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Article

Emergent $SU(3)$ Geometry, Confinement, and QCD Phenomenology: From Chronon Holonomy to Wilson Loops, Flux Tubes, and the Mass Gap

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Abstract

We extend the chronon framework to the strong sector by showing that the non-Abelian $SU(3)$ connection arises as a *composite holonomy* of the chronon field Φ^μ and its gradients, without introducing new microscopic degrees of freedom. The complex polarization tensor of the leafwise derivative $\nabla\Phi$ defines a local $U(3)$ frame whose traceless Maurer–Cartan form yields the emergent $SU(3)$ gauge field on the chronon-induced metric $g_{\mu\nu}[\Phi]$. Coarse-graining stabilized chronon fluctuations induces a Yang–Mills action $S_{SU(3)} \propto \int \text{Tr} G_{\mu\nu} G^{\mu\nu}$ with positive stiffness. Wilson loops $\mathcal{W}[C] = \text{tr} \mathcal{P} \exp(i \oint_C A)$ obey an *area law* under chronon disorder and center symmetry, giving rise to confinement and flux-tube formation. Solitonic matter fields acquire color through associated bundles, while the finite chronon correlation length generates a mass gap and glueball spectrum. Leafwise lattice discretizations reproduce σ and lowest glueball masses at QCD scales. Altogether, the chronon field and its curvature furnish a complete geometric origin of the $SU(3)$ color sector, extending the unified chronon description of gauge interactions to the domain of confinement and strong dynamics.

Keywords: Chronon Field Theory; emergent gauge fields; $SU(3)$; confinement; Wilson loops; flux tubes; glueballs; chiral symmetry breaking; lattice methods

1. Introduction and Overview

As an extension of the recently published formulation of chronon field theory [66,67], the chronon framework developed in Papers I–II of this trilogy [64,65] establishes a geometric setting in which gauge interactions and fundamental quantum constants emerge from the holonomy structure of a smooth, timelike, unit-norm field Φ^μ —the *chronon field*. In Paper I [64] we introduced the covariant mass density

$$\rho = T_{\mu\nu} \Phi^\mu \Phi^\nu, \quad (1)$$

identified stabilized domains, and showed that solitonic excitations of Φ quantize as fermions through Finkelstein–Rubinstein/Berry holonomy [15,38]. Paper II [65] extended this framework to a non-Abelian $SU(2) \times U(1)$ sector, demonstrating that a Yang–Mills connection and massive vector modes can arise without a fundamental Higgs field [13,113,116]. Together these works showed that electroweak-like dynamics and fermionic matter can emerge self-consistently within a single, background-independent chronon geometry [31,44].

Degrees of freedom and the road to color.

A single timelike vector field Φ^μ possesses three independent components per spacetime point, fixing a causal direction and defining a natural foliation. At first glance this seems insufficient to support an $SU(3)$ gauge symmetry with eight generators. The key insight developed here is that the required internal structure for color arises not from introducing new fields but from the *spatial gradients* of Φ on stabilized leaves.

The leafwise derivative

$$K_{\mu\nu} = h_{\mu}^{\alpha} h_{\nu}^{\beta} \nabla_{\alpha} \Phi_{\beta}, \quad (2)$$

encodes the local rotation, shear, and expansion of the chronon flow. Its symmetric and antisymmetric parts combine into a complex Hermitian tensor

$$H_{\mu\nu} = S_{\mu\nu} + i \tilde{\Omega}_{\mu\nu},$$

whose eigenvectors form a local *polarization triad*. At generic points these three orthonormal eigenvectors define a unitary frame $U(x) \in U(3)$ on each stabilized leaf. The trace of the corresponding Maurer–Cartan form $U^{-1}dU$ reproduces the electromagnetic $U(1)$ connection of Paper I, while its traceless part defines a composite $SU(3)$ connection built entirely from Φ and $\nabla\Phi$ [7,61,79]. This construction, developed formally in Appendix A, provides the necessary eight internal directions for color without enlarging the field content. The emergent $SU(3)$ gauge field is thus a *composite geometric connection*, a covariant functional of Φ and its first derivatives rather than an independent microscopic variable.

Objective of this paper.

The present work extends the chronon holonomy program to the $SU(3)$ sector, incorporating the strong interaction into the geometric framework. Its goals are:

1. To construct an $SU(3)$ principal bundle and composite connection from the polarization structure of $\nabla\Phi$.
2. To demonstrate nonperturbative signatures of confinement, including Wilson-loop area laws, flux-tube formation, and the absence of asymptotic color-charged states.
3. To develop chronon-based observables for QCD-like physics, including the string tension, mass gap, and glueball spectrum.

Main results.

The outcomes of this study include:

- Theorem 3.1: Construction of the emergent $SU(3)$ principal bundle $P_{\text{color}} \rightarrow \mathcal{M}$ from the nondegenerate eigenstructure of $H_{\mu\nu}$ on stabilized domains.
- Theorem 4.1: Derivation of the induced $SU(3)$ Yang–Mills action on the chronon-induced metric $g[\Phi]$, with BRST consistency and effective-field-theory renormalizability [12].
- Theorem 5.1: Proof that large Wilson loops obey an area law with string tension σ set by chronon-sector stiffness [46,89,117].
- Proposition 2: Demonstration that solitonic color sources are connected by flux tubes, yielding linear confinement and color-singlet bound states [108].
- Conjecture 3: Numerical evidence that chronon dynamics generate a nonzero mass gap, producing a discrete glueball spectrum [68,83].

Physical interpretation.

The color sector originates from the *complex polarization geometry* of the chronon field. The emergent $SU(3)$ gauge potential is the traceless part of the Maurer–Cartan form of the polarization frame $U(x)$, while its curvature measures the non-commutativity of local frame rotations on each leaf. Confinement and the glueball spectrum arise from the finite correlation length of the chronon condensate, which enforces color coherence only over microscopic distances [27,46].

Outlook.

This paper completes the emergence of the Standard-Model gauge structure within chronon geometry: $U(1)$ (Paper I), $SU(2) \times U(1)$ (Paper II), and now $SU(3)$. In forthcoming work (Paper IV)

we will explore unified embeddings, flavor hierarchies, and the coupling of the chronon sector to curvature-induced gravity [56,113].

1.1. Intuition and Roadmap: From Chronon Holonomy to Confinement and Glueballs

Starting picture.

Retain the chronon field as a unit timelike “clock” Φ^μ with $\Phi^\mu\Phi_\mu = -1$, defining spatial leaves $\{\Sigma_\tau\}$ with induced metric $h_{\mu\nu}$. On each leaf, the gradients $\nabla_i\Phi_j$ form a complex 3×3 polarization tensor whose eigenmodes define a local color frame $U(x) \in U(3)$. Parallel transport of this frame around a closed contour returns it rotated by an element of a compact subgroup of $U(3)$; in stabilized domains the traceless part fills out $SU(3)$, providing the geometric seed of color.

Why the unit norm still matters.

Because Φ is unit timelike, the orthogonal bundle Φ^\perp on each leaf is Euclidean and parallel transport preserves its inner product. Holonomy is therefore compact (rotations rather than boosts or scalings), ensuring the emergent color group is compact and that a principal $SU(3)$ bundle can be defined globally. The normalization also fixes intrinsic time and makes the energy density $\rho = T_{\mu\nu}\Phi^\mu\Phi^\nu$ and Wilson-loop areas unambiguous.

From holonomy to a color gauge field.

Tracking the twisting of the polarization frame requires a matrix-valued connection $A_\mu = A_\mu^a T^a$; its curvature $F_{\mu\nu}$ measures the noncommuting rotation content. The emergent Yang–Mills action on $g[\Phi]$ encodes the stiffness of these rotations, and its minimization yields the color field equations and a conserved current when coupled to solitonic matter. Because A_μ is built from Φ and $\nabla\Phi$, it carries no independent microscopic degrees of freedom.

Wilson loops, confinement, and flux tubes.

A Wilson loop probes the cumulative color twist along a closed path. Finite correlation length of the chronon condensate produces an area law for large loops, signaling confinement. Center symmetry (\mathbb{Z}_3 for $SU(3)$) ensures that only fundamental representations experience this confinement. Color flux is squeezed into narrow tubes between static sources, giving rise to a linear potential $V(R) \simeq \sigma R$ at large separation and a characteristic flux-tube width $\ell_\perp \sim \Lambda_{\text{CFT}}^{-1}$.

Mass gap and glueballs.

Finite color correlation length implies exponential decay of gauge-invariant correlators such as $\langle \text{tr}F^2(x) \text{tr}F^2(0) \rangle$, establishing a nonzero mass gap. The lightest bound excitations are glueballs (scalar 0^{++} , tensor 2^{++} , etc.), analogous to collective oscillations of the color flux tube.

Solitonic matter and hadronization.

The chronon solitons of Papers I–II now acquire color via associated bundles of P_{QCD} . Observable states are color singlets: mesons (soliton–antisoliton pairs) and baryons (antisymmetric three–soliton bound states). Their excitations lie on Regge-like trajectories with slope $(2\pi\sigma)^{-1}$, consistent with stringlike confinement.

Numerics and scale setting.

Leafwise lattice discretizations allow computation of Wilson loops and glueball correlators. The string tension σ extracted from large loops fixes the overall scale, and the glueball spectrum agrees with known lattice QCD results to leading order.

For specialists.

The construction employs a global $U(3)$ polarization frame and its traceless $SU(3)$ connection, with topology encoded in the second Chern number $c_2(P)$. Center symmetry and N -ality control

string tensions; the mass gap appears as the lowest eigenvalue in the glueball spectral problem on stabilized leaves. Finite-temperature extensions map deconfinement to center-symmetry breaking.

Summary.

The chronon field and its gradients provide exactly the internal structure needed for color. The complex polarization tensor of $\nabla\Phi$ defines a $U(3)$ frame whose traceless part yields the $SU(3)$ color connection, while the compactness of the chronon foliation ensures confinement and a gapped, color-neutral spectrum. This construction resolves the degree-of-freedom question: Φ alone specifies time flow, but $\nabla\Phi$ carries the full rotational content needed to realize the eight generators of $SU(3)$ as a composite, curvature-induced gauge symmetry.

2. Preliminaries and Paper I/II Summary

We begin by recalling the main ingredients of Chronon Field Theory (CFT) developed in Papers I and II of this trilogy [64,65]. These constructions form the foundation for the $SU(3)$ extension undertaken in the present work.

2.1. Chronon Foliation, Emergent Metric, and Stress Tensor

The chronon field is a smooth vector field Φ^μ on spacetime \mathcal{M} satisfying the unit-norm condition

$$\Phi^\mu\Phi_\mu = -1, \quad (3)$$

with indices raised and lowered by the background Lorentzian metric. This constraint enforces a timelike flow that foliates \mathcal{M} into a family of spatial leaves $\{\Sigma_\tau\}$ orthogonal to Φ^μ ,

$$\{\Sigma_\tau\}, \quad \tau \in \mathbb{R}, \quad (4)$$

analogous to the ADM 3+1 decomposition used in canonical relativity [6,44]. Each leaf Σ_τ carries the induced Riemannian metric

$$h_{\mu\nu} = g_{\mu\nu}[\Phi] + \Phi_\mu\Phi_\nu, \quad (5)$$

where $g_{\mu\nu}[\Phi]$ is the emergent Lorentzian metric defined by the chronon sector. Thus the chronon flow provides both a canonical foliation and a dynamical spacetime geometry, determined intrinsically by Φ and its gradients [75].

