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Scalar Field and Quintessence in Late-Time Cosmic Expansion

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Abstract

The persistent Hubble tension - marked by a notable disparity between early- and late-universe determinations of the Hubble constant H_0 — poses a serious challenge to the standard cosmological framework. Closely linked to this is the $H_0 - r_d$ tension, which stems from the fact that BAO-based estimates of H_0 are intrinsically dependent on the assumed value of the sound horizon at the drag epoch, r_d. In this study, we construct a scalar field dark energy model within the framework of a spatially flat FLRW model to explore the dynamics of cosmic acceleration. To solve the field equations, we introduce a generalized extension of the standard ΛCDM model that allows for deviations in the expansion history. Employing advanced Markov Chain Monte Carlo techniques, we constrain the model parameters using a comprehensive combination of observational data, including Baryon Acoustic Oscillations, Cosmic Chronometers, and Standard Candle datasets from Pantheon Type Ia Supernovae (SNe Ia), Quasars, and Gamma-Ray Bursts (GRBs). Our analysis reveals a transition redshift from deceleration to acceleration at $z_{\rm tr} = 0.69$, and a present-day deceleration parameter value of $q_0 = -0.64$. The model supports a dynamical scalar field interpretation, with an equation of state parameter satisfying $-1 < \omega_0^{\phi} < 0$, consistent with quintessence behavior, and signaling a deviation from the cosmological constant. While the model aligns closely with the ΛCDM scenario at lower redshifts ($z \lesssim 0.65$), notable departures emerge at higher redshifts ($z \gtrsim 0.65$), offering a potential window into modified early-time cosmology. Furthermore, the evolution of key cosmographic quantities such as energy density ρ^{ϕ}, pressure p^{ϕ}, and the scalar field equation of state highlights the robustness of scalar field frameworks in describing dark energy phenomenology. Importantly, our results indicate a slightly higher value of the Hubble constant H_0 for specific data combinations, suggesting that the model may provide a partial resolution of the current H_0 tension.

Keywords: dark energy; current acceleration of the universe; scalar field cosmological model; parametrization of the Hubble parameter; analysis of cosmographic parameters

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1. Introduction

Einstein's theory of General Relativity (GR) has stood as the foundational framework of gravitational physics and modern cosmology, providing remarkably accurate predictions across a wide range of astrophysical and cosmological phenomena. From the precession of Mercury's perihelion and the deflection of light by massive bodies, to the precise modeling of gravitational lensing and the large-scale dynamics of galaxy clusters, GR has withstood over a century of rigorous experimental and observational scrutiny. Despite its success, applying GR to the universe at cosmological scales reveals profound limitations. Most notably, GR alone cannot account for the observed accelerated expansion of the universe without introducing an exotic energy component with negative pressure—commonly termed dark energy. This enigmatic form of energy, inferred primarily from observations of Type Ia supernovae, Cosmic Microwave Background (CMB) anisotropies, and Baryon Acoustic Oscillations (BAO), remains one of the deepest mysteries in theoretical physics and cosmology [1–3]. These challenges have motivated the exploration of extended theories of gravity and alternative cosmological models that go beyond the standard ΛCDM paradigm.

Observational data from multiple independent cosmological probes — including Type Ia Supernovae (SNe Ia), Cosmic Microwave Background (CMB) anisotropies, and Baryon Acoustic Oscillations (BAO) — have firmly established that the universe is undergoing a phase of accelerated expansion. Within the framework of the standard ΛCDM model, this acceleration is attributed to a cosmological constant (Λ), often interpreted as the vacuum energy density with a constant equation of state $\omega = -1$. Despite the success of the ΛCDM model in fitting a broad range of cosmological data, growing tensions between early- and late-time measurements of key parameters, most notably of the Hubble constant (H_0) and of the sound horizon scale at the drag epoch (r_d), have called into question the completeness of this model. The so-called Hubble tension reflects a significant discrepancy between the local (late-universe) measurements of H_0, such as those based on Cepheid-calibrated supernovae, and the value inferred from the CMB under ΛCDM assumptions. Similarly, the H_0–r_d tension, emerging from BAO data, suggests possible inconsistencies in the assumed physics of the early universe. These anomalies have motivated increasing interest in alternative cosmological models and modifications to general relativity that go beyond the standard paradigm.

Early-universe estimates, particularly those derived from Planck Cosmic Microwave Background (CMB) data under the standard ΛCDM framework, favor a lower value of H_0[3]. In contrast, local measurements based on the Cepheid-calibrated distance ladder, such as those by Riess et al., yield significantly higher values [4,5]. This discrepancy, now exceeding the 5σ threshold in some analyses, has sparked debate about whether the cause lies in unresolved systematic uncertainties or points to the need for new physics beyond the standard cosmological model.

Closely linked to this is the $H_0 - r_d$ tension, which arises due to the dependence of Baryon Acoustic Oscillation (BAO) based determinations of H_0 on the sound horizon at the drag epoch, r_d. In the standard approach, r_d is calculated from early-universe physics assuming well-established pre-recombination conditions. However, recent investigations suggest that modifying early expansion dynamics, introducing additional relativistic species, or altering the gravitational sector could change the value of r_d, thereby affecting BAO-based estimates of H_0 [6,7]. This interconnected set of tensions points to potential gaps in our theoretical understanding of the cosmic expansion history and motivates ongoing exploration of alternative cosmological models.

In the quest to explain the late-time accelerated expansion of the universe, modified gravity theories (MGTs) have emerged as compelling alternatives to the standard ΛCDM paradigm, which relies on a cosmological constant. These MGTs modify the geometric sector of Einstein's field equations and allow for cosmic acceleration without invoking dark energy in the form of a constant vacuum energy density. One of the most well-studied frameworks is $f(R)$ gravity, where the Ricci scalar R in the Einstein-Hilbert action is replaced with a generic function $f(R)$ [8–10]. This extension naturally leads to accelerated expansion under certain functional forms of $f(R)$ and has been successfully tested in various cosmological settings. Similarly, $f(T)$ gravity modifies the gravitational action by replacing curvature with torsion (T), as formulated in the teleparallel equivalent of general relativity [11,12]. Other extensions include $f(R,T)$ gravity, where T denotes the trace of the energy-momentum tensor, introducing explicit matter–geometry coupling [13], and $f(R,L_m)$ gravity, where the Lagrangian density of matter L_m interacts with curvature, enabling novel gravitational dynamics [14]. More recently, $f(Q)$ gravity, based on the non-metricity scalar Q, has garnered attention for its ability to describe cosmological acceleration within the symmetric teleparallel framework [15,16]. Further generalization is achieved in $f(Q,T)$ gravity, where both non-metricity and matter contribute non-trivially to the gravitational Lagrangian [17].

