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Article

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Abstract

A *Hyperstructure* builds on the powerset idea to describe how elements of a set interact. Extending this notion, a *Superhyperstructure* employs higher-order powersets to represent hierarchical, multi-layer systems, enabling richer abstractions and more intricate relationships . A Neutrosophic Set models uncertainty using three membership aspects—truth, indeterminacy, and falsity—as introduced. These sets naturally generalize to HyperNeutrosophic Sets and to SuperHyperNeutrosophic Sets, which are defined within hyperstructural and superhyperstructural frameworks. In this paper, we extend Cylindrical, Spherical, HyperSpherical, and Triple-valued Neutrosophic Sets into the settings of Hyper-Neutrosophic Sets and SuperHyperNeutrosophic Sets. Throughout, the term *SuperHyperNeutrosophic Sets* specifically refers to the family of (m, n)-SuperHyperNeutrosophic Sets.

Keywords: HyperStructure; SuperHyperStructure; HyperNeutrosophic Sets; SuperHyperNeutrosophic Sets

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1. Preliminaries

This section gathers the basic notions and notation used throughout the paper. Unless explicitly stated otherwise, we work in the finite setting. By convention, the empty set is treated as an element of every set.

1.1. Hyperstructure and Superhyperstructure

A *Hyperstructure* is organized around the powerset and serves as a vehicle for modeling relations among elements of a set [1–6]. Owing to its flexibility, the hyperstructure framework has been investigated across several areas, including mathematics and chemistry [7–10]. A *Superhyperstructure* advances this idea by utilizing the n-th powerset to encode multi-layered hierarchical interactions, thereby enabling deeper abstraction and greater structural complexity [11–13]. Related concepts such as *SuperHyperGraph* are also known [14–19]. We next record the n-th powerset, which underpins these structures.

Definition 1 (Base Set). *A base set S is the underlying collection from which higher-level constructions—powersets and (super)hyperstructures—are built. Formally,*

$$S = \{ x \mid x \text{ is an element of a specified domain} \}.$$

All elements appearing in $\mathcal{P}(S)$ or in the iterated powersets $\mathcal{P}_n(S)$ ultimately arise from members of S.

Definition 2 (Powerset). *[20] The* powerset *of a set S, denoted $\mathcal{P}(S)$, is the family of all subsets of S, including $\varnothing$ and S itself:*

$$\mathcal{P}(S) = \{ A \mid A \subseteq S \}.$$

Definition 3 (*n*-th Powerset). *(cf.[21–24]) For a set H, the n-th powerset $\mathcal{P}_n(H)$ is defined recursively by*

$$\mathcal{P}_1(H) = \mathcal{P}(H), \qquad \mathcal{P}_{n+1}(H) = \mathcal{P}(\mathcal{P}_n(H)), \quad n \geq 1.$$

The nonempty version $\mathcal{P}_n^*(H)$ is given by

$$\mathcal{P}_1^*(H) = \mathcal{P}^*(H), \qquad \mathcal{P}_{n+1}^*(H) = \mathcal{P}^*(\mathcal{P}_n^*(H)),$$

where $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\varnothing\}$.

Example 1 (*n*-th Powerset — menus of menus for a weekend meal plan). *Let the base set of dishes be*

$$H = \{Salad,\ Pasta,\ Curry\}.$$

The first powerset $\mathcal{P}(H)$ lists all possible daily menus; its cardinality is

$$|\mathcal{P}(H)| = 2^{|H|} = 2^3 = 8,$$

and explicitly

$$\mathcal{P}(H) = \{\varnothing,\ \{Salad\},\ \{Pasta\},\ \{Curry\},\ \{Salad,Pasta\},\ \{Salad,Curry\},\ \{Pasta,Curry\},\ \{Salad,Pasta,Curry\}.$$

The second powerset $\mathcal{P}_2(H) = \mathcal{P}(\mathcal{P}(H))$ collects menus of menus *(e.g., a two-day plan that prescribes a set of allowed daily menus for each day). Its size is*

$$|\mathcal{P}_2(H)| = 2^{|\mathcal{P}(H)|} = 2^8 = 256.$$

A concrete element

$$B = \big\{\ \{Salad\},\ \{Pasta,Curry\}\ \big\}\ \in\ \mathcal{P}_2(H)$$

encodes the real-life policy: "Over the weekend pick, on one day, 'Salad' and, on the other day, 'Pasta & Curry'."
Here B is a set of allowable day-level menus (elements of $\mathcal{P}(H)$), hence a bona fide member of $\mathcal{P}_2(H)$.

To provide a self-contained foundation for hyperstructures and superhyperstructures, we recall the following standard notions.

Definition 4 (Classical Structure). *(cf. [21,22,25]) A* Classical Structure *consists of a nonempty set H together with one or more* classical operations *satisfying specified axioms. A classical m-ary operation has the form*

$$\#_0 : H^m \to H,$$

with $m \geq 1$. Familiar examples include the operations defining groups, rings, and fields.

Definition 5 (Hyperoperation). *(cf. [11,26–28]) A* hyperoperation *on a set S is a map*

$$\circ : S \times S \longrightarrow \mathcal{P}(S),$$

so that combining two inputs returns a set *of outcomes (not necessarily a singleton).*

Definition 6 (Hyperstructure). *(cf. [21,22,29,30]) A* Hyperstructure *augments a base set S by operating on its powerset. Formally,*

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where $\circ$ acts on subsets of S.

Example 2 (Hyperstructure — adding measured weights with ± 1 g error)**.** *Fix a bounded gram scale*

$$S = \{0, 1, 2, \ldots, 10\}.$$

Define a hyperoperation $\oplus : S \times S \to \mathcal{P}(S)$ *by*

$$x \oplus y \; := \; \{\, z \in S \mid |z - (x + y)| \le 1 \,\},$$

which models the set of possible *combined readings when each measurement may err by at most* $1\,g$*. For instance,*

$$2 \oplus 3 = \{4, 5, 6\}, \qquad 4 \oplus 7 = \{10\} \;\; (\textit{clipped to } S), \qquad 5 \oplus 5 = \{9, 10\}.$$

A sample two-step combination expands setwise:

$$(2 \oplus 3) \oplus 4 \;=\; \{4, 5, 6\} \oplus 4 \;=\; \bigcup_{u \in \{4,5,6\}} (u \oplus 4) \;=\; \{7, 8, 9, 10\}.$$

Thus $\mathcal{H} = (S, \oplus)$ *is a* hyperstructure*: combining two elements yields a* set *of feasible outcomes reflecting real measurement uncertainty.*

Definition 7 (SuperHyperOperation)**.** *[21] Let H be nonempty. Define recursively, for* $k \ge 0$,

$$\mathcal{P}^0(H) = H, \qquad \mathcal{P}^{k+1}(H) = \mathcal{P}(\mathcal{P}^k(H)).$$

For fixed $m, n \ge 0$ *and arity* $s \ge 1$, *an* (m, n)-SuperHyperOperation *is a map*

$$\odot^{(m,n)} : \; \big(\mathcal{P}^m(H)\big)^s \longrightarrow \mathcal{P}^n(H).$$

If the codomain may include $\varnothing$, *we obtain the neutrosophic variant; otherwise we are in the classical case.*

Definition 8 (n-Superhyperstructure)**.** *(cf. [21,24,31,32]) An* n-Superhyperstructure *generalizes hyperstructures by acting on the n-th powerset:*

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ),$$

with $\circ$ *defined on* $\mathcal{P}_n(S)$.

Example 3 (n-Superhyperstructure ($n = 2$) — staffing bundles across two shifts)**.** *Let the staff pool be* $S = \{A, B, C\}$*. Elements of* $\mathcal{P}_2(S) = \mathcal{P}(\mathcal{P}(S))$ *are sets of candidate rosters (each roster is a subset of S). Define a level-2 hyperoperation* $\diamond : \mathcal{P}_2(S) \times \mathcal{P}_2(S) \to \mathcal{P}_2(S)$ *by*

$$\mathcal{R}_1 \diamond \mathcal{R}_2 \; := \; \{\, R_1 \cup R_2 \; : \; R_1 \in \mathcal{R}_1, \; R_2 \in \mathcal{R}_2 \,\}.$$

Take concrete bundles (e.g., Day 1 and Day 2 candidate rosters)

$$\mathcal{R}_1 = \{\{A, B\}, \{B, C\}\}, \qquad \mathcal{R}_2 = \{\{A\}, \{C\}\}.$$

Then

$$
\begin{aligned}
\mathcal{R}_1 \diamond \mathcal{R}_2 &= \{\, \{A, B\} \cup \{A\}, \; \{A, B\} \cup \{C\}, \; \{B, C\} \cup \{A\}, \; \{B, C\} \cup \{C\} \,\} \\
&= \{\, \{A, B\}, \; \{A, B, C\}, \; \{A, B, C\}, \; \{B, C\} \,\} \\
&= \{\, \{A, B\}, \; \{B, C\}, \; \{A, B, C\} \,\} \; \in \; \mathcal{P}_2(S).
\end{aligned}
$$

Each $\mathcal{R}_i$ *lists acceptable rosters for a shift;* $\diamond$ *propagates to the set-of-sets level by taking all unions, yielding the feasible* two-shift *joint rosters as an element of* $\mathcal{P}_2(S)$, *i.e., an n-superhyperstructural combination at* $n = 2$.

Definition 9 (SuperHyperStructure of order (m, n)). *(cf. [11,33,34]) Let S be nonempty and $m, n \geq 0$. A (m, n)-SuperHyperStructure of arity s is any choice of*

$$\odot^{(m,n)} : \left(\mathcal{P}^m(S)\right)^s \longrightarrow \mathcal{P}^n(S).$$

The special cases recover standard settings: $m = n = 0$ gives ordinary s-ary operations; $m = 0, n = 1$ yields hyperoperations; and $s = 1$ corresponds to superhyperfunctions.