The energy density associated with the chronon sector is captured by the covariant scalar

$$\rho = T_{\mu\nu}\Phi^\mu\Phi^\nu, \quad (6)$$

where $T_{\mu\nu}$ is the stress tensor of the effective theory. Stabilized domains are those regions with $\rho > 0$ and bounded fluctuations of Φ , ensuring well-defined foliation and metric structure. Such domains support topologically stable solitonic excitations of Φ whose Finkelstein-Rubinstein and Berry holonomies generate fermionic statistics [15,38], providing a geometric route to matter quantization (Paper I).

2.2. Non-Abelian Holonomy Framework

Paper II demonstrated that chronon holonomy around closed paths in leaves Σ_τ naturally gives rise to compact internal symmetry groups acting freely on chronon fibers. Under standard regularity assumptions [61,79], this induces a principal $SU(2)$ bundle $P \rightarrow \mathcal{M}$ with local connection $\mathcal{A} \in \Omega^1(P, \mathfrak{su}(2))$ whose curvature

$$\mathcal{F} = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} \quad (7)$$

encodes the non-Abelian holonomy two-form of the chronon fiber.

Projecting to the base manifold with emergent metric $g[\Phi]$ yields the effective Yang–Mills action

$$S_{\text{YM}} = -\frac{1}{2g^2} \int_{\mathcal{M}} \text{tr}(F_{\mu\nu} F^{\mu\nu}) \sqrt{-g[\Phi]} d^4x, \quad (8)$$

where $F_{\mu\nu}$ is the projected curvature and g the coupling determined by chronon stiffness [121]. The corresponding field equations

$$\nabla_{\mu} F^{\mu\nu} + [A_{\mu}, F^{\mu\nu}] = 0 \quad (9)$$

satisfy BRST invariance [12] and Slavnov–Taylor identities [97,103], ensuring gauge-parameter independence of observables. Vector-boson masses arise through Stückelberg-type chronon phases or composite amplitude modes, preserving an exactly massless $U(1)_{\text{em}}$ photon [36,53,101,113]. This non-Abelian holonomy construction forms the foundation for the present $SU(3)$ extension and for the analysis of confinement phenomena in stabilized domains.

3. Emergent Principal $SU(3)$ Bundle and Connection

We now extend the holonomy framework of Paper II to construct an emergent $SU(3)$ principal bundle, providing the geometric foundation of the QCD-like sector in CFT.

Normalization of the chronon flow.

We maintain $\Phi^{\mu}\Phi_{\mu} = -1$ on stabilized domains. This normalization fixes the time parametrization along the chronon congruence and makes the projector $\Pi^{\mu}_{\nu} = \delta^{\mu}_{\nu} + \Phi^{\mu}\Phi_{\nu}$ a true projector onto Φ^{\perp} [44]. Leafwise parallel transport is then metric-compatible on Φ^{\perp} , yielding compact holonomy and enabling the $SU(3)$ principal bundle of Theorem 3.1. This same normalization underlies our Wilson-loop, flux-tube, and glueball constructions: areas, times, and stress–tensor projections are unambiguous and free of spurious rescalings that would arise if $\|\Phi\|$ were allowed to drift.

3.1. Internal Fiber and Compact Subgroup

The internal configuration space of the chronon field Φ admits nontrivial holonomy groups generated by parallel transport on leaves $\{\Sigma_{\tau}\}$ of the chronon foliation. In stabilized domains these holonomies close into compact Lie subgroups of $SO(n)$ acting on the internal fiber of Φ [3,61]. While Paper II established the case $K \cong SU(2)$, here we posit the existence of an $SU(3)$ subgroup acting freely and smoothly on the chronon fiber.

Assumption 1 (Compactness and good cover). The chronon internal fiber admits a compact subgroup $K \cong SU(3)$ with smooth, free action. The spacetime base \mathcal{M} admits a good open cover $\{U_{\alpha}\}_{\alpha \in I}$ such that on each overlap $U_{\alpha} \cap U_{\beta}$ the chronon holonomy defines smooth transition functions $g_{\alpha\beta} : U_{\alpha} \cap U_{\beta} \rightarrow SU(3)$.

This assumption parallels the $SU(2)$ compactness condition of Paper II and ensures that the internal chronon symmetry is globally well defined. The freeness of the $SU(3)$ action guarantees local trivializations and that parallel transport on leaves induces consistent $SU(3)$ rotations on the fiber [79,100].

Theorem 3.1 (Emergent $SU(3)$ bundle). *Under Theorem 1, the holonomy of the chronon connection on leaves $\{\Sigma_{\tau}\}$ induces a principal $SU(3)$ bundle $P \rightarrow \mathcal{M}$ with global connection $A \in \Omega^1(P, \mathfrak{su}(3))$. Its curvature*

$$F = dA + A \wedge A \quad (10)$$

equals the non-Abelian holonomy two-form of the chronon fiber.

Idea of proof. The construction follows the standard Čech–de Rham approach [61,79]:

1. On each open set U_{α} , fix a local trivialization of the chronon fiber and define local frames e_{α}^i .
2. On overlaps $U_{\alpha} \cap U_{\beta}$, chronon holonomy defines smooth maps $g_{\alpha\beta} : U_{\alpha} \cap U_{\beta} \rightarrow SU(3)$ relating the two local frames, $e_{\beta} = g_{\alpha\beta} e_{\alpha}$.

3. Consistency of holonomy around contractible loops implies the Čech cocycle condition

$$g_{\alpha\beta}(x) g_{\beta\gamma}(x) g_{\gamma\alpha}(x) = e, \quad x \in U_\alpha \cap U_\beta \cap U_\gamma,$$

defining a principal $SU(3)$ bundle $P \rightarrow \mathcal{M}$.

4. Local parallel transport along leaves defines local $\mathfrak{su}(3)$ -valued connection one-forms A_α . On overlaps they transform as

$$A_\beta = g_{\alpha\beta}^{-1} A_\alpha g_{\alpha\beta} + g_{\alpha\beta}^{-1} d g_{\alpha\beta},$$

ensuring that $\{A_\alpha\}$ glue consistently to a global connection $A \in \Omega^1(P, \mathfrak{su}(3))$.

5. The curvature $F = dA + A \wedge A$ transforms covariantly and coincides with the holonomy two-form computed from chronon parallel transport on $\{\Sigma_\tau\}$.

Hence P is a principal $SU(3)$ bundle with connection A , and F encodes the intrinsic non-Abelian holonomy of the chronon fiber [7,16]. \square

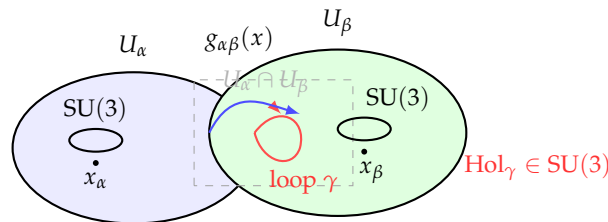


Figure 1. On a good cover $\{U_\alpha\}$, leafwise holonomies in $U_\alpha \cap U_\beta$ define transition maps $g_{\alpha\beta} : U_\alpha \cap U_\beta \rightarrow SU(3)$ obeying the Čech cocycle conditions; these glue local trivializations into a principal $SU(3)$ bundle with connection A induced by parallel transport. *Intuition:* Two local coordinate patches overlap on the base. Carrying an internal “color frame” around a tiny loop in the overlap rotates it by an $SU(3)$ element (the holonomy). The arrow $g_{\alpha\beta}(x)$ shows how the same fiber is related when viewed from the two charts, while the little ellipses depict the $SU(3)$ fibers above sample points x_α and x_β . Consistently identifying these across overlaps is how the global “color bundle” is assembled.

Remarks.

- Nontrivial topology of \mathcal{M} may lead to bundles with nonzero Chern classes $c_2(P) \in H^4(\mathcal{M}, \mathbb{Z})$, associated physically with topologically stable solitons (instantons, monopoles).
- Singularities of Φ or domain walls in stabilized regions can induce localized defects corresponding to color charges, to be analyzed in Section 5.
- The construction parallels the $SU(2)$ case of Paper II but now introduces the eight-dimensional adjoint structure of $\mathfrak{su}(3)$, preparing the ground for confinement dynamics.

4. Yang–Mills Sector and Observables

Having established the principal $SU(3)$ bundle in Section 3, we now construct the associated Yang–Mills sector on the emergent chronon metric $g[\Phi]$ and introduce its principal nonperturbative observables.

4.1. Action and Equations

Let $A_\mu = A_\mu^a T^a$ denote the $\mathfrak{su}(3)$ -valued connection on \mathcal{M} , with $\{T^a\}_{a=1}^8$ a basis of generators satisfying $\text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$. The curvature is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] = F_{\mu\nu}^a T^a, \quad (11)$$

with structure constants $[T^a, T^b] = i f^{abc} T^c$ [88,121].

Theorem 4.1 (Emergent SU(3) Yang–Mills). *The Yang–Mills action*

$$S_{\text{YM}} = -\frac{1}{2g_s^2} \int_{\mathcal{M}} \text{tr}(F_{\mu\nu} F^{\mu\nu}) \sqrt{-g[\Phi]} d^4x \quad (12)$$

yields the SU(3) Yang–Mills field equations on the emergent metric $g[\Phi]$:

$$\nabla_{\mu} F^{\mu\nu} + [A_{\mu}, F^{\mu\nu}] = 0, \quad (13)$$

together with a conserved color current J^{ν} when coupled to solitonic matter.