Parallel to these geometric modifications, scalar field cosmologies provide another well-motivated class of models for understanding both the present cosmic acceleration and the early inflationary epoch [18,19]. In these models, a scalar field ϕ evolves dynamically and interacts with the geometry of spacetime. The associated potential $V(\phi)$ is responsible for producing negative pressure, thereby driving accelerated expansion [18–21]. Among these, the quintessence model is a widely studied scalar field framework that proposes a slowly rolling scalar field as the source of dark energy. It offers significant theoretical advantages, particularly in addressing the fine-tuning and cosmic coincidence problems [20,21]. Unlike a cosmological constant, quintessence permits the energy density of dark energy to vary with time, thus reducing the need for unnatural initial conditions. The evolution of the scalar field may be guided by specially designed potentials, allowing its energy density to naturally approach that of matter at late times, offering a dynamic solution to the coincidence problem. A crucial concept within the quintessence framework is that of tracker fields, introduced in [22], where the scalar field evolves along an attractor solution largely independent of initial conditions. This tracking behavior has received substantial observational support and enhances the model's predictive power. The literature further explores extended quintessence models, including those with non-minimal coupling between the scalar field and gravity [23,24], and those involving non-canonical kinetic terms, such as k-essence or tachyonic fields [25]. The role of time-dependent equation of state (EoS) parameters in scalar–tensor theories has been rigorously examined, offering rich phenomenology in both background dynamics and structure formation [26,27]. Moreover, scalar fields are not limited to cosmological applications. They also play important roles in various astrophysical processes, such as black hole physics, stellar structure, and compact object dynamics. This broad applicability has led to the proliferation of scalar field–based models across both theoretical and observational domains [26–31].

A comprehensive investigation of dynamical dark energy models and modified gravity theories is essential, especially in the context of current and future high-precision cosmological observations. The complex and still poorly understood mechanisms responsible for the late-time acceleration of the universe necessitate rigorous theoretical scrutiny and empirical validation. To facilitate such studies, numerous parametrizations have been developed to effectively describe the evolution of key cosmological quantities, including the Hubble parameter, deceleration parameter, and equation of state (EoS) parameter [32–35]. These parametrizations serve as powerful tools for testing the viability of competing models against observational datasets.

Building on the broader framework of dynamical cosmology, this study adopts a specific parametrization of the Hubble parameter $H(a)$, designed as a generalization to the Λ CDM model. By introducing a physically motivated yet flexible functional form for $H(a)$, we aim to capture the key features of cosmic expansion in a manner that allows direct confrontation with observational data. To place robust constraints on the model, we utilize a comprehensive suite of cosmological datasets, including Baryon Acoustic Oscillations (BAO), Cosmic Chronometers (CC), and Type Ia Supernovae (SNe Ia) from the Pantheon compilation, complemented by measurements from quasars and gamma-ray bursts (GRBs). These diverse observational probes offer independent measurements of the Hubble parameter $H(z)$, enabling a detailed reconstruction of the cosmic expansion history and precise constraints on the deceleration parameter and related cosmological quantities. We perform a thorough statistical analysis using Markov Chain Monte Carlo (MCMC) techniques to explore the parameter space and derive the posterior distributions of the model parameters. A Gaussian prior on the Hubble constant H_0, based on the Riess et al. 2019 measurement (R19), is incorporated to investigate potential tensions between early- and late-time cosmological constraints. Additionally, a covariance matrix analysis is carried out to rigorously account for correlations within the observational datasets, thereby ensuring the statistical robustness and reliability of the inferred cosmological parameters.

The structure of the paper is as follows. In section II, we provide the field equations and in section III, we introduce our parametrization of the Hubbel parameter. The confrontation of the model against observational is done in section IV. In section V, we discuss the cosmographic parameters, and in section VI, the evolution of the scalar field is elucidated upon, with its relevance to quintessence. Finally section VII provided our concluding remarks.

2. Field Equations

Unveiling the true nature of dark energy remains one of the most formidable challenges in modern cosmology. Although the cosmological constant Λ provides a phenomenological explanation for the observed acceleration of the universe, its theoretical limitations, notably the fine-tuning and coincidence problems, have motivated the exploration of dynamic alternatives. Among these, quintessence, a canonical scalar field slowly evolving in time, has emerged as a compelling and theoretically rich candidate for dark energy.

The action for a scalar field minimally coupled to gravity, as proposed in [36], is given by

$$S = \int d^4x\,\sqrt{-g}\left(\frac{1}{2}M_{\rm Pl}^2 R + \mathcal{L}_m + \mathcal{L}_\phi\right), \tag{1}$$

where $\mathcal{L}_m$ denotes the Lagrangian for ordinary matter, and $\mathcal{L}_\phi$ represents the scalar field Lagrangian, defined as

$$\mathcal{L}_\phi = -\frac{1}{2}g^{ij}\partial_i\phi\,\partial_j\phi - V(\phi). \tag{2}$$

In this formulation, g is the determinant of the metric tensor g_{ij}, $M_{\rm Pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass, R is the Ricci scalar, and $V(\phi)$ is the scalar potential responsible for the self-interaction of the field. For physically viable models, the scalar field ϕ is typically assumed to be positive definite. Within the context of dark energy phenomenology, models are broadly categorized as either interacting or non-interacting. Interacting models involve energy exchange between dark energy and cold dark matter [37–39], while non-interacting models assume the independent evolution of matter and dark energy components [40–42]. In this work, we consider a minimally coupled quintessence scenario.

Varying the action (1) with respect to the metric yields the Einstein field equations:

$$R_{ij} - \frac{1}{2}Rg_{ij} = M_{\rm Pl}^{-2}T_{ij}^{(\rm tot)}, \tag{3}$$

where the total energy-momentum tensor is given by the sum of the matter and scalar field contributions:

$$T_{ij}^{(\rm tot)} = T_{ij}^{(m)} + T_{ij}^{(\phi)}. \tag{4}$$

The energy-momentum tensor corresponding to the scalar field ϕ is expressed as

$$T_{ij}^{(\phi)} = \partial_i\phi\,\partial_j\phi - \frac{1}{2}g_{ij}(\partial\phi)^2 - g_{ij}V(\phi), \tag{5}$$

where $(\partial\phi)^2 \equiv g^{ij}\partial_i\phi\,\partial_j\phi$. The scalar field also satisfies the Klein-Gordon equation:

$$\nabla^i\nabla_i\phi - \frac{dV}{d\phi} = 0. \tag{6}$$

To model the large-scale geometry of the universe, we adopt the spatially homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) metric:

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2\right), \tag{7}$$

where $a(t)$ is the scale factor, and $k = +1, 0, -1$ corresponds to closed, flat, and open spatial curvature, respectively. Observations favor a spatially flat geometry ($k = 0$), reducing the metric to:

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2\right). \tag{8}$$

Within this background, the Einstein field equations simplify to the Friedmann equations:

$$3H^2 = M_{\rm Pl}^{-2}\rho^{(\rm tot)}, \tag{9}$$

$$(2q-1)H^2 = M_{\rm Pl}^{-2} p^{(\rm tot)}, \tag{10}$$

where $H = \dot{a}/a$ is the Hubble parameter, and $q = -\ddot{a}/\dot{a}^2$ is the deceleration parameter. The total energy density and pressure are the sum of the matter and scalar field contributions:

$$\rho^{(\rm tot)} = \rho^{(m)} + \rho^{(\phi)}, \quad p^{(\rm tot)} = p^{(m)} + p^{(\phi)}. \tag{11}$$

The energy density and pressure associated with the scalar field are given by:

$$\rho^{(\phi)} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p^{(\phi)} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{12}$$

where $\frac{1}{2}\dot{\phi}^2$ and $V(\phi)$ represent the kinetic and potential energies of the scalar field, respectively.

