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Gravitational and Dark-Matter Concepts that Can Help Explain Cosmic Data

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Abstract

We discuss gravitational concepts and candidate specifications for dark matter that, together, can help explain known ratios of dark-matter effects to ordinary-matter effects and can help explain eras in the rate of expansion of the universe. The ratios pertain to galaxies and galaxy evolution, galaxy clusters, and densities of the universe. The candidate specifications for dark matter reuse, with variations, a set of known elementary particles. Regarding galaxy evolution and the rate of expansion of the universe, we deploy multipole-expansion methods that combine Newtonian gravity, aspects of motions of sub-objects of gravitationally interacting objects, and Lorentz invariance. One outgrowth from our work suggests relationships among some physics constants. Another outgrowth from our work suggests a basis for a candidate specification for quantum gravity.

Keywords: dark matter; elementary particles; gravity; galaxy formation; rate of expansion of the universe; multipole expansions; particle properties; quantum gravity

1. Introduction

In this unit (Unit 1 of this paper), we provide context for and an overview of work that this paper discusses.

Physics includes the following activities.

- Collect data.
- Discuss data.
- Explain phenomena that associate with data.
- Predict data.

Physics includes the following four opportunities.

1. Explain data that associate with the two-element term dark-matter phenomena. (Notions of dark-matter effects date back at least as far as to the 1930s [1,2]).
2. Explain phenomena that associate with a universe that includes more than one galaxy. (One notion of such a universe dates back at least as far as to the discovery of galaxies other than the Milky Way galaxy during the 1920s [3–5]. At least as far back as the 1990s, physics associated the six-word term rate of expansion of the universe with notions of the moving apart from each other of large cosmological objects [6]).
3. Predict elementary particles. (One notion of elementary particles dates back at least as far as to the 1890s discovery of the electron [7]).
4. Discuss properties that physics ascribes to objects or to fields. (One notion regarding properties that pertain to objects features the possible equivalence between the property of inertial mass and the property of gravitational mass. That notion dates back at least as far as to the 1680s [8]).

Physics suggests so-called unsolved problems regarding each one of the four opportunities. Unit 2 of this paper discusses some such unsolved problems.

Our work proposes insight about and steps forward regarding each one of the four opportunities. (Unit 2 of this paper discusses details). For example, the following notions pertain.

1. We propose a class of candidate specifications for dark matter. (The class has bases in notions that dark-matter elementary particles can both be like ordinary-matter elementary particles and underlie stuff that approximately comports with cosmological notions of cold dark matter [9,10]). We discuss specific candidate specifications for dark matter.
2. We suggest that gravitational interactions between pairs of large objects (such as galaxy clusters) associate with changes in the rate of expansion of the universe.
3. We predict dark-matter elementary particles.
4. We discuss gravitational properties of objects. We discuss properties of the gravitational field. We suggest a specification for gravity. (The specification has bases in Newtonian gravity and in the following notions. Cosmological objects include sub-objects that move within the objects. Lorentz invariance pertains regarding perceptions of the sizes of the properties of objects. Newtonian dipole and quadrupole gravitational forces can associate with aspects, such as object-internal kinetic energies of sub-objects, other than spatial distributions of mass. Sub-object-kinetic-energy dipole components of the gravitational interaction associate with gravitational repulsion between objects). We discuss phenomena, including gravitational repulsion, that associate with the specification for gravity.

Units 2 and 3 of this paper propose that, across the four opportunities, our work provides insight about data and can help explain and predict data.

Unit 4.14 of this paper proposes, as an outgrowth from other work in this paper, relationships among some physics constants.

Unit **??** of this paper proposes, as an outgrowth from other work in this paper, a basis for a candidate specification for quantum gravity.

Table 1 discusses acronyms for some key features of our work.

Table 1. Acronyms for some key features of our work.

Acronym	Phrase	Association
IDM	Isomeric dark matter	A class of candidate specifications for dark matter
MULTING	Multi-tier Newtonian gravitation	A basis for describing components of gravitational interactions

Table 2 discusses some conventions that we use in this paper.

Table 2. Conventions that we use in this paper. Generally, assumptions that we posit associate with applications of widely familiar mathematics.

Phrasing	Association
Popular modeling	Aspects that published or posted articles discuss
We posit ...	Assumptions that we make
We suggest ...	Notions that stem from data or that stem from assumptions that we posit
Our work ...	Aspects that this paper develops and discusses
Future modeling	Aspects that future published or posted articles might discuss

Philosophy points to opportunities to discuss relationships between the roles of evidence and the roles of authority [11,12]. For our work, evidence includes data. For our work, extant authority includes assumptions and modeling that associate with popular modeling. We suggest that our suggested changes regarding authority are diverse and tame.

Table 3 summarizes evidence on which we base our work.

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Table 3. Evidence on which we base our work.

Evidence
Ratios of dark-matter effects to ordinary-matter effects
Eras in the rate of expansion of the universe
Associations between elementary-fermion mass states and elementary-fermion flavour (or generation) states
Values of properties (such as mass and charge) of elementary particles

Table 4 summarizes authority that our work suggests.

Table 4. Authority that our work suggests. Known elementary particles associate with ordinary matter.

Authority	Notes	Acronym
Dark-matter elementary particles	Like known elementary particles	IDM
Gravitation, including forces that push away objects	An extension to Newtonian gravity	MULTING
New gravitational properties of objects	Based on the motions of sub-objects	MULTING
Properties that associate with particles or fields	Like aspects of multipole expansions	MULTING
Relationships between physics constants	Based on data	-

2. Methods

In this unit, we do the following. We discuss context, including known data and reusable popular modeling, for our work. We discuss steps that we use to develop the work. We discuss how the steps lead to results that the work suggests. We indicate results that the work suggests, including results that associate with dark matter and results that associate with the rate of expansion of the universe.

Popular modeling has yet to settle on a preferred description for dark matter [13,14]. Popular modeling includes various candidate classes of specifications for dark matter and numerous candidate specifications for dark matter [15]. Some popular modeling suggests that assuming that dark matter features zero-charge fermion particles can help explain data [16]. Some popular modeling candidate specifications for dark matter base dark matter on yet-to-be-found elementary particles [13]. Some popular modeling candidate specifications for dark matter feature copies or near-copies of standard model elementary particles [17,18].

Generally, the following notions pertain regarding research related to dark matter [19–23]. Research focuses on data that popular modeling associates with cosmic microwave background radiation, large-scale structure, baryon acoustic oscillations, supernovae, galaxy rotation curves, gravitational lensing, redshift space distortions, and the Lyman-α forest. Parameters that people use in theories include the dark-matter density parameter, the equation of state parameter, particle properties such as masses and cross-sections, temperature (as in cold, warm, or hot), and parameters regarding interactions between dark matter and baryons or between dark matter and dark energy.

Some popular modeling seeks candidate dark-matter specifications that can help explain the observed $(\approx 5.4) : 1$ ratio of dark-matter density of the universe to ordinary-matter density of the universe [24–29].

Table 5 lists known ratios of dark matter to ordinary matter. (Unit 4.1 of this paper provides references regarding the ratios). The associations with galaxy formation scenarios comport with popular modeling notions of hierarchical structure formation [30].

Table 5. Known ratios of dark-matter stuff to ordinary-matter stuff. For ordinary matter, examples of stuff include unbound subatomic particles (such as electrons that do not associate with atoms or molecules and such as protons that do not associate with atoms), atoms, and stars. The associations with galaxy formation scenarios comport with popular modeling notions of hierarchical structure formation.

Aspect	Ratio	Association with galaxy formation scenarios
Amounts of stuff in some individual galaxies	$(0+):1$	Measures aspects of scenarios
Amounts of stuff in some individual galaxies	$1:(0+)$	Measures aspects of scenarios
Amounts of stuff in some individual spiral galaxies	$(\sim 4):1$	Measures aspects of scenarios
Amounts of stuff in many individual galaxies	$(5+):1$	Measures aspects of scenarios
Amounts of stuff in many individual galaxy clusters	$(5+):1$	Might provide context for scenarios
Densities of the universe	$(\approx 5.4):1$	Might provide context for scenarios

Popular modeling suggests mechanisms that can remove dark matter or ordinary matter from galaxies [31,32]. Popular modeling suggests that such mechanisms might lead to some $(0+):1$ galaxies and to some $1:(0+)$ galaxies. We are not aware of popular modeling that suggests mechanisms or posits principles that suffice to explain all the ratios that Table 5 lists.

We seek to provide a candidate class of dark-matter specifications that can help explain all the ratios that Table 5 lists.

Popular modeling suggests that much dark matter associates with the three-word term cold dark matter [33–35]. Some popular modeling suggests that dark-matter elementary particles are like (or are so-called mirrors of) ordinary-matter elementary particles [17,36–38].

We use the word isomeric and the acronym IDM, as in isomeric dark matter, to associate with our candidate class of dark-matter specifications. We use the word isomeric and the acronym IDM, to associate with each candidate specification that associates with the IDM candidate class of specifications for dark matter. We posit that each IDM candidate specification for dark matter associates with six near copies of the set of known elementary particles. When discussing an IDM candidate specification for dark matter, we use the word isomer to associate with each one of the six near copies of the set of known elementary particles. (We note, as an aside, the following notions regarding the word isomer. Unit **??** of this paper compares our use of the word isomer with other uses of the word isomer. We do not de-emphasize IDM candidate specifications for which we do not specify symmetries that might associate with the six isomers).

Table 6 lists aspects, regarding isomers, that we posit regarding each IDM candidate specification for dark matter that we suggest.

Table 6. Aspects, regarding isomers, that we posit regarding each IDM candidate specification for dark matter that we suggest. Regarding the three-word term much dark matter, our work does not necessarily suggest that nature does not include other types of dark matter. Popular modeling tends to associate the three word term cold dark matter with no electromagnetic interactions. The five-word term approximately with cold dark matter associates with a notion of interactions via magnetic fields, but not necessarily cosmologically significant interactions via electric fields, within stuff that features zero-charge objects (such as analogs to neutrons). The right column states the number of isomers that comport with the left column. Regarding the third row, the following sentences pertain. Some IDM candidate specifications for dark matter associate with the number five. Some IDM candidate specifications for dark matter associate with the number four.

Isomers ...	Number
That underlie ordinary matter	1
That underlie much dark matter	5
That underlie stuff that associates with dark matter and approximately with cold dark matter	5 or 4

Popular modeling suggests two observed multibillion-year eras regarding the rate of expansion of the universe [39–42]. The chronologically first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases. Data and popular modeling might provide hints that the second multibillion-year era might be ending [43,44] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

Generally, the following notions pertain regarding research related to the rate of expansion of the universe [35,45–48]. Research focuses on data that popular modeling associates with supernovae, cosmic microwave background radiation, baryon acoustic oscillations, galaxy clustering and large-scale structure, and gravitational lensing. Parameters that people use in theories include the Hubble constant, baryon density, cold dark matter density, dark energy density, equations of state for dark energy, and measures of the clumpiness of matter.

