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Article

On the Unique Direct Sum Decomposition of the $Cl(1,9)$ Clifford Algebra in a Dynamic-Algebraic Framework

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Abstract

We present a rigorous mathematical proof of the unique direct sum decomposition of the $Cl(1,9)$ Clifford algebra using idempotent elements. This decomposition reveals a rich algebraic structure including a central subalgebra isomorphic to $Cl(1,3)$, corresponding to Minkowski spacetime. The results provide a formal framework potentially relevant to geometric and algebraic approaches in mathematical physics and theoretical cosmology.

Keywords: semisimple Clifford algebra; uniqueness of decomposition; isomorphic spaces; dynamical-algebraic model; inverse portal energy tunneling; phase transition

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1. Introduction

Unlike the Big Bang theory in the Λ CDM [1] framework with the inflationary model, DAM [2–7] is based on the paradigm of a phase transition (a change in the topology of space and the order parameter) from a maternal 10-dimensional space to 4D Minkowski spacetime and residual complementary spaces. In DAM, a “physicalization” of the initial CA $Cl_{1,9}$ is proposed by establishing an equivalence between the law of conservation of the number of basis elements before and after decomposition of the CA $Cl_{1,9}$ and the law of conservation of the total energy of the primordial space. Such a comparison of algebraic and physical global “integrals” sets a hierarchical mechanism for the decomposition-decay of the primordial algebra-space into independent components. In this paper, it is shown that the decomposition of the primordial space corresponding to CA $Cl_{1,9}$, under certain conditions, with high probability has the form of 19 subalgebras, the central one of which is $Cl_{1,3}$, and the residual algebras-complements $Cl_{0,4}$ and $Cl_{0,6}$ have a degenerate time degree of freedom. Some of these subalgebras are isomorphic in structure to the Minkowski space by the sum of signatures $p + q = 1 + 3 = 0 + 4 = 4$, and the other non-isomorphic part has a structure different from our Universe with a total signature $p + q = 0 + 6 \neq 4$. This may explain, for example, (i) the practical difficulty of searching for particles of the dark energy-matter sector (DEMS), which may be completely located in universes that are not isomorphic to ours [2–5]; (ii) the difficulty of calculating critical points of many states of matter (using modern theories of critical phenomena of various kinds, including renormalization group methods), which may be explained by the failure to take into account the interaction with isomorphic to our Universe spaces [3–6]; (iii) the difficulty of describing by standard theories of fission of heavy nuclei, even taking into account shell corrections, the uncharacteristically high experimental probability of spontaneous asymmetric subbarrier fission of heavy nuclei, the width and height of the barrier of which can be significantly reduced by IPET [7], etc.

The Λ CDM model is increasingly being questioned by many astrophysical observations made using the James Webb Space Telescope [8–12]. There are many another alternative models [13–22], but

unlike the recently proposed ones [2–7], none of them establishes a connection between the dynamics of the complete decomposition of semisimple algebras and the physical process of the formation of the Universe and its structure.

Therefore, the goal of this work is to mathematically confirm the maximum likelihood of the complete decomposition of the semisimple associative primary CA $Cl_{1,9}$, provided that the decomposition necessarily includes the simple subalgebra $Cl_{1,3}$, the matrix representation of which characterizes our Universe, and subalgebras whose time-like degenerate degrees of freedom are converted into space-like ones to preserve the global temporal vector alignment.

Note that the methods and approaches used in the works [2–7] are based on (i) the application of the anthropic principle of uniqueness of our Universe, which is mathematically manifested in the central character of the algebra $Cl_{1,3}$ in the decomposition of the algebra $Cl_{1,9}$; (ii) the use of Frobenius theorems [23,24] on the complete decomposition of semisimple algebras into the direct sum of the simple CAs; (iii) the presence of internal symmetry of each of the subalgebras of the finite components of the decomposition. The aim of the work is to obtain a proof of the uniqueness of the decomposition for the mathematical validity of using the components of the complete decomposition of the primordial maternal space to construct DAM, IPET-theory and expand their use for solving numerous nonlinear problems of modern science. The main results of this work are the sequential application of (i) the well-known theorem [23,24] on the isomorphism of the algebra $Cl_{1,9}$ to the supertensor (graded tensor) product of two simple CAs $Cl_{1,3}$ and $Cl_{0,6}$; (ii) the theorem on the isomorphism of such a product to the direct sum of subalgebras obtained by successively reducing the signatures in the decomposition of the primary algebra $Cl_{1,9}$ until all its basis elements are exhausted, and (iii) the theorem on the uniqueness of such a decomposition up to isomorphisms, if a single CA $Cl_{1,3}$ is chosen as the central element that preserves collinearity with respect to the time direction in the primordial CA $Cl_{1,9}$. In addition, we note that the isolation of such a CA $Cl_{1,3}$ is a mandatory element due to the anthropic principle of uniqueness of the Minkowski spacetime in which we live.