The effective equation of state (EoS) parameter for the scalar field is then defined as

$$\omega^{(\phi)} = \frac{p^{(\phi)}}{\rho^{(\phi)}} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{13}$$

In contrast to a cosmological constant ($\omega = -1$), the EoS parameter for quintessence evolves dynamically within the range $-1 < \omega^{(\phi)} < 0$. A value closer to -1 indicates potential-dominated evolution, mimicking a cosmological constant, whereas kinetic dominance leads to higher values of $\omega^{(\phi)}$, resulting in weaker acceleration. The total energy-momentum conservation law, derived from Eqs. (9) and (10), is given by:

$$\dot{\rho}^{(\rm tot)} + 3H\left(\rho^{(\rm tot)} + p^{(\rm tot)}\right) = 0. \tag{14}$$

Finally, using Eqs. (9) and (12), we can isolate the kinetic and potential components of the scalar field in terms of cosmological parameters:

$$\dot{\phi}^2 = 2M_{\rm Pl}^2(q+1)H^2 - \rho^{(m)}, \tag{15}$$

$$V(\phi) = M_{\rm Pl}^2(2-q)H^2 - \frac{1}{2}\rho^{(m)}. \tag{16}$$

These relations allow a reconstruction of the scalar field dynamics based on observational constraints on $H(z)$, $q(z)$, and $\rho^{(m)}(z)$.

3. Parametrization of Hubble Parameter

Parametrizations play a pivotal role in probing the nature of the dark energy-dominated universe in a model-independent framework. By formulating suitable functional forms of key cosmological quantities such as the Hubble parameter or the deceleration parameter, and confronting them with observational data, one can effectively discriminate between viable and non-viable cosmological scenarios. In order to get definite solutions, it is often necessary to introduce such parametrizations. This approach offers a flexible and theoretically agnostic path to understanding late-time cosmic acceleration.

In this work, we adopt a generalized parametric form of the Hubble parameter as a function of the scale factor $a(t)$, given by:

$$H(a) = H_0\left[\alpha\, a^{-\beta} + (1-\alpha)\right], \tag{17}$$

where α and β are free model parameters. This formulation generalizes the standard ΛCDM Hubble rate to:

$$H(a) = H_0\sqrt{\Omega_m a^{-3} + \Omega_\Lambda},$$

and accommodates a broader range of cosmological behaviors, depending on the values of m and n. Parametrizations like Eq. (17) have been widely employed in literature to explore alternatives to ΛCDM in both standard and modified gravity scenarios [43–48].

Transforming Eq. (17) into a redshift-based formulation using the relation $a = \frac{1}{1+z}$ (with $a_0 = 1$ for the present epoch), we obtain:

$$H(z) = H_0\sqrt{\left[\alpha(1+z)^{\beta} + (1-\alpha)\right]}. \tag{18}$$

To understand the dynamics of cosmic expansion, the cosmographic parameters, particularly the deceleration parameter q, provide crucial insight. This parameter helps classify different epochs of cosmic evolution: $q > 0$ signifies a decelerated phase, $-1 < q < 0$ corresponds to power-law acceleration, $q = -1$ denotes exponential (de Sitter) expansion, and $q < -1$ indicates a super-exponential regime. The deceleration parameter is formally defined as:

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right), \tag{19}$$

and, using the parametrized Hubble function in Eq. (18), we derive:

$$q(z) = -1 + \frac{\alpha\beta(1+z)^{\beta}}{2\left[\alpha(1+z)^{\beta} + (1-\alpha)\right]}. \tag{20}$$

4. Datasets and Observational Framework

Baryon Acoustic Oscillations (BAO) refer to the periodic fluctuations imprinted on the photon-baryon plasma during the drag epoch. They arise from acoustic waves in the early universe. These fluctuations are set at the sound horizon scale r_d at the epoch of recombination ($z_d \sim 1060$). They provide a robust cosmological standard ruler for measuring cosmic distances and probing the expansion history of the universe. Unlike constraints from the CMB or Type Ia supernovae (SNe Ia), BAO measurements offer an independent avenue for constraining cosmological parameters [49,50]. The BAO feature is identified across various astrophysical tracers and redshift ranges. The BOSS survey, for example, utilizes the correlation function of Lyman-α absorption lines from quasar spectra, as well as galaxy clustering of luminous red galaxies (LRGs), emission-line galaxies (ELGs), and quasars to detect the BAO signature [51,52]. The BAO scale, observable at multiple redshifts, reflects the integrated expansion of the universe since recombination, making it instrumental for constraining late-time cosmology.

In this analysis, we incorporate BAO measurements from a wide range of sources, including SDSS, WiggleZ, DECaLS, DES, and 6dFGS [53–67]. Additionally, we utilize cosmic chronometer (CC) data [68–71], which provide direct, model-independent measurements of the Hubble parameter $H(z)$ based on the differential aging of galaxies. We also include the Pantheon sample of Type Ia supernovae [1,72–74], complemented by high-redshift probes such as quasars [75] and gamma-ray bursts [76].

To relate these observables to cosmological theory, several distance measures are introduced. The comoving angular diameter distance is defined as:

$$D_M(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}, \tag{21}$$

where $E(z) = H(z)/H_0$ is the dimensionless Hubble parameter. From this, the angular diameter distance $D_A(z) = D_M(z)/(1+z)$ and the Hubble distance $D_H(z) = c/H(z)$ can be derived. A commonly used quantity in BAO analyses is the volume-averaged distance, defined as:

$$D_V(z) = \left[zD_H(z)D_M^2(z)\right]^{1/3}. \tag{22}$$

The sound horizon at the drag epoch, which sets the BAO scale, is given by:

$$r_d = \int_1^{z_d} \frac{c_s(z)}{H(z)}dz, \tag{23}$$

where $c_s(z)$ is the sound speed in the photon-baryon fluid, approximated by:

$$c_s(z) \approx \frac{c}{\sqrt{3\left(1+\frac{3\Omega_b(z)}{4\Omega_\gamma(z)}\right)}}. \tag{24}$$

In practice, BAO measurements yield the angular and redshift projections:

$$\theta_z = \frac{r_d H(z)}{c}, \quad \theta_\ell = \frac{r_d}{(1+z)D_A(z)},$$

which imply that BAO data constrain the combination $r_d H(z)$ rather than r_d and $H(z)$ individually. To disentangle these quantities, an independent constraint on either H_0 or r_d is necessary.

To account for potential correlations among BAO measurements, we perform a covariance analysis as detailed in [77]. The covariance matrix for uncorrelated data points, $C_{aa} = \sigma_a^2$, is modified to include off-diagonal terms:

$$C_{ab} = \sigma_a \sigma_b (1 + \delta_{ab}), \quad \text{for } a \neq b, \tag{25}$$

where $\delta_{ab} = 0.5$ represents a 25% level of random correlation. Our analysis shows that introducing such correlations affects the final parameter estimates by less than 10%, confirming the robustness of treating these measurements as effectively uncorrelated.