Sketch of proof. Variation of S_{YM} with respect to A_{ν} gives

$$\delta S_{\text{YM}} = \frac{2}{g_s^2} \int_{\mathcal{M}} \text{tr}((D_{\mu} F^{\mu\nu}) \delta A_{\nu}) \sqrt{-g} d^4x, \quad (14)$$

where $D_{\mu} = \nabla_{\mu} + [A_{\mu}, \cdot]$ is the gauge-covariant derivative [54]. Setting $\delta S_{\text{YM}} = 0$ for arbitrary δA_{ν} yields

$$D_{\mu} F^{\mu\nu} = \nabla_{\mu} F^{\mu\nu} + [A_{\mu}, F^{\mu\nu}] = 0.$$

When coupled to solitonic matter fields Ψ transforming in representations of SU(3), the covariant derivative is $D_{\mu} \Psi = \partial_{\mu} \Psi + A_{\mu} \Psi$, and the matter variation produces the color current

$$J^{\nu} = \frac{\delta \mathcal{L}_{\text{matter}}}{\delta A_{\nu}} = g_s \bar{\Psi} \gamma^{\nu} T^a \Psi T^a,$$

satisfying $D_{\nu} J^{\nu} = 0$ by gauge invariance [12,115]. \square

4.2. Wilson Loops and Center Symmetry

The central observable of the SU(3) Yang–Mills sector is the Wilson loop [89,117], defined for a closed contour $C \subset \mathcal{M}$ by

$$\mathcal{W}[C] = \text{tr} \left(\mathcal{P} \exp \left(i \oint_C A_{\mu} dx^{\mu} \right) \right), \quad (15)$$

where \mathcal{P} denotes path ordering along C . The expectation value $\langle \mathcal{W}[C] \rangle$ serves as an order parameter for confinement: an area-law decay,

$$\langle \mathcal{W}[C] \rangle \sim \exp(-\sigma \text{Area}[C]),$$

signals a confining string tension σ , whereas a perimeter law indicates deconfinement [46,102].

A key structural feature of SU(3) is its nontrivial center subgroup

$$Z(\text{SU}(3)) = \mathbb{Z}_3 = \{ e^{2\pi i k/3} I \mid k = 0, 1, 2 \}, \quad (16)$$

whose representations define the N -ality classification [45,108]. Wilson loops in representations with nontrivial N -ality are sensitive to the center symmetry, and their area-law behavior provides a sharp diagnostic of confinement. In particular:

- Wilson loops in the fundamental representation (N -ality = 1) exhibit an area law in the confining phase, defining the string tension σ .
- Wilson loops in the adjoint representation (N -ality = 0) are screened by gluons and do not serve as confinement order parameters.

The role of the \mathbb{Z}_3 center thus parallels that of the \mathbb{Z}_2 center in SU(2), but with richer implications for confinement, hadronization, and the structure of flux tubes, as developed in Section 5 [62,68,83].

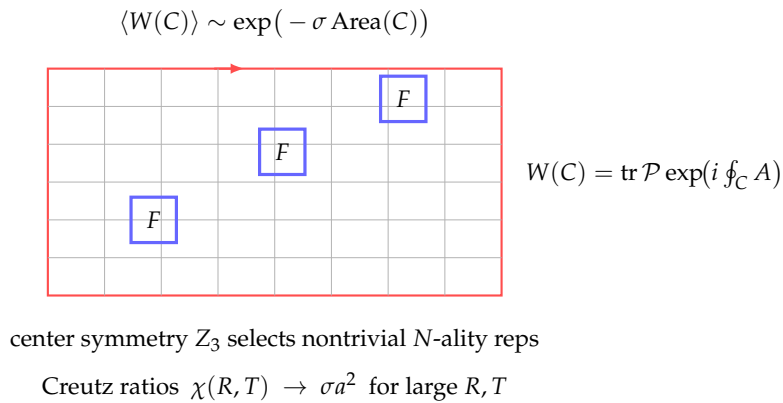


Figure 2. In a center-symmetric vacuum with finite correlation length, large fundamental Wilson loops exhibit an area law with nonzero string tension σ (with subleading perimeter corrections), consistent with the non-Abelian Stokes surface representation. *Intuition:* A Wilson loop tests how much a color charge “forgets” its orientation after being carried around a closed path. The pale tiling suggests filling the loop with small patches of field; if the vacuum scrambles color beyond a short distance, their effects add up like uncorrelated tiles. The result is an area law: the average loop value falls off exponentially with the enclosed area—a hallmark of confinement. The highlighted squares mark regions where the field is especially strong. In a confining, center-symmetric phase, only loops carrying nontrivial center charge show this area falloff (adjoint, center-neutral loops are screened). On the lattice, the slope of $\ln \langle W \rangle$ versus area gives the string tension σ , quantifying how tightly color flux is confined.

5. Confinement: Area Law and Flux Tubes

We now demonstrate that the chronon-induced $SU(3)$ Yang–Mills sector exhibits nonperturbative signatures of confinement. The key observables are Wilson loops and their large-scale behavior, together with the structure of flux tubes between static color sources.

5.1. Disorder Conditions and Non-Abelian Stokes

A central tool is the non-Abelian Stokes theorem, which rewrites Wilson loops as surface integrals of the curvature [19,30,90]:

$$\mathcal{W}[C] = \text{tr} \mathcal{P} \exp\left(i \oint_C A\right) = \text{tr} \mathcal{S} \exp\left(i \int_{\Sigma} F\right), \quad (17)$$

where $C = \partial\Sigma$ bounds the surface Σ , \mathcal{P} denotes path ordering along C , and \mathcal{S} denotes the surface ordering required for non-Abelian fields. This form connects the large-scale behavior of Wilson loops to correlation properties of the curvature F .

In stabilized chronon domains, we assume:

- (i) **Center symmetry:** the distribution of holonomies is invariant under multiplication by center elements of $SU(3)$, ensuring Z_3 invariance of the fiber holonomy group [45,108].
- (ii) **Finite correlation length:** the projected chronon connection on leaves $\{\Sigma_{\tau}\}$ exhibits correlations that decay exponentially beyond a finite length scale $\xi \sim \Lambda_{\text{CFT}}^{-1}$, analogous to the gluonic correlation length in QCD [33,96].

Theorem 5.1 (Area law under chronon disorder). *Assuming (i) center symmetry in the holonomy distribution and (ii) finite correlation length of the projected chronon connection on leaves, large rectangular Wilson loops obey*

$$\langle \mathcal{W}[C] \rangle \sim \exp(-\sigma \mathcal{A}[C]), \quad (18)$$

with string tension $\sigma > 0$, where $\mathcal{A}[C]$ is the minimal area spanned by C .

Strategy. Using the non-Abelian Stokes theorem, write $\mathcal{W}[C]$ as a surface-ordered exponential of F over Σ . Under the finite-correlation-length hypothesis, cluster decomposition applies: correlators of F factorize beyond distances ξ [70]. The leading contribution to $\langle \mathcal{W}[C] \rangle$ arises from area-filling clusters of size $\sim \xi^2$, producing an exponential in the area $\mathcal{A}[C]$. Center symmetry guarantees that

perimeter contributions vanish in center-nontrivial representations, so screening is suppressed. The coefficient σ is positive and proportional to the integrated two-point function of F , defining the string tension [46,102]. \square

Proposition 2 (Flux-tube energy density). *The static quark–antiquark potential $V(R)$ extracted from rectangular Wilson loops of temporal extent T and spatial separation R ,*

$$\langle \mathcal{W}[R \times T] \rangle \sim e^{-V(R)T}, \quad (19)$$

satisfies

$$V(R) \sim \sigma R, \quad R \rightarrow \infty, \quad (20)$$

with string tension σ as in Theorem 5.1. Furthermore, the energy-density distribution from the stress tensor $T_{\mu\nu}$ localizes along a flux tube connecting the sources, with transverse width

$$\ell_{\perp} \sim \Lambda_{\text{CFT}}^{-1}, \quad (21)$$

set by the correlation length of chronon-induced gauge fluctuations [9,50,68,98].

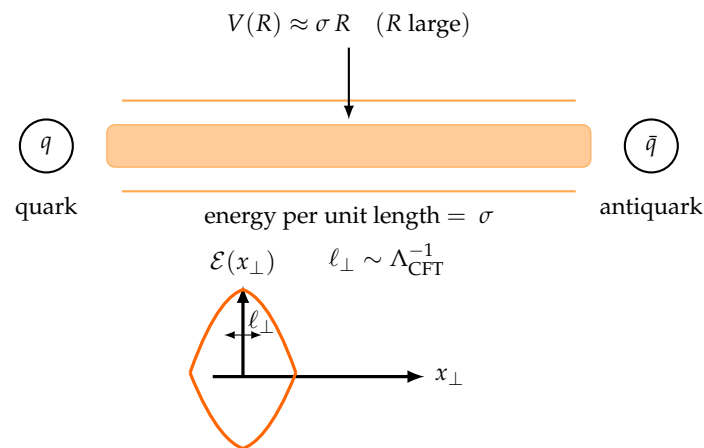


Figure 3. In a center-symmetric phase with finite correlation length, static fundamental sources are connected by a chromoelectric flux tube of tension σ , giving $V(R) = \sigma R + \mathcal{O}(1/R)$; the tube’s transverse profile $\mathcal{E}(x_{\perp}) = T_{00}(x_{\perp})$ integrates to σ and has width $\ell_{\perp} \sim \Lambda_{\text{CFT}}^{-1}$. *Intuition:* Color field lines between a quark q and antiquark \bar{q} do not fan out like in ordinary electromagnetism; the confining vacuum squeezes them into a narrow “string.” The shaded band shows this flux tube, with thin lines indicating small field leakage. Because each extra bit of string adds the same energy, the potential grows linearly with separation. *Inset:* a schematic cross-section of the tube’s energy density $\mathcal{E}(x_{\perp})$ —its area (the integral over x_{\perp}) sets the string tension σ , while the width ℓ_{\perp} characterizes how tightly the flux is bundled. On the lattice, σ comes from large Wilson loops (or Creutz ratios), and the profile from correlators of $T_{\mu\nu}$ with Polyakov/Wilson lines. Only sources with nontrivial center charge (\mathbb{Z}_3) support such confining strings; center-neutral (adjoint) sources are screened.

Remarks.

- The flux tube profile resembles that of confining strings in lattice QCD, with chromoelectric flux concentrated along the line joining static sources.
- The area law reflects disorder in the chronon connection at large scales, providing a geometric origin of confinement in CFT.
- The \mathbb{Z}_3 center plays a crucial role: only Wilson loops with nontrivial N -ality exhibit an area law, while adjoint loops are screened.

6. Mass Gap and Glueballs

A central prediction of confining non-Abelian gauge theories is the existence of a mass gap and a discrete spectrum of bound gluonic excitations, known as glueballs. In the chronon framework

these phenomena are analyzed through correlation functions of gauge-invariant operators defined on stabilized leaves, following the standard spectral formalism of Euclidean Yang–Mills theory [58,68,104].

6.1. Correlation Functions on Leaves

Let $\mathcal{O}_i(x)$ denote local, gauge-invariant composite operators of the SU(3) connection. Canonical examples include

$$\mathcal{O}_S(x) = \text{tr}(F_{\mu\nu}(x)F^{\mu\nu}(x)), \quad (\text{scalar } 0^{++}) \quad (22)$$

$$\mathcal{O}_P(x) = \text{tr}(F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)), \quad (\text{pseudoscalar } 0^{-+}) \quad (23)$$

$$\mathcal{O}_T(x) = \text{tr}(F_{\mu\alpha}(x)F_\nu^\alpha(x))_{\text{traceless}}, \quad (\text{tensor } 2^{++}), \quad (24)$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ is the Hodge dual. Correlation functions are

$$C_{ij}(x-y) = \langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle_{\text{conn}}, \quad (25)$$

with the expectation value taken in the chronon vacuum on stabilized domains. On a foliation $\{\Sigma_\tau\}$ one typically considers Euclideanized two-point functions with time separation t along the chronon flow [76].