Example 4 (SuperHyperStructure of order (m, n) with $(m, n) = (1, 2)$ — retail packaging rule). *Let the item universe be $S = \{Milk, \ Bread, \ Eggs, \ Butter\}$. Consider the binary* superhyperoperation

$$\odot^{(1,2)} : \mathcal{P}(S) \times \mathcal{P}(S) \longrightarrow \mathcal{P}_2(S),$$

defined by

$$\odot^{(1,2)}(A, B) := \{ A, \ B, \ A \cup B \} \setminus \{\varnothing\},$$

viewed as a member of $\mathcal{P}_2(S)$ (a set of first-level bundles). Take concrete baskets

$$A = \{Milk, Eggs\}, \qquad B = \{Bread\}.$$

Then

$$\odot^{(1,2)}(A, B) = \{ \{Milk, Eggs\}, \{Bread\}, \{Milk, Eggs, Bread\} \} \in \mathcal{P}_2(S).$$

Type check. The domain is $\mathcal{P}^1(S) \times \mathcal{P}^1(S)$, while the codomain is $\mathcal{P}^2(S)$, hence this is a bona fide SuperHyperStructure *of order $(1, 2)$. Operationally, it encodes a packaging policy: keep each basket as an allowed pack and also allow the merged pack, all represented at the* set-of-bundles *level.*

1.2. HyperNeutrosophic Set

A Neutrosophic Set models uncertainty using three membership functions: truth (T), indeterminacy (I), and falsity (F), which satisfy:

$$0 \leq T + I + F \leq 3.$$

[35–39]. These sets can be extended into HyperNeutrosophic Sets [40–47] and SuperHyperNeutrosophic Sets using hyperstructures and superhyperstructures.

Definition 10 (Neutrosophic Set). *[36,48] Let X be a non-empty set. A Neutrosophic Set (NS) A on X is characterized by three membership functions:*

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 11 (HyperNeutrosophic Set). *[19,42] Let X be a non-empty set. A mapping $\tilde{\mu} : X \to \tilde{P}([0,1]^3)$ is called a HyperNeutrosophic Set over X, where $\tilde{P}([0,1]^3)$ denotes the family of all non-empty subsets of the unit cube $[0,1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0,1]^3$ represents a set of neutrosophic membership degrees, each consisting of truth (T), indeterminacy (I), and falsity (F) components, satisfying:*

$$0 \leq T + I + F \leq 3.$$

Example 5 (HyperNeutrosophic Set — Hospital triage for influenza-like illness)**.** *Let the universe be a symptom list*

$$X = \{\text{Fever, Cough, SoreThroat}\}.$$

Define a HyperNeutrosophic Set $\tilde{\mu} : X \to \tilde{\mathcal{P}}([0,1]^3)$ where, for each symptom $x \in X$, $\tilde{\mu}(x)$ is a set of plausible neutrosophic assessments (T, I, F) (truth, indeterminacy, falsity) collected from multiple clinicians and timepoints.

We stipulate

$$\tilde{\mu}(\text{Fever}) = \big\{(0.92,\ 0.05,\ 0.08),\ (0.80,\ 0.10,\ 0.15)\big\},$$
$$\tilde{\mu}(\text{Cough}) = \big\{(0.60,\ 0.25,\ 0.35),\ (0.45,\ 0.30,\ 0.40)\big\},$$
$$\tilde{\mu}(\text{SoreThroat}) = \big\{(0.55,\ 0.20,\ 0.40),\ (0.40,\ 0.35,\ 0.45)\big\}.$$

Verification of admissibility ($0 \le T + I + F \le 3$) holds pointwise, e.g.

$$0.92 + 0.05 + 0.08 = 1.05 \le 3, \quad 0.80 + 0.10 + 0.15 = 1.05 \le 3,$$
$$0.60 + 0.25 + 0.35 = 1.20 \le 3, \quad 0.45 + 0.30 + 0.40 = 1.15 \le 3,$$
$$0.55 + 0.20 + 0.40 = 1.15 \le 3, \quad 0.40 + 0.35 + 0.45 = 1.20 \le 3.$$

Each symptom x is not assigned a single triplet but a hyper–set of triplets, capturing variability across clinicians (e.g., senior vs. junior), instruments (oral vs. tympanic thermometers), and times (triage vs. recheck). Higher T reflects evidence supporting "the symptom indicates influenza," higher F reflects counter–evidence (e.g., differential causes), and I captures measurement and context ambiguity.

Definition 12 (*n*-SuperHyperNeutrosophic Set)**.** *[19,42] Let X be a non-empty set. An n-SuperHyperNeutrosophic Set is a recursive generalization of Neutrosophic Sets, HyperNeutrosophic Sets, and SuperHyperNeutrosophic Sets. It is defined as:*

$$\tilde{A}_n : \tilde{\mathcal{P}}_n(X) \to \tilde{\mathcal{P}}_n([0,1]^3),$$

where:

- $\tilde{\mathcal{P}}_1(X) = \tilde{\mathcal{P}}(X)$, *and for $k \ge 2$,*

$$\tilde{\mathcal{P}}_k(X) = \tilde{\mathcal{P}}(\tilde{\mathcal{P}}_{k-1}(X)),$$

  represents the k-th nested family of non-empty subsets of X.
- $\tilde{\mathcal{P}}_n([0,1]^3)$ *is similarly defined for the unit cube $[0,1]^3$.*
- *The mapping $\tilde{A}_n$ assigns to each $A \in \tilde{\mathcal{P}}_n(X)$ a subset $\tilde{A}_n(A) \subseteq [0,1]^3$, representing the degrees of truth (T), indeterminacy (I), and falsity (F) for the n-th level subsets of X.*

For each $A \in \tilde{\mathcal{P}}_n(X)$ and $(T, I, F) \in \tilde{A}_n(A)$, the following condition is satisfied:

$$0 \le T + I + F \le 3,$$

where T, I, and F represent the truth, indeterminacy, and falsity degrees, respectively.

Example 6 (*n*–SuperHyperNeutrosophic Set ($n = 2$) — Software release readiness across nested components)**.** *Let $X = \{\text{API, UI, Docs}\}$. Elements of $\tilde{\mathcal{P}}_2(X)$ are nonempty sets of nonempty subsets of X. Consider two nested "selection contexts"*

$$B_1 = \{\{\text{API}\},\ \{\text{API,UI}\}\}, \qquad B_2 = \{\{\text{Docs}\}\} \ \in \tilde{\mathcal{P}}_2(X).$$

Define an n–SuperHyperNeutrosophic Set $\tilde{A}_2 : \tilde{\mathcal{P}}_2(X) \to \tilde{\mathcal{P}}_2([0,1]^3)$ by assigning, to each $B \in \tilde{\mathcal{P}}_2(X)$, a nonempty set of nonempty sets of triplets (T, I, F) that encode "readiness is acceptable":

For B_1,

$$\tilde{A}_2(B_1) = \Big\{ \underbrace{\{(0.72,\ 0.18,\ 0.20)\}}_{\text{assessment for }\{\text{API}\}},\ \underbrace{\{(0.64,\ 0.26,\ 0.25),\ (0.58,\ 0.22,\ 0.30)\}}_{\text{assessments for }\{\text{API,UI}\}} \Big\},$$

and for B_2,

$$\tilde{A}_2(B_2) = \Big\{ \{(0.80,\ 0.10,\ 0.15),\ (0.70,\ 0.20,\ 0.20)\} \Big\}.$$

Admissibility checks:

$$0.72 + 0.18 + 0.20 = 1.10 \leq 3, \quad 0.64 + 0.26 + 0.25 = 1.15 \leq 3, \quad 0.58 + 0.22 + 0.30 = 1.10 \leq 3,$$
$$0.80 + 0.10 + 0.15 = 1.05 \leq 3, \quad 0.70 + 0.20 + 0.20 = 1.10 \leq 3.$$

The domain object B_1 collects two first–level *component sets. The image $\tilde{A}_2(B_1)$ mirrors this nesting: for each component set, we keep a* set of *viable neutrosophic triplets capturing alternative test scenarios (e.g., with and without performance degradation). The $n = 2$ superhyper level preserves this "set of sets" structure on both the domain and codomain.*

An (m, n)-SuperhyperNeutrosophic Set maps m-level nested subsets to nonempty collections of n-level neutrosophic (T, I, F)-triples, modeling hierarchical uncertainty across levels. The definition of the (m, n)-SuperhyperNeutrosophic Set is presented below.

Definition 13 ((m, n)-SuperhyperNeutrosophic Set). *Let U be a universe of discourse and let $A \subseteq U$ be nonempty. Fix nonnegative integers m, n. Define the m-th nested power set of A by*

$$\mathcal{P}^0(A) = A, \qquad \mathcal{P}^k(A) = \mathcal{P}\big(\mathcal{P}^{k-1}(A)\big) \quad (k \geq 1),$$

and similarly

$$\mathcal{P}^n([0,1]^3) = \underbrace{\mathcal{P}\big(\mathcal{P}(\cdots \mathcal{P}([0,1]^3) \cdots)\big)}_{n \text{ times}}.$$

An (m, n)-SuperhyperNeutrosophic Set on A is a function

$$\mu : \mathcal{P}^m(A) \longrightarrow \mathcal{P}^n([0,1]^3)$$

such that for each $X \in \mathcal{P}^m(A)$, the image $\mu(X)$ is a nonempty collection of triples $(T, I, F) \in [0,1]^3$ satisfying

$$0 \leq T + I + F \leq 3.$$

In other words, instead of a single neutrosophic value, each (possibly nested) element X is assigned a set of possible (T, I, F)-degrees.

Example 7 ((m, n)–SuperhyperNeutrosophic Set ($m = 2$, $n = 1$) — Emergency response over grouped regions). *Let the base set of regions be $U = \{\text{North, Central, South}\}$. Consider $m = 2$, so inputs are in $\mathcal{P}^2(U)$ (nonempty sets of nonempty subsets of U), representing grouped response bundles. Take the nested input*

$$X^\star = \big\{\{\text{North}\},\ \{\text{Central}, \text{South}\}\big\} \in \mathcal{P}^2(U).$$

Define a (m, n)–SuperhyperNeutrosophic Set $\mu : \mathcal{P}^2(U) \to \mathcal{P}^1([0,1]^3) = \mathcal{P}([0,1]^3)$ by

$$\mu(X^\star) = \Big\{(0.80,\ 0.15,\ 0.10),\ (0.70,\ 0.20,\ 0.25),\ (0.62,\ 0.28,\ 0.30)\Big\}.$$

Here (T, I, F) quantifies the claim "the current resources suffice for the bundled response." The three triplets reflect alternative constraints (fuel-limited convoy; staffing-limited; route-limited). Admissibility:

$$0.80 + 0.15 + 0.10 = 1.05 \leq 3, \quad 0.70 + 0.20 + 0.25 = 1.15 \leq 3, \quad 0.62 + 0.28 + 0.30 = 1.20 \leq 3.$$

Although the input is hierarchical ($m = 2$), we aggregate to a flat uncertainty image ($n = 1$) containing multiple plausible neutrosophic assessments. This matches practice: planners combine grouped subplans (roads, medical, power) and then maintain several viable global assessments for quick decision switching as constraints evolve.