The work is structured as follows. Section II presents a brief review of works devoted to the DAM model, the theory of inverse portal energy tunneling IPET, and their practical application in astrophysics, the theory of critical phenomena, and in some aspects of the theory of fission of heavy nuclei.

Section III of the paper deals with the mathematical argumentation of the methodology chosen for the DAM model for the complete decomposition of the initial semisimple associative CA $Cl_{1,9}$, the matrix representation of which corresponds to the chosen initial 10-dimensional space before the phase transition-decay, which had 1 time and 9 space dimensions.

This starting CA $Cl_{1,9}$ decomposed into a single central simple subalgebra $Cl_{1,3}$, in which this time degree of freedom survived, and 18 purely space algebras, 3 of which ($Cl_{0,4}$) are isomorphic to $Cl_{1,3}$ in the sum of signatures, and 15 ($Cl_{0,6}$) are non-isomorphic.

2. Mathematical Formulation of the Dynamic–Algebraic Model and the Concept of Inverse Portal Energy Tunneling

In works [2–7], in contrast to the Λ CDM-formation of the Universe from non-physical “nothing” as from a mathematical object with zero size and zero dimension, a model of phase transition from a previously existing more complex “supermaterial” universe with a higher dimension is developed. Time fluctuation, as a possible cause of the decomposition of the primary universe, is not considered here yet, but we see that in the proposed decomposition, physical time “survived” only in spacetime $Cl_{1,3}$, that is, in our Universe, and in the residual purely Euclidean spatial components the time degree of freedom is degenerate, that is, either absent or “smeared” over spatial coordinates.

Our Minkowski spacetime in the matrix representation of the Clifford algebra $Cl_{1,3}$ is central in the decomposition (see Eq. (1) below). This decomposition consistently reaches a set of direct sums of Clifford algebras (see Eq. (2) below), isomorphic ($Cl_{0,4}$) and non-isomorphic ($Cl_{0,6}$) to $Cl_{1,3}$ spaces with a degenerate time degree of freedom into spatial ones.

Further development of the theory is connected with the fact that spaces after decay of the maternal primordial space, which are isomorphic (equal in the sum of signatures) to our Universe, can interact with it with a very high probability in the boundary zones, the so-called “portals” [3,5–7].

Each such portal has such intermediary properties that are conditioned by the global conservation of energies in neighboring spaces, namely (i) transitive, to ensure the passage of energy flows in any manifestation with the conditions of the rules of smoothness and continuity with a periodic change of the metric in the interspace transition zone and (ii) elastic, to ensure the “reflection” of these flows between these neighboring spaces with different topologies and metrics.

This model of trial energy-information exchange between isomorphic spaces can be of great importance for modeling-describing any critical or nonlinear phenomena observed in our Minkowski spacetime. Therefore, a practical theory of inverse portal energy tunneling (IPET) is built on the basis of DAM and the CA apparatus.

IPET is a universal and fundamental theory in terms of areas of application. An almost sufficient condition for its implementation is a physically correct and justified choice of a macroscopic and experimentally observed quantity that characterizes the approach of the selected studied system to the vicinity of the critical point or to the zone of fundamental change in the properties of this system, such as phase transitions for different types of matter, nuclear fission, etc.

In the works [3,5–7], the practical use of IPET is demonstrated for finding and predicting the critical temperature for phase transitions in five completely different types of matter: water, ordinary hadronic matter, quark-gluon plasma (QGP), colored glass gluon condensate (CGC) and preons.

The only observable quantity in these cases, which characterized the approach of the system to the portal that transforms the topology of Minkowski space, was chosen as the temperature of the system in the vicinity of the critical state (and also, indirectly, the chemical baryon potential). And in the work [7], which is devoted to the subbarrier fission of heavy nuclei, another observable quantity was chosen, namely, the probability of spontaneous low-energy subbarrier fission of a heavy nucleus.

The paradigm of the proposed approach can be partially comparable to the well-known catastrophe theory in mathematics about reaching a bifurcation point, which consists in the unpredictability of the manifestation of the properties of the system after changing the topology of space [25–30].

3. Theorems Concerning the Uniqueness of the Decomposition of the Primordial Algebra in the DAM Framework

So, earlier [2–4] within the framework of DAM a hypothetical phase transition-decay of the 10-dimensional primary space into our Universe and its subspaces-complements was described. This decay of the complete, infinite and continuous initial space, described by the matrix representation of $Cl_{1,9}$, where there were 1 time and 9 spatial degrees of freedom, into 19 independent spaces, which are algebraically represented by the direct sum of 19 independent CAs.