Figures 1–3 collectively illustrate the effectiveness of our model in fitting the current observational data. Figure 1 shows the allowed confidence contours for model parameters, while Figure 2 confirms agreement between our model and $H(z)$ measurements, particularly at low redshifts. Figure 3 highlights the degeneracy between H_0 and r_d, a key factor in the ongoing Hubble tension. The robustness of our model across various datasets reinforces its suitability for late-time cosmological analyses.

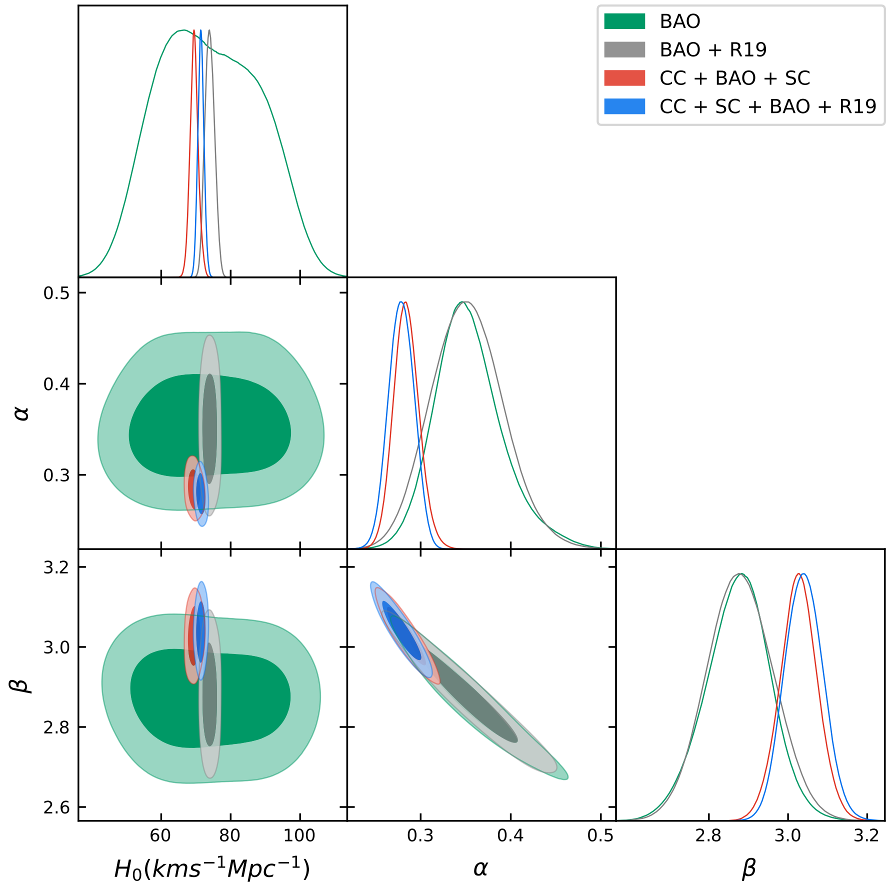

Figure 1. Confidence contours (1σ and 2σ) for model parameters obtained from various datasets.

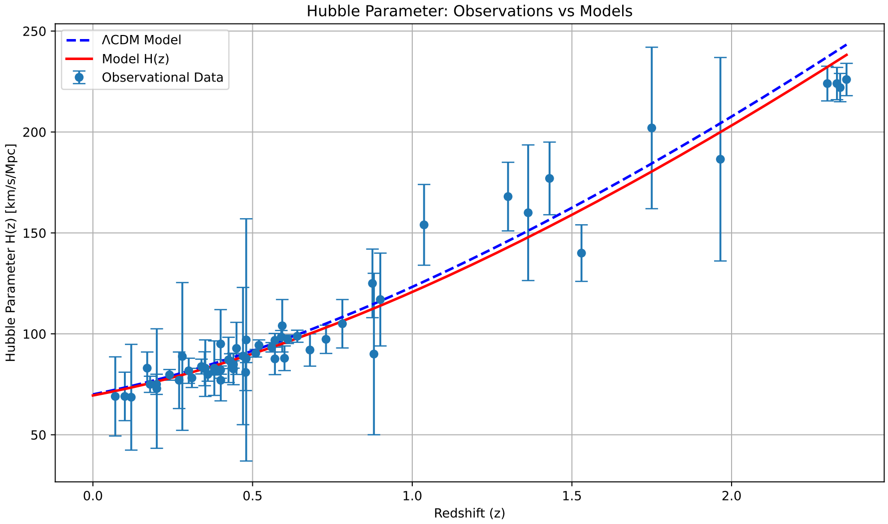

Figure 2. The Hubble parameter $H(z)$ as a function of redshift z for our model (solid line), compared with observational data and the ΛCDM model (dotted line).

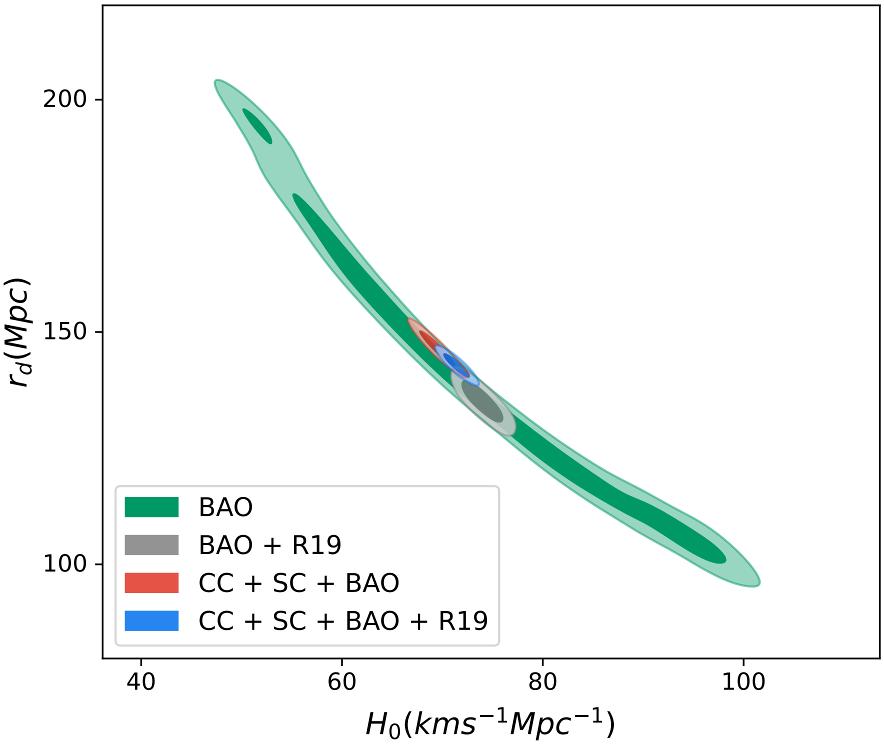

Figure 3. Correlation between the sound horizon scale r_d and the Hubble constant H_0 for various dataset combinations.

