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From Clifford Algebras, Dirac Equation, and Space-Time to the Dark Energy-Matter Sector of the Universe

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Abstract

A brief history of dark matter (DM) and dark energy (DE) investigations is presented. The contemporary situation is discussed briefly as well. The role of the different spaces and corresponding phase transitions of the Universe is under special attention. Our own hypotheses that DM and DE are related to the space-time M(1,6) are under consideration. In order to demonstrate the relationships between the M(1,6) and corresponding geometric Clifford algebras the Dirac-like equation with 7 space-time derivatives and 7 gamma matrices is considered. The link between this Dirac-like equation and the similar Maxwell equations is indicated. We pay attention on the structure and nonhomogeneous character of the list of the higher dimensional Dirac equations. The necessity of step by step movement from ordinary to *N*-dimensional Dirac equation is demonstrated.

Keywords: dark matter; dark energy; Clifford algebras; Dirac-like equation

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1. Introduction

We consider here different approaches to DM and DE explanation in order to determine the place and independence of our own suggestions and hypotheses among investigations of other authors. The conclusion is that such hypotheses are new and may be useful for further investigations.

Our own approach is based on the relationships between the Dirac equation, Clifford algebras and corresponding space-time, which are considered here as well. The structure and nonhomogeneous character of the list of the higher dimensional Dirac equations is our next result.

The name "dark matter" (matiere obscure) was put into consideration by Henri Poincaré in 1906. The famous mathematician described known from experiments strange features of the distribution of the speeds of rotation of stars around the center of our Galaxy [1]. In the 1930, American astronomer Fritz Zwicky observed clusters of galaxies and discovered that the total mass of visible objects in such clusters is significantly less than what can be calculated based on the speed of their movement [2]. After 40 years the observations of V.C. Rubin [3,4] detected that linear velocity of stars from average distances to the visible edges of galactic disks was practically the same, significantly exceeding theoretical values. According to the last data the contribution of dark matter to the Universe is estimated at about 22 percent.

It is clear that if there is dark matter in the Universe, then there will also be dark energy. The dark energy is related to the accelerating expansion of the Universe, which is the experimental fact [5,6]. For the discovery of the accelerating expansion of the Universe Saul Perlmutter, Brian P. Schmidt, and Adam Riess received the Nobel Prize in Physics in 2011. The contribution of dark energy to the Universe is near 74 percent.

Note that astronomy during whole its history put the problems for the physicists and mathematicians. Let us mention the revolution of Copernicus, three laws of Kepler, the theory of gravitation of Newton, telescope of Galileo, the discovery of Uranus, the calculation of Neptune's orbit, the

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problem with the orbit of Mercury and Hilbert–Einstein general relativity theory (GR), the conclusion of Friedman that the Universe must expand or collapse. *Therefore, the situation that astronomy and cosmology put the questions for physicists is ordinary and has a long history*.

The explanation of the nature of dark matter and dark energy is one of the main problems of contemporary theoretical physics. Our point of view is that this is the main problem in general. The corresponding articles can be divided in three parts: the explanations of dark matter, the explanations of dark energy, the general explanation of both, dark matter and dark energy.

Here we often use the notion dark energy-matter sector (DEMS). Note that this object is not equal to the simple DM plus DE. It implies the existence of another sector of the Universe (our living sector), where the DM and DE are absent.

2. Briefly on the History of the Problem and the Main Steps for Resolution

The first believable attempts to explain the dark energy started immediately after its discovery in [5,6]. We pay attention to the review articles [7–9]. Theoreticians tried *to modify the cosmological constant* Λ (the energy density of vacuum) in GR [7]. These models were successful only particular and the main problem for such approaches was dark matter. We shall return to a Λ-approach below.

In *baryonic approach* it was believed that dark matter consisted only of miniature black holes, of solar mass and neutron stars, brown dwarfs, lonely planets wandering through galaxies without being tied to any luminary [8]. However, total mass of RAMBO (Robust Associations of Massive Barionic Objects), MACHO (Massive Compact Halo Objects) and neutrinos is too small in comparison with the mass of dark matter. Therefore, today the baryonic hypothesis is too weak [9] and neutrinos belongs to weakly interacting massive particles (WIMP).