In the section, a proof of the uniqueness of such a decomposition of CA $Cl_{1,9}$ is proposed using a clear hierarchy of gradual application of the following theorems:

Theorem 1 — the basic splitting in the form of a graded tensor product, which is completely isomorphic to the primary CA $Cl_{1,9}$;

Theorem 2 — the uniqueness of the complete decomposition of CA $Cl_{1,9}$ into 19 CAs;

Theorem 3 — naturalness of isolating the unique Minkowski-isomorphic algebra $Cl_{1,3}$ from the group of 4 algebras in the unique decomposition of Theorem 2.

The arguments are supported by numerical dimensionality checks, idempotent structure and, finally, additional physical interpretation.

To describe the dynamics and explain the high probability of such a decomposition of the space $Cl_{1,9}$, for example, in [4] the mathematical apparatus of CA was used, namely:

(i) Frobenius theorems [23,24] on the complete decomposition of an associative semisimple algebra into a direct sum of independent subalgebras;

(ii) the principle of hierarchical sequence of the complete decomposition of the starting algebra with a gradual decrease in the sum of signatures, starting from the residual maximal (0+6) in CA $Cl_{0,6}$, which has the property of orthogonal embedding in $Cl_{1,9}$:

$$Cl_{1,9} \cong Cl_{1,3} \oplus C, \quad Cl_{0,6} \hookrightarrow C \hookrightarrow Cl_{1,9} \quad (1)$$

and initiates the gradual complete decomposition of CA $Cl_{1,9}$ until the number of basic elements from the C residue of the structure is completely exhausted $Cl_{1,3} \oplus C$. This hierarchical process of exhaustion of basis elements proceeds from the maximum integer number of CA $Cl_{0,6}$ that the remainder C can accommodate to the final following form:

$$Cl_{1,9} \cong Cl_{1,3} \oplus \bigoplus_{i=1}^{n_2=3} (Cl_{0,4})_i \oplus \bigoplus_{j=1}^{n_1=15} (Cl_{0,6})_j, \quad (2)$$

where the final result is 4 mutually isomorphic algebras (including $Cl_{1,3}$) with total signature 4, and 15 mutually isomorphic algebras with total signature 6;

(iii) the internal symmetry of the structure of the basis elements of each CA in the decomposition, namely:

– the number of isomorphic CAs with equal sum of signatures ($p + q=4$) is equal to the number of generator vectors in any CA from the group of such isomorphic algebras

$$Cl_{1,3} \oplus \bigoplus_{i=1}^{n_2=3} (Cl_{0,4})_i,$$

and is also equal to the number of 3-vectors; this number is 4;

– the another number of isomorphic CAs with equal sum of signatures ($p + q=6$) is equal to the number of 2-vectors in any CA from from the group of such isomorphic algebras

$$\bigoplus_{j=1}^{n_1=15} (Cl_{0,6})_j,$$

and is also equal to the number of 4-vectors; this number is 15;

(iv) the principle of correspondence of algebraic conservation of the number of all elements of the basis of the primary CA before and after its complete decomposition, and conservation of the total energy of the initial space before and after its decomposition.

To obtain a conclusion about the correctness and uniqueness of the decomposition (2), we first show the validity and completeness of the splitting for a certain Clifford algebra $Cl_{1,9}$ (see Theorem 1 below, [23,24]):

$$Cl_{1,9} \cong Cl_{1,3} \hat{\otimes} Cl_{0,6}, \quad (3)$$

where $\hat{\otimes}$ denotes the \mathbb{Z}_2 -graded tensor product, which preserves the anticommutation relations between odd elements of the constituent algebras.

Let's note the orthogonality between the primes CA $Cl_{1,3}$ and $Cl_{0,6}$ as orthogonal embeddings to $Cl_{1,9}$:

$$Cl_{1,3} \hookrightarrow Cl_{1,9}; \quad Cl_{0,6} \hookrightarrow Cl_{1,9}. \quad (4)$$

Theorem 1 (Theorem on the splitting of any CA). *It is correct for any decomposition $p = p_1 + q_1$, $q = p_2 + q_2$:*

$$Cl_{p,q} \cong Cl_{p_1,q_1} \hat{\otimes} Cl_{p_2,q_2} \quad (5)$$

Let us now apply Theorem 1 to $Cl_{1,9}$. Let's decompose the signature (1+9):

$$p = p_1 + q_1 = 1 + 3, \quad q = p_2 + q_2 = 0 + 6, \quad (6)$$

from where we see the correctness of the expression (3).

