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Quantization of the Cosmic Critical Density

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Abstract

In this paper we study the quantization of the cosmic critical density. Applying the generalized relational expression, we derive a quantized formula for the cosmic critical density and subsequently prove. We compare a graph of the three components of the formula, it reveals that the gravitational quantization term dominates during the very early universe and near the Planck time, suggesting it may be a consequence of a complete theory of quantum gravity. Last we find a primitive function which the Taylor expansion is this quantized formula. Our discussion is intriguing and heuristically valuable.

Keywords: quantization; cosmic critical density; generalized relational expression; formula; Hubble constant; Planck time

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1. Introduction

The cosmic critical density [1] is a key parameter determining the geometric curvature of the universe. The primary motivation for its quantization study stems from two major conflicts: the vacuum energy catastrophe [2] and the coincidence problem [3–9]. Vacuum energy contributions from quantum fluctuations require renormalization, yet face fundamental contradictions [10,11]. Within the framework of loop quantum cosmology, the critical density becomes intricately linked to the discrete structure of spacetime [12]. The string landscape hypothesis leads to a random distribution of cosmic critical density among 10^{500} vacuum solutions, with the anthropic principle selecting habitable values, albeit lacking a dynamical mechanism [13]. Shen. J proposed a spin connection gauge theory where the local Lorentz group acts as the gauge group, preventing vacuum energy from contributing to the field equations and thus avoiding the fine-tuning problem of cosmic critical density, though resulting in third-order differential gravitational field equations [14]. J.S. Peracaula et al proposed that in the very early universe, higher powers of the Hubble constant H would dominate, leading to a period of rapid expansion, with the minimum possible driving term being H^4 [15]. The holographic principle [16] also offers an explanatory path via the multiverse; meanwhile, dynamical field models [17] and precision experiments (e.g., cold atoms, cosmological observations) [18,19] are gradually building bridges for empirical verification.

The structure of this paper is as follows. In Sec. 2, we derive the quantized formula for the cosmic critical density. In Sec. 3, prove the second and third parts of this formula. In Sec. 4, compare the three components of the formula through function graphs. In Sec. 5, find the primitive function. We conclude in Sec. 6.

2. Quantization of Cosmic Critical Density

This section applies the generalized relational expression [20] to derive the quantized formula for the cosmic critical density.

Let's review the generalized relational expression. The basic relationship [20] is

$$A \sim \mathrm{A_P} = [\hbar^{(\delta+\varepsilon+\zeta+\eta)} \mathrm{G}^{(\delta-\varepsilon+\zeta-\eta)} \mathrm{c}^{-(3\delta-\varepsilon+5\zeta-5\eta)} \kappa^{-2\eta} \mathrm{e}^{2\lambda}]^{1/2} \quad (1)$$

Where A is any physical quantity, $[A] = [\mathrm{L}]^{\delta}[\mathrm{M}]^{\varepsilon}[\mathrm{T}]^{\zeta}[\Theta]^{\eta}[\mathrm{Q}]^{\lambda}$ its dimensions, L, M, T, Θ and Q are the dimensions of length, mass, time, temperature and electric charge separately (here we use the LMTΘQ units), $\mathrm{A_P}$ the corresponding Planck scale of A, δ, ε, ζ, η and λ the real number, ħ, G, c, κ and e the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge separately.

The Generalized Relational Expression [20] is

$\prod_{i=1}^{n} A_i^{\alpha_i} \sim \prod_{i=1}^{n} A_{iP}^{\alpha_i}; i = 1, 2, 3\ldots n$ (2)

where A_i is the physical quantity, α_i the real number, and A_{iP} the corresponding Planck scale.

So assuming the cosmic critical density ρ depends solely on the Hubble constant H, we obtain

$\rho H^{\alpha} \sim \rho_P H_P^{\alpha} = \hbar^{-(2+\alpha)/2} G^{-(4+\alpha)/2} c^{5(2+\alpha)/2}$ (3)

where $\rho_P = c^5/\hbar G^2$ is the Planck mass density, $H_P = \sqrt{c^5/\hbar G}$ the Planck Hubble constant.

Setting $2+\alpha = 0$, → $\alpha = -2$, $\hbar^0=1$, and $c^0=1$, that is neglecting the quantum effect and relativistic effect, we give

$\rho \sim H^2/G$

which is $\rho_c = 3H_0^2/8\pi G$ [1], where H_0 is today Hubble constant.

Ordering $4+\alpha = 0$, → $\alpha = -4$, $G^0=1$, it indicates neglecting the gravitational effect, obtain

$\rho \sim \hbar H^4/c^5$ (4)

it represents the quantization of the cosmic critical density.