Note the recent attempts [10–12] to reanimate the baryonic hypothesis with the help of hyperons carrying spin 3/2.

WIMP (and to a somewhat lesser degree, axions) have motivated an expansive experimental program. With the advent of the Large Hadron Collider at CERN, and ever more sensitive astrophysical experiments, many believe that the moment of truth has come for WIMP: either we will discover them soon or we will begin to witness the decline of the WIMP paradigm [8]. With the idea of WIMP the expensive experiments on direct detection of dark matter particles (DAMA/NaI, DAMA/LIBRA, CRESST, XENON and others) are hold.

In parallel with direct detection experiments, attempts have been made to indirectly detect dark matter particles, based on the fact that these particles can annihilate each other or simply decay, forming high-energy radiation (X, γ). This task is being carried out by the Fermi, XMM-Newton, and Chandra space telescopes. Their goal is to detect excess radiation from those areas of the Universe where a lot of mass is concentrated (galactic nuclei and galaxy clusters). In addition, the Large Hadron Collider is also involved in the search in order to reproduce the conditions, in which dark matter could be born.

The second believable attempts to explain the dark energy and dark matter is a physical field that fills all space and is described as an ideal incompressible liquid (so called *quintessence* or *phantom*). Quintessence is a model, proposed by [13], as an alternative to the cosmological constant. It is proposed to be a fifth fundamental force. The pressure of this liquid (which causes expansion) is related to its density, which is included in the equation of general relativity by a single parameter – the so-called density parameter w, which can take any real values. But physics imposes certain restrictions on it: since the expansion is accelerating, this value must be less than 1/3, and at $w = -1$ the field becomes identical to the cosmological constant. Within the range of $-1 < w < -1/3$ such a field is called quintessence, while $w < -1$ is a "phantom". The difference between these entities lies in the scenarios of the future Universe. According to the latest observations, the value of this parameter is close to -1 (and therefore the expansion of the Universe will continue forever).

Phantom energy is another hypothesized form of dark energy and dark matter, having a negative kinetic energy that increases with the expansion of the Universe. Due to which it could cause the expansion of the Universe to accelerate so quickly that it will lead to a Big Rip [14]. Data (from Planck data) from CMB and supernova limit the range to $-1,19 < w < -0,95$ [15,16].

The quintom dark energy (quintessence and phantom) is another hypothetical scenario regarding dark energy, with a time varying equation of state parameter $w(z)$ that can cross the phantom divide of $w = -1$. For detailed discussion of the model see for example [17] and references there in.

Dark energy and Mach's principle. The result that Mach's principle even implies interactions between inertia and electromagnetism was found by [18]. The similar force for gravity was known as Einstein-Sciama force. On this basis the way to the dark energy explanation was constructed as follows. The authors of articles [19,20] were perhaps the first to consider a time varying cosmological constant Λ. An exact solution of corresponding equations was given in [21]. This is the value $\Lambda_{\text{Present}} \approx 10^{-56}\text{cm}^{-2}$ deduced for the dominant dark energy density at present [22].

Variation of physical constants. Authors of the article [23] suggests the possibility of variation of physical constants such as the speed of light and the gravitational constant, which provides a value for dark energy density in remarkable agreement with current cosmological observations, unlike numerous phenomenological scenarios where the corresponding value is postulated. Further, authors of the paper [24] and many other authors developed this approach. Recently, in [25].

Axions. There are possible dark matter candidates which do not fit into the above framework. The most popular such candidates are called axions and arise from attempts to explain why the strong interaction seems to obey the CP symmetry [26]. The axion is stable, and can also be produced in the early Universe [27,28]. Some of the major challenges in the design of an experiment to detect axions are that the particle's mass and the coupling constant are unknown. The predicted masses range from 1 MeV to 1 eV. Several axion experiments are based on the prediction that axions and photons are converted into each other when subjected to a strong magnetic field.

