Article

Not peer-reviewed version

Prospect Theory-Driven Grey Multi-Attribute Decision-Making with Entropy-Based Weighting

Fangyuan Liu and Jinhai Guo *

Posted Date: 30 September 2025

doi: 10.20944/preprints202509.2355.v1

Keywords: multi-attribute decision-making; interval grey number; value function; information entropy

Article

Prospect Theory-Driven Grey Multi-Attribute Decision-Making with Entropy-Based Weighting

Fangyuan Liu and Jinhai Guo *

School of Information and Mathematic, Yangtze University, Jingzhou 434023, China

* Correspondence: xin3fei@21cn.com

Abstract

In order to solve the decision-making problem that the attributive values are internal grey numbers and the attributive weights are unknown, this study develops a novel decision-making method based on prospect theory for environments with interval grey numbers. First, interval grey numbers are employed to characterize uncertain preference information, whereas scenario-specific reference points and distance measures are utilized to compute positive and negative ideal solutions, followed by the construction of a prospect value function. Then, an information entropy optimization method is established to determine optimal attribute weights (with "optimal" explicitly defined as the weight configuration that maximizes decision consistency under entropy constraints), enabling subsequent alternative ranking through comprehensive prospect value evaluation. Finally, the feasibility and effectiveness of the model were empirically validated by applying it to renewable energy investment decision-making. The results conclusively demonstrate that the proposed approach provides an effective solution for complex multi-attribute decision-making under uncertainty, while exhibits substantial practical value in real-world applications.

Keywords: multi-attribute decision-making; interval grey number; value function; information entropy

---

1. Introduction

Multi-attribute decision-making (MADM) serves as a crucial decision-support tool that faces dual challenges of information incompleteness and decision-makers' behavioral preferences in complex environments [1–8] Conventional research often treats information uncertainty processing and behavioral decision analysis as separate domains, failing to adequately address the complexity inherent in real-world decision scenarios. This study develops an innovative decision-making framework by integrating grey system theory, prospect theory, and the entropy weight method to holistically consider both information incompleteness and behavioral characteristics [9–15].

Grey system theory [3] is particularly suitable for handling incomplete information scenarios. Its fundamental component, interval grey numbers [12–15], captures both quantitative and qualitative uncertainty through bounded intervals, providing a mathematical foundation for addressing decision information deficiencies. Complementing this, prospect theory [16,17] from behavioral economics reveals three characteristic decision-making patterns under risk: reference dependence, loss aversion, and diminishing sensitivity - features that are particularly prominent in complex decisions. It is noteworthy that while existing literature [18–20] has begun exploring prospect theory applications in MADM, most studies remain confined to precise information environments and fail to effectively incorporate uncertainty processing methods.

Regarding weight determination, information entropy [21,22] serves as an effective tool for measuring information uncertainty, yet its integration with interval grey numbers remains an understudied area. Particularly when evaluating significantly different alternatives, traditional fixedweight approaches [13] cannot adequately reflect decision-makers' differentiated considerations across alternatives.

To address these theoretical gaps, this study constructs a three-phase integrated framework of "grey number representation - behavioral modification - entropy optimization." The framework first preserves original information integrity through the algebraic operation system of interval grey numbers [15], overcoming information distortion in data-deficient situations. Second, it incorporates prospect theory's dynamic reference point mechanism to quantify psychological expectations as value functions within grey intervals, breaking through the limitation of existing research [18–20] that only applies to precise numerical values. Specifically, we develop a grey-entropy distance-based algorithm for alternative-specific weighting by establishing a bi-objective optimization model that considers both information entropy and solution deviations, thereby achieving dynamic weight adjustment.

The theoretical innovations of this research manifest in three aspects: (1) establishing mapping relationships between interval grey numbers and prospect value functions to enable behavioral parameter calibration under uncertainty for the first time; (2) proposing an improved grey-entropy coupling weight model that enhances weight interpretability through solution discrimination factors; and (3) developing a cumulative prospect-based grey number ranking method that demonstrates higher cognitive consistency in decision outcomes compared to conventional models [13]. This integrated approach not only remedies the artificial separation of uncertainty processing and behavioral analysis in existing research but also provides novel insights for behavioral decision-making in complex environments.

2. Basic Concepts and Definitions

Definition 1. Suppose the decision scheme set is $A=\{A_1,A_2,\cdots,A_m\}$ and the attribute indicator set is $S=\{S_1,S_2,\cdots,S_n\}$. Because the decision information is not represented by precise values, but rather by interval grey numbers, the attribute value of alternative $A_i\ (i=1,\ldots,m)$ under attribute $S_j\ (j=1,\ldots,n)$ is denoted as a non-negative interval grey number: $u_{ij}(\otimes)\in[u_{ij}^L,u_{ij}^U]$ and $u_{ij}^U\ge u_{ij}^L\ge 0$.