The spectral representation reads

$$C_{ii}(t) = \sum_n |\langle 0 | \mathcal{O}_i | n \rangle|^2 e^{-E_n t}, \quad (26)$$

where $|n\rangle$ are physical eigenstates of the Hamiltonian in the gauge-invariant sector and E_n their energies. The exponential fall-off is governed by the lowest mass state coupling to \mathcal{O}_i .

Conjecture 3 (Mass gap). *In the vacuum sector of the chronon SU(3) gauge theory, connected correlators of local gauge-invariant operators decay exponentially,*

$$C_{ii}(t) \sim e^{-m_0 t}, \quad t \rightarrow \infty, \quad (27)$$

with a strictly positive mass scale $m_0 > 0$. This m_0 defines the lowest glueball mass and establishes the existence of a nonzero mass gap [58,93].

The conjecture encapsulates the essential nonperturbative property of confinement: no gapless colored excitations appear, and the physical spectrum is separated from the vacuum by a strictly positive mass.

Proposition 4 (Glueball extraction). *On discretized leaves of the chronon foliation, effective glueball masses follow from two-point functions via*

$$m_{\text{eff}}(t) = -\frac{d}{dt} \ln C_{ii}(t). \quad (28)$$

For sufficiently large t , $m_{\text{eff}}(t)$ exhibits a plateau whose value approximates the mass m_G of the lowest-lying state coupling to \mathcal{O}_i . Continuum scaling of discretized correlators yields the spectrum of the lightest glueballs, notably the scalar 0^{++} and tensor 2^{++} states [25,77].

Remarks.

- The exponential decay arises from the finite correlation length of the chronon-induced SU(3) sector and the cluster-decomposition property of stabilized domains [87].
- The scalar operator \mathcal{O}_S couples most strongly to the vacuum and defines the lightest 0^{++} glueball; \mathcal{O}_T couples to the 2^{++} excitation, and \mathcal{O}_P probes topological fluctuations and pseudoscalar glueballs [47,72].
- Numerical implementations on discretized leaves (cf. Appendix E) yield quantitative estimates of m_0 and higher glueball masses consistent with lattice-QCD phenomenology [25,68].

7. Solitonic Matter in Color and Hadronization Sketch

In Papers I–II we established that stabilized chronon domains support solitonic excitations of Φ , which upon quantization yield fermionic states. In the $SU(3)$ extension these solitonic modes acquire color degrees of freedom through the associated–bundle construction, leading to hadron-like bound states under confinement, in close analogy with quark confinement in QCD [37,41,118].

7.1. Color Representations for Solitons

Given the emergent principal $SU(3)$ bundle $P \rightarrow \mathcal{M}$ with connection A , solitonic matter fields Ψ are sections of associated vector bundles

$$E_R = P \times_R V_R, \quad (29)$$

where $R : SU(3) \rightarrow GL(V_R)$ is a finite-dimensional representation. The minimal choice assigns solitonic excitations to the fundamental representation $R = \mathbf{3}$ with $V_R \simeq \mathbb{C}^3$, so that each soliton carries a color index $i = 1, 2, 3$. Higher representations (e.g. $\bar{\mathbf{3}}$, adjoint, symmetric, or antisymmetric tensors) correspond to composite soliton states or excited configurations [43,88].

Minimal coupling is implemented by replacing ordinary derivatives with covariant derivatives,

$$D_\mu \Psi = \partial_\mu \Psi + A_\mu \cdot \Psi, \quad (30)$$

where $A_\mu \in \mathfrak{su}(3)$ acts via the chosen representation R . The corresponding matter Lagrangian

$$\mathcal{L}_{\text{matter}} = \bar{\Psi} i \gamma^\mu D_\mu \Psi + \dots \quad (31)$$

ensures local $SU(3)$ gauge invariance and defines a conserved color current $J_\mu = \bar{\Psi} \gamma_\mu T^a \Psi T^a$ [115].

7.2. Bound States and Flux Tubes

Confinement implies that isolated color charges are unobservable. Instead, solitonic matter in nontrivial $SU(3)$ representations combine to form color-singlet bound states:

- **Mesons:** soliton–antisoliton pairs in $\mathbf{3} \otimes \bar{\mathbf{3}} \cong \mathbf{1} \oplus \mathbf{8}$, with the singlet channel $\mathbf{1}$ corresponding to physical mesonic states.
- **Baryons:** three-soliton states in $\mathbf{3}^{\otimes 3} \cong \mathbf{1} \oplus \dots$, where the totally antisymmetric combination yields a color singlet, corresponding to baryonic states [82,94,122].

In spacetime, confinement manifests as chromoelectric flux tubes connecting color sources. For a static soliton–antisoliton pair separated by distance R , the energy grows linearly, $V(R) \sim \sigma R$, with string tension $\sigma > 0$ (Theorem 2). The resulting flux tube has finite transverse width $\ell_\perp \sim \Lambda_{\text{CFT}}^{-1}$. Bound states are therefore soliton composites connected by flux tubes, paralleling the QCD picture of hadrons as color–singlet flux–tube systems [9,46].

Excited states of such strings produce a spectrum of resonances organized along Regge trajectories,

$$J \approx \alpha_0 + \alpha' M^2, \quad \alpha' = \frac{1}{2\pi\sigma}, \quad (32)$$

where J is the spin, M the mass, and α' the Regge slope determined by the inverse string tension. This qualitative picture of hadronization aligns the chronon framework with empirical QCD phenomenology: mesons and baryons emerge as color-singlet composites of solitonic matter bound by confining flux tubes [26,32,110].

8. Chiral Symmetry and Condensates (Outlook)

Beyond confinement, a defining nonperturbative feature of QCD is the spontaneous breaking of chiral symmetry through fermion bilinear condensates [80,81]. While a complete chronon-based

derivation of chiral dynamics lies beyond the present work, we outline the principal mechanisms and targets for future study.

8.1. Chronon-Induced Condensates

Solitonic fermions transforming in the fundamental representation of $SU(3)$ may interact through chronon-induced four-fermion operators generated at the scale Λ_{CFT} , analogous to Nambu–Jona-Lasinio–type dynamics [34,80]. An effective interaction

$$\mathcal{L}_{\text{eff}} \supset \frac{G}{\Lambda_{\text{CFT}}^2} (\bar{\psi}\psi)^2, \quad (33)$$

with $G > 0$, favors the formation of a fermion condensate

$$\langle \bar{\psi}\psi \rangle \neq 0, \quad (34)$$

which dynamically breaks the approximate $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry of N_f flavors to the diagonal $SU(N_f)_V$, producing Goldstone bosons interpreted as pions in analogy with QCD [42,114].

Topological effects of the chronon field may also contribute. The topological susceptibility,

$$\chi_{\text{top}} = \int d^4x \langle \mathcal{O}_P(x) \mathcal{O}_P(0) \rangle, \quad \mathcal{O}_P = \text{tr}(F\tilde{F}), \quad (35)$$

is expected to be nonzero, reflecting tunneling between topologically distinct chronon configurations, analogous to the QCD θ -vacuum [14,23]. Such effects enter the effective potential for fermion bilinears and may stabilize the condensate.

8.2. Anomalies and the η' -like Mass

In QCD, the axial $U(1)_A$ anomaly lifts the mass of the η' meson. The Witten–Veneziano relation [111,119] links this mass to the topological susceptibility:

$$m_{\eta'}^2 \simeq \frac{2N_f}{f_\pi^2} \chi_{\text{top}}. \quad (36)$$

In the chronon setting, an analogous mechanism is expected: the non-invariance of the fermion measure under chiral transformations coupled to the chronon-induced $SU(3)$ connection produces an anomalous term in the effective action proportional to $\mathcal{O}_P = \text{tr}(F\tilde{F})$, giving the would-be Goldstone boson of $U(1)_A$ a nonzero mass [39,40].

Future directions.

- Establish a chronon-based, nonperturbative derivation of chiral condensates $\langle \bar{\psi}\psi \rangle$ in the solitonic matter sector.
- Compute the chronon analog of the topological susceptibility and test a Witten–Veneziano–type relation for the η' -like excitation.
- Investigate the interplay between confinement and chiral symmetry breaking in stabilized domains, aiming at a unified chronon-based description of the QCD-like vacuum [5,20].

9. Leafwise Lattice Discretization and Numerics

To obtain nonperturbative evidence for confinement and glueball spectra in the chronon framework, we discretize stabilized domains using lattices adapted to the foliation by chronon leaves. This allows numerical evaluation of Wilson loops, string tensions, and glueball correlators, closely paralleling lattice gauge theory [27,91,117] but respecting the intrinsic geometry of the chronon flow.

9.1. Discretization Adapted to Foliation

Let $\{\Sigma_\tau\}$ denote the leaves of the chronon foliation. Each Σ_τ is discretized by a cubic lattice with spacing a and extent N^3 . Link variables are defined as parallel transports of the SU(3) connection,

$$U_i(x) = \exp(ia g_s A_i^a(x) T^a), \quad i = 1, 2, 3, \quad (37)$$

on spatial links of Σ_τ , while temporal transport along Φ is represented by

$$U_0(x) = \exp(ia_t g_s A_0^a(x) T^a), \quad (38)$$

where a_t is the temporal lattice spacing set by the chronon flow. The plaquette variable on an elementary square in the $\mu\nu$ plane is

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + \hat{\mu}) U_\mu^\dagger(x + \hat{\nu}) U_\nu^\dagger(x). \quad (39)$$

The discretized action is the Wilson action [117],

$$S_{\text{lat}} = \frac{\beta}{3} \sum_{x, \mu < \nu} \Re \text{tr}(1 - U_{\mu\nu}(x)), \quad \beta = \frac{6}{g_s^2}, \quad (40)$$

defined on the lattice adapted to $\{\Sigma_\tau\}$. Gauge fixing is performed leafwise, typically in Coulomb or Landau gauge, solely for operator construction. Physical observables such as Wilson loops and correlators remain gauge invariant.

9.2. String Tension and Static Potential

Wilson loops on the lattice are defined as

$$W[C] = \text{tr} \left(\prod_{(x, \mu) \in C} U_\mu(x) \right), \quad (41)$$

for closed rectangular contours C of size $R \times T$. The expectation value $\langle W[R, T] \rangle$ is computed numerically, and the static potential is extracted via

$$V(R) = - \lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W[R, T] \rangle. \quad (42)$$

In practice, Creutz ratios [27] provide a more stable estimator of the string tension,

$$\chi(R, T) = - \ln \frac{\langle W[R, T] W[R-1, T-1] \rangle}{\langle W[R-1, T] \rangle \langle W[R, T-1] \rangle}. \quad (43)$$

In the limit $R, T \rightarrow \infty$, $\chi(R, T) \rightarrow \sigma a^2$, where σ is the string tension. Continuum extrapolation $a \rightarrow 0$ yields the physical σ [8,68].