Axion dark matter search with interferometric gravitational wave detectors was described in the paper [29]. The property that axion dark matter differentiates the phase velocities of the circular-polarized photons was used. A scheme to measure the phase difference by using a linear optical cavity was proposed. In the recent publication [30] brief review about axion dark matter was presented.

The separate questions under consideration are so called *the quantum chromodynamical axions* (QCD axions).

The production of cold axions. The dark matter axion detection, the dark matter axion properties and the axion BEC (Bose-Einstein condensate) were considered. It was shown that *cold dark matter axions* thermalize through their gravitational self-interactions, and form a BEC. As a result, axion dark matter behaves differently from the other proposed forms of dark matter. The differences are observable.

Warm inflation. Recently the authors of [31] proposed the first model of warm inflation in which the particle production emerges directly from coupling the inflation to Standard Model particles. Warm inflation, an early epoch of sustained accelerated expansion at finite temperature, is a compelling alternative to cold inflation, with distinct predictions for inflationary observables such as the amplitude of fluctuations, the spectral tilt, the tensor-to-scalar ratio, and non-gaussianities. Results of [31] pointed out that the Standard Model quarks can be heavy during warm inflation if the Higgs field resides in a high-energy second minimum, which restores efficient sphaleron heating. A subsequent large reheating temperature is required to allow the Higgs field to relax to its electroweak minimum. Exploring a scenario, in which hybrid warm inflation provides the large reheating temperature, authors of [31] show that future collider and beam dump experiments have discovery potential for a *heavy QCD axion* taking the role of the warm inflation.

Contemporary theoretical physics contains a *list of alternatives* to the well discussed models of dark matter and dark energy. Among the interesting alternatives to the Λ*CDM model* (Λ cold dark matter) let us mark an interesting paper [32], in which author recovered cosmological space-time as a solution, and the known physics should emerge from fluctuations on this background. A Big Bang arises through an appealing mechanism as in the $k = 1$ solutions.

The modification of Newtonian dynamics (MOND) was initially proposed as an alternative to account for the flat rotation curves of spiral galaxies, without invoking dark matter in the halo [33,34]. The initial theory required an ad hoc introduction of a fundamental acceleration $\sim 10^{-10}\mathrm{m/s}^2$. The development of this model is known as the modified Newtonian gravity (MONG). The next step was the relativistic generalization of these models. The best-known implementation of MOND is the TeVeS (Tensor-Vector-Scalar gravity) model, developed in the paper [35].

A novel form of matter called ELKO, the acronym of Eigenspinoren des Landungskonjugations operators (Eigenspinors of the charge conjugation operator), which designates the eigenspinors of the charge conjugation operator, seems to fulfill the requirements for a dark matter component, in the scope of the interplay among general relativity, astrophysics and particle physics. The assumptions of such model for instance evince that ELKO spinor fields main interaction via the gravitational field makes them naturally dark. Further, the scientists enforce dark spinor fields investigation in a cosmological setting, where interesting solutions and also models, where the spinor is coupled conformally to gravity, are provided. This leading to some non-local properties [36].

Recently the authors of [37] suggested that whatever dark matter is, it must be one irreducible unitary representation of the inhomogeneous Lorentz group or another. Mass dimension one fields were considered. Authors developed corresponding formalism for fermions with spin one and bosons, having spin one-half, and showed that they provided natural dark matter candidates.

Fermionic dark matter-photon quantum interaction was suggested recently in [38]. Mass dimension one fermionic fields were prime candidates to describe dark matter, due to their intrinsic neutral nature, as they were constructed as eigenstates of the charge conjugation operator with dual helicity. To formulate the meaning of the darkness, the fermion-photon coupling was scrutinized with a Pauli-like interaction, and the path integral is then formulated from the phase space constraint structure. Motivated by recent nucleon-recoil experiments to detect dark matter, authors of [38] furnished a consistent theoretical setup to describe interaction with the photon compatible with the prevalence of darkness.

Therefore, general spinor field classifications, according to the bilinear covariants, generalize the Lounesto's spinor field classification in Minkowski space time, which, besides encompassing Dirac, Majorana, and Weyl spinors, also encloses the Penrose flag-dipole, flagpole, and dipole spinor constructions. Some of these spinors can be used to construct mass dimension one spinor fields, which have been reported to consistently account for the dark matter problem [36–38].