Table 7 summarizes aspects regarding two known eras in the rate of expansion of the universe and one possibly impending era in the rate of expansion of the universe. Popular modeling suggests that an inflationary epoch preceded the known era of decreasing rate of expansion of the universe [49]. Popular modeling suggests that the transition from the inflationary epoch to the known era of decreasing rate of expansion of the universe occurred less than one second after the so-called Big Bang and that estimates of the time of the transition are model-dependent [50–52].

Table 7. Aspects regarding two known eras in the rate of expansion of the universe and one possibly impending era in the rate of expansion of the universe.

Era	Approximate starting time
Decreasing rate of expansion	Less than one second after the Big Bang
Increasing rate of expansion	Approximately 7.5 to 9 billion years after the Big Bang
Possibly decreasing rate of expansion	Perhaps around or after 14 billion years after the Big Bang

Popular modeling suggests means to help explain the first two eras that Table 7 lists [53]. Popular modeling suggests that such means can depend on computing equations of state and that popular modeling lacks means of determining, from first principles, means to derive equations of state [54,55].

Popular modeling suggests that gravitation is key to the large-scale evolution of the universe [56]. Popular modeling suggests that gravitation is key to galaxy formation [57]. Popular modeling discusses various candidate theories of large-scale gravitation [58–60]. Popular modeling includes theories of gravity that include gravitational repulsion [18,61]. Regarding explaining repulsion via so-called dark energy, popular modeling discusses reasons to find alternatives to invoking the cosmological constant [62]. Popular modeling uses gravitational multipole expansions that feature spatial distributions of mass or energy [63–66].

Original Newtonian gravitation features a monopole term that associates with mutual attraction between two objects that model as pointlike [8].

We suggest extending Newtonian gravitation in a way such that the following notions pertain. (Unit 4.3 of this paper provides details). A dipole term associates with repulsion of one object that models as essentially pointlike away from another object that models as essentially pointlike. A quadrupole term associates with attraction of one object that models as essentially pointlike toward another object that models as essentially pointlike. The bases for the dipole term and the quadrupole term feature the notions that cosmological objects have sub-objects and that the sub-objects can have nonzero velocities within the objects. In popular modeling, Lorentz invariance describes aspects regarding values, perceived by one object, of properties of other objects that might move with respect to the one object [67]. Lorentz invariance plays a role in our extending Newtonian gravity.

Table 8 summarizes aspects regarding the gravitational multipole expansions that we suggest.

Table 8. Aspects regarding the gravitational multipole expansions that we suggest. These aspects pertain regarding modeling that can treat objects as spatially essentially or completely pointlike. Popular modeling notions of a dipole component of gravity treat an object as having non-negligible spatial spread. Depending on the physical circumstances, popular modeling notions of a dipole component of gravity can associate with push or with pull.

Gravitational component	Effects on an object-P of the gravitational field that associates with an object-A
Quadrupole	Pull toward object-A
Dipole	Push away from object-A
Monopole	Pull toward object-A

Unit 4.6 of this paper discusses aspects regarding so-called gravitationally large objects.

We suggest that mechanisms that Table 8 states provide useful insight regarding the eras that Table 7 lists. We posit or suggest that the following notions pertain. Regarding interactions between non-colliding neighboring large objects, the quadrupole component of gravity dominates other components around the starting time for the era of known decreasing rate of expansion of the universe. At the beginning of the era of known decreasing rate of expansion of the universe, the relevant large objects may have been protogalaxies. Later in the era of known decreasing rate of expansion of the universe, the notion of relevant large objects transited, first to galaxies, then to protoclusters, and then to galaxy clusters. Regarding interactions between non-colliding neighboring large objects (such as galaxy clusters), the dipole component of gravity dominates other components around the starting time for the era of known increasing rate of expansion of the universe. Throughout the era of known increasing rate of expansion of the universe, the relevant large objects may have been galaxy clusters.

The first four rows of Table 5 pertain regarding galaxy-formation scenarios. We suggest that the following notions pertain. Each isomer interacts via the quadrupole component of gravity only with itself. The previous sentence underlies the first two rows in Table 5. Each isomer interacts via the dipole component of gravity only with itself and one other isomer. The previous sentence underlies the third row in Table 5. Each isomer interacts via the monopole component of gravity with itself and with the other five isomers. The previous sentence underlies the fourth row in Table 5.

Table 9 summarizes aspects regarding the gravitational multipole expansions that we suggest.

Table 9. Reaches per instance for components of gravity. Each one of the six isomers associates with one instance of the dipole gravitational component.

Gravitational component	Reach per instance	Number of instances
Quadrupole	1 isomer	6
Dipole	2 isomers	3
Monopole	6 isomers	1

The following notions pertain regarding one IDM candidate specification for dark matter.

- We use the notation IDM-1 to associate with the IDM candidate specification for dark matter.
- We posit that counterpart elementary particles have, across the six isomers, identical masses.
- The following notions provide perspective.
  - Some popular modeling suggests that some dark-matter stuff might be like neutrons, except that the dark-matter neutron analogs would not decay (for example, into dark-matter analogs to protons, electrons, and antineutrinos) [38].
  - Popular modeling suggests notions of self-interacting dark matter [13,68].
  - Some popular modeling suggests that some observational results associate with self-interacting dark matter [69–72].
  - Some popular modeling points to possible benefits of considering that some dark matter is self-interacting dark matter [73–76] .
- Unit **??** of this paper discusses IDM-1.
- For each one of four dark-matter isomers, the following notions pertain. The rest energy of the flavour-1 charged lepton exceeds the difference between the rest energy of an ordinary-matter neutron and the rest energy of an ordinary-matter proton. Units **??** and **??** of this paper suggest that the neutron counterparts that associate with stuff that associates with the isomer are prevalent and do not decay.
- Those four dark-matter isomers approximately comport with popular modeling notions of cold dark matter. The other dark-matter isomer does not necessarily comport with popular modeling notions of cold dark matter and might comport with popular modeling notions of self-interacting dark matter.
- IDM-1 comports with the integer four in the last row of Table 6.

We suggest that various means exist for generating, compared to IDM-1, other IDM candidate specifications for dark matter. The following are examples of such means.

- Relax the IDM-1 constraint that counterpart elementary particles have identical masses. (Some popular modeling suggests that dark matter might include elementary particles that both are similar to ordinary-matter elementary particles and have masses that differ from the masses of the counterpart ordinary-matter elementary particles [77]).
- Change three items, other than the OM item, in the lepton-flavours column of Table 11 (in Unit **??** of this paper) so that the lepton-flavours column includes all six permutations of the integers one, two, and three.
- Change items, other than the OM item, in one or both handedness columns of Table 11.
- Change items, other than the OM item, in the quark-generations column of Table 11.

We use the notation IDM-2 to associate with one IDM candidate specification for dark matter. Unit 4.11 of this paper discusses IDM-2. IDM-2 comports with the integer five in the last row of Table 6.

We suggest that choosing between IDM-1, IDM-2, and other IDM candidate specifications for dark matter might depend on data that science has yet to gather or on data that popular modeling has not yet analyzed sufficiently to help regarding making such a choice.

Unit 4.1 of this paper discusses known data that might help qualify or rule out (IDM or other) candidate specifications for dark matter.

Unit 4.13 of this paper discusses somewhat known data and possible future data that might help qualify or rule out (IDM or other) candidate specifications for dark matter.

Units 4.8 and **??** of this paper discuss symmetries or approximate symmetries that might associate, for some IDM candidate specifications, with the six isomers.

Table 10 summarizes aspects regarding non-gravitational interactions.

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Table 10. Reaches per instance for components of non-gravitational interactions. The word suggested associates with the notion that much data associates with a reach per instance of one for ordinary-matter interactions. The acronym TBD (as in the three-word phrase to be determined) associates with the notion that there might not be enough data to enable suggesting the reach per instance and the number of instances. Regarding TBD items, our notions of six isomeric sets of elementary particles might associate with reaches per instance of one isomer and numbers of instances of six.

Interaction	Reach per instance	Number of instances	Note
Electromagnetic interaction	1 isomer	6	Suggested
Weak interaction	TBD	TBD	TBD
Strong interaction	TBD	TBD	TBD

We suggest that our work posits principles and suggests both mechanisms and means that can help explain and predict cosmic phenomena and cosmic data. For example, the following sentences pertain. The notions of IDM, isomers, and reaches per instance associate with principles. Dipole gravitational push and reaches per instance associate with mechanisms. Modeling based on the principles and mechanisms associates with means.

Unit 4.20 of this paper suggests opportunities for future research. Each one of the following words or phrases associates with opportunities for at least one of observational research, experimental research, and theoretical research. Isomers. Dipole push components of gravitational interactions. Dipole gravitational properties of objects. Quadrupole pull components of gravitational interactions. Isomeric dark matter.

3. Results

In this unit, we summarize results, including results that associate with dark matter and results that associate with the rate of expansion of the universe, that Unit 2 of this paper develops.

Our work suggests insight regarding observed ratios (including the ratios that Table 5 lists) of dark-matter effects to ordinary-matter effects.

Our work suggests insight regarding at least two inflection times (including the inflection times that Table 7 lists) regarding the rate of expansion of the universe.

Our work suggests insight (including insight to which Table 9 alludes) about gravity.

Our work posits a new class of specifications for dark-matter elementary particles and suggests some details (including details that Units **??** and 4.11 of this paper discuss) about a few candidate specifications.

Unit 4 of this paper discusses aspects of how future data and future work can help hone in on details that Unit 2 of this paper does not discuss.

4. Discussion

In this unit, we provide details that associate with aspects that Units 2 and 3 of this paper discuss or anticipate.

4.1. Ratios of Dark-Matter Effects to Ordinary-Matter Effects

In this unit, we do the following. We discuss ratios of dark-matter effects to ordinary-matter effects. We provide references regarding the ratios. We summarize our suggested explanations for the ratios.

Each one of the following items notes an approximate ratio of dark-matter stuff to ordinary-matter stuff and suggests an explanation for how nature came to comport with the ratio. For ratios that pertain to galaxies, the items discuss aspects regarding the masses of relevant types of galaxies. Across the galaxy-related items, the galaxy-formation scenarios that we suggest seem to be consistent with each other and with respect to relevant mass ranges. In general, the masses of galaxies range from about 10^6 solar masses to about 3×10^{13} solar masses [78,79].