Let us now check the conservation of dimension. It is known that the dimension CA:

$$\dim Cl_{p,q} = 2^{p+q}. \quad (7)$$

Therefore: $\dim Cl_{1,9} = 2^{1+9} = 1024$, $\dim Cl_{1,3} = 2^{1+3} = 16$, $\dim Cl_{0,6} = 2^{0+6} = 64$, from which we see the correctness of the product of dimensions:

$$\dim Cl_{1,3} = 16, \quad \dim Cl_{0,6} = 64 \quad \Rightarrow \quad 16 \times 64 = 1024 = \dim Cl_{1,9}. \quad (8)$$

Thus, the dimensions coincide, which confirms isomorphism (3), which can be constructed by constructing generators for $Cl_{1,3}$, $Cl_{0,6}$ and $Cl_{1,9}$. Let's first build the generators to $Cl_{1,3}$: $\gamma_0, \gamma_i (i = 1, 2, 3)$, then we will have:

$$\gamma_0^2 = +1, \quad \gamma_i^2 = -1 \quad (i = 1, 2, 3), \quad \{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}, \quad \eta = \text{diag}(+1, -1, -1, -1). \quad (9)$$

Similarly for $Cl_{0,6}$ denote the generators $e_i (i = 1, 2, 3, 4, 5, 6)$, then we will have:

$$e_i^2 = -1, \quad \{e_i, e_j\} = -2\delta_{ij}. \quad (10)$$

Finally, we will construct the generators for $Cl_{1,9}$ in the following form:

$$\Gamma_\mu = \begin{cases} \gamma_\mu \otimes 1, & \mu = 0, 1, 2, 3, \\ 1 \otimes e_i, & i = 1, \dots, 6. \end{cases} \quad (11)$$

They satisfy Clifford's relation:

$$(\gamma_0 \otimes 1)^2 = +1, \quad (\gamma_i \otimes 1)^2 = -1, \quad (1 \otimes e_j)^2 = -1. \quad (12)$$

Let us also consider an alternative approach (via matrices) for checking dimensionality. The known isomorphisms have the form:

$$Cl_{1,3} \cong M_2(\mathbb{H}) \quad \text{and} \quad Cl_{0,6} \cong M_8(\mathbb{R}), \quad (13)$$

where \mathbb{H} stands for the quaternions.

Dimensional check:

$$\dim M_2(\mathbb{H}) = 4 \times 4 = 16. \quad (14)$$

Then:

$$Cl_{1,9} \cong M_2(\mathbb{H}) \otimes M_8(\mathbb{R}) \cong M_{16}(\mathbb{H}), \quad (15)$$

from where we have:

$$\dim M_{16}(\mathbb{H}) = 16^2 \times 4 = 1024. \quad (16)$$

Thus, the final conclusion is that we do indeed formally have an isomorphism (3), which is consistent with the decomposition (5) of Theorem 1, in which the subalgebras $Cl_{1,3}$ and $Cl_{0,6}$ are mutually orthogonal due to the commutation rules of their generators.

The next steps to prove the uniqueness of the decomposition of the primary CA are to formalize the isomorphism of the graded tensor product (3) and the direct sum (2) using idempotents. Let us consider the equivalence of the tensor product and the direct sum through idempotents.

In the framework of semisimple associative CA, the Wedderburn-Artin theorem [23,24] allows the decomposition:

$$\mathcal{A} \cong \bigoplus_{i=1}^n \mathcal{A}_i, \quad (17)$$

where \mathcal{A}_i - simple algebras that are orthogonal to each other. For such a decomposition, there is a system of orthogonal idempotents e_i , such as:

$$e_i^2 = e_i, \quad e_i e_j = 0 \quad (i \neq j), \quad \sum_{i=1}^n e_i = 1. \quad (18)$$

Then:

$$\mathcal{A}_i = \mathcal{A} e_i, \quad (19)$$

and the algebra itself is decomposed as:

$$\mathcal{A} \cong \bigoplus_{i=1}^n \mathcal{A} e_i. \quad (20)$$

Then it is natural to assume that if we have:

$$\mathcal{A} \cong \mathcal{B} \hat{\otimes} \mathcal{C}, \quad (21)$$

where \mathcal{C} decomposed into a direct sum $\bigoplus_{i=1}^n \mathcal{C}_i$, then:

$$\mathcal{A} \cong \mathcal{B} \hat{\otimes} \left(\bigoplus_{i=1}^n \mathcal{C}_i \right) \cong \bigoplus_{i=1}^n (\mathcal{B} \hat{\otimes} \mathcal{C}_i). \quad (22)$$

This fact is the basis for the transition from the hierarchical tensor product in (3) to the final direct sum of 19 algebras (2).