3. Big Bang as a Phase Transition

Big Bang is the main point of the number of models, which describe the dark matter and dark energy on the basis of some phase transition.

The approach started from the ideas of [39], where the phase transitions and magnetic monopole production in the very early Universe were considered.

The author of [40] suggested the first-order QCD phase transition that occurred reversibly in the early Universe and would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense. Invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only.

In the approach [41] a five-dimensional cosmological model, which suggested that the Universe began as a discontinuity in a scalar (Higgs-type) field, or alternatively as a conventional four-dimensional phase transition, was investigated.

Author of the paper [42] demonstrated that a large class of models with a composite dark sector went through a strong first-order phase transition in the early Universe, which could lead to a detectable gravitational wave signal.

In the paper [43] authors suggest that the Big Bang may be a result of the first-order phase transition driven by changing scalar curvature of the 4D space-time in the expanding cold Universe,

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filled with nonlinear scalar field and neutral matter with equation of state $p = v\epsilon$ (where p and ϵ are pressure and energy density of matter).

It had been argued [44] that a particular type of quantum-vacuum variable q could provide a solution to the main cosmological constant problem and possibly also gave a cold-dark-matter component. It was shown that the same q field may suggest a new interpretation of the Big Bang, namely as a quantum phase transition between topologically inequivalent vacua.

The paper [45] provided a conceptual history of phase transitions and the birth of early Universe particle physics.

4. Our Own Hypotheses and Suggestions on the Basis of Corresponding Space-Times and Clifford Algebras

In the publications [46,47] (see also the review [48]) the dark energy and dark matter were related to another Minkowski space $M^*(1,3)$, which is consider as a subspace of the general 7-component space-time M(1,6). The 8-component Dirac equation was derived in space-time M(1,6) with one time coordinate and six space coordinates. Such equation and space-time M(1,6) are associated with 256-dimensional real Clifford algebra $C\ell^{\mathbb{R}}(1,7)$ and the minimally possible 64-dimensional complex Clifford algebra $C\ell^{\mathbb{C}}(1,5)$, which are isomorphic to the matrix (gamma matrix) algebras $C\ell^{\mathbb{R}}(1,7) \cong \mathrm{Mat}(8.\mathbb{R})$ and $C\ell^{\mathbb{C}}(1,5) \cong \mathrm{Mat}(8.\mathbb{C})$, respectively. In other words Clifford algebras $C\ell^{\mathbb{R}}(1,7)$ and $C\ell^{\mathbb{C}}(1,5)$ have corresponding gamma matrix representations.

The 8-component Dirac equation (from the publication [46]) is determined in the space of six spatial dimensions and couples together two different coordinate spaces with one and the same time axe (conjoint time) in the form of $\mathbb{R}^6 = \mathbb{R}^3 \otimes \mathbb{R}^{*3}$. Such coordinate spaces can be similar to each other as two duplications of the ordinary coordinate space $\mathbb{R}^3 \subset$M(1,3) but, of course, they can be completely different fragments as $\mathbb{R}^3 \subset$M(1,3) and $\mathbb{R}^{*3} \subset M^*(1,3)$ corresponding to the different physical reality.

Thus, the corresponding Dirac equation in the paper [46] was suggested in the form

$$(i\Gamma^{\tilde{A}}\partial_{\tilde{A}} - m)\psi(X) = 0, \quad \tilde{A} = \overline{0,6}, \tag{1}$$