5. Cosmographic Parameters

To extract model-independent information about the universe's kinematic evolution, we analyze the cosmographic parameters, viz., the deceleration, jerk, and snap, which characterize the expansion history without assuming a specific gravitational theory. These parameters are derived directly from derivatives of the scale factor $a(t)$ and are instrumental in probing the nature of cosmic acceleration.

5.1. Deceleration Parameter

The deceleration parameter $q(z)$ quantifies the rate of cosmic acceleration and was defined in equation (19). Figure 4 depicts the reconstructed evolution of $q(z)$. Our analysis reveals a transition redshift $z_{\text{tr}} \approx 0.68$, marking the epoch at which the universe shifted from decelerated to accelerated expansion. This aligns closely with the ΛCDM prediction $z_{\text{tr}} \approx 0.67$ (Planck 2018). At the present epoch ($z = 0$), the best-fit value $q_0 \approx -0.59$ also agrees well with the standard value $q_0 \approx -0.55$, supporting the validity of our parameterization. Small deviations may reflect dataset-specific systematics or manifestations of the H_0 tension, an ongoing debate in modern cosmology.

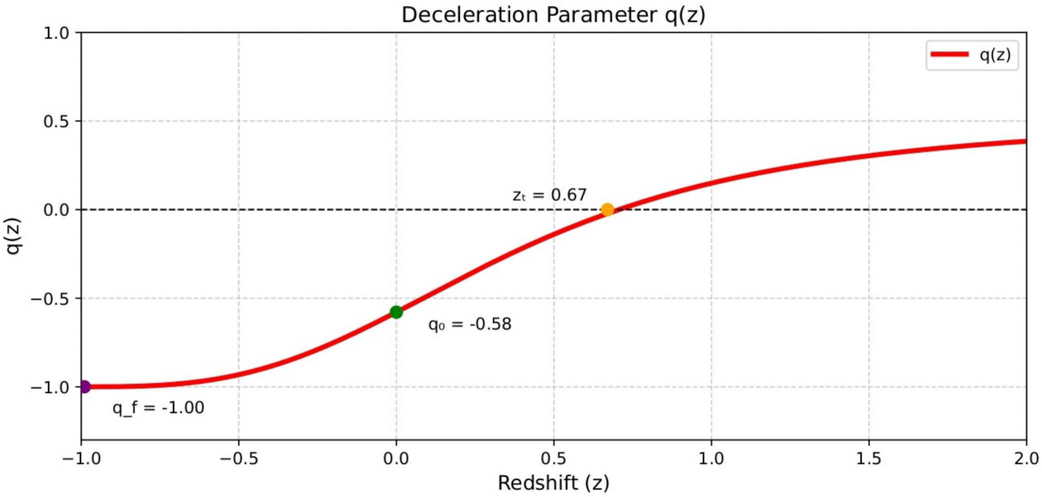

Figure 4. Reconstructed deceleration parameter $q(z)$. The transition from deceleration to acceleration occurs around $z_{\text{tr}} \approx 0.68$.

5.2. Jerk Parameter

The jerk parameter $j(z)$ measures the third-order time derivative of the scale factor and provides a test for the ΛCDM model, for which j is a constant and $j = 1$. It is defined by:

$$j(t) = \frac{\dddot{a}}{aH^3} = q(2q+1) + (1+z)\frac{dq}{dz}.$$

as a function of time t, and as a function of the redshift z, it is:

$$j(z) = \left[-1 + \frac{\alpha\beta(1+z)^\beta}{2\left[\alpha(1+z)^\beta + (1-\alpha)\right]}\right]\left[1 + \frac{\alpha\beta(1+z)^\beta}{\left[\alpha(1+z)^\beta + (1-\alpha)\right]}\right] + \left[\frac{\alpha\beta^2(1+z)^\beta(1+z)^\beta}{2\left[\alpha(1+z)^\beta + (1-\alpha)\right]^2}\right]$$

As shown in Figure 5, our model predicts $j(z) > 1$, with the difference from the ΛCDM model decreasing with time. This deviation suggests the presence of a time-varying dark energy component or modification to general relativity. Specifically, we obtain $j_0 \approx 1.01$ at $z = 0$, indicating a slight departure from the constant ΛCDM value. In the future, $j(z)$ approaches the ΛCDM value 1 as $z \to -1$.

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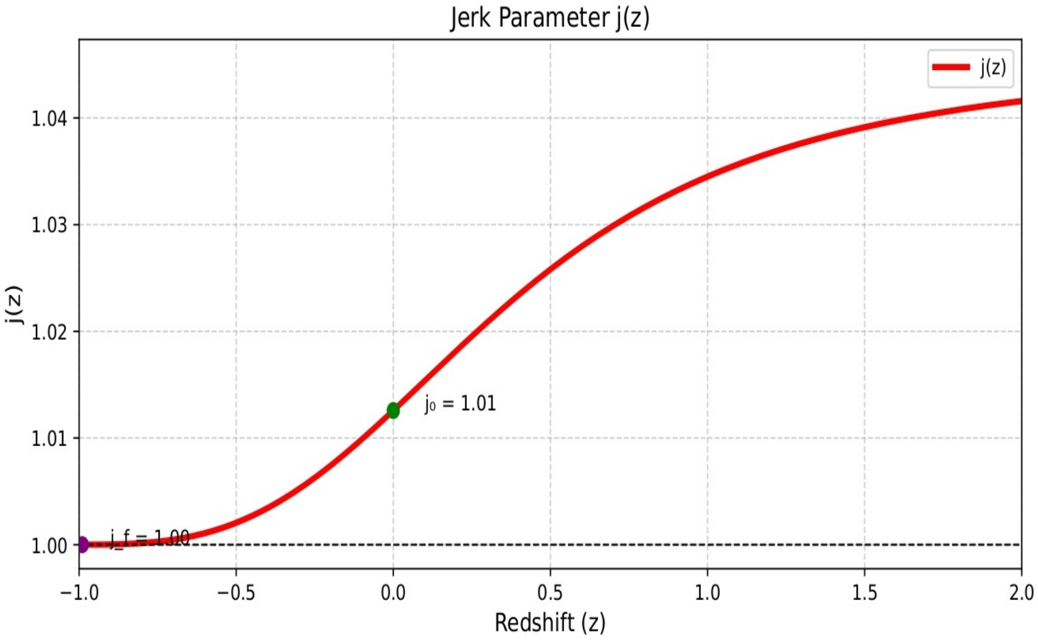

Figure 5. Redshift evolution of the jerk parameter $j(z)$. The slight deviation from the ΛCDM value ($j = 1$) suggests possible non-standard cosmological dynamics.