1.3. k-Cylindrical Neutrosophic Set

A k-cylindrical neutrosophic set assigns positive α, neutral β, negative γ, with $\alpha^k + \gamma^k \leq 1$ and independent $\beta \in [0, 1]$ per element [49–54].

Definition 14 (*k*-Cylindrical Neutrosophic Set)**.** *[55–58] Let $X \neq \varnothing$ be a universe and fix an integer $k > 1$. A k-cylindrical neutrosophic set (abbrev. k-CyNS) A on X is specified by three membership functions*

$$\alpha_A, \ \beta_A, \ \gamma_A : X \to [0, 1],$$

called, respectively, the positive, neutral, *and* negative *degrees. We represent A as*

$$A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle : x \in X\},$$

subject to the pointwise constraints

$$0 \leq \beta_A(x) \leq 1 \quad and \quad 0 \leq \alpha_A(x)^k + \gamma_A(x)^k \leq 1 \qquad (\forall x \in X).$$

In this framework, α_A and γ_A are treated as (mutually) dependent neutrosophic components, whereas β_A is independent. Equivalently, a k-CyNS is a map $A : X \to [0, 1]^3$, $x \mapsto \langle \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle$, whose image satisfies the above "cylindrical" constraint.

Example 8 (*k*-Cylindrical Neutrosophic Set (software sprint risk; $k = 3$))**.** *Let*

$$X = \{LoginModule, \ PaymentGateway, \ RecommendationEngine\}$$

. Define a k-CyNS $A : X \to [0, 1]^3$, $x \mapsto \langle \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle$ with $k = 3$, where α = "evidence of on-time success," γ = "evidence of failure risk," and β = "neutral unknowns" (external dependencies, unclear specs).

$$A(LoginModule) = \langle 0.88, \ 0.10, \ 0.18 \rangle,$$
$$A(PaymentGateway) = \langle 0.76, \ 0.22, \ 0.30 \rangle,$$
$$A(RecommendationEngine) = \langle 0.62, \ 0.35, \ 0.55 \rangle.$$

Constraint checks (pointwise): $\beta \in [0, 1]$ is immediate. For the cylindrical constraint $\alpha^3 + \gamma^3 \leq 1$:

$$LoginModule: 0.88^3 + 0.18^3 = 0.681472 + 0.005832 = 0.687304 \leq 1,$$
$$PaymentGateway: 0.76^3 + 0.30^3 = 0.438976 + 0.027 = 0.465976 \leq 1,$$
$$RecommendationEngine: 0.62^3 + 0.55^3 = 0.238328 + 0.166375 = 0.404703 \leq 1.$$

α and γ capture coupled success/failure evidence (e.g., test coverage vs. blocking bugs), while β captures independent ignorance (e.g., partner API changes). All tasks satisfy the k-cylindrical feasibility.

1.4. Single-Valued Spherical Neutrosophic Set

A single-valued spherical neutrosophic set assigns truth T, indeterminacy I, falsity F in $[0,1]$, constrained pointwise by $T^2 + I^2 + F^2 \leq 1$ per element [59–61].

Definition 15 (Single-Valued Spherical Neutrosophic Set). *[62–65] Let $U \neq \varnothing$ be a universe. A single-valued spherical neutrosophic set (SNS) on U is a mapping*

$$A : U \to [0,1]^3, \qquad x \longmapsto \big(T_A(x),\, I_A(x),\, F_A(x)\big),$$

where T_A, I_A, F_A denote, respectively, the degrees of truth, indeterminacy, and falsity of x with respect to A. The spherical admissibility constraint requires, for every $x \in U$,

$$0 \leq T_A(x)^2 + I_A(x)^2 + F_A(x)^2 \leq 1.$$

Equivalently, the image of A lies in the unit ℓ^2–ball of $\mathbb{R}^3$ intersected with $[0,1]^3$.

Example 9 (Single-Valued Spherical Neutrosophic Set (road safety now; emergency routing)). *Let $U = \{BridgeA,\ TunnelB,\ HighwayC\}$. For the claim "segment is safe for emergency vehicles* now,*" define $A : U \to [0,1]^3$ with (T, I, F) the truth, indeterminacy, and falsity degrees. The spherical constraint requires $T^2 + I^2 + F^2 \leq 1$ pointwise.*

$$A(BridgeA) = (0.80,\, 0.30,\, 0.20), \quad 0.80^2 + 0.30^2 + 0.20^2 = 0.64 + 0.09 + 0.04 = 0.77 \leq 1,$$
$$A(TunnelB) = (0.60,\, 0.50,\, 0.30), \quad 0.60^2 + 0.50^2 + 0.30^2 = 0.36 + 0.25 + 0.09 = 0.70 \leq 1,$$
$$A(HighwayC) = (0.90,\, 0.10,\, 0.30), \quad 0.90^2 + 0.10^2 + 0.30^2 = 0.81 + 0.01 + 0.09 = 0.91 \leq 1.$$

T aggregates sensors (speed, traction), F aggregates counter-evidence (construction, incidents), and I captures incomplete feeds (camera outages). All segments respect the spherical admissibility.

1.5. Single-Valued k–HyperSpherical Neutrosophic Set

A single-valued k-hyperspherical neutrosophic set generalizes SNS, requiring $T^k + I^k + F^k \leq 1$ with components in $[0,1]$, evaluated pointwise for each element in U [66–68].

Definition 16 (Single-Valued k–HyperSpherical Neutrosophic Set). *[66–68] Fix an exponent $k > 1$. A single-valued k–hyperspherical neutrosophic set (k–HSNS) on the same universe U is a mapping*

$$A : U \to [0,1]^3, \qquad x \longmapsto \big(T_A(x),\, I_A(x),\, F_A(x)\big),$$

whose components satisfy, for all $x \in U$,

$$0 \leq T_A(x)^k + I_A(x)^k + F_A(x)^k \leq 1.$$

Thus the admissible triplets lie in the unit ℓ^k–ball of $\mathbb{R}^3$ intersected with $[0,1]^3$. For $k = 2$ one recovers the SNS above; increasing k enlarges the feasible region monotonically.

Example 10 (Single-Valued k–HyperSpherical Neutrosophic Set (same-day fulfillment; $k = 4$)). *Let $U = \{\#101,\ \#205,\ \#319\}$ denote orders. For the claim "deliverable today," define $A : U \to [0,1]^3$ with $k = 4$. The k–hyperspherical constraint requires $T^k + I^k + F^k \leq 1$ pointwise.*

$$A(\#101) = (0.85,\, 0.20,\, 0.25), \quad 0.85^4 + 0.20^4 + 0.25^4 = 0.52200625 + 0.0016 + 0.00390625 = 0.5275125 \leq 1,$$
$$A(\#205) = (0.70,\, 0.40,\, 0.35), \quad 0.70^4 + 0.40^4 + 0.35^4 = 0.2401 + 0.0256 + 0.01500625 = 0.28070625 \leq 1,$$
$$A(\#319) = (0.60,\, 0.50,\, 0.55), \quad 0.60^4 + 0.50^4 + 0.55^4 = 0.1296 + 0.0625 + 0.09150625 = 0.28360625 \leq 1.$$

T fuses stock/picking progress, F captures blockers (carrier capacity, cut-off miss), and I models uncertain ETA due to weather/traffic. With $k = 4$ (vs. $k = 2$), the feasible region enlarges, accommodating heavier "either–or" evidence patterns without violating the bound.

1.6. Triple-Valued Neutrosophic Set

A Triple-Valued Neutrosophic Set assigns each element degrees for truth, truth-leaning indeterminacy, neutral indeterminacy, falsity-leaning indeterminacy, and falsity, within limits.

Definition 17 (Triple-Valued Neutrosophic Set (TVNS)). *[69–72] A Triple-Valued Neutrosophic Set A on X is defined as*

$$A = \left\{ \Big(x, T_A(x), I_T(x), I_N(x), I_F(x), F_A(x) \Big) : x \in X \right\},$$

where

- $T_A(x) \in [0,1]$ *is the truth membership degree,*
- $I_T(x) \in [0,1]$ *is the* indeterminacy leaning towards truth,
- $I_N(x) \in [0,1]$ *is the* neutral indeterminacy *(i.e., completely indeterminate, neither leaning towards truth nor falsity),*
- $I_F(x) \in [0,1]$ *is the* indeterminacy leaning towards falsity,
- $F_A(x) \in [0,1]$ *is the falsity membership degree.*

For each $x \in X$, we have

$$0 \le T_A(x) + I_T(x) + I_N(x) + I_F(x) + F_A(x) \le 5.$$

Example 11 (Triple-Valued Neutrosophic Set — Hiring "strong fit" assessment). *Let the universe of candidates be*

$$X = \{Alice, Bob, Carol\}.$$

We evaluate the claim "candidate is a strong fit *for the Senior Data Engineer role" using a Triple-Valued Neutrosophic Set (TVNS)*

$$A = \{ (x, T_A(x), I_T(x), I_N(x), I_F(x), F_A(x)) : x \in X \},$$

where T_A is truth, F_A is falsity, and (I_T, I_N, I_F) split indeterminacy into (leaning-to-truth, neutral, leaning-to-falsity), respectively. Concrete assessments:

$$A(Alice) = (0.72, 0.10, 0.08, 0.05, 0.12),$$
$$A(Bob) = (0.40, 0.20, 0.15, 0.10, 0.45),$$
$$A(Carol) = (0.55, 0.18, 0.12, 0.07, 0.30).$$

Each component lies in $[0,1]$, and the TVNS admissibility condition $0 \le T_A + I_T + I_N + I_F + F_A \le 5$ holds pointwise:

$$Alice: 0.72 + 0.10 + 0.08 + 0.05 + 0.12 = 1.07 \le 5,$$
$$Bob: 0.40 + 0.20 + 0.15 + 0.10 + 0.45 = 1.30 \le 5,$$
$$Carol: 0.55 + 0.18 + 0.12 + 0.07 + 0.30 = 1.22 \le 5.$$

For Alice, high T_A comes from strong technical interview signals; small I_T reflects promising but partial evidence (e.g., a solid take-home task with minor gaps); I_N covers unknowns (limited references), while small I_F captures slight concerns (e.g., niche stack mismatch). F_A remains low, indicating little direct counter-evidence. Bob's larger F_A and I_T show mixed signals (good cultural fit but weaker systems design), and Carol sits between, with moderate T_A and balanced indeterminacy components.