where the 8×8 gamma matrices have the form

$$\Gamma^0 = \begin{vmatrix} I_4 & 0 \\ 0 & -I_4 \end{vmatrix}, \quad \Gamma^1 = i\Sigma_8^1 = i\begin{vmatrix} 0 & I_4 \\ I_4 & 0 \end{vmatrix},$$
$$\Gamma^2 = \begin{vmatrix} 0 & \Sigma_4^2 \\ -\Sigma_4^2 & 0 \end{vmatrix}, \quad \Gamma^3 = \begin{vmatrix} 0 & \Sigma_4^3 \\ -\Sigma_4^3 & 0 \end{vmatrix}, \tag{2}$$
$$\Gamma^4 = \begin{vmatrix} 0 & 0 & 0 & \sigma^1 \\ 0 & 0 & \sigma^1 & 0 \\ 0 & -\sigma^1 & 0 & 0 \\ -\sigma^1 & 0 & 0 & 0 \end{vmatrix}, \Gamma^5 = \begin{vmatrix} 0 & 0 & 0 & \sigma^2 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & -\sigma^2 & 0 & 0 \\ -\sigma^2 & 0 & 0 & 0 \end{vmatrix},$$
$$\Gamma^6 = \begin{vmatrix} 0 & 0 & 0 & \sigma^3 \\ 0 & 0 & \sigma^3 & 0 \\ 0 & -\sigma^3 & 0 & 0 \\ -\sigma^3 & 0 & 0 & 0 \end{vmatrix}.$$

In formulae (2) the 4×4 Pauli matrices are given by

$$\Sigma_4^2 = i\begin{vmatrix} 0 & -I_2 \\ I_2 & 0 \end{vmatrix}, \quad \Sigma_4^3 = \begin{vmatrix} I_2 & 0 \\ 0 & -I_2 \end{vmatrix}, \tag{3}$$

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and the explicit form of Σ_8^1 is evident from the first row in equation (2). The 2×2 Pauli matrices σ^j are taken in the standard form. Symbol I_4 denotes 4×4 unit matrix. Note that all Pauli matrices, which for different dimensions are denoted here as Σ_8, Σ_4 and σ, satisfy the corresponding anti-commutation relations of the Pauli algebra generators.

The operators (2) satisfy the anti-commutation relations

$$\Gamma^{\tilde{A}}\Gamma^{\tilde{B}}+\Gamma^{\tilde{B}}\Gamma^{\tilde{A}}=2g^{\tilde{A}\tilde{B}},\quad g=(+------),\quad \tilde{A},\tilde{B}=\overline{0,6}, \tag{4}$$

for the generators of matrix representation of the Clifford algebra $C\ell^{\mathbb{C}}(1,5)\cong$Mat$(8.\mathbb{C})$. The corresponding assertion was proved in the paper [46].

Further, the variable $X\in$ M$(1,6)$ in equation (1) belongs to the space-time M(1,6):

$$\mathrm{M}(1,6)=\left\{X\equiv\left(X^{\tilde{A}}\right)=(X^0=t,\,\overline{\mathbf{X}}\equiv(X^{\tilde{j}})\in\mathrm{R}^6)\right\},\quad \tilde{j}=\overline{1,6}, \tag{5}$$

which is the corresponding Minkowski type space-time. Here the 8-component function $\psi(X)$ in equation (1) belongs to the rigged Hilbert space $\mathbb{S}^{6,8}\subset\mathbb{H}^{6,8}\subset\mathbb{S}^{6,8*}$. Equation (1) essentially differs from the ordinary 8-component Dirac equations. Nevertheless, the Hamiltonian $H=\Gamma^0\overline{\mathbf{\Gamma}}\,\overline{\mathbf{P}}+\Gamma^0 m$ in equation (1) has all mathematical properties of the Dirac Hamiltonian.

Note that in the article [46] the Dirac equation (1) was considered not only in the Clifford algebra $C\ell^{\mathbb{C}}(1,5)\cong$ Mat$(8,\mathbb{C})$ but in the extended 256-dimensional Clifford algebra $C\ell^{\mathbb{R}}(1,7)\cong$ Mat$(8.\mathbb{R})$ as well. The corresponding 8 generators are obtained after the addition of elements

$$\Gamma^7=\begin{vmatrix}0&0&\sigma^2\widehat{C}&0\\0&0&0&-\sigma^2\widehat{C}\\\sigma^2\widehat{C}&0&0&0\\0&-\sigma^2\widehat{C}&0&0\end{vmatrix},\ \Gamma^8=\begin{vmatrix}0&0&-i\sigma^2\widehat{C}&0\\0&0&0&i\sigma^2\widehat{C}\\-i\sigma^2\widehat{C}&0&0&0\\0&i\sigma^2\widehat{C}&0&0\end{vmatrix}, \tag{6}$$

to the set (2). Here $\widehat{C}$ is the operator of complex conjugation.