5.3. Snap Parameter

The snap parameter $s(z)$ probes the fourth derivative of the scale factor and offers insights into higher-order corrections in the expansion history. It is given by:

$$s(z) = -\left[-1 + \frac{3\alpha\beta(1+z)^{\beta}}{2\left[\alpha(1+z)^{\beta} + (1-\alpha)\right]}\right]$$

This parameter is especially relevant in precision cosmology and helps to distinguish fine deviations from the ΛCDM scenario. Figure 6 presents the evolution of $s(z)$. Whilst our results broadly follow the trend expected from the ΛCDM model at lower redshifts, they also exhibit potential departures at $z < 0$, hinting at richer dynamical features beyond the standard paradigm. These deviations, though subtle, could signal evolving dark energy or higher-order gravitational effects.

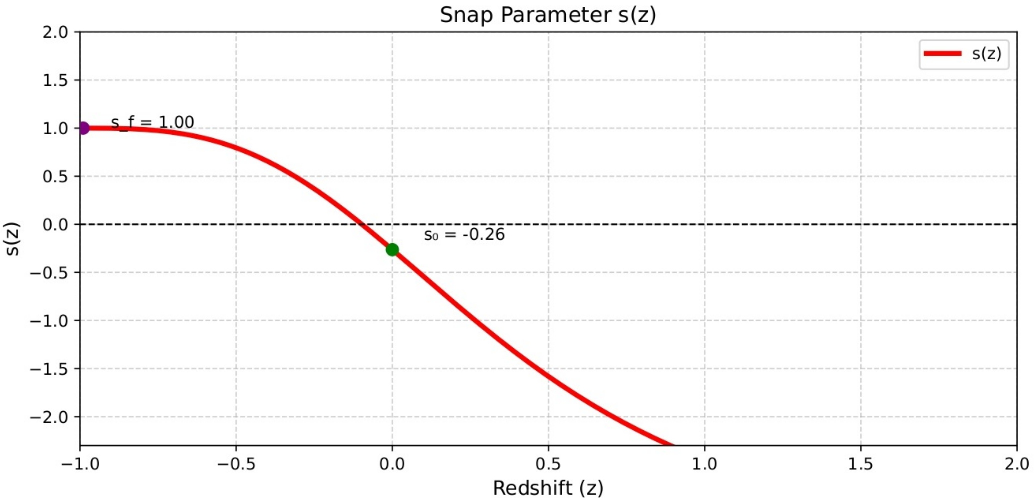

Figure 6. Reconstruction of the snap parameter $s(z)$ over redshift. The trajectory generally aligns with ΛCDM at low redshifts, with possible divergence as we go back in time.

6. Physical Characteristics of Quintessence as a Dark Energy Source: Cosmic Evolution

The equations of motion for a minimally coupled quintessence field can be derived using Eqs. (9) and (10), yielding expressions for the energy density and pressure of the scalar field as:

$$M_{\rm Pl}^{-2}\rho^{\phi} = 3H^2 - M_{\rm Pl}^{-2}\rho^{m}, \tag{26}$$

$$M_{\rm Pl}^{-2}p^{\phi} = (2q-1)H^2, \tag{27}$$

under the assumption of pressureless matter ($p^m = 0$). We consider a two-component, non-interacting cosmic fluid: matter and the quintessence scalar field. The respective conservation equations take the form:

$$\dot{\rho}^m + 3H\rho^m = 0, \qquad \dot{\rho}^{\phi} + 3H(\rho^{\phi} + p^{\phi}) = 0. \tag{28}$$

Solving the matter conservation equation leads to

$$\rho^m = \rho_0 a^{-3} = \rho_0(1+z)^3, \tag{29}$$

where $\rho_0 = 3M_{\rm Pl}^2 H_0^2 \Omega_0^m$ denotes the present-day matter density. Substituting this back yields:

$$\rho^m = 3M_{\rm Pl}^2 H_0^2 \Omega_0^m (1+z)^3. \tag{30}$$

Using Eq. (18), the scalar field energy density and pressure can be explicitly written as:

$$M_{\rm Pl}^{-2}H_0^{-2}\rho^{\phi}(z) = 3\Big[\alpha(1+z)^{\beta} + 1 - \alpha\Big] - \rho_0(1+z)^3, \tag{31}$$

$$M_{\rm Pl}^{-2}H_0^{-2}p^{\phi}(z) = \left[-3 + \frac{\alpha\beta(1+z)^{\beta}}{\big[\alpha(1+z)^{\beta} + 1 - \alpha\big]}\right]\Big[\alpha(1+z)^{\beta} + 1 - \alpha\Big]. \tag{32}$$

In the figures below, we plot the energy density, pressure and equation of state of the scalar field.

Figures 7–9 collectively demonstrate the viability of quintessence as a dark energy candidate. We see that the energy density is a positive monotonically decreasing function. The pressure is negative at late times, consistent with cosmological acceleration. Notably, the present-day value of the equation of state parameter is $\omega_0^{\phi} \approx -0.8$, consistent with observational expectations for a slowly evolving dark energy component. In future, $\omega_0^{\phi} \to -1$, coinciding with the value for the ΛCDM model. It is notable that latest observations from DESI DR2 [78] seem to indicate an evolving equation of state.

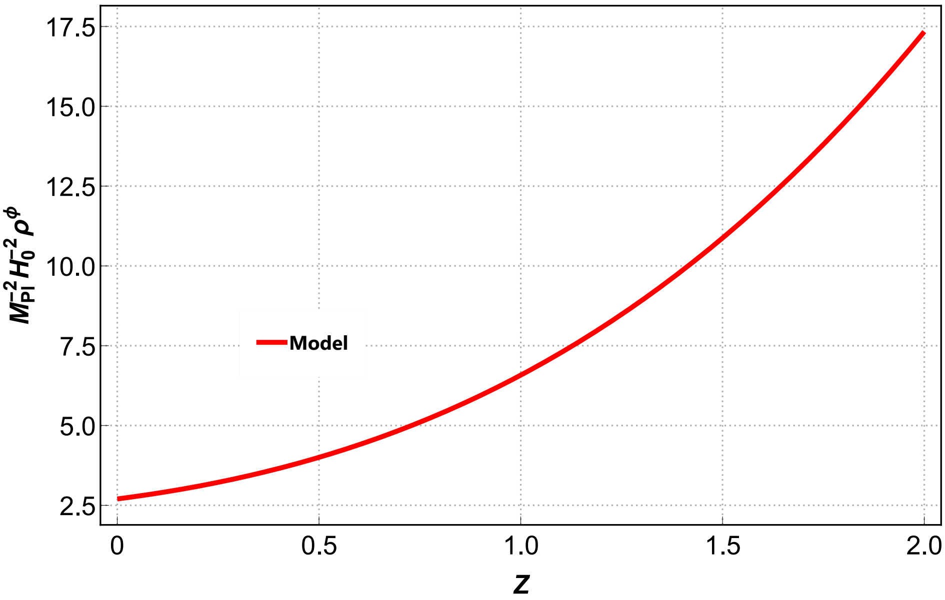

Figure 7. Normalized scalar field energy density $\rho^{\phi}(z)$ reconstructed using best-fit parameters from observational datasets.