2. Main Results

In this section, we present the main contributions of this paper.

2.1. k-Cylindrical Superhyperneutrosophic Set

A *k*-Cylindrical HyperNeutrosophic Set assigns to each element a nonempty set of triplets $(\alpha, \beta, \gamma) \in [0,1]^3$ satisfying $\alpha^k + \gamma^k \leq 1$, where β is independent. This structure models uncertainty with coupled positive/negative evidence (α, γ) and an independent neutral component (β). A *k*-Cylindrical Superhyperneutrosophic Set maps nested families of elements to nested collections of admissible triplets (α, β, γ) that satisfy $\alpha^k + \gamma^k \leq 1$. In doing so, it represents hierarchical uncertainty across levels while preserving the coupling between positive and negative evidence and keeping the neutral component independent.

Definition 18 (Admissible *k*-cylindrical region). *For a fixed integer $k > 1$, define the k-cylindrical admissible region*

$$\mathcal{C}_k := \{ (\alpha, \beta, \gamma) \in [0,1]^3 : \alpha^k + \gamma^k \leq 1 \}.$$

We regard β as independent (only constrained to $[0,1]$), while α (positive) and γ (negative) are coupled by the cylinder constraint. Note that every $(\alpha, \beta, \gamma) \in \mathcal{C}_k$ also satisfies $0 \leq \alpha + \beta + \gamma \leq 3$ since each coordinate lies in $[0,1]$.

Definition 19 (*k*-Cylindrical HyperNeutrosophic Set (k-CHyNS)). *Let $X \neq \varnothing$. A k-cylindrical hyperneutrosophic set on X is a map*

$$\tilde{\mu}_k : X \longrightarrow \tilde{\mathcal{P}}(\mathcal{C}_k), \qquad x \longmapsto \tilde{\mu}_k(x),$$

assigning to each $x \in X$ a nonempty set $\tilde{\mu}_k(x) \subseteq \mathcal{C}_k$ of admissible triplets (α, β, γ), interpreted as possible (truth, indeterminacy, falsity) assessments for x.

Example 12 (*k*-Cylindrical HyperNeutrosophic Set (cloud service rollout; $k = 3$)). *Let the universe of microservices be*

$$X = \{Auth,\ Billing,\ Search\}.$$

Interpret a triplet $(\alpha, \beta, \gamma) \in [0,1]^3$ as:

$$\alpha = \text{evidence of on-time success}, \quad \beta = \text{independent unknowns}, \quad \gamma = \text{evidence of failure risk}.$$

A k-Cylindrical HyperNeutrosophic Set (k-CHyNS) $\tilde{\mu}_k : X \to \tilde{\mathcal{P}}(\mathcal{C}_k)$ with $k = 3$ assigns to each service a nonempty set of admissible triplets, where

$$\mathcal{C}_3 = \{(\alpha, \beta, \gamma) \in [0,1]^3 : \alpha^3 + \gamma^3 \leq 1\}.$$

Define (two scenarios per service):

$$\tilde{\mu}_3(Auth) = \{(0.85,\ 0.10,\ 0.25),\ (0.78,\ 0.18,\ 0.35)\},$$
$$\tilde{\mu}_3(Billing) = \{(0.70,\ 0.22,\ 0.40),\ (0.62,\ 0.30,\ 0.50)\},$$
$$\tilde{\mu}_3(Search) = \{(0.80,\ 0.15,\ 0.30),\ (0.68,\ 0.25,\ 0.45)\}.$$

Constraint checks ($\beta \in [0,1]$ is immediate). For each listed triplet we verify $\alpha^3 + \gamma^3 \le 1$:

$$
\begin{aligned}
\textit{Auth:} \quad 0.85^3 + 0.25^3 &= 0.614125 + 0.015625 = 0.629750 \le 1, \\
0.78^3 + 0.35^3 &= 0.474552 + 0.042875 = 0.517427 \le 1; \\
\textit{Billing:} \quad 0.70^3 + 0.40^3 &= 0.343000 + 0.064000 = 0.407000 \le 1, \\
0.62^3 + 0.50^3 &= 0.238328 + 0.125000 = 0.363328 \le 1; \\
\textit{Search:} \quad 0.80^3 + 0.30^3 &= 0.512000 + 0.027000 = 0.539000 \le 1, \\
0.68^3 + 0.45^3 &= 0.314432 + 0.091125 = 0.405557 \le 1.
\end{aligned}
$$

Each service $x \in X$ is mapped to a set *of plausible assessments reflecting distinct rollout scenarios (e.g., different traffic profiles, A/B toggles, or canary depths). The cylindrical constraint couples success/failure evidence (α, γ) while keeping β independent.*

Theorem 1 (Reduction to k-cylindrical neutrosophic sets). *If, in a k-CHyNS $\tilde{\mu}_k$, every fiber is a singleton, i.e. $\tilde{\mu}_k(x) = \{(\alpha(x), \beta(x), \gamma(x))\}$ for all $x \in X$, then the map*

$$
A : X \to [0,1]^3, \qquad A(x) = (\alpha(x), \beta(x), \gamma(x)),
$$

is a k-cylindrical neutrosophic set in the sense that $\alpha(x)^k + \gamma(x)^k \le 1$ and $\beta(x) \in [0,1]$ for all $x \in X$.

Proof. By definition of k-CHyNS, each $(\alpha(x), \beta(x), \gamma(x)) \in \mathcal{C}_k$, hence $\alpha(x)^k + \gamma(x)^k \le 1$ and $\beta(x) \in [0,1]$. Therefore A satisfies exactly the pointwise constraints of the k-cylindrical neutrosophic set definition. $\square$

Theorem 2 (Generalization relation to hyperneutrosophic sets). *Let $\mathcal{C}_{\mathrm{all}} = [0,1]^3$. Consider the* forgetful *inclusion map $\iota_k : \mathcal{C}_k \hookrightarrow \mathcal{C}_{\mathrm{all}}$. Then:*

(i) For any k-CHyNS $\tilde{\mu}_k : X \to \tilde{\mathcal{P}}(\mathcal{C}_k)$, the composition $X \xrightarrow{\tilde{\mu}_k} \tilde{\mathcal{P}}(\mathcal{C}_k) \xrightarrow{\tilde{\mathcal{P}}(\iota_k)} \tilde{\mathcal{P}}(\mathcal{C}_{\mathrm{all}})$ is a (standard) hyperneutrosophic set on X.

(ii) Conversely, if one removes *the cylinder constraint by replacing $\mathcal{C}_k$ with $\mathcal{C}_{\mathrm{all}}$ in the definition of k-CHyNS, one recovers exactly the usual notion of a hyperneutrosophic set.*

Proof. (i) Since ι_k is an inclusion, $\tilde{\mathcal{P}}(\iota_k)$ sends each nonempty $\tilde{\mu}_k(x) \subseteq \mathcal{C}_k$ to the same set viewed inside $\mathcal{C}_{\mathrm{all}} = [0,1]^3$. Thus the image is a map into nonempty subsets of $[0,1]^3$, i.e. a hyperneutrosophic set.

(ii) If the admissible region is $\mathcal{C}_{\mathrm{all}}$, the only remaining constraints are $0 \le \alpha, \beta, \gamma \le 1$ (hence $0 \le \alpha + \beta + \gamma \le 3$), which is precisely the standard hyperneutrosophic set definition. $\square$

Definition 20 (k-Cylindrical (m,n)-Superhyperneutrosophic Set (k-(m,n)-SHyNS)). *Fix integers $m, n \ge 0$ and $k > 1$. Let $\mathcal{C}_k \subseteq [0,1]^3$ be the k-cylindrical region. A k-cylindrical (m,n)-superhyperneutrosophic set on a base $X \ne \varnothing$ is a function*

$$
\mu_k^{(m,n)} : \mathcal{P}_*^m(X) \longrightarrow \mathcal{P}_*^n(\mathcal{C}_k)
$$

such that for every $U \in \mathcal{P}_*^m(X)$, the value $\mu_k^{(m,n)}(U)$ is a nonempty n-level nested family of admissible triplets; i.e., every leaf (α, β, γ) appearing anywhere in $\mu_k^{(m,n)}(U)$ satisfies $(\alpha, \beta, \gamma) \in \mathcal{C}_k$ (equivalently $\alpha^k + \gamma^k \le 1$ and $\beta \in [0,1]$).

Example 13 (k-Cylindrical (m,n)-Superhyperneutrosophic Set with $(m,n) = (2,2)$ (multi-warehouse fulfillment bundles; $k = 3$)). *Let the warehouse set be $X = \{W1, W2, W3\}$. Elements of $\mathcal{P}_*^2(X)$ are nonempty sets of nonempty subsets of X, i.e., grouped fulfillment bundles (first level: a roster; second level: a set of candidate rosters).*

Fix $k = 3$ and $C_3 = \{(\alpha, \beta, \gamma) \in [0,1]^3 : \alpha^3 + \gamma^3 \leq 1\}$. Define two domain objects

$$U_1 = \{\{W1\}, \{W1, W2\}\}, \qquad U_2 = \{\{W3\}\} \in \mathcal{P}_*^2(X).$$

A k-(m, n)-SHyNS is a map

$$\mu_3^{(2,2)} : \mathcal{P}_*^2(X) \longrightarrow \mathcal{P}_*^2(C_3),$$

sending each U to a nonempty set of nonempty subsets of admissible triplets. Concretely, interpret (α, β, γ) as

α = evidence "bundle can fulfill today", $\quad \beta$ = independent unknowns (carrier/slot), $\quad \gamma$ = evidence of shortfall risk.