- $(\approx 0) : 1$ – Amounts of stuff in observed or optically observable solar systems [80,81]. We suggest that gravitational quadrupole attraction of ordinary-matter stuff leads to known solar systems that contain essentially only ordinary-matter stuff.
- $1 : (\approx 0)$ – Amounts of stuff in solar-system-like objects that contain essentially only dark matter. (As far as we are aware, there are no reports of solar-system-like objects that contain essentially only dark matter). We suggest that gravitational quadrupole attraction of dark-matter stuff might lead to solar-system-like objects that contain essentially only dark matter.
- $(0+) : 1$ – Amounts of stuff in some individual galaxies [82–89]. We suggest that gravitational quadrupole attraction of ordinary-matter stuff (such as solar-system stuff) leads to early galaxies that contain essentially only ordinary-matter stuff. We suggest that some of those early ordinary-matter galaxies survive intact today and associate with some known ordinary-matter galaxies. Today's ordinary-matter galaxies have masses that range from about 2×10^8 solar masses to about 6×10^8 solar masses [90] and might range from about 10^8 solar masses to about 10^9 solar masses [91].
- $1 : (0+)$ – Amounts of stuff in some individual galaxies [72,92–102]. We suggest that gravitational quadrupole attraction of dark-matter stuff leads to early galaxies that contain essentially only dark-matter stuff. We suggest that some of those early dark-matter galaxies survive intact today and associate with some known dark-matter galaxies. Today's dark-matter galaxies have masses that range from about 10^7 solar masses to about 10^8 solar masses [103].
- $(\sim 4) : 1$ – Amounts of stuff in some individual spiral galaxies [104,105]. We suggest that some of today's galaxies that contain four times as much dark-matter stuff as ordinary-matter stuff resulted from ordinary-matter galaxies that first repelled (via gravitational dipole repulsion) from their neighborhoods some ordinary-matter stuff and essentially all of one-fifth of dark-matter stuff and second accreted (via gravitational monopole attraction) nearby ordinary-matter stuff and nearby dark-matter stuff. Spiral galaxies have masses that range from about 10^8 solar masses to about 10^{12} solar masses [106,107].
- $(5+) : 1$ – Amounts of stuff in many individual galaxies [92,108]. We suggest that gravitational monopole attraction leads to known recent galaxies that contain about (actually, generally somewhat more than) five times as much dark-matter stuff as ordinary-matter stuff. We suggest that many of today's galaxies that contain five times as much dark-matter stuff as ordinary-matter stuff resulted from mergers of smaller galaxies. Today's most massive galaxies tend to contain five (actually, generally five-plus) times as much dark-matter stuff as ordinary-matter stuff.
- $(5+) : 1$ – Amounts of stuff in many individual galaxy clusters [108–112]. We suggest that gravitational attraction between galaxies leads to known recent galaxy clusters that contain about (actually, generally somewhat more than) five times as much dark-matter stuff as ordinary-matter stuff. Today's galaxy clusters tend to contain five (actually, generally five-plus) times as much dark-matter stuff as ordinary-matter stuff.
- $(5+) : 1$ – Densities of the universe [113]. We suggest that the $5 : 1$ ratio of dark-matter isomers to ordinary-matter isomers underlies the presence in the universe of approximately five times as much dark-matter stuff as ordinary-matter stuff. (Unit **??** of this paper discusses notions about the plus in the $(5+) : 1$ ratio of densities of the universe).

4.2. Uses Of The Word Isomer

In this unit, we compare our use of the word isomer with other uses of the word isomer.

Our use of the word isomer can associate with notions of symmetries, including chirality (or mirror image) symmetry, and with notions of approximate symmetries. Chemistry uses the word isomer regarding the notion that molecules can be mirror images of each other.

Our use of the word isomer does not directly associate with some other uses of the word isomer, for example regarding alternative geometric arrangements (other than uses related to chiral symmetry) of atoms within molecules or regarding long-lived excited states of atomic nuclei.

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We are not aware of attempts (to parallel, in elementary-particle physics, nuclear-physics notions of isomers and thereby ..). to use the word isomer in conjunction with the three flavour states of ordinary-matter charged leptons or the three generation-states of similarly-charged ordinary-matter quarks. Our work might be able to embrace such uses of the word isomer; however, our work does not presently use the word isomer for such purposes.

4.3. MULTING: Multipole Gravity for Objects that Model as Having Sub-Objects

In this unit, we develop our notions of gravitational monopole pull, gravitational dipole push, and gravitational quadrupole pull.

We discuss interactions between a nonzero-mass object-A and a nonzero-mass object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as gravitational fields and electromagnetic fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

We review aspects of seventeenth century Newtonian gravity.

Equation (1), Equation (2), and Equation (3) describe aspects regarding the motion of object-P [8]. G is the gravitational constant. m_{oA} is the mass of object-A. Mass is a scalar property. $m_{oA} > 0$ pertains. m_{oP} is the mass of object-P. $m_{oP} > 0$ pertains. r is the 3-vector distance that object-P is away from object-A. $\bigtriangledown$ is the gradient operator. $\bigtriangledown$ produces a 3-vector field from a scalar field. F_{oP} is the force that object-P feels. (We note, as an aside, that object-P might sense effects of that force via an accelerometer that associates with object-P). In equations such as Equation (1), V is a scalar field. Popular modeling associates with V the word potential. In equations such as Equation (2), r^{n_V} denotes the n_V-th power of the magnitude of the 3-vector r. For $n_V = 1$, the force attracts (or pulls) object-P toward object-A.

$$G m_{oA} m_{oP} (-\nabla V) = F_{oP} \tag{1}$$

$$V = -1/r^{n_V} \tag{2}$$

$$n_V = 1 \tag{3}$$

Equation (4) and Equation (5) describe aspects of the magnitude of the force, that object-P feels, as a function of $|r|$. We associate the four-word term monopole component of force with Equation (5). For $n_F = 2$, the force attracts (or pulls) object-P toward object-A.

$$|F_{oP}| \propto 1/|r|^{n_F} \tag{4}$$

$$n_F = 2 \tag{5}$$

The left-hand side of Equation (1) does not involve a velocity of object-P, a velocity of object-A, a velocity of object-A relative to object-P, or a velocity of object-P relative to object-A. With respect to recent popular modeling, one can consider that m_{oA} associates with the rest mass of object-A. With respect to recent popular modeling, one can consider that m_{oP} associates with the rest mass of object-P.

The left-hand side of Equation (1) exhibits a notion that, regarding object-A, rest masses add. For example, if object-A consists of two co-located sub-objects, the rest mass of object-A is the sum of the rest masses of the two sub-objects.

Popular modeling that includes Lorentz invariance can focus on energies instead of masses.

Equation (6) recasts Equation (1) in terms of the rest energy $E_{0,oA}$ of object-A, the rest energy $E_{0,oP}$ of object-P, and c, the speed of light.

$$(G/c^4)E_{0,oA}E_{0,oP}(-\nabla V) = F_{oP} \tag{6}$$

Regarding object-A, Lorentz invariance implies that object-P senses an object-A energy E_{oA} that is $\gamma = (1-(|v|/c)^2)^{-1/2}$ times the rest energy $E_{0,oA}$ of object-A [67]. Here, v denotes the velocity (relative to object-P) of object-A. v is a 3-vector. Popular modeling associates $(\gamma - 1)E_{0,oA}$ with $P_{oA}c$, in which P_{oA} denotes the magnitude of the momentum of object-A. Equation (7) and Equation (8) pertain.

$$(E_{oA})^2 = (m_{oA}c^2)^2 + (P_{oA}c)^2 \tag{7}$$

$$E_{0,oA} = m_{oA}c^2 \tag{8}$$

For $|v|/c \ll 1$, Equation (9) associates with the energy beyond the energy that associates with the rest mass of object-A that object-P associates with the object-A. For $|v|/c \ll 1$, $(1/2)m_{oA}|v|^2$ associates with popular modeling notions of object-A kinetic energy.

$$E_{0,oA}(\gamma - 1) = E_{0,oA}((1-(|v|/c)^2)^{-1/2} - 1) \approx (1/2)E_{0,oA}(|v|/c)^2 = (1/2)m_{oA}|v|^2 \tag{9}$$

We consider a case in which the only energies that object-P can infer about object-A associate with E_{oA} and P_{oA}. The following notions pertain. $n_F = 2$ pertains. In effect, object-P infers the following. E_{oA} associates with gravitational pull (on object-P) that associates with object-A. $P_{oA}c$ associates with $|v| > 0$ and with gravitational push (on object-P) that associates with object-A. The arithmetic combination of the gravitational pull that associates with E_{oA} and the gravitational push that associates with $P_{oA}c$ associates with gravitational pull that associates with $E_{0,oA}$ (as in, for example, Equation (6)). In effect, the gravitational push (on object-P) that associates with the nonzero motion of object-A associates with diluting the gravitational pull (on object-P) that associates with E_{oA}.

So far, we have de-emphasized the notion that, at least for cosmological objects-A, object-A can have nonzero-mass sub-objects that can move with respect to the center of energy (or, the center of mass) of object-A. We now consider motions of nonzero-mass sub-objects of object-A.

We use the symbol $E_{00,oA}$ to refer to the ground-state energy, in the rest frame that associates with object-A, of object-A. Here, the two-word term ground state refers to a lowest-energy state with respect to the first-tier nonzero-mass sub-objects of object-A. Here, the two-word term first tier excludes nonzero-mass sub-sub-objects (as in sub-objects of sub-objects).

We consider a case for which $P_{oA} = 0$ and object-A, as perceived by object-P, has nonzero spin (as in nonzero object-internal angular momentum) S_{oA}. S_{oA} is a 3-vector. We assume that S_{oA} associates with the motions of nonzero-mass sub-objects of object-A. $P_{oA} = 0$ implies that $v = 0$. The following notions pertain.

- The notions of dipole and $n_F = 3$ pertain regarding gravitational effects, on object-P, of the nonzero spin S_{oA} of object-A. (We note, as an aside, that one might consider a notion that spin is to gravitation as magnetic moment is to electromagnetism. Unit **??** of this paper discusses aspects of magnetic moments that associate with moving nonzero-charge sub-objects of an object-A).
- Based on the nonzero spin that associates with object-A, object-P perceives that E_{oA} exceeds $E_{00,oA}$. (We note, as an aside, that one might consider a notion that it takes energy to spin up, from zero spin, object-A).
- Paralleling the previous case (for which $E_{oA} - E_{0,oA}$ associates with $n_F = 2$ and with $|v| > 0$), for this $n_F = 3$ case, the gravitational effects (on object-P) that associate with $E_{oA} - E_{00,oA}$ associate with gravitational push.
- We consider 3-vectors r that share one direction (that is, the 3-vectors are parallel to each other). For a sufficiently large magnitude of the separation $|r|$ between object-A and object-P, the gravitational pull that associates with $E_{00,oA}$ and with $n_F = 2$ dominates the total relevant gravitational push on

object-P. Keeping the direction of r the same and decreasing the size of $|r|$ results mathematically in a range of $|r|$ for which the relevant gravitational push (on object-P) can dominate the gravitational pull (on object-P) that associates with $E_{00,oA}$ and with $n_F = 2$. Within the range of $|r|$ for which push dominates mathematically, if object-A and object-P are not too close together (that is, for example, if object-A and object-P are not colliding), the net gravitational effect of object-A on object-P associates with gravitational push of object-P away from object-A.