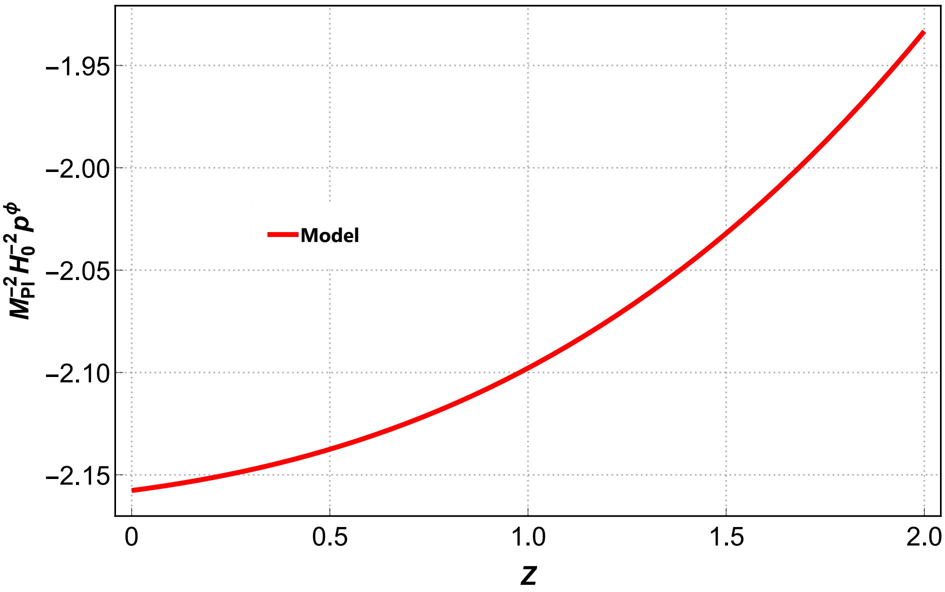

Figure 8. Normalized pressure $p^{\phi}(z)$ showing negative values across redshifts, essential for driving cosmic acceleration.

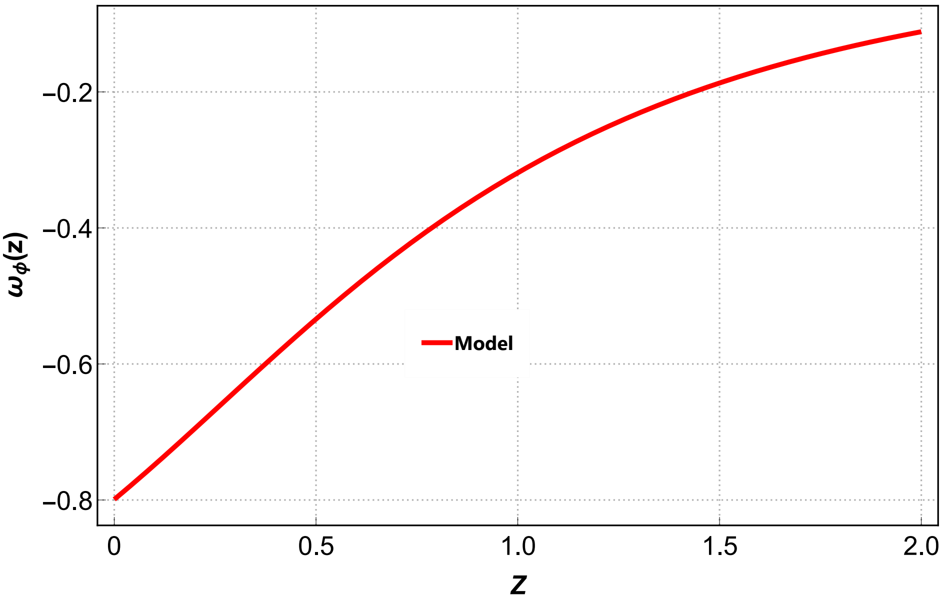

Figure 9. Evolution of the scalar field equation of state $\omega^{\phi}(z) = p^{\phi}/\rho^{\phi}$, remaining within the quintessence range $-1 < \omega^{\phi} < 0$.

Kinetic and Potential Terms of the Scalar Field

The dynamics of the quintessence field are determined by its kinetic energy $\dot{\phi}^2$ and potential $V(\phi)$. Using Eqs. (15), (16), and (29), we express these quantities as:

$$\dot{\phi}^2 = 2M_{\rm Pl}^2 H_0^2 \frac{\alpha\beta(1+z)^{\beta}}{2\left[\alpha(1+z)^{\beta}+1-\alpha\right]}\left[\alpha(1+z)^{\beta}+1-\alpha\right] - \rho_0(1+z)^3, \tag{33}$$

$$V(z) = M_{\rm Pl}^2 H_0^2 \left[1 - \frac{\alpha\beta(1+z)^{\beta}}{2\left[\alpha(1+z)^{\beta}+1-\alpha\right]}\right]\left[\alpha(1+z)^{\beta}+1-\alpha\right] - \frac{\rho_0}{2}(1+z)^3. \tag{34}$$

To reconstruct the potential $V(\phi)$ in physically meaningful terms, we transform the redshift-dependent quantities into scalar field-dependent ones. By numerically integrating $\dot{\phi}$ via:

$$\frac{d\phi}{dz} = \frac{\dot{\phi}}{(1+z)H(z)},$$

we obtain $\phi(z)$, which enables us to plot the parametric form $V(\phi)$.

The monotonic evolution of $\phi(z)$ in Figure 10 confirms the scalar field's regular behavior across cosmic time, ensuring physical consistency. The resulting potential $V(\phi)$ in Figure 11 shows a smooth, decreasing profile that flattens at early times - characteristic of viable quintessence models. This flattening allows the scalar field to slow-roll, mimicking a near-constant equation of state as observed in Figure 9, thus providing a natural mechanism for late-time acceleration.

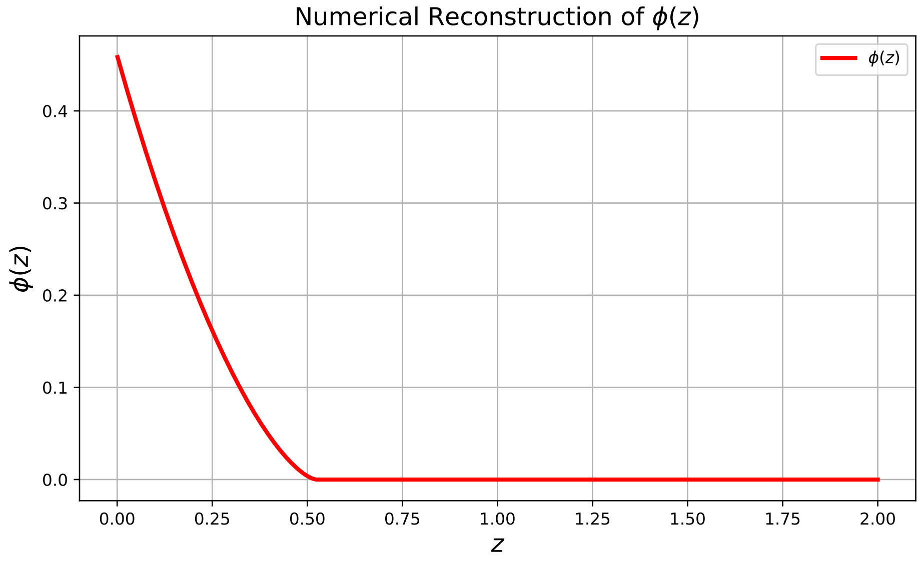

Figure 10. Evolution of the scalar field $\phi(z)$ reconstructed via numerical integration.