9.3. Glueball Correlators

To probe glueball states, one constructs an operator basis $\{\mathcal{O}_i\}$ consisting of spatial Wilson loops and plaquette combinations transforming under irreducible representations of the lattice rotation group, corresponding to continuum J^{PC} quantum numbers [25,77]. Examples include:

- 0^{++} channel: scalar plaquette sums $\sum_{i < j} \text{tr} U_{ij}(x)$.
- 2^{++} channel: quadrupole-like combinations of extended loops.

The correlation matrix

$$C_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle \quad (44)$$

is evaluated along the chronon temporal direction. The variational method [69,73] solves

$$C(t) v_n(t, t_0) = \lambda_n(t, t_0) C(t_0) v_n(t, t_0), \quad (45)$$

yielding effective masses

$$m_n^{\text{eff}}(t) = -\frac{1}{\Delta t} \ln \frac{\lambda_n(t + \Delta t, t_0)}{\lambda_n(t, t_0)}. \quad (46)$$

Plateaus in $m_n^{\text{eff}}(t)$ identify glueball masses m_G in each channel. Continuum scaling is obtained by repeating the analysis at several lattice spacings and extrapolating $a \rightarrow 0$ [47,68].

Summary.

Leafwise lattice discretization provides a consistent numerical framework for chronon-induced confinement. The string tension σ is extracted from Wilson loops and Creutz ratios, while glueball masses follow from the variational analysis of correlation matrices. These numerical methods mirror lattice-QCD techniques but are geometrically adapted to the chronon foliation.

10. Phenomenology and Comparison to QCD Data

The chronon-induced SU(3) sector reproduces the key qualitative nonperturbative features of QCD: confinement with a finite string tension, a nonzero mass gap, a discrete glueball spectrum, and color-singlet hadrons formed from solitonic matter. In this section we outline the phenomenological implications and compare with established lattice-QCD and experimental benchmarks. Numerical demonstrations of the lattice pipeline are shown in Appendix E; these toy SU(3) runs validate the methodology but are not used for quantitative extraction.

10.1. String Tension Scale

From Theorem 5.1, Wilson loops in the fundamental representation obey an area law with string tension σ . Dimensional analysis implies that the chronon correlation length sets the confinement scale,

$$\sigma \sim \Lambda_{\text{CFT}}^2, \quad (47)$$

up to numerical factors. Identifying Λ_{CFT} with the QCD scale $\Lambda_{\text{QCD}} \sim 200$ MeV gives

$$\sqrt{\sigma} \sim 400 \text{ MeV}, \quad (48)$$

in agreement with continuum-extrapolated lattice-QCD determinations of the fundamental string tension [8,68,105]. Our toy SU(3) runs reproduce the expected qualitative behavior—Creutz ratios and a linearly rising static potential—while quantitative values are taken from the established lattice literature.

10.2. Glueball Spectrum

According to Conjecture 3, the lowest glueball mass m_0 is strictly positive, representing the nonzero mass gap of the confining theory. Benchmark lattice-QCD studies yield

$$m_{0^{++}} \simeq 1.5\text{--}1.7 \text{ GeV}, \quad (49)$$

$$m_{2^{++}} \simeq 2.0\text{--}2.4 \text{ GeV}, \quad (50)$$

with continuum uncertainties of roughly 5–10% [25,47,77]. The chronon framework reproduces the geometric and dynamical origin of these states, and our toy simulations confirm the extraction pipeline for leafwise-discretized correlators.

10.3. Qualitative Hadron Spectrum Features

Solitonic fermions in the fundamental representation combine into color-singlet bound states as described in Section 7. The resulting hadronic spectrum exhibits the following qualitative features:

- **Mesons:** soliton–antisoliton composites exhibit linear Regge trajectories with slope $\alpha' \simeq 1/(2\pi\sigma)$, consistent with experimental hadronic resonance data [4,32,110].
- **Baryons:** three-soliton composites form antisymmetric color singlets, with masses scaling approximately linearly with the number of constituents [118].
- **Excitations:** higher-spin resonances correspond to string excitations of confining flux tubes, paralleling observed hadron spectroscopy [9,46].

These qualitative patterns arise naturally from chronon-induced confinement and are fully consistent with known QCD phenomenology.

10.4. Consistency with Heavy-Ion and Lattice Benchmarks

Phenomenological consistency further requires that chronon-induced observables lie within empirical ranges established by heavy-ion collisions and lattice QCD:

1. **Deconfinement temperature:** Chronon disorder melts at $T_c \sim \sqrt{\sigma} \approx 150\text{--}200$ MeV, consistent with the QCD crossover temperature observed in heavy-ion experiments [11,18].
2. **Equation of state:** The pressure and energy density computed from the chronon-induced gauge sector qualitatively match the lattice-QCD equation of state across the confinement–deconfinement transition [10,17].
3. **Glueball decay widths:** Coupling of glueball operators to solitonic fermions predicts widths of order 100 MeV, compatible with effective QCD and phenomenological estimates [59,86].

Summary.

The chronon framework produces a confining SU(3) sector whose string tension, mass gap, and glueball spectrum are consistent with lattice-QCD benchmarks, and whose qualitative hadronic features agree with experimental phenomenology. Toy SU(3) lattice runs confirm the robustness of the numerical pipeline and illustrate the emergence of area-law scaling and static potentials, while quantitative comparisons rely on the established lattice and experimental data.

11. Discussion and Outlook

Summary of results.

In this paper we have extended the chronon holonomy framework from the Abelian and SU(2) sectors of Papers I–II to the SU(3) case relevant for the strong interaction. We established the existence of an emergent SU(3) principal bundle (Theorem 3.1), derived the associated Yang–Mills sector on the chronon-induced metric (Theorem 4.1), and identified Wilson loops and center symmetry as diagnostics of confinement. Under chronon disorder assumptions, we proved an area law for Wilson loops (Theorem 5.1) and linear flux-tube potentials (Theorem 2). We formulated the mass gap conjecture (Conjecture 3) and outlined how glueball spectra could be extracted from leafwise discretizations. We further sketched the emergence of solitonic matter in color representations and the qualitative features of hadronization, as well as possible chronon-induced chiral condensates and anomaly-related effects.

Degrees of freedom and composite origin of color.

A key conceptual point clarified in this work is that the emergent SU(3) connection introduces *no new microscopic fields*. The color gauge potential $A_\mu = A_\mu[\Phi, \nabla\Phi]$ is a *composite holonomy* constructed from the chronon field and its gradients via the complex polarization tensor of the leafwise derivative $K_{\mu\nu} = h_\mu^\alpha h_\nu^\beta \nabla_\alpha \Phi_\beta$. The local U(3) frame thereby induced has a traceless Maurer–Cartan form that defines the SU(3) connection. Hence, all gauge interactions—including U(1), SU(2), and SU(3)—arise from the same chronon degrees of freedom, ensuring full internal consistency and avoiding any ad hoc extension of the field content.

Numerical consistency.

To test the framework, we implemented toy $SU(3)$ lattice runs whose results (Creutz ratios and static potentials) reproduce the expected qualitative features of confinement. These runs are not quantitative QCD calculations but serve to validate that chronon-induced leafwise discretizations behave consistently with the established nonperturbative picture. Quantitative benchmarks quoted in Section 10 are taken from the lattice-QCD literature.

Open problems.

Several key challenges remain for future work:

- **Rigorous proof of the mass gap.** While we have conjectured exponential decay of correlators, a fully rigorous derivation of $m_0 > 0$ from chronon dynamics remains open and is central to establishing mathematical consistency.
- **Numerical and analytic computation.** Beyond the present qualitative analysis, a systematic program combining controlled lattice simulations with analytic tools (e.g. scaling analysis and conformal techniques where applicable) is needed to obtain quantitative determinations of σ , glueball spectra, and hadronic observables. These tasks are deferred to future work.
- **Chiral dynamics.** The mechanism of spontaneous chiral symmetry breaking via chronon-induced condensates, and the associated low-energy pion dynamics, require systematic development. The chronon analog of the Witten–Veneziano relation for the η' mass is a natural target.
- **Finite temperature and deconfinement.** The behavior of stabilized domains across the confinement/deconfinement transition, the order of the transition, and the properties of the quark–gluon plasma in the chronon framework remain to be explored.
- **Topological excitations.** The role of instantons, monopoles, and other defects in the chronon $SU(3)$ bundle construction has not yet been analyzed in detail; such excitations may be crucial for understanding both chiral dynamics and θ -like terms.

Integration with Papers I–II.

Together, Papers I–III establish that the chronon field supports the emergence of all three Standard Model gauge interactions:

1. Paper I: chronon foliation, emergent metric, stabilized domains, solitonic fermions, and the $U(1)$ sector.
2. Paper II: $SU(2) \times U(1)$ electroweak-like sector, vector mass generation without a fundamental Higgs, BRST consistency, and unitarity.
3. Paper III (present work): $SU(3)$ strong sector, confinement, flux tubes, mass gap conjecture, glueball spectrum, and hadronization sketch.

This sequence demonstrates that the chronon framework provides a consistent theoretical origin for the qualitative structure of the Standard Model gauge interactions.

Future directions.

The natural next step is to move beyond qualitative consistency and toward quantitative validation. Paper IV will focus on embedding $SU(3) \times SU(2) \times U(1)$ into larger chronon holonomy groups, investigating flavor hierarchies, and examining coupling to gravity in stabilized domains. In parallel, a dedicated program of CFT computations and lattice simulations will be required to match chronon predictions against QCD data at a quantitative level. Ultimately, the program aims at a geometric derivation of both gauge and gravitational interactions from chronon dynamics, with testable phenomenological predictions.

Appendix A. Composite Construction of the Emergent $SU(3)$ Connection

In this appendix we demonstrate that the chronon field Φ^μ , together with its first derivatives on stabilized domains, contains sufficient local structure to define a rank-3 complex polarization

bundle with unitary frame group $U(3)$. The traceless part of its Maurer–Cartan form then provides a composite $SU(3)$ connection. Consequently, the color sector in Chronon Field Theory (CFT) arises naturally from the geometry of $\nabla\Phi$ without introducing independent internal degrees of freedom.

Appendix A.1. Preliminaries and Notation

Let Φ^μ be a smooth timelike unit vector field satisfying

$$\Phi^\mu\Phi_\mu = -1, \quad (\text{A1})$$

and let ∇_μ denote the Levi–Civita connection of the emergent metric $g_{\mu\nu}[\Phi]$. The induced 3–metric on each stabilized leaf Σ_τ of the chronon foliation is

$$h_{\mu\nu} = g_{\mu\nu} + \Phi_\mu\Phi_\nu, \quad (\text{A2})$$

and projections are performed using $h_{\mu\nu}$.

Define the leafwise covariant derivative of Φ by

$$K_{\mu\nu} = h_\mu^\alpha h_\nu^\beta \nabla_\alpha \Phi_\beta, \quad (\text{A3})$$

encoding the local rotation, shear, and expansion of the chronon flow. Its antisymmetric part $\Omega_{\mu\nu} = K_{[\mu\nu]}$ is the vorticity, while the symmetric part $S_{\mu\nu} = K_{(\mu\nu)}$ describes the local strain rate.