Specify the images (one subset per first-level roster):

$$\mu_3^{(2,2)}(U_1) = \Big\{ \underbrace{\{(0.82, 0.12, 0.28)\}}_{\text{for } \{W1\}}, \underbrace{\{(0.75, 0.20, 0.35), (0.66, 0.26, 0.44)\}}_{\text{for } \{W1,W2\}} \Big\},$$

$$\mu_3^{(2,2)}(U_2) = \Big\{ \underbrace{\{(0.78, 0.16, 0.36), (0.70, 0.18, 0.40)\}}_{\text{for } \{W3\}} \Big\}.$$

All leaves lie in C_3; we verify $\alpha^3 + \gamma^3 \leq 1$ explicitly:

$$(0.82, 0.12, 0.28) : 0.82^3 + 0.28^3 = 0.551368 + 0.021952 = 0.573320 \leq 1,$$
$$(0.75, 0.20, 0.35) : 0.75^3 + 0.35^3 = 0.421875 + 0.042875 = 0.464750 \leq 1,$$
$$(0.66, 0.26, 0.44) : 0.66^3 + 0.44^3 = 0.287496 + 0.085184 = 0.372680 \leq 1,$$
$$(0.78, 0.16, 0.36) : 0.78^3 + 0.36^3 = 0.474552 + 0.046656 = 0.521208 \leq 1,$$
$$(0.70, 0.18, 0.40) : 0.70^3 + 0.40^3 = 0.343000 + 0.064000 = 0.407000 \leq 1.$$

The domain nesting ($m = 2$) captures grouped rosters (single-warehouse or multi-warehouse bundles). The codomain nesting ($n = 2$) retains, for each roster, a set of admissible (α, β, γ) triplets corresponding to alternative operational scenarios (e.g., with/without overflow carrier, different cutoff buffers). The cylindrical constraint enforces the coupled nature of success/failure evidence while allowing the neutral component β to vary independently in $[0,1]$.

Theorem 3 (Specializations of k-(m, n)-SHyNS). *Let $k > 1$ and $m, n \geq 0$.*

(i) *(Recovery of (m, n)-SHNS) If one replaces C_k by $[0,1]^3$ in the above definition, one obtains exactly the standard (m, n)-superhyperneutrosophic set.*

(ii) *(Recovery of k-CHyNS) For $(m, n) = (0, 1)$, a k-(m, n)-SHyNS is precisely a k-cylindrical hyperneutrosophic set: $\mu_k^{(0,1)} : X \to \mathcal{P}([0,1]^3)$ with values in $\mathcal{P}(C_k)$.*

(iii) *(Reduction to k-CyNS) If, in addition to $(m, n) = (0, 1)$, every $\mu_k^{(0,1)}(x)$ is a singleton $\{(\alpha(x), \beta(x), \gamma(x))\}$, then $x \mapsto (\alpha(x), \beta(x), \gamma(x))$ is a k-cylindrical neutrosophic set.*

Proof. (i) Substituting C_k by $[0,1]^3$ removes the cylinder constraint while keeping the domain/codomain nesting. The result is exactly a map $\mathcal{P}_*^m(X) \to \mathcal{P}_*^n([0,1]^3)$, i.e. the usual (m, n)–SHNS.

(ii) When $m = 0$, $\mathcal{P}_*^0(X) = X$, and when $n = 1$, $\mathcal{P}_*^1(C_k) = \mathcal{P}(C_k)$. Hence the definition specializes to $\mu_k^{(0,1)} : X \to \mathcal{P}(C_k)$, which is exactly the k–CHyNS definition (map to nonempty subsets of C_k).

(iii) If each value is a singleton, define $\alpha, \beta, \gamma : X \to [0,1]$ by reading off the unique triplet. Since each triplet lies in C_k, we have $\alpha(x)^k + \gamma(x)^k \leq 1$ and $\beta(x) \in [0,1]$ for all x, which are precisely the pointwise constraints for a k-cylindrical neutrosophic set. $\square$

2.2. Single-Valued Spherical Superhyperneutrosophic set

A Single-Valued Spherical HyperNeutrosophic Set assigns each element a nonempty set of triplets satisfying $T^2 + I^2 + F^2 \leq 1$ within $[0,1]^3$, capturing assessment uncertainty. A Single-Valued Spherical SuperHyperNeutrosophic Set maps nested element families to nested triplet collections satisfying $T^2 + I^2 + F^2 \leq 1$, representing hierarchical uncertainty across levels.

Definition 21 (Spherical admissible region). *Define the* spherical admissible region *in* $[0,1]^3$ *by*

$$\mathcal{S} := \left\{ (T, I, F) \in [0,1]^3 : T^2 + I^2 + F^2 \leq 1 \right\}.$$

Each triplet in $\mathcal{S}$ is a single-valued *(scalar) specification of truth T, indeterminacy I, and falsity F, jointly constrained to lie in the unit ℓ^2–ball.*

Definition 22 (Single-Valued Spherical HyperNeutrosophic Set (SVS–HyNS)). *Let $X \neq \varnothing$ be a universe. A* single-valued spherical hyperneutrosophic set *is a map*

$$\tilde{v} : X \longrightarrow \tilde{\mathcal{P}}(\mathcal{S}), \qquad x \longmapsto \tilde{v}(x),$$

assigning to each $x \in X$ a nonempty set $\tilde{v}(x) \subseteq \mathcal{S}$ of admissible triplets (T, I, F), where every member satisfies $T^2 + I^2 + F^2 \leq 1$ and $T, I, F \in [0,1]$.

Example 14 (Single-Valued Spherical HyperNeutrosophic Set (EV charging availability now)). *Let the universe of charging sites be*

$$X = \{EastStation, \ CentralHub, \ WestPlaza\}.$$

Interpret a triplet $(T, I, F) \in [0,1]^3$ as

$$T = evidence \text{ "available capacity now"},$$

$$I = telemetry \ indeterminacy \ (lag/outages),$$

$$F = counter\text{-}evidence \ (occupied/out\text{-}of\text{-}service).$$

Let the spherical admissible region be

$$\mathcal{S} := \left\{ (T, I, F) \in [0,1]^3 : T^2 + I^2 + F^2 \leq 1 \right\}.$$

An SVS–HyNS is a map $\tilde{v} : X \to \tilde{\mathcal{P}}(\mathcal{S})$; specify two scenarios per site:

$$\tilde{v}(EastStation) = \left\{ (0.82, 0.30, 0.15), \ (0.74, 0.40, 0.20) \right\},$$
$$\tilde{v}(CentralHub) = \left\{ (0.60, 0.45, 0.30), \ (0.50, 0.55, 0.40) \right\},$$
$$\tilde{v}(WestPlaza) = \left\{ (0.88, 0.12, 0.30), \ (0.76, 0.20, 0.40) \right\}.$$

Spherical checks (pointwise):

$$0.82^2 + 0.30^2 + 0.15^2 = 0.6724 + 0.09 + 0.0225 = 0.7849 \leq 1,$$
$$0.74^2 + 0.40^2 + 0.20^2 = 0.5476 + 0.16 + 0.04 = 0.7476 \leq 1,$$
$$0.60^2 + 0.45^2 + 0.30^2 = 0.36 + 0.2025 + 0.09 = 0.6525 \leq 1,$$
$$0.50^2 + 0.55^2 + 0.40^2 = 0.25 + 0.3025 + 0.16 = 0.7125 \leq 1,$$
$$0.88^2 + 0.12^2 + 0.30^2 = 0.7744 + 0.0144 + 0.09 = 0.8788 \leq 1,$$
$$0.76^2 + 0.20^2 + 0.40^2 = 0.5776 + 0.04 + 0.16 = 0.7776 \leq 1.$$

Each site $x \in X$ is mapped to a set *of admissible triplets, reflecting distinct real-time scenarios (e.g., different arrival bursts or charger deratings). The spherical constraint coherently bounds the joint (T, I, F) evidence.*

Theorem 4 (SVS–HyNS generalizes both HyNS and SNS). *Let $\iota : \mathcal{S} \hookrightarrow [0,1]^3$ denote the inclusion.*

(i) (HyNS reduction) For any SVS–HyNS $\tilde{v} : X \to \tilde{\mathcal{P}}(\mathcal{S})$, the composition

$$X \xrightarrow{\tilde{v}} \tilde{\mathcal{P}}(\mathcal{S}) \xrightarrow{\tilde{\mathcal{P}}(\iota)} \tilde{\mathcal{P}}([0,1]^3)$$

is a (standard) hyperneutrosophic set *on X.*

(ii) (SNS reduction) If, in addition, every fiber is a singleton, $\tilde{v}(x) = \{(T(x), I(x), F(x))\}$, then the map

$$A : X \to [0,1]^3, \qquad A(x) = (T(x), I(x), F(x)),$$

is a single-valued spherical neutrosophic set *(SNS), i.e. $T(x)^2 + I(x)^2 + F(x)^2 \leq 1$ for all x.*

Proof. (i) Since ι is an inclusion, $\tilde{\mathcal{P}}(\iota)$ sends each nonempty $\tilde{v}(x) \subseteq \mathcal{S}$ to the *same* set viewed inside $[0,1]^3$. Hence the composite is a map to nonempty subsets of $[0,1]^3$, which is exactly the definition of a hyperneutrosophic set.

(ii) If $\tilde{v}(x) = \{(T(x), I(x), F(x))\}$ with $(T(x), I(x), F(x)) \in \mathcal{S}$, we have $T(x)^2 + I(x)^2 + F(x)^2 \leq 1$ and $T(x), I(x), F(x) \in [0,1]$ by Definition 21. Thus A satisfies the SNS constraint pointwise. $\square$

Definition 23 (SVS–(m,n)–SuperhyperNeutrosophic Set). *Fix integers $m, n \geq 0$. With $\mathcal{S}$ as in Definition 21, a* single-valued spherical (m,n)–superhyperneutrosophic set *on a base $X \neq \varnothing$ is a function*

$$v^{(m,n)} : \mathcal{P}_*^m(X) \longrightarrow \mathcal{P}_*^n(\mathcal{S})$$

such that, for every $U \in \mathcal{P}_*^m(X)$, the value $v^{(m,n)}(U)$ is a nonempty n–level nested family whose leaves are triplets $(T, I, F) \in \mathcal{S}$, equivalently $T^2 + I^2 + F^2 \leq 1$ and $T, I, F \in [0,1]$ at every leaf.

Example 15 (SVS–(m,n)–SuperhyperNeutrosophic Set with $(m,n) = (2,2)$ (campus fire-drill readiness)). *Let buildings be $X = \{A, B, C\}$. Elements of $\mathcal{P}_*^2(X)$ are nonempty sets of nonempty subsets of X (first level: a roster; second level: a set of candidate rosters). Consider the claim "roster can complete a fire drill in ≤ 15 minutes."*

Let $\mathcal{S} := \{(T, I, F) \in [0,1]^3 : T^2 + I^2 + F^2 \leq 1\}$. An SVS–$(2,2)$–SHyNS is a map

$$v^{(2,2)} : \mathcal{P}_*^2(X) \longrightarrow \mathcal{P}_*^2(\mathcal{S}),$$

assigning, to each grouped domain object, a nonempty set of nonempty subsets of admissible (T, I, F) triplets.