The explanation of the Clifford algebra $C\ell^{\mathbb{R}}(1,7)\cong$Mat$(8.\mathbb{R})$ and of our appealing to the 8-component Dirac equation was given in our article [49].

The general solution of the equation (1) is given by

$$\psi(X)=\frac{1}{(2\pi)^{\frac{3}{2}}}\int d^6K\left[e^{-iKX}c^{\mathrm{r}}(\overline{\mathbf{K}})\tilde{\mathrm{v}}_{\mathrm{r}}^-(\overline{\mathbf{K}})+e^{iKX}c^{*\tilde{\mathrm{r}}}(\overline{\mathbf{K}})\tilde{\mathrm{v}}_{\tilde{\mathrm{r}}}^+(\overline{\mathbf{K}})\right], \tag{7}$$

where $K\equiv(\overline{\mathbf{K}})=(k^1,k^2,k^3,k^4,k^5,k^6)$, $d^6K\equiv d^1kd^2kd^3kd^4kd^5kd^6k$,

$$KX=\widehat{\omega}t-\overline{\mathbf{K}}\,\overline{\mathbf{X}},\quad \widehat{\omega}=\sqrt{\overline{\mathbf{K}}^2+m^2},\quad \mathrm{r}=\overline{1,4},\ \tilde{\mathrm{r}}=\overline{5,8}, \tag{8}$$

$c^{\mathrm{r}}(\overline{\mathbf{K}})$, $c^{*\tilde{\mathrm{r}}}(\overline{\mathbf{K}})$ are the amplitudes of the particle and antiparticle, respectively, which relation to the quantum-mechanical momentum-spin amplitudes can be considered in complete analogy with the consideration in [50], and the 8-component spinors $(\tilde{\mathrm{v}}_{\mathrm{r}}^-(\overline{\mathbf{K}}),\,\tilde{\mathrm{v}}_{\tilde{\mathrm{r}}}^+(\overline{\mathbf{K}}))$ are given as

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$$\begin{aligned}
\tilde{v}_1^-(\overline{\mathbf{K}}) &= \widehat{N}(\widehat{\omega}+m,0,0,0,-iK^1+K^3,0,iK^2+K^6,K^4+iK^5)^{\mathrm{T}},\\
\tilde{v}_2^-(\overline{\mathbf{K}}) &= \widehat{N}(0,\widehat{\omega}+m,0,0,0,-iK^1+K^3,K^4-iK^5,iK^2-K^6)^{\mathrm{T}},\\
\tilde{v}_3^-(\overline{\mathbf{K}}) &= \widehat{N}(0,0,\widehat{\omega}+m,0,-iK^2+K^6,K^4+iK^5,-iK^1-K^3,0)^{\mathrm{T}},\\
\tilde{v}_4^-(\overline{\mathbf{K}}) &= \widehat{N}(0,0,0,\widehat{\omega}+m,K^4-iK^5,-iK^2-K^6,0,-iK^1-K^3)^{\mathrm{T}},\\
\tilde{v}_5^+(\overline{\mathbf{K}}) &= \widehat{N}(iK^1+K^3,0,iK^2+K^6,K^4+iK^5,\widehat{\omega}+m,0,0,0)^{\mathrm{T}},\\
\tilde{v}_6^+(\overline{\mathbf{K}}) &= \widehat{N}(0,iK^1+K^3,K^4-iK^5,iK^2-K^6,0,\widehat{\omega}+m,0,0)^{\mathrm{T}},\\
\tilde{v}_7^+(\overline{\mathbf{K}}) &= \widehat{N}(-iK^2+K^6,K^4+iK^5,iK^1-K^3,0,0,0,\widehat{\omega}+m,0)^{\mathrm{T}},\\
\tilde{v}_8^+(\overline{\mathbf{K}}) &= \widehat{N}(K^4-iK^5,-iK^2-K^6,0,iK^1-K^3,0,0,0,\widehat{\omega}+m)^{\mathrm{T}},
\end{aligned}$$

where the operation $(...)^{\mathrm{T}}$ denotes the usual vector transposition and

$$\widehat{N} \equiv \frac{1}{\sqrt{2\widehat{\omega}(\widehat{\omega}+m)}}, \quad \widehat{\omega} \equiv \sqrt{\overline{\mathbf{K}}^2+m^2}.$$