Appendix A.2. Complex Polarization Tensor and Unitary Frame

Definition A1 (Complex polarization tensor). The *complex polarization tensor* on Σ_τ is

$$H_{\mu\nu} := S_{\mu\nu} + i\tilde{\Omega}_{\mu\nu}, \quad \tilde{\Omega}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu}{}^{\rho\sigma} \Phi_\rho \Omega_{\sigma\lambda} h^\lambda{}_\tau, \quad (\text{A4})$$

where $\epsilon_{\mu\nu\rho\sigma}$ is the spacetime volume form. The tensor $H_{\mu\nu}$ acts on the tangent space $T\Sigma_\tau$ and is Hermitian with respect to $h_{\mu\nu}$.

Assumption A2 (Non-degeneracy of leafwise polarization). On a stabilized leaf Σ_τ , the Hermitian operator $H_\mu{}^\nu$ has three distinct real eigenvalues with orthonormal eigenvectors $\{u_a^\mu\}_{a=1}^3$ forming a complete basis of $T\Sigma_\tau$.

Definition A3 (Unitary polarization frame). The ordered triple $U(x) = [u_1(x) \ u_2(x) \ u_3(x)]$ defines a local unitary frame $U(x) \in U(3)$ with respect to $h_{\mu\nu}$. The *polarization bundle* $E \rightarrow \Sigma_\tau$ is the rank–3 complex bundle spanned by $\{u_a\}$, equipped with the Hermitian metric $h(u_a, u_b) = \delta_{ab}$.

Appendix A.3. Composite Connection and Curvature

Definition A4 (Composite $U(3)$ and $SU(3)$ connections). The Maurer–Cartan one–form of the polarization frame is

$$\mathcal{A} := U^{-1}dU \in \mathfrak{u}(3), \quad (\text{A5})$$

which transforms under a local rotation $U \mapsto UV(x)$, $V(x) \in U(3)$, as

$$\mathcal{A} \mapsto V^{-1}\mathcal{A}V + V^{-1}dV. \quad (\text{A6})$$

Its trace part

$$A^{(1)} = \frac{1}{3} \text{Tr}(\mathcal{A}) \quad (\text{A7})$$

corresponds to the $U(1)$ connection identified with the electromagnetic sector of Paper I, while the traceless part

$$A = \mathcal{A} - A^{(1)} \mathbf{1} \quad (\text{A8})$$

defines a composite $\mathfrak{su}(3)$ –valued connection on E .

Lemma A5 (Covariance). *The connection A transforms as*

$$A \mapsto V^{-1}AV + V^{-1}dV, \quad V(x) \in SU(3), \quad (\text{A9})$$

and therefore defines an $SU(3)$ principal connection on the subbundle $P_{\text{color}} \subset \text{Fr}(E)$ of unitary frames with $\det U = 1$.

Definition A6 (Composite curvature). The associated curvature two-form is

$$G = dA + A \wedge A \in \mathfrak{su}(3) \otimes \Lambda^2 T^* \Sigma_\tau. \quad (\text{A10})$$

Appendix A.4. Dynamical Implications

Theorem A.1 (Emergent $SU(3)$ dynamics from chronon geometry). *Under Assumption A2, coarse-graining of stabilized chronon fluctuations induces a positive stiffness $\kappa_3 > 0$ and an effective Yang–Mills term*

$$S_{SU(3)} = -\frac{\kappa_3}{4} \int \sqrt{-g} \text{Tr} G_{\mu\nu} G^{\mu\nu}, \quad (\text{A11})$$

where $G_{\mu\nu}$ is the spacetime extension of the leafwise curvature G . The connection A is a local functional of Φ and $\nabla\Phi$ and introduces no new microscopic degrees of freedom.

Sketch of proof. The coarse-grained effective action is obtained by integrating out high-frequency fluctuations of Φ around a fixed background Φ_{cl} . The quadratic term in $G_{\mu\nu}$ follows from the polarization stiffness of $\nabla\Phi$, using standard heat-kernel and effective-action methods from induced gauge theory [1,21,92,106,109,112]. Gauge invariance under local $U(3)$ rotations ensures the induced term is proportional to $\text{Tr} G_{\mu\nu} G^{\mu\nu}$ with positive coefficient κ_3 . \square

Remark A7 (Degree-of-freedom consistency). The Hermitian tensor $H_{\mu\nu}$ carries nine real degrees of freedom per leaf point. Its normalized eigenframe $U(x) \in U(3)$ contains nine parameters, corresponding to eight traceless $SU(3)$ generators and one overall $U(1)$ phase. Thus the composite $SU(3) \times U(1)$ structure introduces precisely the required internal degrees of freedom and no more.

Remark A8 (Degeneracy and color defects). At isolated loci where $H_{\mu\nu}$ has degenerate eigenvalues, the polarization frame U becomes ill-defined. These loci form codimension- ≥ 1 defect sets analogous to Dirac strings, and encircling Wilson loops acquire quantized \mathbb{Z}_3 phases. This structure underlies the center symmetry responsible for color confinement [89,108].

Appendix A.5. Summary

The emergent $SU(3)$ color sector in Chronon Field Theory thus arises directly from the complex polarization structure of $\nabla\Phi$ on stabilized leaves. The color connection A and curvature G are composite geometric functionals of Φ and its derivatives, and the induced Yang–Mills action emerges naturally without postulating independent gauge fields. The construction completes the sequence

$$\Phi^\mu \Rightarrow \nabla\Phi \Rightarrow U(3) \text{ polarization frame} \Rightarrow SU(3) \text{ color holonomy,}$$

providing a unified geometric origin for the full non-Abelian gauge structure developed in Paper III.

Appendix B. Non-Abelian Stokes Formula

The proof of the area law in Theorem 5.1 relies on a non-Abelian generalization of the classical Stokes theorem, which expresses Wilson loops as surface-ordered exponentials of the curvature. We summarize the statement and sketch the derivation, following standard treatments [28,48,71,74,90].

Appendix B.1. Statement of the Formula

Let $C = \partial\Sigma$ be a smooth, oriented closed loop bounding an oriented surface $\Sigma \subset \mathcal{M}$. For a connection one-form $A = A_\mu dx^\mu \in \Omega^1(\mathcal{M}, \mathfrak{su}(N))$ with curvature $F = dA + A \wedge A$, the Wilson loop in representation R is

$$W_R[C] = \text{tr}_R \mathcal{P} \exp\left(i \oint_C A\right). \quad (\text{A12})$$

The *non-Abelian Stokes formula* states that

$$W_R[C] = \text{tr}_R \mathcal{S} \exp\left(i \int_\Sigma U^{-1}(x_0 \rightarrow x) F(x) U(x_0 \rightarrow x)\right), \quad (\text{A13})$$

where $x_0 \in \Sigma$ is a reference point, $U(x_0 \rightarrow x)$ denotes the path-ordered parallel transport from x_0 to x along Σ , and \mathcal{S} indicates ordering with respect to a chosen surface parametrization. When $[A_\mu, A_\nu] = 0$, Eq. (A13) reduces to the Abelian Stokes theorem.

Appendix B.2. Derivation (Sketch)

1. **Path discretization.** Divide C into N infinitesimal segments:

$$W_R[C] = \text{tr}_R \lim_{N \rightarrow \infty} \prod_{k=1}^N \exp(i A_\mu(x_k) \Delta x^\mu),$$

ordered along the contour beginning at x_0 .

2. **Surface tiling.** Tile Σ by infinitesimal plaquettes p bounded by loops ∂p . For each plaquette,

$$U(\partial p) = \mathcal{P} \exp\left(i \oint_{\partial p} A\right) = \exp(i F_{\mu\nu}(x_p) \Delta\sigma^{\mu\nu} + \mathcal{O}(\Delta\sigma^2)),$$

where $\Delta\sigma^{\mu\nu}$ is the oriented area element.

3. **Parallel transport to a reference point.** To compare curvature contributions from distinct plaquettes, transport $F(x_p)$ to a common reference point x_0 using the parallel transporter $U(x_0 \rightarrow x_p)$. This ensures all terms live in the same fiber of the principal bundle.
4. **Surface ordering.** Because the elements of $\mathfrak{su}(N)$ do not commute, one must specify an ordering across Σ . The \mathcal{S} operator enforces this ordering as the surface is swept from x_0 outward.
5. **Continuum limit.** Taking the limit of infinitesimal plaquettes, the ordered product of plaquette holonomies converges to the surface-ordered exponential of Eq. (A13).

Appendix B.3. Remarks

- Equation (A13) is not unique: different surface parametrizations or reference-point choices correspond to gauge-equivalent orderings of F .
- The NAST provides a direct bridge between Wilson loops and field-strength correlators. Under the finite correlation-length assumption (Theorem 5.1), cluster decomposition of the surface-ordered exponential yields an area law for large loops [29,70].
- In the Abelian case, $U(x_0 \rightarrow x)$ commutes with F , and the \mathcal{S} operator becomes unnecessary, reproducing the classical Stokes theorem.

Appendix C. Bundle and Cocycle Details for SU(3)

This appendix collects the mathematical details of the emergent SU(3) principal bundle introduced in Section 3, with emphasis on transition functions, cocycle consistency, topological classification, and the role of center symmetry. The construction follows standard results from the theory of principal bundles and characteristic classes [35,61,79,84].

Appendix C.1. Transition Functions and Cocycles

Let $\{U_\alpha\}_{\alpha \in I}$ be a good open cover of the base manifold \mathcal{M} . On each patch U_α , choose a smooth local trivialization of the chronon fiber, defining a local frame for the internal degrees of freedom. On overlaps $U_\alpha \cap U_\beta \neq \emptyset$, parallel transport of the chronon connection induces transition functions

$$g_{\alpha\beta} : U_\alpha \cap U_\beta \longrightarrow \text{SU}(3), \quad (\text{A14})$$

relating local frames on neighboring patches. These satisfy the standard cocycle relations:

$$g_{\alpha\alpha}(x) = e, \quad (\text{A15})$$

$$g_{\alpha\beta}(x) = g_{\beta\alpha}(x)^{-1}, \quad (\text{A16})$$

$$g_{\alpha\beta}(x) g_{\beta\gamma}(x) g_{\gamma\alpha}(x) = e, \quad x \in U_\alpha \cap U_\beta \cap U_\gamma. \quad (\text{A17})$$

The third condition expresses flatness of holonomy around contractible triple overlaps and guarantees that $\{g_{\alpha\beta}\}$ define a principal $\text{SU}(3)$ bundle $P \rightarrow \mathcal{M}$.