Then, the attribute vector of A_i is denoted as $u_i(\otimes)=(u_{i1}(\otimes),u_{i2}(\otimes),\cdots,u_{im}(\otimes)),i=1,2,\cdots,n$.

The decision matrix is expressed as:

$$R=\left(u_{ij}(\otimes)\right)_{m\times n}=\begin{bmatrix}[u_{11}^L,u_{11}^U] & [u_{12}^L,u_{12}^U] & \cdots & [u_{1n}^L,u_{1n}^U]\\ [u_{21}^L,u_{21}^U] & [u_{22}^L,u_{22}^U] & \cdots & [u_{2n}^L,u_{2n}^U]\\ \vdots & \vdots & \ddots & \vdots\\ [u_{m1}^L,u_{m1}^U] & [u_{m2}^L,u_{m2}^U] & \cdots & [u_{mn}^L,u_{mn}^U]\end{bmatrix} \quad (1)$$

Definition 2. Let the weight assigned by the decision-maker to attribute A_i under alternative S_j be an interval grey number $w_{ij}(\otimes)(i=1,2,\cdots,m$, $j=1,2,\cdots,n)$, $\omega_{ij}(\otimes)\in\left[w_{ij}^L,w_{ij}^U\right]$ $0\leqslant w_{ij}^L\leqslant w_{ij}^U\leqslant 1$.

Then, the interval grey weight matrix composed of $w_{ij}(\otimes)$ is expressed as:

$$W=\left(w_{ij}(\otimes)\right)_{m\times n}=\begin{bmatrix}[w_{11}^L,w_{11}^U] & [w_{12}^L,w_{12}^U] & \cdots & [w_{1n}^L,w_{1n}^U]\\ [w_{21}^L,w_{21}^U] & [w_{22}^L,w_{22}^U] & \cdots & [w_{2n}^L,w_{2n}^U]\\ \vdots & \vdots & \ddots & \vdots\\ [w_{m1}^L,w_{m1}^U] & [w_{m2}^L,w_{m2}^U] & \cdots & [w_{mn}^L,w_{mn}^U]\end{bmatrix} \tag{2}$$

Definition 3. Given the differing dimensions across attributes, it is necessary to normalize the elements in the interval-valued decision matrix. For benefit-type attributes, normalization was performed as follows:

$$[x_{ij}^L,x_{ij}^U]=\left[u_{ij}^L\Big/\sqrt{\sum_{i=1}^m\left(u_{ij}^U\right)^2},\,u_{ij}^U\Big/\sqrt{\sum_{i=1}^m\left(u_{ij}^L\right)^2}\right],i=1,\ldots,m;j=1,\ldots,n \tag{3}$$

For cost-type attributes, the normalization is performed as follows:

$$[x_{ij}^L,x_{ij}^U]=\left[\frac{1}{u_{ij}^U}\Big/\sqrt{\sum_{i=1}^m\left(\frac{1}{u_{ij}^L}\right)^2},\,\frac{1}{u_{ij}^L}\Big/\sqrt{\sum_{i=1}^m\left(\frac{1}{u_{ij}^U}\right)^2}\right],i=1,\ldots,m;j=1,\ldots,n \tag{4}$$

Let the normalized decision matrix after dimensionless processing be denoted as:

$$X=\left(x_{ij}(\otimes)\right)_{m\times n}=\begin{bmatrix}[x_{11}^L,x_{11}^U] & [x_{12}^L,x_{12}^U] & \cdots & [x_{1n}^L,x_{1n}^U]\\ [x_{21}^L,x_{21}^U] & [x_{22}^L,x_{22}^U] & \cdots & [x_{2n}^L,x_{2n}^U]\\ \vdots & \vdots & \ddots & \vdots\\ [x_{m1}^L,x_{m1}^U] & [x_{m2}^L,x_{m2}^U] & \cdots & [x_{mn}^L,x_{mn}^U]\end{bmatrix} \tag{5}$$

where all elements are non-negative interval grey numbers defined in [0, 1].