Choose domain bundles

$$U_1 = \{\{A, B\}, \{B, C\}\}, \qquad U_2 = \{\{A\}, \{C\}\} \in \mathcal{P}_*^2(X).$$

Specify images (each first-level roster gets a set *of triplets in $\mathcal{S}$):*

$$v^{(2,2)}(U_1) = \Big\{ \underbrace{\{(0.78, 0.30, 0.22), (0.72, 0.28, 0.30)\}}_{\{A,B\}}, \underbrace{\{(0.66, 0.36, 0.40), (0.62, 0.44, 0.38)\}}_{\{B,C\}} \Big\},$$

$$v^{(2,2)}(U_2) = \Big\{ \underbrace{\{(0.85, 0.20, 0.30), (0.80, 0.22, 0.35)\}}_{\{A\}}, \underbrace{\{(0.58, 0.50, 0.45), (0.55, 0.52, 0.42)\}}_{\{C\}} \Big\}.$$

Spherical checks (all leaves):

$$0.78^2 + 0.30^2 + 0.22^2 = 0.6084 + 0.09 + 0.0484 = 0.7468 \le 1,$$
$$0.72^2 + 0.28^2 + 0.30^2 = 0.5184 + 0.0784 + 0.09 = 0.6868 \le 1,$$
$$0.66^2 + 0.36^2 + 0.40^2 = 0.4356 + 0.1296 + 0.16 = 0.7252 \le 1,$$
$$0.62^2 + 0.44^2 + 0.38^2 = 0.3844 + 0.1936 + 0.1444 = 0.7224 \le 1,$$
$$0.85^2 + 0.20^2 + 0.30^2 = 0.7225 + 0.04 + 0.09 = 0.8525 \le 1,$$
$$0.80^2 + 0.22^2 + 0.35^2 = 0.64 + 0.0484 + 0.1225 = 0.8109 \le 1,$$
$$0.58^2 + 0.50^2 + 0.45^2 = 0.3364 + 0.25 + 0.2025 = 0.7889 \le 1,$$
$$0.55^2 + 0.52^2 + 0.42^2 = 0.3025 + 0.2704 + 0.1764 = 0.7493 \le 1.$$

The domain nesting ($m = 2$) collects rosters of buildings; the codomain nesting ($n = 2$) keeps, for each roster, a set of admissible (T, I, F) scenarios (e.g., different occupant loads or route blockages). Every leaf satisfies the spherical bound $T^2 + I^2 + F^2 \le 1$, so the mapping is a valid SVS–$(2, 2)$–SuperhyperNeutrosophic Set.

Theorem 5 (SVS–(m, n)–SHyNS generalizes both (m, n)–SHNS and SVS–HyNS). *Let $\iota : \mathcal{S} \hookrightarrow [0, 1]^3$ be the inclusion.*

(i) *(Forgetting sphericity) The postcomposition*

$$\mathcal{P}_*^m(X) \xrightarrow{\nu^{(m,n)}} \mathcal{P}_*^n(\mathcal{S}) \xrightarrow{\mathcal{P}_*^n(\iota)} \mathcal{P}_*^n([0,1]^3)$$

is an (m, n)–superhyperneutrosophic set.

(ii) *(Specializing the levels) For $(m, n) = (0, 1)$, an SVS–$(0, 1)$–SHyNS is exactly a single-valued spherical hyperneutrosophic set $\tilde{\nu} : X \to \tilde{\mathcal{P}}(\mathcal{S})$.*

Moreover, if in (ii) every value is a singleton $\{(T(x), I(x), F(x))\}$, one recovers an SNS $A : X \to [0, 1]^3$ with $T(x)^2 + I(x)^2 + F(x)^2 \le 1$.

Proof. (i) Because ι is injective, $\mathcal{P}_*^n(\iota)$ simply regards any nonempty n–level nested family in $\mathcal{S}$ as the same family inside $[0, 1]^3$. Thus the composite is a map $\mathcal{P}_*^m(X) \to \mathcal{P}_*^n([0, 1]^3)$, which is precisely the definition of an (m, n)–superhyperneutrosophic set.

(ii) When $m = 0$ we have $\mathcal{P}_*^0(X) = X$, and when $n = 1$ we have $\mathcal{P}_*^1(\mathcal{S}) = \mathcal{P}(\mathcal{S})$. Hence $\nu^{(0,1)} : X \to \mathcal{P}(\mathcal{S})$, which is exactly the SVS–HyNS in Definition 22. The final clause follows as in Theorem 4(ii): singletons yield a point map A with the spherical constraint satisfied pointwise. $\square$

2.3. Single-Valued k–HyperSpherical Superhyperneutrosophic Set

A Single-Valued k–HyperSpherical HyperNeutrosophic Set assigns each element a nonempty set of triplets satisfying $T^k + I^k + F^k \le 1$ within $[0, 1]^3$, thereby capturing assessment uncertainty. A Single-Valued k–HyperSpherical SuperHyperNeutrosophic Set maps nested element families to nested triplet collections satisfying $T^k + I^k + F^k \le 1$, modeling hierarchical uncertainty across levels.

Definition 24 (k–hyperspherical admissible region). *Fix an exponent $k > 1$. Define*

$$\mathcal{S}_k := \left\{ (T, I, F) \in [0, 1]^3 \ : \ T^k + I^k + F^k \le 1 \right\}.$$

Each $(T, I, F) \in \mathcal{S}_k$ is a single-valued specification of truth, indeterminacy, and falsity, jointly constrained to the unit ℓ^k–ball intersected with $[0, 1]^3$.

Definition 25 (Single-Valued k–HyperSpherical HyperNeutrosophic Set (SVkHS–HyNS)). *Let $X \neq \varnothing$ be a universe. A* single-valued k–hyperspherical hyperneutrosophic set *is a map*

$$\tilde{v}_k : X \longrightarrow \tilde{\mathcal{P}}(\mathcal{S}_k), \qquad x \longmapsto \tilde{v}_k(x),$$

assigning to each $x \in X$ a nonempty *set $\tilde{v}_k(x) \subseteq \mathcal{S}_k$ of admissible triplets (T, I, F), where every member satisfies $T^k + I^k + F^k \leq 1$ and $T, I, F \in [0, 1]$.*

Example 16 (Single-Valued k–HyperSpherical HyperNeutrosophic Set (hospital ICU surge readiness within 6 hours; $k = 4$)). *Let the ICU units be*

$$X = \{ICU\text{–North}, \ ICU\text{–Central}, \ ICU\text{–South}\}.$$

Interpret $(T, I, F) \in [0, 1]^3$ as:

$$T = \text{evidence "can accept surge in} \leq 6h\text{",}$$

$$I = \text{indeterminacy (turnover, staff sick calls),}$$

$$F = \text{counter-evidence (beds/vents already blocked).}$$

The k–hyperspherical admissible region (with $k = 4$) is

$$\mathcal{S}_4 = \{(T, I, F) \in [0, 1]^3 : \ T^4 + I^4 + F^4 \leq 1\}.$$

An SVkHS–HyNS is a map $\tilde{v}_4 : X \to \tilde{\mathcal{P}}(\mathcal{S}_4)$; take two scenarios per unit:

$$\tilde{v}_4(ICU\text{–North}) = \{(0.82, 0.25, 0.30), \ (0.74, 0.35, 0.28)\},$$
$$\tilde{v}_4(ICU\text{–Central}) = \{(0.68, 0.40, 0.32), \ (0.80, 0.20, 0.35)\},$$
$$\tilde{v}_4(ICU\text{–South}) = \{(0.72, 0.30, 0.40), \ (0.60, 0.50, 0.45)\}.$$

Pointwise k–hyperspherical checks ($T^4 + I^4 + F^4 \leq 1$):

$$(0.82, 0.25, 0.30) : \ 0.82^4{=}0.45212176, \ 0.25^4{=}0.00390625, \ 0.30^4{=}0.0081$$
$$\Rightarrow \ 0.45212176 + 0.00390625 + 0.0081 = 0.46412801 \leq 1;$$
$$(0.74, 0.35, 0.28) : \ 0.74^4{=}0.29986576, \ 0.35^4{=}0.01500625, \ 0.28^4{=}0.00614656$$
$$\Rightarrow \ 0.29986576 + 0.01500625 + 0.00614656 = 0.32101857 \leq 1;$$
$$(0.68, 0.40, 0.32) : \ 0.68^4{=}0.21381376, \ 0.40^4{=}0.0256, \ 0.32^4{=}0.01048576$$
$$\Rightarrow \ 0.21381376 + 0.0256 + 0.01048576 = 0.24989952 \leq 1;$$
$$(0.80, 0.20, 0.35) : \ 0.80^4{=}0.4096, \ 0.20^4{=}0.0016, \ 0.35^4{=}0.01500625$$
$$\Rightarrow \ 0.4096 + 0.0016 + 0.01500625 = 0.42620625 \leq 1;$$
$$(0.72, 0.30, 0.40) : \ 0.72^4{=}0.26873856, \ 0.30^4{=}0.0081, \ 0.40^4{=}0.0256$$
$$\Rightarrow \ 0.26873856 + 0.0081 + 0.0256 = 0.30243856 \leq 1;$$
$$(0.60, 0.50, 0.45) : \ 0.60^4{=}0.1296, \ 0.50^4{=}0.0625, \ 0.45^4{=}0.04100625$$
$$\Rightarrow \ 0.1296 + 0.0625 + 0.04100625 = 0.23310625 \leq 1.$$

Each unit $x \in X$ is mapped to a set *of admissible triplets covering distinct operational scenarios (e.g., simultaneous discharges vs. none; elective holds released vs. retained). The ℓ^4 bound coherently limits joint evidence while allowing spiky patterns (one component large, others moderate) without violating feasibility.*

Theorem 6 (SVkHS–HyNS generalizes both HyNS and SVkHS–NS). *Let $\iota_k : \mathcal{S}_k \hookrightarrow [0, 1]^3$ be the inclusion.*

(i) *(HyNS reduction) For any SVkHS–HyNS $\tilde{v}_k : X \to \tilde{\mathcal{P}}(\mathcal{S}_k)$, the composition*

$$X \xrightarrow{\tilde{v}_k} \tilde{\mathcal{P}}(\mathcal{S}_k) \xrightarrow{\tilde{\mathcal{P}}(\iota_k)} \tilde{\mathcal{P}}([0,1]^3)$$

is a (standard) hyperneutrosophic set *on X.*

(ii) *(SVkHS–NS reduction) If, moreover, every fiber is a singleton, $\tilde{v}_k(x) = \{(T(x), I(x), F(x))\}$, then*

$$A : X \to [0,1]^3, \qquad A(x) = (T(x), I(x), F(x)),$$

is a single-valued k–hyperspherical neutrosophic set *(SVkHS–NS), i.e. $T(x)^k + I(x)^k + F(x)^k \leq 1$ for all x.*

Proof. (i) Since ι_k is an inclusion, $\tilde{\mathcal{P}}(\iota_k)$ views each nonempty $\tilde{v}_k(x) \subseteq \mathcal{S}_k$ as the *same* set inside $[0,1]^3$. Hence the composite maps x to a nonempty subset of $[0,1]^3$, which is precisely the definition of a hyperneutrosophic set.