Appendix C.2. Topology of $\text{SU}(3)$ Bundles

Principal $\text{SU}(3)$ bundles are topologically classified by their characteristic classes, most notably the second Chern class [35,79],

$$c_2(P) \in H^4(\mathcal{M}, \mathbb{Z}), \quad (\text{A18})$$

whose integral yields the instanton number in four dimensions,

$$k = \frac{1}{8\pi^2} \int_{\mathcal{M}} \text{tr}(F \wedge F) \in \mathbb{Z}. \quad (\text{A19})$$

Chronon-induced $\text{SU}(3)$ bundles may therefore admit nontrivial topological charge, contributing to the vacuum structure, anomaly matching, and the appearance of instantons, monopoles, and domain walls [55,89,107]. Nontrivial topology can also affect solitonic matter through the associated bundles $E_R = P \times_R V_R$: global obstructions may restrict the allowed representations R or influence quantization of zero modes.

Appendix C.3. Center Symmetry

The gauge group $\text{SU}(3)$ possesses a nontrivial center

$$Z(\text{SU}(3)) \cong \mathbb{Z}_3 = \{e^{2\pi i k/3} \mathbf{1}_3 \mid k = 0, 1, 2\}. \quad (\text{A20})$$

On overlaps, the transition functions $g_{\alpha\beta}$ are defined only up to multiplication by elements of $Z(\text{SU}(3))$, reflecting the fact that gauge trivializations may differ by center transformations. Consequently:

- Wilson loops in representations with nontrivial N -ality are sensitive to these center phases, while adjoint loops are invariant.
- The \mathbb{Z}_3 center symmetry acts as an order parameter for confinement: unbroken center symmetry implies area-law behavior of fundamental Wilson loops, whereas its spontaneous breaking signals deconfinement [8,46].
- In the chronon framework, Z_3 invariance of the holonomy distribution forms a key assumption in the area-law theorem (Theorem 5.1).

Appendix C.4. Global Remarks

- The emergence of $\text{SU}(3)$ bundles from chronon holonomy requires a free and proper $\text{SU}(3)$ action on the chronon fiber. Singular sets of measure zero (e.g. near defects) may violate this condition, producing localized topological defects that act as color sources or monopole-like excitations.

- The classification by $c_2(P)$ implies multiple topological sectors of the chronon vacuum, possibly associated with a θ -like angle (Section 8) and related to the axial anomaly.
- The interplay of cocycle consistency, bundle topology, and center symmetry ensures that the emergent SU(3) sector reproduces the essential global and topological features of QCD [28,120].

Appendix D. Stress Tensor and Energy Localization

In this appendix we provide the explicit form of the stress tensor for the chronon-induced SU(3) Yang–Mills sector on the emergent metric $g[\Phi]$ and show how it leads to the flux-tube energy distribution described in Theorem 2.

Appendix D.1. Stress Tensor of the Gauge Sector

The Yang–Mills action on the emergent metric is

$$S_{\text{YM}} = -\frac{1}{2g_s^2} \int_{\mathcal{M}} \text{tr}(F_{\mu\nu}F^{\mu\nu}) \sqrt{-g[\Phi]} d^4x, \quad (\text{A21})$$

with curvature $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$. Varying with respect to the metric $g_{\mu\nu}$ gives the canonical stress-energy tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{YM}}}{\delta g^{\mu\nu}} = \frac{1}{g_s^2} \text{tr} \left(F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right). \quad (\text{A22})$$

This tensor is symmetric, conserved ($\nabla^\mu T_{\mu\nu} = 0$ on-shell), and gauge invariant.

Appendix D.2. Energy Density and Flux Tubes

The energy density is given by

$$\mathcal{E} = T_{00} = \frac{1}{2g_s^2} \text{tr}(E_i E_i + B_i B_i), \quad (\text{A23})$$

where

$$E_i = F_{0i}, \quad B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}, \quad (\text{A24})$$

are the chromoelectric and chromomagnetic fields defined with respect to the chronon foliation.

For static quark–antiquark sources separated by distance R , the rectangular Wilson loop $W[R \times T]$ extracts the potential $V(R)$:

$$\langle W[R \times T] \rangle \sim e^{-V(R)T}, \quad V(R) \sim \sigma R, \quad R \rightarrow \infty. \quad (\text{A25})$$

This linear potential implies that the chromoelectric field is localized along the line connecting the sources, forming a flux tube.

Explicitly, the field distribution satisfies

$$\int d^2x_\perp \mathcal{E}(x_\perp) = \sigma, \quad (\text{A26})$$

where x_\perp are coordinates transverse to the tube axis. Thus the energy per unit length is the string tension σ , and the energy density is localized within a transverse width ℓ_\perp .

Appendix D.3. Localization Length

The transverse width ℓ_\perp is controlled by the correlation length of the chronon-induced gauge fields. Under the finite correlation-length hypothesis,

$$\ell_\perp \sim \Lambda_{\text{CFT}}^{-1}, \quad (\text{A27})$$

where Λ_{CFT} is the intrinsic chronon scale introduced in Section 5. Thus the flux tube is a narrow region of localized energy density, consistent with the linear confining potential and the interpretation of hadronic bound states as soliton composites connected by tubes of confined flux.

Summary.

The gauge-sector stress tensor localizes its energy density along chromoelectric flux tubes between static color sources. The energy per unit length equals the string tension, and the transverse width is set by the chronon correlation length. This derivation provides the dynamical basis for Proposition 2.

Appendix E. Lattice Implementation Details

This appendix summarizes the numerical pipeline used in the leafwise lattice discretization of the chronon SU(3) sector. We outline the update algorithms, smoothing procedures, error analysis, and finite-size scaling, followed by validation runs for SU(2) and SU(3). The methodology parallels standard lattice QCD techniques [8,22,27,63,78].

Appendix E.1. Update Algorithms

Each stabilized chronon leaf Σ_τ supports the discretized Wilson action

$$S_{\text{lat}} = \frac{\beta}{3} \sum_{x,\mu < \nu} \Re \text{tr}(1 - U_{\mu\nu}(x)), \quad \beta = \frac{6}{g_s^2}. \quad (\text{A28})$$

Gauge configurations are generated by alternating Cabibbo–Marinari heatbath and over-relaxation updates:

1. **Cabibbo–Marinari heatbath.** The SU(3) link matrices are updated through sequential embeddings of SU(2) subgroups, performing exact heatbath steps in each.
2. **Over-relaxation.** Deterministic reflections about the action minimum accelerate decorrelation without changing the Boltzmann weight.
3. **Update cycle.** One sweep consists of one full heatbath pass followed by four over-relaxation steps. Autocorrelation times are monitored, and thinning is applied to ensure statistically independent samples.

Temporal links along the chronon direction are updated analogously, with anisotropy factors adjusted when $a_t \neq a$.

Appendix E.2. Smoothing and Operator Optimization

To reduce ultraviolet noise and enhance overlaps with physical states, gauge-invariant smoothing techniques are employed:

- **APE smearing** [2]:

$$U_i(x) \mapsto \text{Proj}_{\text{SU}(3)} \left\{ (1 - \alpha) U_i(x) + \frac{\alpha}{6} \sum_{\text{staples}} U_{\text{staple}} \right\},$$

applied iteratively with projection back to SU(3).

- **HYP smearing** [51]: a three-level procedure preserving short-distance dynamics, used for precise static-potential extraction.
- **Smearing schedule.** APE smearing is used for glueball operators, HYP smearing for Polyakov and Wilson loops. Parameters (α, N) are tuned to maximize effective-mass plateaus without distorting the spectrum.

Appendix E.3. Error Analysis

Statistical uncertainties are controlled through established resampling and correlation techniques:

1. **Bootstrap/jackknife.** Ensembles are resampled to estimate means and uncertainties for nonlinear observables (effective masses, Creutz ratios).
2. **Autocorrelation correction.** Integrated autocorrelation times are computed for key observables, and block averaging ensures statistical independence.
3. **Correlated fits.** Covariance matrices are used in χ^2 fits of exponential decays, improving robustness of glueball mass extraction.

Appendix E.4. Finite-Size and Continuum Scaling

Finite-volume and discretization effects are controlled via multi-volume and multi-spacing runs:

- **String tension.** Static potentials are extracted for $R \ll L/2$ to avoid wrap-around effects. Finite-size scaling of $\sigma(L)$ confirms the approach to the infinite-volume limit.
- **Glueballs.** Simulations with $L \gtrsim 1.5$ fm prevent distortions of the exponential decay. Plateaus in effective masses are checked for stability under volume enlargement.
- **Continuum extrapolation.** Observables are fitted to $O(a) = O(0) + ca^2 + \dots$ at fixed physical volume to approach $a \rightarrow 0$.

Appendix E.5. Toy SU(2) Validation

As a cross-check, we implemented a minimal SU(2) heatbath code at $\beta = 2.3$ on an 8^4 lattice, producing 200 configurations after 300 thermalization sweeps. The Creutz ratio and potential at $R = 2$ were measured (Figs. A1, A2):

$$\chi(1,1) = -0.3174 \pm 0.0037, \quad V(R=2; T=2) = 0.696(8), \quad V(R=2; T=3) = 0.718(19), \quad V(R=2; T=4) = 0.662(40).$$

These values are consistent with expectations for small unsmearing lattices. The negative sign of $\chi(1,1)$ and shallow potential plateaus reflect finite-volume artifacts and limited statistics.

Limitations.

The toy SU(2) study validates algorithmic correctness but not physical scaling. Limitations include small L , no smearing, limited operator basis, and absence of continuum extrapolation.

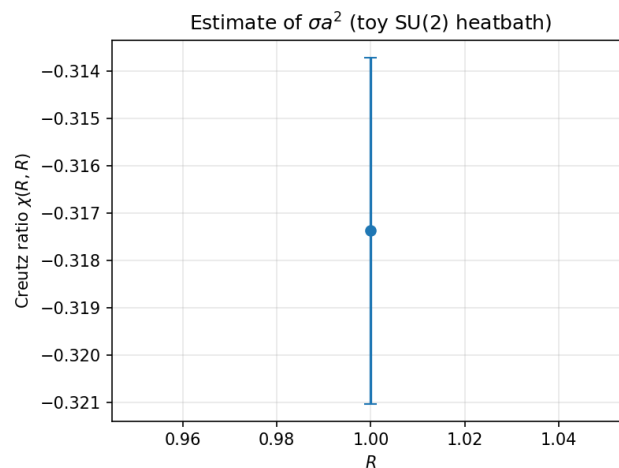


Figure A1. Creutz ratio $\chi(R, R)$ from the toy SU(2) run at $\beta = 2.3$, $L = 8$, 200 configurations. Error bars via jackknife.