Theorem 2 (On the uniqueness of the decomposition of $Cl_{1,9}$). *Let $Cl_{1,9}$ be a CA over a field of characteristic $p \neq 2$. Then, if there exists a decomposition:*

$$Cl_{1,9} \cong Cl_{1,3} \hat{\otimes} Cl_{0,6} \cong \bigoplus_{i=1}^{19} \mathcal{A}_i \quad (23)$$

where each \mathcal{A}_i is a simple orthogonal subalgebra, and the number of basis elements is preserved, then this decomposition is unique up to isomorphism:

$$\forall j, \exists \mathcal{A}_j \cong Cl_{p_j, q_j}, \quad (24)$$

where the set (p_j, q_j) matches one of the fixed 19 signature variants:

$$Cl_{1,9} \cong \bigoplus_{i=1}^4 (Cl_{1,3})_i \oplus \bigoplus_{j=1}^{15} (Cl_{0,6})_j, \quad (25)$$

which preserve the sum of basis elements. The construction of idempotents is carried out over a field of characteristic $p \neq 2$, which ensures the decomposability of quadratic forms.

Remark: The phrase in the formulation of Theorem 2, "over a field of characteristic $p \neq 2$," means that we are considering standard number fields \mathbb{R} , \mathbb{C} , and \mathbb{Q} , where anticommutative relations make sense and Clifford algebra (CA) is defined correctly. This is a technical condition necessary to preserve properties used in proofs (such as idempotent decomposition, orthogonal products, etc.). In

most physical and algebraic applications, the field of real numbers is used \mathbb{R} and \mathbb{C} , which have the characteristic $p = 0$, i.e. $p \neq 2$.

Proof. As for the decomposition of an associative semisimple algebra, according to the Wedderburn–Artin theorem, any finite-dimensional semisimple associative algebra \mathcal{A} over an algebraically closed field is isomorphic to a direct sum of matrix algebras over division algebras:

$$\mathcal{A} \cong \bigoplus_{i=1}^k M_{n_i}(D_i), \quad (26)$$

where each D_i is a finite-dimensional division algebra over the base field. If the field is algebraically closed, then each $D_i \cong \mathbb{F}$, and thus

$$\mathcal{A} \cong \bigoplus_{i=1}^k M_{n_i}(\mathbb{F}). \quad (27)$$

where D_i - division rings. In context CA over \mathbb{R}, \mathbb{C} (which have characteristic 0, corresponding to $p \neq 2$) these D_i are reduced to \mathbb{R}, \mathbb{C} or \mathbb{H} , and CA \mathcal{A} is a semisimple algebra. Each of the three cases centers \mathbb{R}, \mathbb{C} or \mathbb{H} corresponds to exactly one series of simple Clifford algebras according to the Bott periodicity table [12,13] (see Table 1 for $Cl_{p+8,q} \cong Cl_{p,q} \otimes M_{16}(\mathbb{R})$).

Table 1. Classification of Real Clifford Algebras (mod 8)

$p - q \bmod 8$	Clifford Algebra Type
0	$M_{2^k}(\mathbb{R})$
1	$M_{2^k}(\mathbb{R}) \oplus M_{2^k}(\mathbb{R})$
2	$M_{2^k}(\mathbb{R})$
3	$M_{2^{k-1}}(\mathbb{C})$
4	$M_{2^{k-1}}(\mathbb{H})$
5	$M_{2^{k-1}}(\mathbb{H}) \oplus M_{2^{k-1}}(\mathbb{H})$
6	$M_{2^{k-1}}(\mathbb{H})$
7	$M_{2^{k-1}}(\mathbb{C})$

Here $k = (p + q)/2$ - is the degree of dimensionality of the spinor space.

To ensure mathematical consistency in the decomposition of $Cl_{1,9}$ via graded tensor products, we analyze its spinor representation. This representation plays a key role in justifying the transition from a graded tensor product to a direct sum of subalgebras. It guarantees correct dimensional matching and confirms that each subalgebra contributes faithfully to the structure of the full spinor space.

The CA $Cl_{1,9}$ admits the following decomposition as a graded tensor product:

$$Cl_{1,9} \simeq Cl_{1,3} \hat{\otimes} Cl_{0,6}. \quad (28)$$

This decomposition is valid because $p_1 - q_1 - (p_2 - q_2) \equiv 0 \pmod{2}$ for $Cl_{1,3}$ and $Cl_{0,6}$.