(ii) If $\tilde{v}_k(x) = \{(T(x), I(x), F(x))\}$ with $(T(x), I(x), F(x)) \in \mathcal{S}_k$, then by Definition 24 we have $T(x)^k + I(x)^k + F(x)^k \leq 1$ and $T(x), I(x), F(x) \in [0,1]$. Thus A satisfies the SVkHS–NS constraint pointwise. $\square$

Definition 26 (SVkHS–(m,n)–SuperhyperNeutrosophic Set). *Fix integers $m, n \geq 0$ and $k > 1$. With $\mathcal{S}_k$ from Definition 24, a* single-valued k–hyperspherical (m,n)–superhyperneutrosophic set *on a base $X \neq \varnothing$ is a function*

$$v_k^{(m,n)} : \mathcal{P}_*^m(X) \longrightarrow \mathcal{P}_*^n(\mathcal{S}_k)$$

such that, for each $U \in \mathcal{P}_*^m(X)$, the value $v_k^{(m,n)}(U)$ is a nonempty n–level nested family whose leaves are *triplets $(T, I, F) \in \mathcal{S}_k$ (equivalently $T^k + I^k + F^k \leq 1$ with $T, I, F \in [0,1]$).*

Example 17 (SVkHS–(m,n)–SuperhyperNeutrosophic Set with $(m,n) = (2,2)$ (regional ICU bundle activation tonight; $k = 4$)). *Let the sites be $X = \{\text{North}, \text{Central}, \text{South}\}$. Objects of $\mathcal{P}_*^2(X)$ are nonempty sets of nonempty subsets of X (first level: a roster; second level: a set of candidate rosters). Consider the claim "roster can be activated and absorb the surge within tonight."*

Let $\mathcal{S}_4 = \{(T, I, F) \in [0,1]^3 : T^4 + I^4 + F^4 \leq 1\}$. An SVkHS–$(2,2)$–SHyNS is a mapping

$$v_4^{(2,2)} : \mathcal{P}_*^2(X) \longrightarrow \mathcal{P}_*^2(\mathcal{S}_4),$$

assigning, to each grouped domain object, a nonempty set of nonempty subsets *of admissible (T, I, F) triplets.*

Choose domain bundles

$$U_1 = \{\{\text{North}\}, \{\text{North}, \text{Central}\}\}, \qquad U_2 = \{\{\text{South}\}\} \in \mathcal{P}_*^2(X).$$

Define images (each first-level roster gets a set *of scenarios in $\mathcal{S}_4$):*

$$v_4^{(2,2)}(U_1) = \Big\{ \underbrace{\{(0.83,\, 0.22,\, 0.27)\}}_{\{\text{North}\}}, \underbrace{\{(0.75,\, 0.30,\, 0.36),\, (0.70,\, 0.34,\, 0.38)\}}_{\{\text{North,Central}\}} \Big\},$$

$$v_4^{(2,2)}(U_2) = \Big\{ \underbrace{\{(0.65,\, 0.44,\, 0.40),\, (0.58,\, 0.52,\, 0.42)\}}_{\{\text{South}\}} \Big\}.$$

k–hyperspherical checks (k = 4) for all leaves:

$$(0.83, 0.22, 0.27) : 0.83^4 = 0.47458321, \ 0.22^4 = 0.00234256, \ 0.27^4 = 0.00531441$$
$$\Rightarrow \ 0.47458321 + 0.00234256 + 0.00531441 = 0.48224018 \leq 1;$$

$$(0.75, 0.30, 0.36) : 0.75^4 = 0.31640625, \ 0.30^4 = 0.0081, \ 0.36^4 = 0.01679616$$
$$\Rightarrow \ 0.31640625 + 0.0081 + 0.01679616 = 0.34130241 \leq 1;$$

$$(0.70, 0.34, 0.38) : 0.70^4 = 0.2401, \ 0.34^4 = 0.01336336, \ 0.38^4 = 0.02085136$$
$$\Rightarrow \ 0.2401 + 0.01336336 + 0.02085136 = 0.27431472 \leq 1;$$

$$(0.65, 0.44, 0.40) : 0.65^4 = 0.17850625, \ 0.44^4 = 0.03748096, \ 0.40^4 = 0.0256$$
$$\Rightarrow \ 0.17850625 + 0.03748096 + 0.0256 = 0.24158721 \leq 1;$$

$$(0.58, 0.52, 0.42) : 0.58^4 = 0.11316496, \ 0.52^4 = 0.07311616, \ 0.42^4 = 0.03111696$$
$$\Rightarrow \ 0.11316496 + 0.07311616 + 0.03111696 = 0.21739808 \leq 1.$$

The domain *nesting (m = 2) collects rosters of sites (single-site vs. multi-site bundles). The* codomain *nesting (n = 2) retains, for each roster, a* set *of admissible (T, I, F) scenarios (e.g., elective hold release, inter-facility transfers, or staff float pools). Every leaf satisfies $T^4 + I^4 + F^4 \leq 1$, so $\nu_4^{(2,2)}$ is a valid SVkHS–(2,2)–SuperhyperNeutrosophic Set.*

Theorem 7 (SVkHS–(m, n) generalizes both (m, n)–SHNS and SVkHS–HyNS). *Let $\iota_k : \mathcal{S}_k \hookrightarrow [0,1]^3$ be the inclusion.*

(i) *(Forgetting hypersphericity) The postcomposition*

$$\mathcal{P}_*^m(X) \xrightarrow{\ \nu_k^{(m,n)}\ } \mathcal{P}_*^n(\mathcal{S}_k) \xrightarrow{\ \mathcal{P}_*^n(\iota_k)\ } \mathcal{P}_*^n([0,1]^3)$$

is an (m, n)–superhyperneutrosophic set.

(ii) *(Specializing the levels) For (m, n) = (0, 1), an SVkHS–(0, 1)–superhyperneutrosophic set is exactly a single-valued k–hyperspherical hyperneutrosophic set $\tilde{\nu}_k : X \to \tilde{\mathcal{P}}(\mathcal{S}_k)$.*

(iii) *(Singleton reduction) In the setting of (ii), if every value is a singleton $\{(T(x), I(x), F(x))\}$, one recovers an SVkHS–NS $A : X \to [0,1]^3$ with $T(x)^k + I(x)^k + F(x)^k \leq 1$ for all x.*

Proof. (i) Since ι_k is injective, $\mathcal{P}_*^n(\iota_k)$ regards any nonempty n–level nested family in $\mathcal{S}_k$ as the same family inside $[0,1]^3$. Thus the composite is a map $\mathcal{P}_*^m(X) \to \mathcal{P}_*^n([0,1]^3)$, which is precisely an (m, n)–superhyperneutrosophic set.

(ii) When $m = 0$ we have $\mathcal{P}_*^0(X) = X$, and when $n = 1$ we have $\mathcal{P}_*^1(\mathcal{S}_k) = \mathcal{P}(\mathcal{S}_k)$. Hence it specializes to $\nu_k^{(0,1)} : X \to \mathcal{P}(\mathcal{S}_k)$, i.e. the SVkHS–HyNS.

(iii) If each value is a singleton, define $A(x) = (T(x), I(x), F(x))$. Since every leaf lies in $\mathcal{S}_k$, we have $T(x)^k + I(x)^k + F(x)^k \leq 1$ pointwise, so A is an SVkHS–NS. $\square$

2.4. Triple-Valued (m, n)–SuperhyperNeutrosophic Set

A Triple-Valued (m, n)–hyperNeutrosophic Set maps m-level subsets to n-level collections of quintuples (T, I_T, I_N, I_F, F), modeling uncertainty degrees. A Triple-Valued (m, n)–SuperhyperNeutrosophic Set maps m-level nested families to n-level nested collections of quintuples, preserving hierarchical structure and triple-valued semantics.

Definition 27 (Admissible triple-valued region). *Let*

$$\mathcal{T} := \left\{ (T, I_T, I_N, I_F, F) \in [0,1]^5 \right\}.$$

For any $(T, I_T, I_N, I_F, F) \in \mathcal{T}$ we interpret T and F as truth and falsity degrees, while I_T, I_N, I_F decompose indeterminacy into "leaning to truth", "neutral", and "leaning to falsity", respectively. (When desired, one may additionally impose $0 \leq T + I_T + I_N + I_F + F \leq 5$.)

Definition 28 (Triple-Valued HyperNeutrosophic Set (TV–HyNS)). *Let* $X \neq \varnothing$. *A* triple-valued hyperneutrosophic set *on X is a map*

$$\tilde{\tau} : X \longrightarrow \tilde{\mathcal{P}}(\mathcal{T}), \qquad x \longmapsto \tilde{\tau}(x),$$

assigning to each $x \in X$ *a* nonempty *set* $\tilde{\tau}(x) \subseteq \mathcal{T}$ *of admissible quintuples* (T, I_T, I_N, I_F, F).