The n-dimensional non-negative interval grey number vector:

$$x^+(\otimes)=\left(x_1^+(\otimes),\ x_2^+(\otimes),\cdots,\ x_n^+(\otimes)\right) \tag{6}$$

is called the grey positive ideal point and is the n-dimensional non-negative interval grey number vector.

$$x^-(\otimes)=\left(x_1^-(\otimes),\ x_2^-(\otimes),\cdots,\ x_n^-(\otimes)\right) \tag{7}$$

is called the grey negative ideal point, where:

$$x_j^+(\otimes)=\left[a_j^L,a_j^U\right]=\left[\max_i x_{ij}^L,\max_i x_{ij}^U\right],(j=1,\ldots,n) \tag{8}$$

$$x_j^-(\otimes)=\left[b_j^L,b_j^U\right]=\left[\min_i x_{ij}^L,\min_i x_{ij}^U\right],(j=1,\ldots,n) \tag{9}$$

Definition 4. As a valid metric in metric spaces, Euclidean distance satisfies the following three fundamental properties: non-negativity, homogeneity, and triangle inequality. In grey space, when grey numbers degenerate to whitened values, the grey distance reduces to Euclidean distance. Thus, Euclidean distance serves as a special case of grey distance.

Consider two n-dimensional interval grey number vectors:

$\mathbf{a}(\otimes)=(a_1(\otimes),a_2(\otimes),\cdots,a_m(\otimes))$,

$\mathbf{b}(\otimes)=(b_1(\otimes),b_2(\otimes),\cdots,b_m(\otimes))$,

where $a_j(\otimes)\in[a_j^L,a_j^U]$, $b_j(\otimes)\in[b_j^L,b_j^U]$, $(j=1,\ldots,n)$.

 doi:10.20944/preprints202509.2355.v1

The distance between the two n-dimensional interval grey number vectors $\mathbf{a}(\otimes)$ and $\mathbf{b}(\otimes)$ is defined as follows:

$$d(\mathbf{a}(\otimes),\mathbf{b}(\otimes))=\frac{\sqrt{2}}{2}\sqrt{\sum_{j=1}^{n}\left[\left(a_j^L-b_j^L\right)^2+\left(a_j^U-b_j^U\right)^2\right]} \tag{10}$$

Then, the distances from each alternative to the positive and negative ideal points are calculated as:

$$D_i^+=\left|x_j^+-x_{ij}\right|=\frac{\sqrt{2}}{2}\sqrt{\sum_{j=1}^{n}\left[\left(x_{ij}^L-b_j^L\right)^2+\left(x_{ij}^U-b_j^U\right)^2\right]} \tag{11}$$

$$D_i^-=\left|x_{ij}-x_j^-\right|=\frac{\sqrt{2}}{2}\sqrt{\sum_{j=1}^{n}\left[\left(x_{ij}^L-a_j^L\right)^2+\left(x_{ij}^U-a_j^U\right)^2\right]} \tag{12}$$

3. Decision-Making Model Based on Prospect Theory

Prospect Theory

The prospect value is jointly determined by a value function and decision weights, expressed as:

$$V=\sum_{i=1}^{n}v(x_i)\pi(p_i) \tag{13}$$

Here, V is the prospect value, $v(x)$ is the value function that represents the subjectively perceived value formed by the decision maker, and $\pi(p_i)$ is the decision weight, an evaluative function assigned by the decision maker. If the attribute value provided by the decision maker is greater than the positive ideal point, the decision maker may perceive a gain, where the magnitude of the gain is determined by the distance between the attribute value and the positive ideal point. Similarly, the magnitude of the loss can be determined by the distance between the attribute value and the negative ideal point. Based on the function proposed by Tversky and Kahneman [17] to measure the decision-maker's attitude toward gains and losses, the prospect value function formula for each attribute of the alternative A_i is as follows:

$$v\left(x_{ij}\right)=\begin{cases}\left(x_{ij}-x_j^-\right)^\alpha, x_{ij}\geq x_r\\ -\lambda\left(x_j^+-x_{ij}\right)^\beta, x_{ij}<x_r\end{cases} \tag{14}$$

Let $v^+\left(x_{ij}\right)=\left(x_{ij}-x_j^-\right)^\alpha$ denote the gain prospect function, representing a positive prospect value, and $v^-\left(x_{ij}\right)=-\lambda\left(x_j^+-x_{ij}\right)^\beta$ ($0\leq\alpha,\beta\leq 1$) denote the loss prospect function, representing a negative prospect value. Where α and β are the gain- and loss-correlation coefficients, respectively. λ is a risk-aversion parameter; λ > 0. Ensuring that losses are steeper than gains. α = 0.89, β = 0.92, and λ = 2.25 [16].

3.2. Determining Optimal Attribute Weights

For a MADM problem with partially unknown weight information and interval grey numbers as attribute values, let the decision attribute weight vector for each alternative be $w=(w_1,w_2,\cdots,w_n)$ or $\sum_{j=1}^{n}w_j=1$.

For each alternative A_i, the comprehensive prospect value of each alternative should be maximized. Therefore, the optimal weights can be computed using information entropy.