- The distance $|r|$ below which push can dominate pull varies depending on the angle between r and the axis that associates with the spin of object-A. Unit **??** of this paper discusses similar results for electromagnetic phenomena and the effects of magnetic moments (of objects-A) for which the motions of nonzero-charge sub-objects of objects-A generate the magnetic moments of objects-A. (We note, as an aside, that we do not try to explore similarities and differences between gravitoelectromagnetism [114–116] and our work regarding gravitational multipole expansions). We posit that similarities between such electromagnetic phenomena and the gravitational phenomena that we are discussing suffice to make the case that the gravitational dipole push at a distance $|r|$ from object-A is stronger for circumstances in which the direction of r is similar to the direction of the axis that associates with the spin of object-A than for circumstances in which the direction of r is closer to perpendicular to the axis that associates with the spin of object-A. In general, the following two sentences pertain. For a positive value for $|r|$, the relevant gravitational push effects on object-P decrease with increases (within the range of zero radians to $\pi/2$ radians) in the angle between r and the axis that associates with the spin of object-A. For a positive value for $|r|$, the relevant gravitational push effects on object-P increase with increases (within the range of $\pi/2$ radians to π radians) in the angle between r and the axis that associates with the spin of object-A.

Including and beyond the case we just discussed, the following notions pertain for a case in which the only energies that object-P can infer about object-P associate with E_{oA}, P_{oA}, and the velocities (within object-A, relative to the center of mass of object-A) of nonzero-mass sub-objects of object-A. The velocities of the nonzero-mass sub-objects associate with gravitational $n_F = 3$ dilution of the gravitational pull effects that associate with E_{oA}. Some energy related to the velocities of nonzero-mass sub-objects might associate with object-A exhibiting nonzero spin (or object-internal angular momentum). Some energy related to the velocities of nonzero-mass sub-objects might associate with object-A exhibiting nonzero temperature. The additional energy that is related to the velocities of nonzero-mass sub-objects does not vary based on the distance between object-A and object-P.

We turn our attention to the notion of possible quadrupole gravitational effects.

We consider a thought experiment.

The left-hand side of Equation (1) is invariant (except regarding an overall change of sign) with respect to exchanging m_{oA} and m_{oP}. We suggest that modeling object-A as having no moving nonzero-mass sub-objects and object-P as having some moving nonzero-mass sub-objects associates with notions of (at least some) gravitational dipole push.

We suggest that modeling each one of object-A and object-P as having some moving nonzero-mass sub-objects associates with over-counting velocity-related aspects that associate with gravitational dipole push.

We suggest that modeling each one of object-A and object-P as having some moving nonzero-mass sub-objects associates with gravitational quadrupole (as in $n_F = 4$) pull.

We posit that the thought experiment associates with the following notions. Adding, to modeling that associates with gravitational 2^{n_F-2}-pole, a new type of motion yields modeling that associates with gravitational $2^{(n_F-2)+1}$-pole and with the dilution of effects that associate with gravitational 2^{n_F-2}-pole effects. For $n_F \geq 2$, even-integer values of n_F associate with gravitational pull. For $n_F \geq 3$, odd-integer values of n_F associate with gravitational push.

We move forward from the thought experiment and other notions that we discuss above.

We suggest that modeling can include a recursive (or hierarchical) notion that nonzero-mass objects can model as having nonzero-mass sub-objects.

We associate the acronym MULTING (with MULTING being an acronym for the three-element phrase multi-tier Newtonian gravitation) with the following notions. (We note, as an aside, that, for the producing of an output audio signal, the word multing can refer to the partitioning, before further processing and then the final production of the output signal, of an input audio signal into component audio signals. Such a partitioning of signals might have parallels to notions of decomposing gravitational fields into components that associate with specific values of n_F).

- The only relevant gravitational property of object-P is the object-P passive property of rest energy (or, equivalently, rest mass). Here, the rest energy of object-P does not necessarily associate with a ground-state energy of object-P.
- Object-A monopole gravity associates with the rest energy of object-A.
- Object-A monopole associates with $n_F = 2$ and 2^{n_F-2}-pole.
- Effects of object-A monopole gravity associate with gravitational pull of object-P toward object-A. (The notion of pull pertains, independently of the velocity of object-A relative to object-P. The magnitude, as perceived by object-P, of the pull varies, based on Lorentz invariance, with the velocity of object-A relative to object-P).
- For an n_F that exceeds two, the step from a value of one less than n_F to the value of n_F can associate with the following.
  - Adding one sub- to the relevant type of sub-objects of object-A and, thereby, adding one so-called tier of sub-objects.
  - If n_F is odd, changing from associating with gravitational pull to associating with gravitational push.
  - If n_F is even, changing from associating with gravitational push to associating with gravitational pull.

We turn our attention to some features of the discussion above.

Regarding each one of object-A and object-P, the discussion above features rest energies of nonzero-mass sub-objects and motion-related (or, generally, kinetic) energies of nonzero-mass sub-objects but not potential energies that might affect the motions of nonzero-mass sub-objects within the object.

Regarding each one of object-A and object-P, the discussion above tends to associate with modeling based on approximating spatial distributions of stuff as being spatially pointlike. Popular modeling features notions of multipole expansions that feature stationary, non-changing, non-pointlike distributions of mass. Unit 4.5 of this paper discusses examples of popular modeling multipole expansions for spatial distributions of mass. We suggest that, with respect to our framework, popular modeling that features multipole expansions regarding spatial distributions of mass associates with $n_F = 2$. We suggest that, for circumstances in which gravitational $n_F > 2$ aspects dominate gravitational $n_F = 2$ aspects, modeling can consider that popular modeling multipole expansions that associate with spatial distributions of mass for object-A are not necessarily significantly relevant.

Discussion above focuses on notions of pull and push. Discussion above de-emphasizes notions of torque.

Discussion above focuses on gravitational interactions that do not necessarily associate with the following circumstances. Collisions between objects. Mergers of objects. Splitting of objects into objects that associate with lesser sums of rest masses. Transfers of nonzero-mass stuff between objects. Changes of internal states of objects.

4.4. *Popular Modeling that Pertains for Electromagnetic Interactions between Two Charged Objects of which One Object has a Nonzero Magnetic Moment that Associates Solely with the Motions of Charged Sub-Objects*

In this unit, we discuss the notion that, for electromagnetic pull or push effects, on one nonzero-charge object, that associate with motions of charges within a second object, the magnitude of the effects depends on the angle between the vector that associates with the separation of the two objects and the vector that associates with the magnetic moment of the second object.

Equation (10) is an aspect of popular modeling for two-object electromagnetism [117,118]. ε_0 denotes the vacuum electric permittivity. q_{oA} is the charge of object-A. Charge is a scalar property. q_{oP} is the charge of object-P. Equation (2) and Equation 3 pertain.

$$-(1/(4\pi\varepsilon_0))q_{oA}q_{oP}(-\nabla(V)) = F_{oP} \tag{10}$$

Equation (10) has similarities to Equation (1).

We consider the following thought experiment.

We assume that a nonzero-charge object-A does not move relative to the position of a nonzero-charge object-P.

We consider a sub-object (of object-A) that has a rest charge q_0 and that moves (relative to each one of object-A and object-P) with a 3-vector velocity v. We define γ^* by $\gamma^* \equiv 1 + (\gamma - 1)$, in which $\gamma = (1 - (|v|/c)^2)^{-1/2}$ is the Lorentz factor. The first term (as in the first 1) in γ^* associates with $v = 0$. The second term (as in $\gamma - 1$) in γ^* associates with $|v| > 0$. Popular modeling associates the scalar potential (perceived by object-P) with a constant multiplied by $q_0\gamma^*/|r|$, in which r denotes a 3-vector distance from the sub-object. Popular modeling associates the vector potential (perceived by object-P) with a constant multiplied by $q_0\gamma^*(v/c)/|r|$. Based on aspects that are compatible with popular modeling, we replace the motion-related portion ($q_0(\gamma - 1)/|r|$) of the scalar potential with a vector-potential term that is proportional to $q_0 \int^t (\gamma - 1)v/|r|^2 dt$, in which t denotes the temporal coordinate and $\int^t dt$ associates with an indefinite integral with respect to t. (The relevant popular modeling equation is $E = -\bigtriangledown \phi - \partial A/\partial t$, in which E is the 3-vector electric field, $\bigtriangledown$ is the spatial-gradient operator, ϕ is the scalar electromagnetic scalar potential, A is the 3-vector electromagnetic vector potential, and t is the temporal coordinate [118]. Our work, in effect, replaces the motion-related, as perceived by object-P, portion of ϕ with a motion-related, as perceived by object-P, component of A that does not contribute to the magnetic field perceived by object-P. For $|v|/c \ll 1$ and the substitution of t for $\int^t dt$, $q_0(\gamma - 1)vt/|r|^2 \approx q_0(v|v|/2)(t/(c|r|^2))$. The motion-related portion of ϕ would have associated with a push or pull component of force on object-P. Thus, the replacement motion-related component of A should associate with a push or pull component of force on object-P. A vector representation that comports with $q_0(v|v|/2)(t/(c|r|^2))$ can comport with a notion of $v \times (v \times r)$, which is parallel or antiparallel to r. $\times$ denotes the cross product).

We consider cases for which, within object-A, all the moving charge associates with one sign (either positive or negative) of charge and for which moving charged sub-objects orbit the center of mass with the same angular velocity of charge.

For one case, the axis that associates with the magnetic moment of object-A is parallel or antiparallel to r. In this case, each $v \times (v \times r)$ associates with an effect for which the magnitude associates with $|v|^2$.

For another case, the axis that associates with the magnetic moment of object-A is perpendicular to r. In this case, each $v \times (v \times r)$ associates with an effect for which the magnitude associates with $(|v|)(|v|(1 - (v/|v|) \cdot (r/|r|))$, in which the symbol $\cdot$ denotes the vector dot product. For each sub-object, $0 \leq (|v|)(|v|(1 - (v/|v|) \cdot (r/|r|)) \leq |v|^2$. For each one of some sub-objects, $(|v|)(|v|(1 - (v/|v|) \cdot (r/|r|)) < |v|^2$.

For a positive value for $|r|$ and these two cases and similar cases, the relevant effects on object-P decrease with increases (within the range of zero to $\pi/2$ radians) in the angle between r and the axis that associates with the magnetic moment of object-A.

4.5. Popular Modeling Gravitational Multipole Expansions that Pertain to Spatial Distributions of Mass

In this unit, we discuss the notion that, in popular modeling, dipole gravitational contributions can, depending on circumstances, dilute monopole gravitational pull or add to monopole gravitational pull.

Popular modeling regarding gravitation points to situations in which dipole contributions dilute monopole contributions and to situations in which dipole contributions augment monopole contributions.

For example, consider an object-A that models as being two separated, equal-mass pointlike sub-objects. From the perspective of another object, object-P, that lies along a line that runs through the center-of-mass of object-A and is perpendicular to the line that runs through the two sub-objects, each sub-object is farther away than is the center-of-mass of object-A. Object-P senses less gravitational pull than object-P would sense if the two sub-objects of object-A existed at the center-of-mass point of object-A.