For the corresponding spinor spaces, we have:

$$S_{1,3} \cong \mathbb{R}^4, \quad S_{0,6} \cong \mathbb{R}^8. \quad (29)$$

Then the spinor representation of the product has dimension:

$$S_{1,3} \otimes S_{0,6} \cong \mathbb{R}^{32}. \quad (30)$$

The spinor space of the CA $Cl_{1,9}$ therefore has dimension 32. Since $Cl_{1,9}$ has signature $(1,9)$, meaning $1 + 9 = 10$ dimensions, the total dimension of the algebra is:

$$\dim(Cl_{1,9}) = 2^{10} = 1024. \quad (31)$$

The algebra $Cl_{1,9}$ is semisimple, and according to the periodicity of real CAs, we have:

$$Cl_{1,9} \cong Mat_{32}(\mathbb{R}). \quad (32)$$

This follows from the fact that $1 - 9 = -8 \equiv 0 \pmod{8}$, and in this case the CA is isomorphic to a full matrix algebra over \mathbb{R} . The spinor representation is faithful, as every element of $Cl_{1,9}$ acts nontrivially on the 32-dimensional spinor space $S_{1,9} \cong \mathbb{R}^{32}$.

Note that while $Cl_{0,6}$ itself is not simple (it decomposes as $Cl_{0,6} \cong Mat_8(\mathbb{R}) \oplus Mat_8(\mathbb{R})$), its spinor representation remains irreducible of dimension 8, and the graded tensor product structure is preserved in the decomposition of $Cl_{1,9}$.

The graded tensor product structure $Cl_{1,3} \hat{\otimes} Cl_{0,6}$ provides a natural way to construct the basis elements of $Cl_{1,9}$.

As the next step in the proof, we apply the theory of idempotents. For such a decomposition, there exist pairwise orthogonal central idempotents e_j , such as $e_j e_k = \delta_{jk}$ and $\sum e_j = 1$. Algebra \mathcal{A} decays as a direct sum of ideals $\mathcal{A} \cong \bigoplus_{i=1}^k A e_i$, where each $A e_i$ is simple algebra. This allows us to go from a hierarchical tensor product to a direct sum of 19 algebras (25).

Let us now emphasize the equivalence of the tensor product and the direct sum. We know from (21) that if $A \cong B \otimes C$ and C decomposed into a direct sum $\bigoplus_{i=1}^n C_i$, then $\mathcal{A} \cong B \otimes (\bigoplus_{i=1}^n C_i) \cong \bigoplus_{i=1}^n (B \otimes C_i)$.

This property allows us to represent the decomposition (25), which is a direct sum of 19 independent CAs, as the result of the hierarchical decomposition of the starting algebra $Cl_{1,9}$.

Let us now note the fact of preserving the number of basis elements. The principle of correspondence of algebraic preservation of the number of all elements of the basis of the primary $Cl_{1,9}$ before and after its complete decomposition is fundamental. This means that the dimension of the initial algebra $2^{10}=1024$ is stored in the sum of the dimensions of the subalgebras that arise in the decomposition (25). For example, checking the dimensions in expressions (14) and (16) confirms the correctness.

Let us emphasize the property of nesting and orthogonality of generators. The principle of hierarchical sequence of the complete decomposition of the starting algebra with a gradual decrease in the sum of signatures and the property of orthogonal nesting are key. The orthogonality of the subalgebras $Cl_{1,3}$ and $Cl_{0,6}$ in the decomposition (17) due to the commutation of their generators is a necessary condition for the application of the decomposition theorems. Let us note the uniqueness of the CA classification. For each (p,q) CA $Cl_{p,q}$ is isomorphic to one of the 8 basic types of matrix algebras over \mathbb{R}, \mathbb{C} or \mathbb{H} (see Table 1).

This classification, known as Bott periodicity, guarantees that for a given signature there exists a unique up to isomorphism simple CA. Since the decomposition (25) leads to 19 fixed variants of signatures, and each of these signatures corresponds to a unique (up to isomorphism) simple CA, the decomposition into these components is itself unique up to isomorphism.

Conclusion: The uniqueness of the complete decomposition of CA $Cl_{1,9}$ in the context of the DAM model is guaranteed by the combination of the Wedderburn-Artin theorem, which provides a decomposition into simple components, and the unique classification of simple CAs by their signatures. The conservation of the number of basis elements and the orthogonality property of generators are internal confirmations of the correctness of this decomposition. The set of 19 fixed signatures uniquely defines these simple subalgebras up to isomorphism, thereby proving the uniqueness of the decomposition (25).

Thus, the uniqueness of the complete decomposition of CA in the context of the DAM model is guaranteed by:

- i) the Wedderburn-Artin theorem (see (17) and [23,24]);
- ii) the use of orthogonal idempotents;
- iii) the preservation of the number of basis elements;
- iv) the property of nesting and orthogonality of generators;

v) the decomposition in the form (25). \square

The next step is to isolate only one algebra $Cl_{1,3}$ from the group $\bigoplus_{i=1}^4 Cl_{1,3}$ in decomposition (25).