Taking $\alpha = -6$, find

$\rho \sim \hbar^2 G H^6/c^{10}$ (5)

which represents the gravitational quantization of the cosmic critical density. Thus

$\rho \sim H^2/G + \hbar H^4/c^5 + \hbar^2 G H^6/c^{10} \sim \rho_{I} + \rho_{II} + \rho_{III}$ (6)

This is the quantized formula for the cosmic critical density. As $\hbar \to 0$, $\rho \sim H^2/G$, consistent with the result from general relativity.

3. Proof of ρ_{II} and ρ_{III}

This section proves the quantized formula (6) for the cosmic critical density.

3.1. Proof of ρ_{II}

In quantum field theory, the vacuum energy density for a free scalar field is

$\rho = 0.5\int_0^{k_{max}} \frac{d^3k}{(2\pi)^3} \hbar\omega_k$

where $E = \hbar\omega_k/2$ is the zero-point energy [21], ω_k the frequency, and k the wavenumber. For a massless scalar field, $\omega_k = ck$. In three-dimensional momentum space, $d^3k = 4\pi k^2 dk$, substituting them into the above equation, we obtain

$\rho = \hbar c k_{max}^4/16\pi^2$

If the cutoff wavenumber k_{max} is chosen as the Hubble radius $R_H = c/H$, then the minimum wavelength $\lambda_{min} = R_H = c/H$, and the maximum wavenumber is $k_{max} = 2\pi/\lambda_{min} = 2\pi H/c$. Substituting into the above formula, we get

$\rho_{II} = \pi^2 \hbar H^4/c^5$ (7)

Proof completes.

3.2. Proof of ρ_{III}

Similarly, assuming

$\rho = 2\pi \int_0^{k_{max}} \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 G\omega_k^3}{c^5}$

where $E = 2\pi\hbar^2 G\omega_k^3/c^5$ [22]. Substituting $\omega_k = ck$, $d^3k = 4\pi k^2 dk$, and $k_{max} = 2\pi/\lambda_{min} = 2\pi H/c$ into above equation, we find

$\rho_{III} = 8\pi^5 \hbar^2 G H^6/3c^{10}$ (8)

Thus, we obtain

$\rho = 3H^2/8\pi G + \pi^2 \hbar H^4/c^5 + 8\pi^5 \hbar^2 G H^6/3c^{10}$ (9)

As $\hbar \to 0$, $\rho = 3H^2/8\pi G$.

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4. Comparison of ρ_{I}, ρ_{II} and ρ_{III}

This section compares the three terms in Eq. (9) via $\lg\rho - \lg t$ function graphs.

As the universe expands, H gradually decreases. However, since Eq. (9) involves powers 2, 4, and 6 of H, a direct comparison is inconvenient. Therefore, we substitute $H = 1/t$, where t is the age of the universe, yielding

$$\lg\rho=\lg(3/8\pi G t^2+\pi^2\hbar/c^5t^4+8\pi^5\hbar^2G/3c^{10}t^6)=\lg(\rho_{\mathrm{I}}+\rho_{\mathrm{II}}+\rho_{\mathrm{III}}) \quad (10)$$

Plotting the $\lg\rho - \lg t$ function graph gives

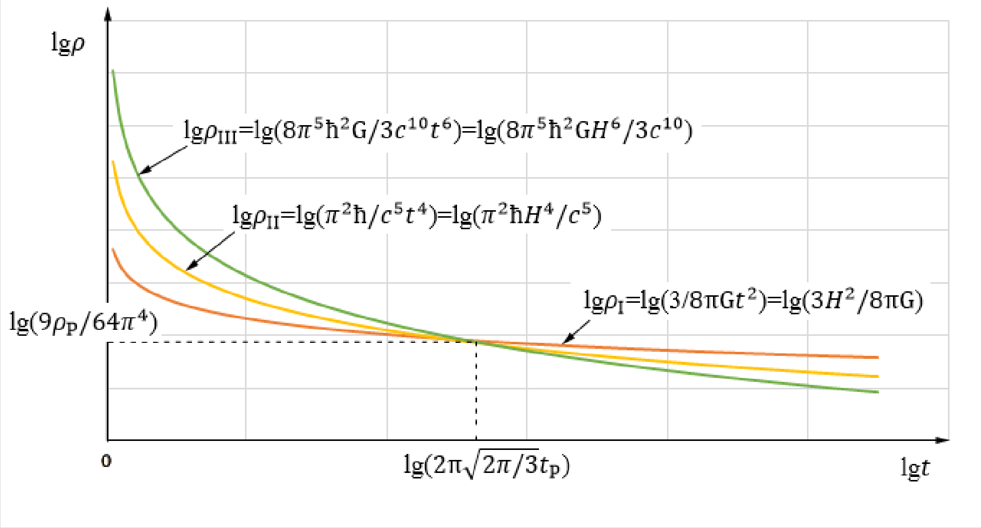

Figure 1. Schematic diagram showing the evolution of ρ_{I}, ρ_{II}, and ρ_{III} with cosmic time t. ρ_{I} ($\sim 1/t^2$) dominates at late times, ρ_{II} ($\sim 1/t^4$) is significant at intermediate times, and ρ_{III} ($\sim 1/t^6$) dominates at very early times near t_{P}.