However, if object-P lies along a line that runs through the two sub-objects of object-A and is farther away from the center-of-mass of object-A than is each sub-object of object-A, object-P senses more gravitational pull than object-P would sense if the two sub-objects of object-A existed at the center-of-mass point of object-A.

From such notions, one might conclude that whether there is net dipole dilution of gravitational forces or net dipole augmentation of gravitational forces depends (for two objects) on details and (across several two-object interactions) on statistics.

4.6. Gravitationally Large Objects

In this unit, we propose objects that might be acceptable objects for exploring the extent to which gravitational interactions between large objects affect the rate of expansion of the universe.

Generally, the following statements associate with today's galaxy clusters [119,120]. The shapes are somewhat spherical or, more precisely, ellipsoidal. Diameters of 1 to 3 Mpc (as in megaparsecs, with one megaparsec being about 3.3 million light years) pertain. Lengths of around 20 to 90 Mpc associate with typical separations between neighboring galaxy clusters.

Generally, the following statements associate with today's filaments [121,122]. The shapes are threadlike or somewhat cylindrical. Lengths of tens to 150 Mpc pertain.

Popular modeling suggests that many galaxy clusters are located at places where two or more filaments overlap [123,124].

Because filaments overlap, we de-emphasize trying to associate individual filaments with notions of distinct objects.

We are not aware of popular modeling that has proposed and cataloged enough individual objects that would be larger than filaments to allow meaningfully exploring the extent to which gravitational interactions between such individual objects might impact the rate of expansion of the universe.

We suggest that, for times during which galaxy clusters exist, one can consider that galaxy clusters are relevant large objects for exploring the extent to which gravitational interactions between large objects affect the rate of expansion of the universe.

Popular modeling suggests that today's galaxy clusters evolved from earlier protoclusters [125–128]. Popular modeling suggests that some protoclusters started to form no later than one billion years after the Big Bang. Popular modeling suggests that transitions from protoclusters to galaxy clusters started about two-thirds of one billion years after the Big Bang to two billion years after the Big Bang [125,129,130]. Popular modeling suggests that transitions from protoclusters to galaxy clusters ended about three billion years after the Big Bang to seven billion years after the Big Bang [125,128].

Popular modeling suggests that some galaxies evolved from earlier protogalaxies [131–133]. Popular modeling suggests that some protogalaxies started to form no later than 200 million years after the Big Bang to 500 million years after the Big Bang.

We suggest that, to the extent that gravitational interactions between large objects can provide insight regarding the rate of expansion of the universe, the following notion pertains. Starting from some time after the Big Bang, the notion of most-appropriate large objects transits from protogalaxies to galaxies to protoclusters to galaxy clusters.

4.7. The IDM-1 Member Of The IDM Class Of Candidate Specifications For Dark Matter

In this unit, we define and discuss the IDM-1 member of the IDM class of candidate specifications for dark matter.

We use the acronym SESI (as in significantly-electromagnetically-self-interactive) to describe aspects of the stuff that associates with the ordinary-matter isomer of elementary particles.

We use the acronym MESI (as in marginally-electromagnetically-self-interactive) to describe aspects of the stuff that associates with four dark-matter isomers of elementary particles.

Table 11 discusses a numbering scheme for the posited IDM-1 six isomers, specifications for the one OM (as in ordinary-matter) isomer and the five DM (as in dark-matter) isomers, and aspects of the stuff that associates with each isomer. (Unit 4.8 of this paper discusses a possible doublet symmetry or doublet approximate symmetry that might associate with handedness. Unit **??** of this paper discusses a possible triplet approximate symmetry that might associate with changes, across isomers, between the orderings of quark generations and lepton flavours. Unit **??** of this paper suggests means by which the stuff that associates with each one of the four MESI isomers evolves to feature neutron-like analogs and proposes that the neutron-like analogs do not decay significantly into proton-like analogs and electron-like analogs).

Table 11. A numbering scheme for the posited IDM-1 six isomers, specifications for the one OM (as in ordinary-matter) isomer and the five DM (as in dark-matter) isomers, and aspects of the stuff that associates with each isomer. The symbol $l_{is.pr}$ denotes the isomer-pair number. The symbol l_{is} denotes the isomer number. The masses of counterpart elementary particles are, across the sets of elementary particles, the same. Handedness associates with the handedness that popular modeling would associate with the elementary particles that exhibit handedness. For each row, the quark generations column assigns the three generation numbers in order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the generation. (The following pertain for the ordinary-matter isomer. Generation-1 associates with the up quark and the down quark. Generation-2 associates with the charm quark and the strange quark. Generation-3 associates with the top quark and the bottom quark). For each row, the lepton flavours column assigns the three flavor numbers in order of increasing mass for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau). The notion that, for four isomers, the lepton-flavours order does not match the quark-generations order associates with our notions of a possible symmetry or approximate symmetry regarding charged-lepton flavours and charge-lepton masses. The stuff column identifies stuff made from the isomer that associates with the table row as OM, as in ordinary matter, or DM, as in dark matter. The acronym SESI abbreviates our term significantly-electromagnetically-self-interactive. The stuff that associates with DM (SESI) interacts electromagnetically with itself on a par with OM stuff interacting electromagnetically with OM stuff. The acronym MESI abbreviates our term marginally-electromagnetically-self-interactive. The stuff that associates with MESI interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-net-charge objects that are analogs to ordinary-matter neutrons or that are comprised of analogs to ordinary-matter neutrons.

$l_{is.pr}$	l_{is}	Quark handedness	Quark generations	Lepton flavours	Lepton handedness	Stuff
0	0	Left	1, 2, 3	1, 2, 3	Left	OM (SESI)
0	3	Right	1, 2, 3	1, 2, 3	Right	DM (SESI)
1	1	Left	1, 2, 3	3, 1, 2	Left	DM (MESI)
1	4	Right	1, 2, 3	3, 1, 2	Right	DM (MESI)
2	2	Left	1, 2, 3	2, 3, 1	Left	DM (MESI)
2	5	Right	1, 2, 3	2, 3, 1	Right	DM (MESI)

The stuff that associates with the SESI dark-matter isomer comports with the popular modeling acronym SIDM (as in self-interacting dark matter). The stuff that associates with each MESI dark-matter isomer might approximately comport with the popular modeling acronym CDM (as in cold dark matter).

IDM-1 comports with the integer four in the last row of Table 6.

4.8. A Possible Doublet Symmetry or Doublet Approximate Symmetry

In this unit, we posit the existence of a doublet symmetry or doublet approximate symmetry that associates with data about ordinary-matter fermion elementary particles.

We suggest that each one of the following notions about ordinary matter is not incompatible with the notion of a doublet symmetry or doublet approximate symmetry that would associate with isomers and would differentiate some isomers from other isomers.

- Popular modeling associates the one-element term left-handed with the set of known elementary particles [134,135]. For this notion regarding a doublet, some dark-matter isomers might associate with the three-element term right-handed elementary particles.
- Popular modeling discusses matter-antimatter asymmetry (which is also known as baryon asymmetry) [136]. For this notion regarding a doublet, one dark-matter isomer might underlie stuff that enables considering, in an adequately broad context, notions of matter-antimatter symmetry.

We posit that the symmetry or approximate symmetry associates with, for some dark-matter isomers, associations between fermion elementary particles and handedness that differ from the associations between fermion elementary particles and handedness that associate with the ordinary-matter isomer.

We do not try to fully characterize the possible symmetry or approximate symmetry.

4.9. A Possible Triplet Approximate Symmetry

In this unit, we posit the existence of a triplet approximate symmetry that associates with data about ordinary-matter fermion elementary particles.

We suggest that each one of the following notions about ordinary matter is not incompatible with the notion of a triplet symmetry or triplet approximate symmetry that would associate with isomers and would differentiate some isomers from other isomers.

- Popular modeling discusses neutrino oscillations [137,138]. Popular modeling suggests that the three neutrino flavour-eigenstates do not fully align with the three neutrino mass-eigenstates [139–141]. For this notion regarding a triplet, we suggest that the mismatches associate with a triplet approximate symmetry.
- Equations (11), (12), (13), and (14) pertain regarding the masses of the three charged leptons. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau. Similar equations (with $k = 0, +1,$ and $+2$, σ_k as per Equation (12), and δ as per Equation (13)) pertain regarding the geometric-mean masses for the three quark generations (Table 3.9.10 in [142] or Table 14 in [143]). We suggest that the notion that δ is somewhat small (compared to one) but not zero might associate with an approximate but not exact symmetry.

$$k = 0, +2, \text{ and } +3, \text{ for charged-lepton flavour-1, flavour-2, and flavor-3, respectively} \tag{11}$$

$$\sigma_k = 0, +1, -1, \text{ and } 0, \text{ for } k = 0, 1, 2, \text{ and } 3, \text{ respectively} \tag{12}$$

$$\delta \approx 0.03668 \tag{13}$$

$$m_k / m_e \approx (m_\tau / m_e)^{(1/3)(k + \sigma_k \delta)} \tag{14}$$

- The weak interaction associates with interactions in which charged leptons change flavour and with interactions in which quarks change generation [144]. We suggest that mechanisms that underlie the changes of flavour and the changes of generation associate with an approximate symmetry.

We posit that the approximate symmetry associates with, for some dark-matter isomers, associations between lepton flavours and quark generations that differ from the associations between lepton flavours and quark generations that associate with the ordinary-matter isomer.

We do not try to fully characterize the possible approximate symmetry.

4.10. The Evolution Of MESI (As In Marginally-Electromagnetically-Self-Interactive) dark-matter Stuff

In this unit, we discuss how the evolution of MESI (as in marginally-electromagnetically-self-interactive) stuff leads to MESI stuff that features stable counterparts to ordinary-matter-stuff neutrons.

For the stuff that associates with each one of the six isomers, a ground-state singly-charged baryon that includes exactly three generation-3 quarks would be more massive than the counterpart, within the same-isomer stuff, ground-state zero-charge baryon that includes exactly three generation-3 quarks. For example, for ordinary-matter-isomer stuff, a ground-state nonzero-charge baryon that includes just two tops and one bottom would have a larger mass than would a ground-state zero-charge baryon that includes just one top and two bottoms. Popular modeling suggests that, for ordinary matter, W bosons play key roles regarding the decay of generation-3 baryons, such as possible generation-3 baryons to which the previous sentence alludes, into ground-state generation-1 baryons, namely the neutron and the proton [145]. Per Table 11, MESI-isomer flavour-3 charged leptons would be less massive than ordinary-matter flavour-3 charged leptons. When generation-3 quark states are much populated, the stuff that associates with a MESI-isomer would convert more charged baryons to zero-charge baryons than would the stuff that associates with the ordinary-matter isomer. Eventually, regarding the stuff that associates with the MESI-isomer, interactions that entangle multiple MESI-isomer W bosons would result in the stuff that associates with the MESI-isomer having more counterparts to ordinary-matter-stuff neutrons and fewer counterparts to ordinary-matter-stuff protons than does the stuff that associates with the ordinary-matter isomer. The sum of the mass of a MESI-isomer counterpart to the ordinary-matter proton and the mass of a MESI-isomer flavour-1 charged lepton would exceed the mass of a MESI-isomer counterpart to the ordinary-matter neutron. Compared to ordinary-matter neutrons, MESI-isomer neutrons would scarcely decay.