Theorem 3 (On the uniqueness of Minkowski spacetime of type $Cl_{1,3}$). *In the structure of the complete decomposition of the Clifford algebra $Cl_{1,9}$, according to Theorem 2, for the direct sum of 19 orthogonal subalgebras, there remains only one (up to isomorphism) direct term of type $Cl_{1,3}$, which contains a single time generator. The remaining three $Cl_{1,3}$ subalgebras degenerate into a group of three isomorphic subalgebras $Cl_{0,4}$, consisting only of spatial generators:*

$$Cl_{1,9} \cong \bigoplus_{i=1}^4 (Cl_{1,3})_i \oplus \bigoplus_{j=1}^{15} (Cl_{0,6})_j \cong Cl_{1,3} \oplus \bigoplus_{i=1}^3 (Cl_{0,4})_i \oplus \bigoplus_{j=1}^{15} (Cl_{0,6})_j. \quad (33)$$

Let us analyze the formulation of Theorem 3 before the formal proof. Let us note in passing that the need for the algebraic form (33) is indirectly dictated by the anthropic principle, which requires only one unique CA $Cl_{1,3}$, which implements a single structure with one time direction, so the time generator e_0 with the property $e_0^2 = +1$ can be present in only one direct term. Physically, only this unique term $Cl_{1,3}$ is the only subalgebra that models our 4-dimensional Minkowski spacetime, while the remaining 18 subalgebras can be interpreted as: (i) external spaces that do not intersect with our Universe; (ii) internal spaces (symmetries, quark or lepton spaces); (iii) hidden dimensions (compact or holographic); (iv) CGC fields, preons, etc.

Let us now detail the transition from Theorem 2 to Theorem 3 not only from physical (the need to fulfill the anthropic principle of uniqueness of the 4-dimensional Minkowski spacetime), but also from strictly mathematical considerations. We have from Theorem 2 expression (25), however, the physical and algebraic structure of the decomposition indicates that among the group of algebras $\bigoplus_{i=1}^4 Cl_{1,3}$ there should be only one that preserves the valid time direction, so let's consider 4 mathematical motivations for such a selection:

(i) *Generalized minimum time criterion:*

Let's assume that all 4 time generators give a common time vector:

$$E_0 = e_0^{(1)} + e_0^{(2)} + e_0^{(3)} + e_0^{(4)}. \quad (34)$$

If we consider only one direction of this time vector $e_0^{(1)}$, collinear to the time direction of the primary CA $Cl_{1,9}$, then the rest form a space orthogonal to it:

$$E_0 \perp e_0^{(2)}, e_0^{(3)}, e_0^{(4)} \Rightarrow \bigoplus_{i=1}^3 (Cl_{0,4})_i = Cl_{0,4}^{\oplus 3}. \quad (35)$$

This means that only one copy of $Cl_{1,3}$, with active time, remains, while the other three copies are converted into $Cl_{0,4}$, because time there no longer plays its role — the signature becomes Euclidean rather than pseudo-Euclidean.

(ii) *Center of algebras and symmetry:*

$$Z(Cl_{1,3}) \cong \mathbb{C}, \quad (36)$$

$$Z(Cl_{0,4}) \cong \mathbb{R} \oplus \mathbb{R}. \quad (37)$$

If we apply the time automorphism $t \rightarrow -t$, then only one $Cl_{1,3}$ preserves center invariance.

(iii) *Phase transition-decay in DAM:*

In the process of phase transition-decay of the initial $Cl_{1,9}$ with a common time vector, only one $Cl_{1,3}$ preserves the Lorentzian metric, while the other three transition to the Euclidean $Cl_{0,4}$.

(iv) *Spinor stability:*

Only one $Cl_{1,3}$ of the four supports the 4-component structure of Dirac spinors, the others are projected onto $SO(4)$ -degenerate spinors.

(v) *Spinor representation theory:*

Only the algebra $Cl_{1,3}$ admits a faithful and irreducible representation of Dirac spinors in \mathbb{C}^4 with Lorentzian signature. The 4-dimensional complex spinor space naturally arises from the isomorphism:

$$Cl_{1,3} \otimes \mathbb{C} \cong M_4(\mathbb{C}), \quad (38)$$

which ensures the existence of a 4-dimensional irreducible module, known as the Dirac spinor.

In contrast, the degenerate algebras $Cl_{0,4}$ admit only real representations:

$$Cl_{0,4} \cong \mathbb{H}(2), \quad (39)$$

whose spinor modules are quaternionic and do not carry Lorentz symmetry. Moreover, $Cl_{0,4}$ cannot support a chirality structure or time-reversal symmetry essential for physical spinors.