Clearly, when $t=2\pi\sqrt{2\pi/3}t_{\mathrm{P}}$ (i.e., $H = H_{\mathrm{P}}\sqrt{3/2\pi}/2\pi$), $\rho_{\mathrm{I}}=\rho_{\mathrm{II}}=\rho_{\mathrm{III}}=9\rho_{\mathrm{P}}/64\pi^4$, where $H_{\mathrm{P}}=1/t_{\mathrm{P}}$ and $t_{\mathrm{P}}=\sqrt{\hbar G/c^5}$ is the Planck time.

When $t > 20\pi\sqrt{2\pi/3}t_{\mathrm{P}}$, $\rho_{\mathrm{II}}/\rho_{\mathrm{I}} < 0.01$, $\rho_{\mathrm{III}}/\rho_{\mathrm{II}} < 0.01$, allowing ρ_{II} and ρ_{III} to be neglected, we obtain $\rho=3H^2/8\pi G$.

When $0 < t < 0.2\pi\sqrt{2\pi/3}t_{\mathrm{P}}$, $\rho_{\mathrm{I}}/\rho_{\mathrm{II}} < 0.01$, $\rho_{\mathrm{II}}/\rho_{\mathrm{III}} < 0.01$, allowing ρ_{I} and ρ_{II} to be neglected, give $\rho=8\pi^5\hbar^2GH^6/3c^{10}$.

This indicates that the gravitational quantization term dominates from the very beginning of the universe until approximately 0.91 times the Planck time, suggesting that this may be a result of a complete theory of quantum gravity. Therefore, the cosmic critical density today differs from that during the very early universe and near the Planck time.

5. Primitive Function

If we regard Eq. (9) as the Taylor expansion of a function near $H = 0$, it is

$$\rho=\frac{3H^2}{8\pi G(1-\frac{8\pi^3\hbar G H^2}{3c^5})}=\frac{3H^2}{8\pi G(1-\frac{8\pi^3H^2}{3H_P^2})},\quad 0 < H < H_{\mathrm{P}}\sqrt{3/2\pi}/2\pi \qquad (11)$$

As ħ → 0, $\rho=3H^2/8\pi G$ also.

Writing the coefficients of the last two terms of Eq. (9) in ratio form, we get

$$\frac{\pi^2\hbar}{c^5}=\frac{3}{8\pi G}\left(\frac{8\pi^3\hbar G}{3c^5}\right),\quad \frac{8\pi^5\hbar^2G}{3c^{10}}=\frac{3}{8\pi G}\left(\frac{8\pi^3\hbar G}{3c^5}\right)^2$$

Then Eq. (9) can be rewrite as

$$\rho=\frac{3H^2}{8\pi G}(1+\beta H^2+\beta^2H^4+\cdots),\quad \beta=\frac{8\pi^3\hbar G}{3c^5}$$

The geometric series is

$$\frac{1}{1-\beta H^2}=1+\beta H^2+\beta^2H^4+\cdots$$

Therefore, the primitive function is Eq. (11)

When $H = \mathrm{H_P}\sqrt{3/2\pi}/2\pi$, $\rho \to \infty$, Eq. (11) has singularity, so it needs to be analyzed and extended to the rang of $[\mathrm{H_P}\sqrt{3/2\pi}/2\pi, 0)$.

From Eq. (11), obtain

$$H^2 = \frac{8\pi G\rho}{3(1+\frac{64\pi^4\hbar G^2\rho}{9c^5})} = \frac{8\pi G\rho}{3(1+\frac{64\pi^4\rho}{9\rho_P})} \qquad (12)$$

It is different from the model that writes quantum corrections as $H^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_{crit}}\right)$ [23], and the density coupled theory that writes the effective gravitational constant as $G_{eff} = G(1 + \alpha/2)$ [24]. But there is not enough experimental and observational data to prove G being variable.

6. Conclusion

In this paper we have studied the quantization of the cosmic critical density. Applying the generalized relational expression [1], we derived a quantized formula. This formula includes the result from general relativity, a quantization term, and a gravitational quantization term. It reduces to the general relativity result when quantum effects are neglected. Then we prove the latter two terms. We compared the three components via $\lg\rho - \lg t$ graphs, it showed that the gravitational quantization term dominates during the very early universe until approximately 0.91times the Planck time, suggesting it is a consequence of a complete theory of quantum gravity. Note Eq. (9) is the Taylor expansion of primitive function Eq. (11) near $H = 0$ which is analyzed and extended to the rang of $[\mathrm{H_P}\sqrt{3/2\pi}/2\pi, 0)$. Our discussion is intriguing and provides heuristic inspiration for developing a complete theory of quantum gravity.

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