Some popular modeling suggests that some dark matter might associate with stable counterparts to ordinary-matter neutrons [38].

4.11. The IDM-2 Member of the IDM Class of Candidate Specifications for Dark Matter

In this unit, we define and discuss the IDM-2 member of the IDM class of candidate specifications for dark matter.

IDM-2 is like IDM-1, except that the following notions pertain (for IDM-2) for the second row in Table 11.

- The mass of the flavour-1 charged lepton associates with $k = +1$ and $\sigma_k = +1$ in Equations (12) and (14).
- The mass of the flavour-1 charged lepton is sufficiently large that the charged lepton does not enable decays of neutron-like particles.
- MESI (and not SESI) pertains.

IDM-2 comports with the integer five in the last row of Table 6.

4.12. The Pluses In $(5+):1$ Ratios Of Dark-matter effects To Ordinary-Matter Effects

In this unit, we indicate that our work might help provide explanations for at least some portions of the pluses in some observed $(5+):1$ ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that $(5+):1$ ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, for many galaxy clusters, and for densities of the universe.

Our work does not necessarily rule out the possibility that some portions of (or the entireties of) the pluses in the $(5+):1$ ratios of dark-matter presence to ordinary-matter presence associate with axions, with other unfound elementary particles, or with other popular modeling suggestions regarding the nature of dark matter.

The following notions indicate that, for IDM-1 dark matter, our work might explain all or some of the amounts that underlie the pluses in the $(5+):1$ ratios of dark-matter presence to ordinary-matter

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presence. Our work proposes that the stuff that associates with either one of the two SESI isomers (one of which associates with ordinary matter and one of which associates with dark matter) associates with more electromagnetic energy than does the stuff that associates with any one of the four MESI isomers (each of which associates with dark matter). Table 9 provides an example of a reach-6 interaction component. Our work proposes that, at least early in the history of the universe, reach-6 interaction components or reach-3 interaction components might have enabled (via electromagnetic or other means) flows of electromagnetic energy between isomer-pairs. The net flows could have resulted in each MESI isomer having more stuff than each SESI isomer has. This notion of more stuff might explain all or some of the amounts that underlie the pluses in the $(5+) : 1$ ratios of dark-matter presence to ordinary-matter presence.

This paper does not discuss the extent to which similar notions (regarding $(5+) : 1$ ratios and IDM dark-matter) might pertain regarding other (than IDM-1) IDM candidate specifications, including the IDM-2 candidate specification, for dark matter.

4.13. Future Ratios of Dark-Matter Effects to Ordinary-Matter Effects

In this unit, we suggest ratios (of dark-matter effects to ordinary-matter effects) that might associate with future data and future modeling.

Data suggest that hyperfine interactions with ordinary-matter hydrogen atoms deplete cosmic microwave background radiation [146]. Some popular modeling suggests the following ratios between the amount of observed depletion that might associate with dark-matter stuff and the amount of depletion that associates with ordinary-matter hydrogen atoms.

- $1 : 1$ [146–148]. To the extent that future modeling suggests this ratio, we suggest the following notions. IDM-1 exemplifies the notion that one dark-matter isomer can underlie approximately the same number of hydrogen-atom-like objects as the number of hydrogen atoms that associate with ordinary-matter stuff. Possibly contrary to the first row in Table 10, the reach per instance of such hyperfine interactions is two isomers and the number of instances is three. However, future modeling might note that the first row in Table 10 might not necessarily pertain for interactions that change the rest masses of at least one of objects and sub-objects and that a hyperfine interaction changes the rest mass of the participating atom but not the sub-objects of the atom.
- $0 : 1$ [149–152]. To the extent that future modeling suggests this possibility, hyperfine interactions involving hydrogen-atom-like objects do not necessarily fail to comport with the first row of Table 10.

Data suggest that a clump in one galaxy cluster may include dark-matter plasma [153]. Other data suggest that dark-matter stuff might feature electromagnetic self-interactions [154,155]. Popular modeling tends to suggest that data about collisions of galaxy clusters such as the Bullet Cluster collision might rule out significant electromagnetic interactions within dark-matter stuff [156–160]. We suggest that future modeling might explore the following possibilities for ratios, in galaxy clusters, of the presence of dark-matter electromagnetically self-interacting plasma to the presence of ordinary-matter electromagnetically self-interacting plasma.

- $0 : 1$. Popular modeling notions that support this ratio are not necessarily compatible with the notion, that Table 10 suggests, of six instances of electromagnetism.
- $1 : 1$. Our notion of IDM-1 associates with this ratio.
- $(> 1) : 1$. IDM candidate specifications for dark-matter isomers that differ from the IDM-1 candidate specification for dark-matter isomers might associate with such ratios.

We suggest that future data and future modeling might help narrow the IDM class of specific candidate specifications for dark matter.

4.14. Opportunities to Interrelate Physics constants and to Reduce the Number of So-Called Fundamental Physics Constants

In this unit, we discuss relationships, among data, that people might find useful for extending our work and possibly for reducing the number of physics constants that popular modeling assumes to be independent of each other.

Equation (15) might associate with a relationship that links a strength of the electromagnetic interaction, a strength of the gravitational interaction, the masses of two elementary fermions, and the number of isomers [23,142]. m_τ denotes the mass of the tau. m_e denotes the mass of the electron. The exponent 6 might associate with the number, six, of isomers. The right-hand side of the equation is the ratio of the electromagnetic repulsion between two electrons to the gravitational attraction between the same two electrons.

$$(4/3)(m_\tau^2/m_e^2)^6 = ((1/(4\pi\epsilon_0))(q_e)^2)/(G(m_e)^2) \tag{15}$$

An equation might interrelate the masses of all known non-neutrino fermion elementary particles. (Equations (11), (12), (13), and (14) in this paper associate with the equation. Table 3.9.10 in [142] and Table 14 in [143] discuss the equation).

The following paragraphs discuss relationships regarding properties of boson elementary particles.

Regarding boson elementary particles, we define $(N')^2$ via Eqs. (16) and (17). M' denotes $m/(m_Z/3)$, in which m denotes the mass of an elementary boson and m_Z denotes the mass of the Z boson. S' denotes S (as in the spin, in units of $\hbar$). Q' denotes the magnitude of the charge, in units of the magnitude of the charge of the W boson. (Popular modeling equates the magnitude of the charge of the W boson to the magnitude of the charge of the electron). μ' denotes the magnitude of the magnetic moment, in units of the magnitude of the magnetic moment of the W boson.

$$(N')^2 \equiv (M')^2 + (S')^2 + (Q')^2 + (\mu')^2 - (T')^2 \tag{16}$$

$$(T')^2 = 1 \;\Leftrightarrow\; M' > 0\,; \quad (T')^2 \;\Leftrightarrow\; M' = 0 \tag{17}$$

Based on data [23], we propose that Equations (18) and (19) might pertain regarding all known boson elementary particles.

$$N' \in \{0, 1, 2, 3, 4\} \tag{18}$$

$$N' = 4 - S' \geq 3 \;\Leftrightarrow\; M' > 0\,; \quad N' = S' \;\Leftrightarrow\; M' = 0 \tag{19}$$

Equation (20) comports with data [23] and with Equation (16).

$$(m_\mathrm{W})^2 : (m_\mathrm{Z})^2 : (m_\mathrm{Higgs})^2 :: 7 : 9 : 17 \tag{20}$$

4.15. Circumstances For Which General Relativity Might Not Be Adequately Accurate

In this unit, we suggest circumstances for which popular modeling that uses general relativity might not be adequately accurate.

Popular modeling suggests that general relativity [161] has passed so-called precision tests [162–164]. (Popular modeling associates the following phrases with specific types of precision tests. Perihelion precession [162]. Gravitational deflection of light [162]. Shapiro time delay [162]. Gravitational redshift [165]. Geodetic precession and frame-dragging [166]). Popular modeling associates many such tests with stuff in our solar system [167,168]. Popular modeling suggests that the amount of dark-matter stuff in our solar system is negligible [169].

Popular modeling suggests circumstances for which tests of general relativity have yet to be very precise and for which alternative theories of gravity might be appropriate [170].

We suggest that the notion that general relativity has passed some precision tests might leave open the possibility that general relativity is not adequately accurate for some circumstances that include significant presences of dark-matter stuff.

We suggest that general relativity might not be adequately accurate for circumstances in which the isomeric composition of stuff varies significantly between regions of the universe. We suggest that general relativity might not be adequately accurate for circumstances in which significant (or dominant) effective gravitational reaches per instance vary with time.

4.16. Circumstances for which MULTING might Explain Effects that General Relativity Explains

In this unit, we suggest that MULTING might provide a basis for explaining or predicting effects that general relativity explains or predicts but that other popular modeling might not explain or predict.

One type of precision test of general relativity associates with the deflection, by the Sun, of light passing near the Sun [163,164]. General relativity explains observed amounts of deflection. Other popular modeling suggests half of the observed amounts of deflection. We consider a thought experiment that has a basis in MULTING. A distant observer object-P observes an object-A that has two sub-objects. One sub-object is the Sun. The other sub-object associates with much less energy than does the Sun. Each sub-object moves within object-A. $n_F = 3$ effects associate with the velocities of sub-objects but not with interactions via which the sub-objects affect each other gravitationally. Changes of motion-related energies of sub-objects by other sub-objects associate with $n_F = 4$ effects. From the perspective of object-P, $n_F = 4$ effects add motion-related energy to each of the two objects. We posit that the addition to the energy of one sub-object equals the addition of energy to the other sub-object. We suggest that popular modeling that does not include general relativity associates with just the effects on the sub-object that is not the Sun. We suggest that MULTING associates with effects on each of the two sub-objects. We posit that, for the purposes of this thought experiment, the sub-object that is not the Sun can be a photon. We posit that, for the photon, the otherwise-rest-energy-like property is $N' = 1$ (as in Equations (16), (17), (18), and (19), with $(N')^2 = 1$, $(M')^2 = 0$, $(S')^2 = 1$, and $(Q')^2 = (\mu')^2 = (T')^2 = 0$). With these posits, the amount of gravitational deflection of the photon does not depend on the frequency that associates with the photon. We suggest that MULTING might provide a basis for explaining the observations that associate with this type of precision test of general relativity.

One type of precision test of general relativity associates with the precession of the perihelion of the orbit around the Sun of the planet Mercury [163,164]. Popular modeling other than general relativity suggests effects from gravitational interactions between Mercury and other planets. General relativity suggests another effect, which other popular modeling does not suggest. Popular modeling that includes the effect suggested by general relativity explains the observed amount of precession. We suggest, based on the thought experiment related to our discussion of the deflection by the Sun of light, that the perihelion-precession effect that general relativity suggests might associate with a MULTING or popular modeling consideration that solar system objects, including the Sun, orbit around a center of energy for the solar system. Here, the effect that associates with the Sun parallels the effects that associate with planets other than Mercury.