Thus, out of four candidate subalgebras $Cl_{1,3}^{(i)}$, only one can host physically meaningful spinor representations consistent with the Lorentz group $SO(1,3)$ and the associated covering group $Spin(1,3) \cong SL(2, \mathbb{C})$.

This provides a strong representational argument in favor of singling out one unique $Cl_{1,3}$ subalgebra as the carrier of spacetime and spinorial degrees of freedom, while the others project to purely spatial forms.

All the given variants motivate the same transition:

$$\bigoplus_{i=1}^4 (Cl_{1,3})_i \Rightarrow Cl_{1,3} \oplus \bigoplus_{i=1}^3 (Cl_{0,4})_i. \quad (40)$$

This is the mathematical basis of Theorem 3 on the uniqueness of the single selection $Cl_{1,3}$ among the four candidates. Expression (40) is the result of the structural degeneration of the pseudometric due to the mutual orthogonalization of the time direction under the action of the surrounding spatial background $\bigoplus_{j=1}^{15} Cl_{0,6}$.

Proof. It is known from Theorem 2 that the following is true (25):

$$Cl_{1,9} \cong \bigoplus_{i=1}^4 (Cl_{1,3})_i \oplus \bigoplus_{j=1}^{15} (Cl_{0,6})_j,$$

then we show that in the group of algebras with the sum of signatures $1+3=4$, one needs to isolate one direct term $Cl_{1,3}$ that preserves the temporal degree of freedom aligned with the time direction of the primordial algebra $Cl_{1,9}$, while the remaining three are isomorphically mapped into fully spatial direct terms $Cl_{1,3} \oplus \bigoplus_{i=1}^3 (Cl_{0,4})_i$ due to a structural constraint on the space of time-like generators.

Let us denote in each $Cl_{1,3}^{(i)}$ its time vector:

$$e_0^{(i)} \oplus Cl_{1,3}^{(i)}, \quad (e_0^{(i)})^2 = +1. \quad (41)$$

Consider the subspace spanning all time generators:

$$V := \text{span}_{\mathbb{R}} e_0^{(1)}, e_0^{(2)}, e_0^{(3)}, e_0^{(4)}. \quad (42)$$

Since the global signature of the primary algebra $Cl_{1,9}$:

$$Cl_{1,9} \Rightarrow \text{one } (+1), \text{ nine } (-1), \quad (43)$$

then in the entire space V only one linearly independent time vector E_0 is admissible, such that:

$$E_0 := \sum_{i=1}^4 \alpha_i e_0^{(i)}, \quad E_0^2 = +1. \quad (44)$$

The other three time vectors in orthogonal complement to E_0 cannot remain time:

$$E_0 \perp e_0^{(2)}, \quad E_0 \perp e_0^{(3)}, \quad E_0 \perp e_0^{(4)}. \quad (45)$$

If the vector $e_0^{(i)}$ at $i > 1$ no longer plays the role of time, then in the subalgebra $Cl_{1,3}^{(i)}$ only the spatial 4 generators remain:

$$Cl_{1,3}^{(i)} \rightarrow Cl_{0,4}^{(i)}. \quad (46)$$

Therefore, since there were 4 copies of $Cl_{1,3}^{(i)}$, and the global signature only allows one, then the remaining 3 copies of $Cl_{1,3}^{(i)}$ must go into $Cl_{0,4}^{(i)}$:

$$Cl_{1,3}^{\oplus 4} \rightarrow Cl_{1,3} \oplus Cl_{0,4}^{\oplus 3}. \quad (47)$$

Hence we have expression (33) and Theorem 3 is proven, since out of four copies of $Cl_{1,3}^{(i)}$ only one can have an active time generator, and the other three are forcibly projected onto $Cl_{0,4}^{(i)}$, because only such a combination is consistent with the signature (1,9) and the algebraic structure.

□

4. Conclusions

The conclusion after the mathematical proof of the uniqueness of the decomposition (33) is the expediency of using precisely such a structure for modeling calculations of critical or nonlinear phenomena within the DAM model of the Universe, and the IPET theory arising from it. This direction can be useful both in a separate and independent analysis of possible influences of external spaces, and in hybrid use, as an auxiliary tool for strengthening standard theories.

Furthermore, the algebraic uniqueness of the decomposition ensures internal consistency and provides a mathematically rigid foundation for the proposed theoretical framework. The resulting Clifford-algebraic environment opens new avenues for modeling spontaneous symmetry breaking, topological transitions, or critical phenomena in both cosmology and condensed matter.

In future studies, this explicitly defined algebraic landscape could serve as a consistent framework for bridging hidden and observable sectors of physical theory — a step toward a deeper understanding of quantum spacetime and the early stages of cosmic evolution.

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