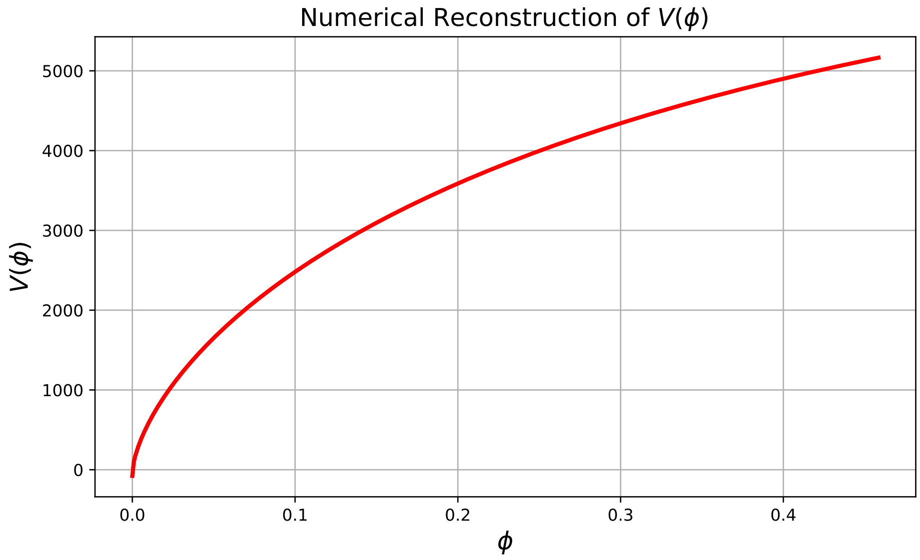

Figure 11. Reconstructed scalar potential $V(\phi)$, exhibiting a monotonic flattening behavior consistent with slow-roll quintessence dynamics.

7. Conclusion

In this work, we studied the late time acceleration of the universe via a scalar field cosmological model. Scalar fields provide a well-motivated class of models which enable us to understand the current acceleration of the universe. Such fields have several advantages over the ΛCDM model. The model exbibits quintessence at late times, thus reducing the need for unnatural initial conditions. Specially chosen potentials, like the one here, allows the energy density of the scalar field to evolve and approach the matter density at late times, offering a solution to the coincidence problem. By a strategic choice of the Hubble parameter, we capture the key features that allow confrontation with observations.

After introducing the field equations entailing the scalar field, we adopted the parametrisation of the Hubble parameter as in equation 17, which included the parameters H_0, α and β. We were then able to find $H(z)$ and $q(z)$. The latest observational datasets allowed a determination of these parammeters, as well as the sound horizon r_d at the drag epoch. The results from Table 1 show good agreement with those of the ΛCDM model. The cosmographic parameters, such as the deceleration, jerk and snap parameters were then studied and plotted against the redshift in Figures 4–6. We note from Figure 4 that $q_0 = -0.58$, and $z_t = 0.67$, in excellent agreement with the ΛCDM model. The jerk parameter shows a small deviation from the ΛCDM value of $j = 1$ in the past, but asymptotically approaches it in the future. The snap parameter aligns well with the ΛCDM curve at low redshifts, especially into the future, and deviates slightly going back into the past.

Table 1. 95% CL constraints on the cosmological parameters for the ΛCDM model and our model for different combinations of the datasets.

MCMC Results						
Model	Parameters	Priors	BAO	BAO+R19	CC+BAO+SC	CC+SC+BAO+R19
ΛCDM Model	H_0	[50,100]	$69.08^{+4.39}_{-6.25}$	$73.90^{+1.35}_{-2.78}$	$69.85^{+1.25}_{-2.38}$	$71.61^{+1.00}_{-1.93}$
	Ω_m	[0,1]	$0.25^{+0.02}_{-0.06}$	$0.25^{+0.02}_{-0.06}$	$0.26^{+0.01}_{-0.02}$	$0.26^{+0.01}_{-0.03}$
	Ω_Λ	[0,1]	$0.73^{+0.02}_{-0.03}$	$0.73^{+0.02}_{-0.06}$	$0.72^{+0.00}_{-0.01}$	$0.72^{+0.00}_{-0.01}$
	r_d	[100,200]	$149.50^{+10.03}_{-15.21}$	$139.44^{+2.91}_{-5.88}$	$146.54^{+2.59}_{-5.01}$	$143.29^{+2.21}_{-4.35}$
	$r_d/r_{d,fid}$	[0.9,1.1]	$1.00^{+0.06}_{-0.10}$	$0.94^{+0.02}_{-0.03}$	$0.99^{+0.01}_{-0.03}$	$0.96^{+0.01}_{-0.03}$
Proposed Model	H_0	[50,100]	$73.74^{+5.61}_{-2.70}$	$73.98^{+1.35}_{-2.34}$	$69.48^{+1.15}_{-2.28}$	$71.43^{+0.87}_{-1.61}$
	α	[0,0.6]	$0.353^{+0.033}_{-0.062}$	$0.351^{+0.04}_{-0.06}$	$0.284^{+0.013}_{-0.027}$	$0.278^{+0.01}_{-0.02}$
	β	[2,4]	$2.873^{+0.07}_{-0.17}$	$2.879^{+0.07}_{-0.16}$	$3.02^{+0.04}_{-0.09}$	$3.03^{+0.04}_{-0.08}$
	r_d	[100,200]	$140.66^{+31.47}_{-38.49}$	$135.03^{+2.86}_{-5.66}$	$146.54^{+2.32}_{-4.90}$	$142.79^{+1.63}_{-3.37}$
	$r_d/r_{d,fid}$	[0.9,1.1]	$1.005^{+0.07}_{-0.1}$	$0.99^{+0.06}_{-0.09}$	$0.996^{+0.06}_{-0.09}$	$1.003^{+0.06}_{-0.09}$

We then considered the quintessence field to be minimally coupled, and pressure-less matter. The matter and scalar field were taken to be conserved independently, leading to expressions for the scalar density and pressure. These were plotted against redshift, as well as the equation of state. These exhibited the familiar characteristics of a monotonically decreasing energy density, negative pressure at late times, and evolving equation of state which approaches the ΛCDM value of -1 in the future. The kinetic energy and potential of the scalar field were then determined, and numerically plotted. The field starts off being nearly constant at early times, and starts increasing at late times from around $z = 0,5$.

Taken together, all the results reinforce the consistency of this scalar field model with present-day observations and its potential to act as a physically motivated alternative to the cosmological constant, and also with the potential to solve some of the issues with the ΛCDM model, e.g., the fine tuning and coincidence problems.

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