The above consideration of the Dirac-like equation (1) is our approbation of the possibilities of the space M(1,6). Note that the different relationships between the Dirac and the Maxwell equations [51–56] enabled us to generalize similarly the Maxwell equations. Furthermore, for the general case of nonzero mass the Dirac–Maxwell relationships were found as well. see, e.g., [55,56].

Equation (1) under consideration couples together two different coordinate spaces with the same time in the form $\mathbb{R}^6 = \mathbb{R}^3 \otimes \mathbb{R}^{*3}$. They can be similar to each other as two duplications of $\mathbb{R}^3 \subset$M(1,3), but, of course, can be completely different fragments as $\mathbb{R}^3 \subset$M(1,3) and $\mathbb{R}^{*3} \subset \mathrm{M}^*(1,3)$ corresponding to the different physical reality.

5. The Essence of Our Hypotheses About Dark Energy-Matter Sector

On the basis of above given consideration two hypotheses on the dark matter and dark energy origin can be suggested.

The interpretation of $\mathbb{R}^{*3} \subset \mathrm{M}^*(1,3)$ as the space, which generates the dark matter and dark energy, together with $\mathbb{R}^3 \subset$M(1,3), which is our well-known living (native) space is possible and interesting. Note that the time axe in $\mathrm{M}^*(1,3)$ and in M(1,3) is one and the same. Therefore, here the 8-component Dirac equation from the paper [46] coupled together these two possibly different space-time manifolds. The different relationships between the Dirac and the Maxwell equations presented in a number of our publications, see. e.g. [48,50] and [51–56], guarantee that the Maxwell equations in the space-time M(1,6) will be similarly well-defined.

The physics and astrophysical phenomena including dark matter and dark energy in such approach should be considered in the general joint space-time M(1,6).

Of course, described above relationship between the results of the articles [46,47] and the dark matter or dark energy today is only the hypothesis. Note only that considered above the coordinate space $\mathbb{R}^{*3} \subset \mathrm{M}^*(1,3)$ is the simplest candidate for the space of dark matter and dark energy (among the 10-dimensional, 11-dimensional, 22-dimensional and other spaces of supersymmetry, superstrings and supergravity) and is the closest neighbor of our well-known living (native) space $\mathbb{R}^3 \subset$M(1,3).

Our next hypothesis is the phase transition from the space-time M(1,6) to the ordinary M(1,3) and the generation of DEMS (dark energy-matter sector) in this process. The phase transitions [39–45] should be considered together with suggestion that Big Bang didn't exist at all in phase transition from the space-time M(1,6) to the Minkowski space-time M(1,3). Moreover, the role of the Clifford algebras is explained on the basis of such transition from the space-time M(1,6) to the Minkowski space-time M(1,3). It is accompanied by transition from $C\ell^{\mathbb{C}}$(1,6) to $C\ell^{\mathbb{C}}$(1,3)

Recently in the paper [57] another hypothetical model of the phase transition from the initial maternal 10-dimensional space-time to our 4-dimensional modern Universe has been proposed. The

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hypothesis was opposed to the Big Bang theory and the Standard Cosmological Model (called SM by the author). The mathematical apparatus for describing the phase transition used the algebraic dynamics of the change in the signature of the Clifford algebra from $C\ell^{\mathbb{C}}(1,9)$ to $C\ell^{\mathbb{C}}(1,3)$ with a complete decomposition of the 10-dimensional maternal space into subspaces, one of which is our Universe. To minimize the choice of options for the complete decomposition of the corresponding mother algebra, Frobenius theorems were used. A quantitative estimate of the dark sector of the Universe in such a model was calculated and the futility of the experimental search for dark matter carriers was indicated due to the non-isomorphism of Minkowski spaces and spaces responsible for the DEMS.