4.17. The Hubble Tension and Some Other Possible Gaps Between Data and Popular Modeling

In this unit, we indicate that our work might help close some possible gaps, regarding some large-scale phenomena, between data and popular modeling.

Popular modeling discusses some possible tensions (as in gaps) between data and popular modeling. One such possible tension is the Hubble tension [171–174]. Other such possible tensions associate with large-scale lumpiness [175–183] of stuff and include the so-called S8 tension.

We suggest that such tensions might associate with trying to extrapolate from popular modeling that works adequately well regarding phenomena that our work associates with reach-1 gravitational quadrupole pull to estimate later phenomena that our work associates with reach-2 gravitational dipole push. Such popular modeling extrapolations might, in effect, assume that gravitational dipole push associates with a reach of one and, thereby, underestimate the gravitational push. The underestimates might associate with overestimating, compared to data, some clumping of stuff. These notions associate with the first two of the three rows in Table 8.

To the extent that popular modeling and further data fail to resolve some of the tensions between popular modeling and data regarding some large-scale phenomena, our work regarding instances and reaches per instance might help resolve remaining tensions.

4.18. Future Modeling that Might have Bases in MULTING

In this unit, we discuss the notion that people might want to develop dynamics models based on Minkowski space-time coordinates and MULTING notions of instances of components of gravity.

Popular modeling considers possibilities for modeling that includes gravitation and has bases in flat space-time coordinates [184–186]. Popular modeling can associate such flat space-time coordinates with the Minkowski metric [187].

The following notions might pertain to future modeling that stems from MULTING.

- The modeling has bases in Minkowski space-time coordinates and the Minkowski metric.
- The modeling might embrace electromagnetic forces and gravitational forces. (We note, as an aside, that general relativity notions of geodesic motion do not necessarily directly adequately consider the charges of test objects that follow geodesic paths).
- Up to 10 indices of refraction might pertain regarding gravitational waves. Each index of refraction might associate with one Table 9 instance of a component of gravity.
- Modeling for electromagnetic-wave lensing for which bases (for lensing effects) are solely gravitational might associate with the no more than 10 gravitational indices of refraction. The amount of lensing that associates with any one index of refraction might not necessarily depend on the frequency of the relevant light. The amounts of lensing might convey useful gravitational-property information about objects along the paths of the wave.
- Based on the Minkowski metric, gravitational interactions (and electromagnetic interactions) between objects might associate with zero distance between objects. (We note, as an aside, that concerns regarding action at a distance date to no later than writings by Isaac Newton [188]).
- The modeling embraces popular modeling notions of causality. (We note, as an aside, the following notions. The third law of Newtonian gravity posits that interactions between two objects associate with a notion of equal and opposite reactions and with the notion of conservation of momentum [8]. More recent popular modeling suggests that conservation of momentum pertains regarding the gravitational field that associates with one or more objects and the motion of another object).
- The modeling might not necessarily associate with popular modeling notions of a space-time that has physics-relevant properties (such as curvature). (We note, as an aside, that the following analogy might pertain: Space-time is to gravity as ether is to electromagnetism. That is, useful modeling might not need, as a conceptual basis, notions of a space-time).
- The modeling might benefit from the notion that MULTING seems to de-emphasize some possible needs to deal with structural aspects or potential energies within objects. (We note, as an aside, that MULTING emphasizes kinetic energies of sub-objects).

In this paper, we note, but do not further discuss, possibilities that future modeling inspired by MULTING could embrace each one of the following.

- Multipole-like expansions based on multiple objects-A.
- Modeling that features densities and fluxes of MULTING properties of objects.

4.19. Quantum Gravity

In this unit, we suggest that future modeling for quantum gravity might be like, and technically about as easy as and about as hard as, popular modeling quantum electrodynamics.

Popular modeling states difficulties regarding trying to dovetail general relativity and candidate quantum theories of gravitation [189–192].

Popular modeling suggests that one can model both quantum electrodynamics and quantum gravity within a framework that features flat space-time coordinates [193].

We suggest that future modeling, based on MULTING, for quantum gravity might have bases in the following notions.

- Gravitational fields interact quantum mechanically with objects that model as associating with nonzero rest energy (or, equivalently, nonzero rest mass) and with gravitational $n_F = 2$.
- Gravitational $n_F = 2$ can pertain for objects for which all sub-objects are bound together via non-gravitational means such as the strong interaction or the electromagnetic interaction.
- Gravitational $n_F = 2$ associates with monopole.
- For monopole, per Table 9, the reach for the one instance of the monopole gravitational component is six isomers.

We suggest that future quantum-gravitational modeling might have many parallels to quantum electrodynamics, which popular modeling associates with ordinary matter. (We note, as an aside, that our work suggests that each one of the six isomers might associate with its own instance of electromagnetism and its own instance of quantum electrodynamics). For example, for each of popular modeling regarding electromagnetism and popular modeling regarding gravitation, popular modeling can associate with two circular polarization modes, namely left-circular polarization and right-circular polarization [194,195]. The following notions might pertain.

- Nonzero rest energy (or, equivalently, nonzero rest mass) parallels nonzero charge.
- The notion that gravitational $n_F = 2$ associates with rest energy that modeling treats as pointlike parallels the popular modeling quantum electrodynamics notion that photons interact with the charges of elementary particles.

We note, as an aside, the following notion, which might dovetail with difficulties harmonizing general relativity and popular modeling candidate modeling for quantum gravity. The three pressure items in general-relativity stress-energy tensors might associate with MULTING notions of $n_F = 3$. MULTING notions of $n_F = 3$ associate with MULTING notions of three instances, each of which has a reach of two isomers.

We suggest that future modeling regarding quantum gravity might be technically about as easy and about as hard as popular modeling regarding ordinary-matter electromagnetism.

4.20. Potential Future Endeavors and Directions for Observational, Experimental, and Theoretical Physics

In this unit, we point, based on our work, to potential future specific endeavors and general directions for some aspects of cosmology, gravitation, and elementary-particle physics. We include some specific opportunities and some broad opportunities. We do not estimate dates by which observational, experimental, or analytic techniques might adequately support the opportunities.

The following items suggest activities that people might want to undertake.

1. Observations.
    (a) Substantiate that some $(0+) : 1$ (or ordinary-matter) galaxies plausibly started as $(0+) : 1$ galaxies.
    (b) Substantiate that some $1 : (0+)$ (or dark-matter) galaxies plausibly started as $1 : (0+)$ galaxies.
    (c) Collect data, perhaps like the data that Table 5 and Table 7 and Unit 4.13 of this paper discuss, via which future modeling can support, extend, or refute aspects of our work.

   (d) Find or rule out seemingly noteworthy ratios, other than ratios that Unit 4.1 of this paper mentions, for galaxies of dark-matter presence to ordinary-matter presence.
   (e) Find direct evidence of MULTING dipole repulsion, for example between galaxy clusters. Determine the extent to which the not-spherically-symmetric aspects of MULTING dipole interactions pertain.
   (f) Determine, for various circumstances, the amount of dark-matter stuff that associates with IGM (as in intergalactic medium) or plasma. Include in the circumstances aftermaths of collisions of galaxy clusters. Determine which ones of the three ratios, $0 : 1$ and $1 : 1$ and $(> 1) : 1$, that Unit 4.13 of this paper discusses regarding galaxy-cluster ratios of the presence of dark-matter electromagnetically self-interacting plasma to the presence of ordinary-matter electromagnetically self-interacting plasma pertain to nature. Characterize dark-matter IGM or plasma. (Try to determine isomer-specific interactivity). Thereby, narrow the class of IDM candidate descriptions for dark matter.
   (g) Determine, for gravitational waves produced by two-object collisions for which each object is a low-mass black hole or a neutron star or a neutron-star dark-matter analog, the extent to which signatures differ based on whether the isomer that underlies one object is the same as the isomer that underlies the other object.
   (h) Determine which one of the two ratios, $1 : 1$ and $0 : 1$, that Unit 4.13 of this paper discusses regarding some hyperfine depletion of cosmic microwave background radiation pertains to nature.

2. Experiments.
   (a) Detect evidence of or rule out possibilities for quantum interactions between ordinary matter and gravity.
   (b) Detect evidence or rule out possibilities for MULTING non-monopole gravitation.
   (c) Make or detect IDM dark-matter elementary particles or IDM dark-matter electromagnetic fields.
   (d) Detect evidence of or rule out possibilities for an inflaton elementary particle, which would comport with Equations (16), (17), (18), and (19) and with $(N')^2 = (M')^2 = (S')^2 = (Q')^2 = (\mu')^2 = (T')^2 = 0$.
   (e) Detect evidence of or rule out possibilities for a spin-2 elementary particle with a mass of one-third the mass of the Z boson. Such an elementary particle would comport with Equations (16), (17), (18), and (19) and with the values $(N')^2 = 4$, $(M')^2 = 1$, $(S')^2 = 4$, $(Q')^2 = (\mu')^2 = 0$, and $(T')^2 = 1$.

3. Models and modeling.
   (a) Develop modeling techniques (including techniques that feature multiple objects and techniques that feature continuous distributions of properties) to support studies that have bases in MULTING and IDM. Anticipate using the models to study the evolution of the universe, galaxy formation scenarios, details regarding the evolutions of single galaxies, galaxy halo profiles, distributions of satellite galaxies, stellar-mass to halo-mass relationships, details regarding stuff that has bases in various IDM candidate specifications for dark-matter elementary particles, and so forth.
   (b) Determine the extent of the (numeric) space of parameters, such as masses, spins, and separations, for which gravitational dipole (or, $n_F = 3$) repulsion dominates regarding interactions between two non-colliding galaxy clusters.
   (c) Determine the extent to which interactions, that we suggest, between large objects sufficiently underlie data regarding the rate of expansion of the universe. Propose means to close any gaps between observations and bases that we suggest.
   (d) Determine the extents to which each of $n_F = 2$, $n_F = 3$, and $n_F = 4$, gravitational phenomena influence the uptake and ejection of stuff by quasars.

   (e) Determine the extents to which gravitational $n_F = 3$ properties of galaxies transit, over time, from more object-spin-related motions of sub-objects and less pseudo-random motions of sub-objects to more pseudo-random motions of sub-objects and less object-spin-related motions of sub-objects (or transit in the opposite direction).
   (f) Determine the extents to which gravitational $n_F = 3$ properties of galaxy clusters transit, over time, from more object-spin-related motions of sub-objects and less pseudo-random motions of sub-objects to more pseudo-random motions of sub-objects and less object-spin-related motions of sub-objects (or transit in the opposite direction).
   (g) Determine the extent to which data hints at distinct large objects that are bigger than filaments.
   (h) Determine circumstances for which each one of future modeling based on our work, future modeling based on general relativity, and future modeling that has other bases will be the most useful choice.