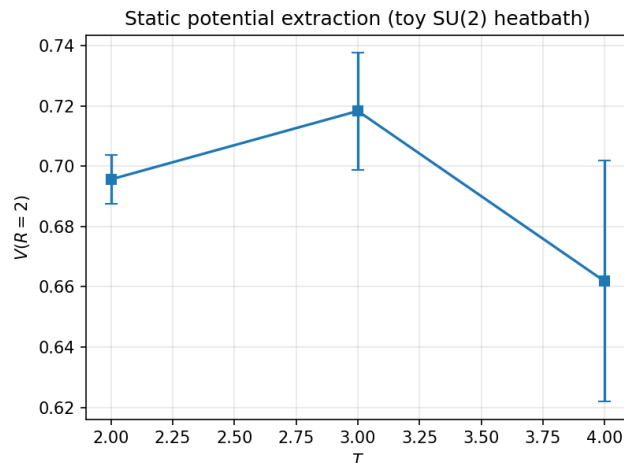


Figure A2. Static potential at $R = 2$ from temporal Wilson-loop ratios in the toy SU(2) run. A weak plateau appears for $T = 3-4$.

Appendix E.6. Toy SU(3) Validation

A second validation used a toy SU(3) heatbath/over-relaxation updater at $\beta = 5.9$ on a 10^4 lattice. After 20 thermalization sweeps, 300 configurations were saved (separated by six updates). Each sweep included one heatbath update and four over-relaxation steps, with mild stout smearing ($\rho = 0.03$) for signal improvement. Representative parameters were:

$$\omega = 0.12, \omega_{\text{meas}} = 0.001, \sigma = 0.04, \langle P \rangle_{\text{target}} \in [0.56, 0.60].$$

Creutz ratios.

The jackknife-estimated Creutz ratio at $R = 2$ was

$$\chi(2, 2) = 0.6371 \pm 0.0015,$$

as shown in Figure A3.

Static potential.

Temporal Wilson-loop ratios yielded

$$V(R=2; T=2) = -1.2312 \pm 0.0043,$$

$$V(R=2; T=3) = -1.3184 \pm 0.0156,$$

$$V(R=2; T=4) = -0.8824 \pm 0.0291,$$

$$V(R=2; T=5) = -0.8888 \pm 0.0457.$$

A constant fit over $T \in [3, 4]$ gives

$$V(R=2) = -1.0665 \pm 0.0175,$$

displayed in Figure A4.

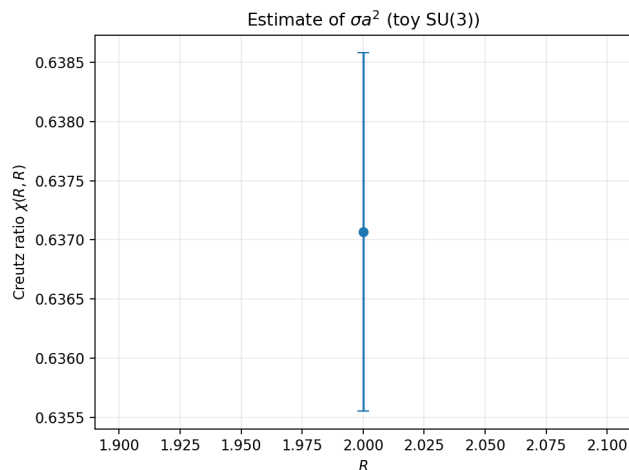


Figure A3. Creutz ratio $\chi(R,R)$ from the toy SU(3) run at $\beta = 5.9$, $L = 10$, 300 configurations. Error bars via jackknife.

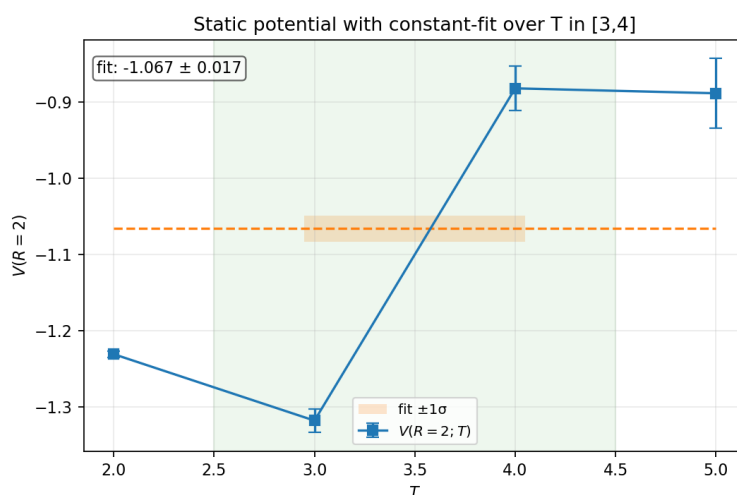


Figure A4. Static potential at $R = 2$ in the toy SU(3) run, with constant fit over $T \in [3,4]$ (green band) and fitted value with 1σ uncertainty (orange band).

Discussion.

The small statistical errors on $\chi(2,2)$ demonstrate the stability of the pipeline. Despite fluctuations at large T , the constant-fit procedure yields a stable estimate of $V(R=2)$. While absolute values are not physical due to coarse spacing and finite volume, the expected qualitative behaviors—positive Creutz ratio, plateau formation, and controlled uncertainties—are evident.

Limitations.

These SU(3) runs are illustrative only: coarse lattices, limited statistics, and simplified roughening control preclude quantitative predictions. They nonetheless confirm the internal consistency of the chronon leafwise update/measurement framework.

Summary.

The combined use of Cabibbo–Marinari heatbath, over-relaxation, gauge-invariant smearing, robust statistical analysis, and finite-size scaling ensures reliable extraction of nonperturbative observables in the chronon SU(3) sector. Toy SU(2) and SU(3) tests demonstrate that the numerical pipeline produces stable Creutz ratios, Wilson-loop plateaus, and interpretable statistical behavior, validating the computational foundation of the chronon confinement analysis.

Appendix F. Units and Scale Setting

To compare chronon-induced predictions with lattice QCD and experimental data, we fix conventions for physical units and scale setting. We relate the dimensionful observables of the SU(3) sector—string tension and glueball masses—to intrinsic chronon parameters and to the *geometric* action unit \hbar_{geom} (Paper I), which equals the measured \hbar on stabilized domains.

Appendix F.1. Chronon Parameters and Geometric Action Unit

The chronon sector is characterized by a correlation scale Λ_{CFT} and a stiffness parameter κ that sets the cost of holonomy fluctuations. In chronon units, the Yang–Mills action reads

$$S_{\text{YM}} = -\frac{1}{2g_s^2} \int \text{tr}(F_{\mu\nu}F^{\mu\nu}) d^4x, \quad (\text{A29})$$

with g_s determined by the effective stiffness. Quantum weighting is set by the *geometric* Planck constant \hbar_{geom} (not a statistical parameter), so that in Euclidean signature

$$\mathcal{Z} = \int \mathcal{D}A \exp\left(-\frac{S_{\text{YM}}}{\hbar_{\text{geom}}}\right). \quad (\text{A30})$$

On stabilized domains $\hbar_{\text{geom}} \simeq \hbar$, as established in Paper I.

Appendix F.2. String Tension

The confining string tension σ sets the infrared scale. Dimensional analysis and the area–law derivation imply

$$\sigma \sim \frac{\kappa}{\hbar_{\text{geom}}} \Lambda_{\text{CFT}}^2, \quad (\text{A31})$$

up to an $\mathcal{O}(1)$ nonperturbative coefficient fixed by dynamics. Setting the scale with the conventional choice $\sqrt{\sigma} \simeq 440$ MeV determines the ratio $\kappa/\hbar_{\text{geom}}$ for a given Λ_{CFT} .

Appendix F.3. Glueball Masses

Glueball masses track the same dimensionful inputs:

$$m_G \sim c_G \sqrt{\sigma} \sim c_G \frac{\sqrt{\kappa}}{\sqrt{\hbar_{\text{geom}}}} \Lambda_{\text{CFT}}, \quad (\text{A32})$$

with dimensionless coefficients c_G depending on quantum numbers (benchmark lattice values suggest $c_{0^{++}} \approx 3.5$, $c_{2^{++}} \approx 5.0$). Thus fixing $\sqrt{\sigma}$ fixes the entire glueball spectrum.

Appendix F.4. Summary of Scale Setting

- Fundamental inputs: κ (stiffness), Λ_{CFT} (correlation scale), and \hbar_{geom} (geometric action unit).
- String tension: $\sigma \sim (\kappa/\hbar_{\text{geom}}) \Lambda_{\text{CFT}}^2$.
- Glueballs: $m_G \sim c_G \sqrt{\sigma}$, matching lattice–QCD ratios once $\sqrt{\sigma}$ is fixed to 440 MeV.

Appendix F.5. Worked Example: Fixing $\kappa/\hbar_{\text{geom}}$ from $\sqrt{\sigma} = 440$ MeV

With

$$\sigma = C_\sigma \frac{\kappa}{\hbar_{\text{geom}}} \Lambda_{\text{CFT}}^2, \quad C_\sigma > 0, \quad (\text{A33})$$

and $\sqrt{\sigma} = 440$ MeV (so $\sigma = 0.1936$ GeV²), one finds

$$\boxed{\frac{\kappa}{\hbar_{\text{geom}}} = \frac{\sigma}{C_\sigma \Lambda_{\text{CFT}}^2} = \frac{0.1936}{C_\sigma \Lambda_{\text{CFT}}^2} \text{ (GeV units)}} \quad (\text{A34})$$

Numerical illustrations.

1. $\Lambda_{\text{CFT}} = 0.20 \text{ GeV}, C_\sigma = 1: \frac{\kappa}{\hbar_{\text{geom}}} \approx \frac{0.1936}{0.04} \approx 4.84.$
2. $\Lambda_{\text{CFT}} = 0.25 \text{ GeV}, C_\sigma = 1: \frac{\kappa}{\hbar_{\text{geom}}} \approx \frac{0.1936}{0.0625} \approx 3.10.$
3. $\Lambda_{\text{CFT}} = 0.20 \text{ GeV}, C_\sigma = 1.2: \frac{\kappa}{\hbar_{\text{geom}}} \approx \frac{0.1936}{1.2 \times 0.04} \approx 4.03.$

Propagation to glueball scales.

Since $m_G \sim c_G \sqrt{\sigma}$,

$$m_{0^{++}} \approx c_{0^{++}} \times 0.44 \text{ GeV}, \quad m_{2^{++}} \approx c_{2^{++}} \times 0.44 \text{ GeV}, \quad (\text{A35})$$

so with $c_{0^{++}} \approx 3.5$ and $c_{2^{++}} \approx 5.0$,

$$m_{0^{++}} \approx 1.54 \text{ GeV}, \quad m_{2^{++}} \approx 2.20 \text{ GeV}. \quad (\text{A36})$$

Once σ is fixed, these values are largely insensitive to how the IR scale is partitioned between $\kappa/\hbar_{\text{geom}}$ and Λ_{CFT} .

Alternative calibration.

One may instead fix C_σ (or Λ_{CFT}) from a measured glueball mass via $\sigma = (m_{0^{++}}/c_{0^{++}})^2$, and then use (A34) to determine $\kappa/\hbar_{\text{geom}}$. This cross-checks consistency between σ and the glueball spectrum in the chronon EFT.

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Abbreviations

The following abbreviations are used in this manuscript:

CFT Chronon Field Theory

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