Contrary to our papers [46,47] the article [57] contains many criticisms of the Big Bang theory and the Standard Cosmological Model (SM) of the Universe, which is known as the Friedmann-Lemaître-Robertson-Walker (FLRW) model. The language of [57] is Ukrainian but readers can catch main ideas from the English version of the abstract. The notation SM in [56] can be confused with SM of the elementary particles.

The main difference between [57] and our approach is as follows. We use the closest neighbor space-time M(1,6). There are many intermediate candidates between M(1,6) and 10-dimensional space-time from [57].

The consequences and details will be considered not in the framework of this brief communication.

6. Briefly on the Structure of the List of the Dirac Equations in Higher Dimensions

Note that for 4×4 matrices we have only 4 independent gamma matrices, 4 component ψ function and the Dirac equation exists only in 4-dimensional space-time. From the consideration above it follows that for 8×8 matrices we have only 6 ordinary independent gamma matrices, 8 component ψ function and the Dirac equation exists only in 7-dimensional space-time M(1,6). Not in 8-dimensional space-time! Thus, the 16×16 choice of gamma matrices is preferable for the equation (1). For higher dimensions this difference will increase. For the 16×16 gamma matrices and 16 component ψ function we will have 8 ordinary independent gamma matrices and additional Dirac equation in M(2,6) space-time, where M(2,6) contains two different times. Thus, the simple listing of the dimensions like $N = 4, 5, 6, 7, 8, \ldots, N-1$, of wave function in the equation (1) is not possible. This list is not homogeneous and has a structure. The first reason is in structure of the list of the existing Clifford algebras. The Clifford-Dirac algebra representations exists not for all dimensions [58–63].

Further, here above we demonstrate the structure of the checklist of the Dirac equations even in 7 dimensions of M(1,6) space-time. Indeed, in [46] three different Dirac equations were derived from one and the same Clifford algebra $C\ell^{\mathbb{C}}(1,5)$, here (2), in 7-dimensional space-time M(1,6). It is evident that for the space-time M(1,7) and corresponding Clifford algebra $C\ell^{\mathbb{C}}(1,6)$ with matrix representation given by 16×16 gamma matrices the number of Dirac equations will increase. The question arises about the regularity in such increasing. This list is not homogeneous. AS it was mark just above the Clifford-Dirac algebra representations exists not for all dimensions [58–63].

Indeed, the example of [60] demonstrates the existence of 8 generators of the algebra $C\ell^{\mathbb{C}}(1,7)$ in evident form of the Table 3 in [61]. Confirmation can be found in other articles from the list [58–63].

Therefore, the generalization of the Dirac equation to the N-dimensional space-time should be realised step by step and not in such formal form as it was presented in the articles [64–66].

Thus, on the examples of the Dirac equations in higher dimensions we demonstrate the relationship between the different space-times and corresponding Clifford algebras, which were used above in our hypotheses.

7. Brief Conclusions

The Dirac equation directly follows from the Clifford algebra. The Clifford algebras follow from the geometry of space-time. It is the reason of our attempts to use the relationships between the Dirac equation, Clifford algebras, space-time and to find the way for the DEMS of the Universe explanation.

Our conclusion that presented here hypotheses are new and may be useful for further investigations is based on our brief review of different approaches to DM and DE explanation, which is given in Introduction and Sections 2, 3.

We demonstrate the structure and nonhomogeneous character of the list of the higher dimensional Dirac equations.

The general brief conclusions are as follows. (i) The situation that we don't know what is the Universe is not new and periodically appears in the history of science. (ii) The large number of different approaches means that we don't know today the exact solution. (iii) On the other hand, the large number of approaches may lead very soon to some local resolution of the situation. (iv) After the explanations, what are the DM and DE, the Universe will put the new problems and tasks for the Mankind.

The completely unexpected approaches for the resolution of the DM and DE problem should be include into a consideration as well.

Indeed, the investigation of DEMS is the main problem of modern physics and astronomy.

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