral components of antimatter elementary fermions associate, via the so-called the $SU(2)_L$ gauge group, with doublets and with interactivity via the weak interaction. Right-chiral components of matter elementary fermions and left-chiral components of antimatter elementary fermions associate, via the so-called the $SU(2)_L$ gauge group, with singlets and with no interactivity via the weak interaction.

- Popular modeling associates the technical aspects with the weak interaction and with a breaking of PC, as in parity and charge, symmetry by the weak interaction.
- We posit that popular modeling might consider that the following notions pertain for the counterpart-to-ordinary-matter dark-matter isomer. Right-chiral components of antimatter elementary fermions and left-chiral components of matter elementary fermions associate, via an $SU(2)_R$ gauge group, with doublets and with interactivity via the weak interaction. Left-chiral components of antimatter elementary fermions and right-chiral components of matter elementary fermions associate, via the $SU(2)_R$ gauge group, with singlets and with no interactivity via the weak interaction. Popular modeling would associate these technical aspects with the weak interaction and with a breaking of PC, as in parity and charge, symmetry.
- Possibly, popular modeling would associate the combination of the ordinary-matter-isomer aspects and the counterpart-to-the-ordinary-matter dark-matter aspects with a symmetry related to the weak interaction.
- Possibly, regarding the counterpart-to-the-ordinary-matter dark-matter stuff, nature would produce more antimatter stuff than matter stuff and popular modeling would associate the combination of the ordinary-matter-isomer aspects and the counterpart-to-the-ordinary-matter dark-matter isomer aspects with notions of matter-antimatter symmetry.
- The counterpart-to-the-ordinary-matter dark-matter stuff would include hydrogen-like atoms. While popular modeling regarding electromagnetism can feature two (as in left and right) orthogonal circular-polarization modes (or can feature two orthogonal linear-polarization modes), popular modeling notions of electromagnetic interactions do not necessarily depend on weak-interaction notions of left-chiral and right-chiral. One can leave to observational work the question as to whether counterpart-to-the-ordinary-matter dark-matter-stuff hydrogen-like atoms can absorb light that ordinary-matter stuff emitted.

We note, as an aside, that discussion above features specific (direct or implicit) reuses, from the ordinary-matter isomer to the counterpart-of-the-ordinary-matter dark-matter isomer, of dual-pairs such as matter and antimatter, more-prevalent and less-prevalent, $SU(2)_L$ and $SU(2)_R$, and positive charge and negative charge. Possibly, the number of dual-pairs is sufficiently large that alternative (to the choices we discuss above) choices (regarding some choices regarding some pairs) regarding the counterpart-to-the-ordinary-matter dark-matter isomer could be appropriate.

A.7. Possible Approximate Symmetry Regarding the Factor of Three (That Associates with the Number, Six, of Isomers)

This unit discusses aspects related to the factor of three (that associates with the number, six, of isomers) and to matches or mismatches, across isomers, between charged lepton flavours and quark generations.

We propose that each one of the following notions about ordinary matter is not incompatible with the notion of an approximate symmetry that would associate with and differentiate the three isomer-pairs.

- The three neutrino flavour eigenstates do not equal the three neutrino mass eigenstates. We propose that the mismatch associates with an approximate symmetry.
- Eqs. (12), (13), (14), and (15) pertain regarding the masses of the three charged leptons. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau. Similar equations (with $k = 0, +1, \text{ and } +2$) pertain regarding the geometric-mean masses for the three quark generations (Table 3.9.10 in [60] or Table 14 in [61]). We note, as an aside, that the notion that δ is not zero might associate with an approximate symmetry.

$$k = 0, +2, \text{ and } +3, \text{ for charged-lepton flavour-1, flavour-2, and flavor-3, respectively} \quad (12)$$

$$\sigma_k = 0, +1, -1, \text{ and } 0, \text{ for } k = 0, 1, 2, \text{ and } 3, \text{ respectively} \quad (13)$$

$$\delta \approx 0.03668 \quad (14)$$

$$m_k/m_e \approx (m_\tau/m_e)^{(1/3)(k+\sigma_k\delta)} \quad (15)$$

- The weak interaction associates with interactions in which charged leptons change flavour and with interactions in which quarks change generation. We propose that the mismatch associates with an approximate symmetry.

We propose that the approximate symmetry associates with, for some isomers, associations between lepton flavours and quark generations that differ from the associations between lepton flavours and quark generations that associate with the ordinary-matter isomer.

We do not try to fully characterize the possible approximate symmetry.

A.8. The Evolution of MEA (as in Marginally-Electromagnetically-Active) Dark-Matter Stuff

This unit discusses how the evolution of MEA (as in marginally-electromagnetically-active) stuff leads to MEA stuff that features stable counterparts to ordinary-matter-stuff neutrons.

For the stuff that associates with each one of the six isomers, a ground-state singly-charged baryon that includes exactly three generation-3 quarks would be more massive than the counterpart, within the same-isomer stuff, ground-state zero-charge baryon that includes exactly three generation-3 quarks. For example, for ordinary-matter-isomer stuff, a ground-state nonzero-charge baryon that includes just two tops and one bottom would have a larger mass than would a ground-state zero-charge baryon that includes just one top and two bottoms. Popular modeling suggests that, for ordinary matter, W bosons play key roles regarding the decay of generation-3 baryons, such as possible generation-3 baryons to which the previous sentence alludes, into ground-state generation-1 baryons, namely the neutron and the proton [111]. Per Table 2, MEA-isomer flavour-3 charged leptons would be less massive than ordinary-matter flavour-3 charged leptons. When generation-3 quark states are much populated, the stuff that associates with an MEA-isomer would convert more charged baryons to zero-charge baryons than would the stuff that associates with the ordinary-matter isomer. Eventually, regarding the stuff that associates with the MEA-isomer, interactions that entangle multiple MEA-isomer W bosons would result in the stuff that associates with the MEA-isomer having more counterparts to ordinary-matter-stuff neutrons and fewer counterparts to ordinary-matter-stuff protons than does the stuff that associates with the ordinary-matter isomer. The sum of the mass of an MEA-isomer counterpart to the ordinary-matter proton and the mass of an MEA-isomer flavour-1 charged lepton would exceed the mass of an MEA-isomer counterpart to the ordinary-matter neutron. Compared to ordinary-matter neutrons, MEA-isomer neutrons would scarcely decay.

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