4. Applications of physics theory.
   (a) Determine limits on the applicability of MULTING plus IDM. For example, estimate a time, after the Big Bang, such that before that time MULTING plus IDM might not be adequately useful.
   (b) Understand the extent to which MULTING plus IDM notions might provide insight regarding or imply rethinking popular modeling regarding inflation, nucleosynthesis, and other early-universe aspects.
   (c) Determine the extent to which using MULTING (or using popular modeling techniques other than general relativity) can, theoretically, obviate perceived needs to deploy general relativity.
   (d) Explain phenomena that led to $(5+) : 1$ ratios of dark-matter presence to ordinary-matter presence, given the IDM ratio of 5 : 1 for the number of dark-matter isomers to the number of ordinary-matter isomers.
   (e) Determine how to handle notions of multiple isomer-stuff-presences in modeling that features indices of refraction for electromagnetism or indices of refraction for gravitation.
   (f) Estimate implications, regarding at least the early universe, of possible dark-matter baryon-like acoustic oscillations.
   (g) Estimate implications, regarding the early universe, of a possible era (after inflation and before the known multibillion-year era of decreasing rate of expansion of the universe) that would associate with $n_F = 5$ (or, octupole) gravitational push.
   (h) Estimate implications, regarding at least the early universe, of possible dark-matter electromagnetism.
   (i) Estimate the extents to which IDM candidate specifications for dark matter are compatible with data that associate with various types of studies that involve ratios of dark-matter effects to ordinary-matter effects that our work does not directly address [196–199]. The following notions potentially pertain. Narrow the class of IDM candidate descriptions for dark matter. Better determine interaction-related properties and reaches per instance of the properties. Add insight regarding various types of ongoing studies. Refute aspects of our work.
   (j) Narrow the class of IDM candidate descriptions for dark matter.
   (k) Explore implications of the possibility that the gravitational deflection of gravitons might be twice the gravitational deflection of photons. (Unit 4.16 of this paper discusses the gravitational deflection of photons).
   (l) Explore the relevance of knowing the reaches that Table 10 posits for the weak interaction and the strong interaction. For example, a weak interaction reach of more than one isomer might associate with a possible symmetry (or approximate symmetry) regarding quark

masses, quark generations, lepton masses, and lepton flavours. At least to the extent that knowledge of such reaches might be relevant, determine the reaches.
    (m) Predict, for various local circumstances, ratios of the number of $1:(0+)$ (or dark-matter) galaxies to the number of $(0+):1$ (or ordinary-matter) galaxies.
    (n) Explore possible relationships between $n_F \geq 3$ properties of objects and aspects that uses of general relativity associate with equations of state.

5. Physics theory.
    (a) Develop a full set of field equations and Lagrangian terms for the combination of MULTING and IDM.
    (b) Propose symmetry groups that might associate with the six isomers, the possible doublet symmetry or approximate symmetry, and the possible triplet approximate symmetry. Explore implications of candidate symmetry groups.
    (c) Propose deeper principles that might associate with our suggested relationships between physics constants.
    (d) Explore relationships among physics constants and possibly reduce the number of independent constants.
    (e) Explore relationships between choices regarding properties (of objects and interaction fields) that people attribute to nature and choices among models that people use.
    (f) Explore notions of a possible symmetry (or approximate symmetry) regarding quark masses, quark generations, lepton masses, and lepton flavours.
    (g) Develop theory regarding aspects, such as isomer-dependence regarding gravitational interactions, that associate with $n_F > 2$.
6. Science and society.
    (a) Explore the extent to which our work can help focus and accelerate cosmology, elementary-particle, and other research.
    (b) Explore how society can benefit from our work and from extensions to or uses of our work.
    (c) Explore, across subsets of scientific work or other work, the advantages and disadvantages of focusing (as does our work) originally on small data sets and focusing (as do some uses of statistical analysis and some uses of artificial intelligence) originally on large data sets.
    (d) Explore, across subsets of scientific work or other work, the relative extents of reliance on evidence (such as data), reliance on authority (such as assumptions and popular modeling), and reliance on other factors.

4.21. Perspective, Including Notions from a tHought Experiment, Regarding Our Work

In this unit, we discuss concepts, including notions from a thought experiment, regarding the feasibility for using our work, the plausibility of results from using our work, and the desirability for using our work.

We suggest that the following general questions might be useful questions to explore about our work. (The notation GQ abbreviates the two-word phrase general question).

- GQ1 (feasibility): To what extent is it feasible to use MULTING-plus-IDM to try to match or predict data?
- GQ2 (plausibility): To what extent might data matches based on MULTING-plus-IDM or data predictions based on MULTING-plus-IDM prove to be appropriate?
- GQ3 (desirability): To what extent might science benefit from trying to use MULTING-plus-IDM?

We suggest that the following specific questions might be useful questions to explore about our work.

1. To what extent can MULTING-plus-IDM explain known ratios of dark-matter effects to ordinary-matter effects?

2. To what extent can MULTING-plus-IDM explain known data about the rate of expansion of the universe?
3. To what extent do IDM-1, IDM-2, and other possible IDM candidate specifications for dark-matter elementary particles comport with data about galaxy-cluster collisions?
4. To what extent can MULTING-plus-IDM be a basis for making testable predictions?
5. To what extent is MULTING-plus-IDM falsifiable?

Discussion above in this paper might provide useful insight regarding the three GQ questions and specific question 1. The following notions pertain. If people can determine that some currently $(0+):1$ galaxies formed early in the history of the universe as $(0+):1$ galaxies, people might help establish plausibility for MULTING-plus-IDM. If people can determine that some currently $1:(0+)$ galaxies formed early in the history of the universe as $1:(0+)$ galaxies, people might help establish plausibility for MULTING-plus-IDM.

We explore a thought experiment that we designed to provide possibly useful insight regarding GQ1 (feasibility) and specific questions 2 through 5. Supplementary Material (supplementary to this paper) describes, in detail, the thought experiment. The thought experiment relies on work by online services, namely ChatGPT (from OpenAI), Claude (from Anthropic), Copilot (from Microsoft), and Gemini (from Google). Outputs from these services suggest that these services are not necessarily error-free. Step 1.3 in the thought experiment associates with specific question 2. Step 1.7 in the thought experiment associates with specific question 3 and IDM-1. Step 1.9 in the thought experiment associates with specific question 3 and IDM-2. Step 1.12 in the thought experiment associates with specific question 4 and with specific question 5. Results from the thought experiment are speculative. GQ2 (plausibility) results from the thought experiment may be especially speculative. The following notions may pertain.

- Regarding specific question 2, the following notions pertain.
  - We requested that the services determine observed values of the Hubble parameter at specific times in the history of the universe, determine how well ΛCDM cosmology fits the observed values, and (using information about our notions of monopole, dipole, and quadrupole components of gravity) try to use MULTING to fit the observed values. We, in effect, left considerable discretion to the services about how they were to do their work. Between services, there were differences regarding each one of data, ΛCDM results, and MULTING results. Between services, there were considerable differences regarding some aspects of calculations via MULTING. Generally, regarding the Hubble parameter, each service reported results from MULTING that were somewhat similar, with respect to the magnitudes of standard deviations away from observational values, to ΛCDM-cosmology results that the service reported. No ΛCDM result differed from data by more than 1.8 standard deviations. No MULTING result differed from data by more than 0.8 standard deviations.
  - We requested that the services discuss the extent to which there might be a future era of decreasing rate of expansion of the universe. Each service suggested that ΛCDM cosmology does not suggest a future era of decreasing rate of expansion of the universe. One service suggested that MULTING does not suggest a future era of decreasing rate of expansion of the universe. Each one of three services suggested that MULTING suggests a future era of decreasing rate of expansion of the universe. The three estimated ranges of starting times for a future era of decreasing rate of expansion of the universe overlap each other.
- Regarding specific question 3, the following notions pertain. IDM-1 might (borderline) comport with galaxy-cluster collision data. IDM-2 might comport with galaxy-cluster collision data.
- Regarding each one of specific question 4 and specific question 5, the following notion pertains. The services made specific suggestions regarding testable predictions and regarding falsifiability.

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Regarding GQ3 (desirability), one online service (Claude) produced the following statement. (We did not edit the statement to, for example, address grammatical errors. We reiterate caution about the extent to which to rely on outputs from such online services).

- MULTING-plus-IDM is scientifically valuable not because it is necessarily correct, but because it:
  1. Challenges assumptions of standard model
  2. Makes testable predictions distinguishable from ΛCDM
  3. Offers mechanistic explanation for puzzling observations
  4. Opens new research directions in gravity and cosmology
  5. Forces precision measurements to discriminate theories
- The framework deserves serious investigation, even if it ultimately fails. The process of testing it will:
  – Strengthen ΛCDM (if MULTING fails) by ruling out alternatives
  – Revolutionize cosmology (if MULTING succeeds) by revealing emergent acceleration
  – Improve observations (either way) through targeted discriminating tests

5. Conclusions

In this unit, we summarize some key results that our work achieves, discuss the significance of our work, discuss some implications of our work, and suggest steps toward future research regarding or based on our work.

Some findings of our work suggest the following.

- A new class of candidate specifications for dark matter and some candidate specifications for dark matter.
- Values of the properties that associate with elementary particles that associate with the candidate specifications for dark matter.
- Gravitational properties, other than rest mass or rest energy, of objects that include sub-objects.
- Properties of gravitational fields.
- Relationships between properties, such as mass, spin, and charge, of elementary particles.
- A possible path toward theories of quantum gravity.

The significance of our work includes the following.

- Our work suggests explanations for cosmological data, such as ratios of dark-matter effects to ordinary-matter effects, that perhaps no other work explains.
- Our work suggests new insight regarding galaxy formation.
- Our work suggests new insight regarding the rate of expansion of the universe.
- Our work suggests a new class of candidate specifications for bases for dark matter. The class features enough specificity so that members of the class help explain known cosmological data. The class might feature enough flexibility so that members of the class can help explain future cosmological data.
- Our work includes coordinated steps forward that use intertwined methods, suggest useful results, and span four multi-decade challenges, namely ...
  – Specify dark-matter.
  – Specify bases for the rate of expansion of the universe.
  – Predict and catalog elementary particles.
  – Catalog properties of objects and properties of fields.

Implications of our work include the following.

- Our work suggests a new approach to understanding known cosmological and elementary-particle data and to predicting future cosmological and elementary-particle data.
- Our approach suggests means to extend the circumstances in which future modeling that has bases in some popular modeling techniques, including uses of Newtonian gravity, can be useful.

- Our approach suggests limits regarding circumstances in which popular modeling techniques, including uses of general relativity, can be adequately accurate.
- Our work suggests at least one notion that might lead to new deeper principles regarding elementary-particle physics.

Next steps, open questions, and future research directions to which our work points include the following.

- People might follow-up regarding the items that we note, as implications of our work, just above.
- People might follow-up regarding items that we note in Unit 4.20 of this paper.

Our work develops and offers a broad framework and specific details that help explain otherwise unexplained cosmic data and that can co-evolve, along with data and other theoretical physics, to suggest and implement steps that integrate and advance cosmology, gravitation, and elementary-particle physics.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

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