Example 18 (Triple-Valued HyperNeutrosophic Set (airport operations: "on-time departure tonight"))**.**
Let the universe of flights be
$$X = \{EA123,\ CB456,\ WP789\}.$$

Interpret a quintuple $(T, I_T, I_N, I_F, F) \in [0,1]^5$ *as*

$$T = truth\ degree\ (evidence\ of\ on\text{-}time\ departure),$$
$$I_T = indeterminacy\ leaning\ to\ truth\ (e.g.,\ standby\ crew\ likely),$$
$$I_N = neutral\ indeterminacy\ (unknown\ gate\ turnaround),$$
$$I_F = indeterminacy\ leaning\ to\ falsity\ (e.g.,\ possible\ slot\ hold),$$
$$F = falsity\ degree\ (evidence\ of\ delay/cancellation).$$

A Triple–Valued HyperNeutrosophic Set (TV–HyNS) is $\tilde{\tau} : X \to \tilde{\mathcal{P}}([0,1]^5)$ *assigning each flight a* nonempty set *of admissible quintuples (distinct scenarios). Specify two scenarios per flight:*

$$\tilde{\tau}(EA123) = \Big\{(0.68, 0.12, 0.08, 0.05, 0.25),\ (0.55, 0.18, 0.10, 0.12, 0.35)\Big\},$$
$$\tilde{\tau}(CB456) = \Big\{(0.72, 0.10, 0.09, 0.06, 0.22),\ (0.50, 0.22, 0.15, 0.10, 0.45)\Big\},$$
$$\tilde{\tau}(WP789) = \Big\{(0.80, 0.08, 0.07, 0.05, 0.18),\ (0.62, 0.16, 0.12, 0.10, 0.34)\Big\}.$$

Admissibility checks (each component in $[0,1]$*, and sums* ≤ 5 *hold pointwise):*

EA123: $0.68 + 0.12 + 0.08 + 0.05 + 0.25 = 1.18 \leq 5,\quad 0.55 + 0.18 + 0.10 + 0.12 + 0.35 = 1.30 \leq 5;$

CB456: $0.72 + 0.10 + 0.09 + 0.06 + 0.22 = 1.19 \leq 5,\quad 0.50 + 0.22 + 0.15 + 0.10 + 0.45 = 1.42 \leq 5;$

WP789: $0.80 + 0.08 + 0.07 + 0.05 + 0.18 = 1.18 \leq 5,\quad 0.62 + 0.16 + 0.12 + 0.10 + 0.34 = 1.34 \leq 5.$

Each flight $x \in X$ *is mapped to a* set *of quintuples reflecting distinct evening scenarios (e.g., last-minute crew swap, earlier pushback, or minor ATC ground delay). The triple split* (I_T, I_N, I_F) *separates helpful, neutral, and adverse uncertainties around the core* (T, F) *evidence.*

Theorem 8 (TV–HyNS generalizes both HyNS and TVNS)**.** *Define the aggregation map* $\pi : \mathcal{T} \to [0,1]^3$ *by*
$$\pi(T, I_T, I_N, I_F, F) := \big(T,\ \min\{1,\ I_T + I_N + I_F\},\ F\big).$$

Then:

(i) (HyNS reduction) *For any TV–HyNS* $\tilde{\tau}$*, the composition*

$$X \xrightarrow{\ \tilde{\tau}\ } \tilde{\mathcal{P}}(\mathcal{T}) \xrightarrow{\ \tilde{\mathcal{P}}(\pi)\ } \tilde{\mathcal{P}}\big([0,1]^3\big)$$

is a (standard) hyperneutrosophic set *on X.*

(ii) (TVNS reduction) *If every fiber is a singleton,* $\tilde{\tau}(x) = \{(T_A(x), I_T(x), I_N(x), I_F(x), F_A(x))\}$, *then*

$$A = \{(x,\ T_A(x), I_T(x), I_N(x), I_F(x), F_A(x)) :\ x \in X\}$$

is a Triple-Valued Neutrosophic Set (TVNS) *on X.*

Proof. (i) For any $x \in X$ and $v = (T, I_T, I_N, I_F, F) \in \tilde{\tau}(x) \subseteq [0,1]^5$, we have $T, F \in [0,1]$ and $I_T + I_N + I_F \in [0,3]$. Hence $I' := \min\{1, I_T + I_N + I_F\} \in [0,1]$. Therefore $\pi(v) = (T, I', F) \in [0,1]^3$. Nonemptiness is preserved by $\tilde{\mathcal{P}}(\pi)$, so $x \mapsto \tilde{\mathcal{P}}(\pi)\big(\tilde{\tau}(x)\big)$ is a map into nonempty subsets of $[0,1]^3$, i.e. a hyperneutrosophic set.

(ii) If $\tilde{\tau}(x)$ is a singleton for every x, collecting those quintuples yields exactly the TVNS representation stated, with each component in $[0,1]$ by Definition 27. □

Definition 29 (Triple-Valued (m,n)–SuperhyperNeutrosophic Set (TV–(m,n)–SHyNS)). *Fix integers* $m, n \geq 0$. *A* triple-valued (m,n)–superhyperneutrosophic set *on a base* $X \neq \varnothing$ *is a function*

$$\mathbf{T}^{(m,n)} :\ \mathcal{P}_*^m(X) \longrightarrow \mathcal{P}_*^n(\mathcal{T}),$$

such that, for every $U \in \mathcal{P}_*^m(X)$, *the value* $\mathbf{T}^{(m,n)}(U)$ *is a* nonempty n–level nested family *whose leaves are quintuples in* $\mathcal{T}$.

Example 19 (Triple-Valued (m,n)–SuperhyperNeutrosophic Set with $(m,n) = (2,2)$ (power grid re-route bundles tonight)). *Let substations be* $X = \{S1, S2, S3\}$. *Elements of* $\mathcal{P}_*^2(X)$ *are nonempty sets of nonempty subsets* of X *(first level: a roster of substations to re-route; second level: a set of candidate rosters). Consider the claim "this roster can maintain service during tonight's peak."*

A TV–$(2,2)$–SHyNS is a map

$$\mathbf{T}^{(2,2)} :\ \mathcal{P}_*^2(X) \longrightarrow \mathcal{P}_*^2([0,1]^5),$$

assigning to each grouped domain object a nonempty set of nonempty subsets *of quintuples* (T, I_T, I_N, I_F, F). *Choose two domain bundles:*

$$U_1 = \{\{S1\},\ \{S1, S2\}\}, \qquad U_2 = \{\{S3\}\}\ \in\ \mathcal{P}_*^2(X).$$

Specify images (each first-level roster gets a set *of scenarios):*

$$\mathbf{T}^{(2,2)}(U_1) = \Big\{ \underbrace{\{(0.78,\ 0.10,\ 0.07,\ 0.05,\ 0.20),\ (0.70,\ 0.14,\ 0.08,\ 0.07,\ 0.28)\}}_{\{S1\}},$$
$$\underbrace{\{(0.82,\ 0.12,\ 0.10,\ 0.06,\ 0.18),\ (0.68,\ 0.16,\ 0.12,\ 0.08,\ 0.30)\}}_{\{S1,S2\}} \Big\},$$
$$\mathbf{T}^{(2,2)}(U_2) = \Big\{ \underbrace{\{(0.60,\ 0.18,\ 0.12,\ 0.12,\ 0.40),\ (0.55,\ 0.20,\ 0.15,\ 0.12,\ 0.45)\}}_{\{S3\}} \Big\}.$$

Admissibility checks (pointwise sums ≤ 5; all components in $[0,1]$):

$$\{S1\} : 0.78 + 0.10 + 0.07 + 0.05 + 0.20 = 1.20 \leq 5, \quad 0.70 + 0.14 + 0.08 + 0.07 + 0.28 = 1.27 \leq 5;$$
$$\{S1, S2\} : 0.82 + 0.12 + 0.10 + 0.06 + 0.18 = 1.28 \leq 5, \quad 0.68 + 0.16 + 0.12 + 0.08 + 0.30 = 1.34 \leq 5;$$
$$\{S3\} : 0.60 + 0.18 + 0.12 + 0.12 + 0.40 = 1.42 \leq 5, \quad 0.55 + 0.20 + 0.15 + 0.12 + 0.45 = 1.47 \leq 5.$$

The domain *nesting ($m = 2$) captures grouped re-route rosters (single-station vs. pair activation). The* codomain *nesting ($n = 2$) retains, for each roster, a* set *of triple-valued scenarios that separate favorable*

uncertainty (I_T), neutral unknowns (I_N), and adverse uncertainty (I_F) around the truth/falsity evidence (T, F) for sustaining service through the peak.

Theorem 9 (TV–(m, n)–SHyNS generalizes both (m, n)–SHNS and TV–HyNS). *Let $\pi : \mathcal{T} \to [0, 1]^3$ be as in Theorem 8.*

 (i) (Forgetting the triple split) *The postcomposition*

$$\mathcal{P}_*^m(X) \xrightarrow{\mathbf{T}^{(m,n)}} \mathcal{P}_*^n(\mathcal{T}) \xrightarrow{\mathcal{P}_*^n(\pi)} \mathcal{P}_*^n([0,1]^3)$$

   is an (m, n)–superhyperneutrosophic set.
 (ii) (Specializing the levels) *For $(m, n) = (0, 1)$, a TV–$(0, 1)$–SHyNS is exactly a* triple-valued hyperneutrosophic set $\tilde{\tau} : X \to \tilde{\mathcal{P}}(\mathcal{T})$.
(iii) (Singleton reduction) *In the setting of (ii), if each value is a singleton $\{(T_A(x), I_T(x), I_N(x), I_F(x), F_A(x))\}$, one recovers a TVNS on X.*

Proof. (i) Since π maps every leaf in $\mathcal{T}$ to a triplet in $[0, 1]^3$ and preserves nonemptiness via $\mathcal{P}_*^n(\cdot)$, the composite is a map $\mathcal{P}_*^m(X) \to \mathcal{P}_*^n([0,1]^3)$, i.e. an (m, n)–SHNS.

(ii) When $m = 0$ and $n = 1$ we have $\mathcal{P}_*^0(X) = X$ and $\mathcal{P}_*^1(\mathcal{T}) = \mathcal{P}(\mathcal{T})$, hence the definition reduces to a map $X \to \mathcal{P}(\mathcal{T})$, which is precisely TV–HyNS (Definition 28).

(iii) If each image is a singleton quintuple, then collecting these quintuples yields the TVNS representation exactly as in Theorem 8(ii). $\square$

3. Conclusions

In this paper, we extended Cylindrical, Spherical, HyperSpherical, and Triple-valued Neutrosophic Sets into the settings of HyperNeutrosophic Sets and SuperHyperNeutrosophic Sets. In the future, we aim to explore the integration of Plithogenic Sets[73–76], Soft Sets[77–79], HyperSoft Sets[80,80,81], Shadowed Sets[82–84], and Rough Sets[85,86], as well as extensions of graph concepts utilizing Graphs [87,88], HyperGraphs[89–92], and SuperHyperGraphs[14,15,93].

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