Based on Definition 2, let:

$$W^L = \begin{bmatrix} w_{11}^L & w_{12}^L & \cdots & w_{1n}^L \\ w_{21}^L & w_{22}^L & \cdots & w_{2n}^L \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1}^L & w_{m2}^L & \cdots & w_{mn}^L \end{bmatrix} \quad (15)$$

$$W^U = \begin{bmatrix} w_{11}^U & w_{12}^U & \cdots & w_{1n}^U \\ w_{21}^U & w_{22}^U & \cdots & w_{2n}^U \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1}^U & w_{m2}^U & \cdots & w_{mn}^U \end{bmatrix} \quad (16)$$

Then:

$$R = W^L \times \left(diag\left(sum\left(W^L\right)\right)\right)^{-1} \quad (17)$$

$$E = \frac{-sum\left(R \times \log\left(R\right)\right)}{\log\left(m\right)} \quad (18)$$

$$F = 1 - E \quad (19)$$

and:

$$\pi^+\left(w^L\right) = \frac{F}{sum\left(F\right)} = \left(w_1^+, w_2^+, \cdots, w_n^+\right) \quad (20)$$

and:

$$\pi^-\left(w^U\right) = \left(w_1^-, w_2^-, \cdots, w_n^-\right) \quad (21)$$

Let $\pi^+\left(w^L\right) = \left(w_1^+, w_2^+, \cdots, w_n^+\right)$, $\pi^-\left(w^U\right) = \left(w_1^-, w_2^-, \cdots, w_n^-\right)$ be the optimal solution to be determined, where $\sum_{j=1}^{n} w_j^+ = 1$, $\sum_{j=1}^{n} w_j^- = 1$.

Then, the optimal comprehensive prospect value for alternative A_i is given by:

$$V_i = \sum_{j=1}^{n} v^+(x_{ij})\pi^+(w_j^L) + \sum_{j=1}^{n} v^-(x_{ij})\pi^-(w_j^U) \quad (22)$$

By ranking the optimal comprehensive prospect values $V_i \ (i = 1, \ldots, m)$ of all alternatives in descending order, we obtain the complete ranking of alternatives and identify the optimal solution.

4. Procedure

Step 1: Construct the interval grey number decision matrix $R = \left(u_{ij}\left(\otimes\right)\right)_{m\times n}$ based on the MADM problem, and then normalize it to obtain the standardized decision matrix $X = \left(x_{ij}\left(\otimes\right)\right)_{m\times n}$.

Step 2: Determine ideal positive and negative points. Calculate the distance between each alternative's attributes and $x_j^+(\otimes)$ and $x^-(\otimes)$ using Equations (10) and (11). The positive and negative prospect matrices for each alternative are then computed using (Equation (14)).

Step 3: With the goal of maximizing the comprehensive prospect value, calculate the optimal weights for gains $\pi^+\left(w^U\right) = \left(w_1^+, w_2^+, \cdots, w_n^+\right)$ and losses $\pi^-\left(w^L\right) = \left(w_1^-, w_2^-, \cdots, w_n^-\right)$ using the information entropy method (Equations (17)–(20)).

 doi:10.20944/preprints202509.2355.v1

Step 4: Substitute $\pi^{+}\left(w^{L}\right)=\left(w_{1}^{+},w_{2}^{+},\cdots,w_{n}^{+}\right)$, $\pi^{-}\left(w^{U}\right)=\left(w_{1}^{-},w_{2}^{-},\cdots,w_{n}^{-}\right)$ into $V_i=\sum_{j=1}^{n}v^{+}(x_{ij})\pi^{+}(w_j^{L})+\sum_{j=1}^{n}v^{-}(x_{ij})\pi^{-}(w_j^{U})$ to compute the optimal comprehensive prospect value V_i $(i=1,\ldots,m)$, for each alternative. All alternatives were ranked in descending order of V_i to obtain the final optimal ranking.

5. Example

Here, we refer to data from Reference [1], considering a venture capital firm evaluating five potential investment projects: A_1, A_2, A_3, A_4, and A_5. Six risk-based criteria were assessed: market risk (S_1); technological risk (S_2), management risk (S_3), environmental risk (S_4), production risk (S_5), financial risk (S_6).

1. All six attributes are cost-type criteria, with evaluation scores ranging from one (low risk) to five (high risk). Decision-makers assessed each project against these criteria, providing interval-valued scores for each attribute. The resulting decision matrix is expressed as follows:

Table 1. Information for the decision-making.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	$[2,4]$	$[3,4]$	$[2,3]$	$[2,3]$	$[2,3]$	$[4,5]$
A_2	$[3,4]$	$[2,3]$	$[3,4]$	$[3,4]$	$[2,4]$	$[2,3]$
A_3	$[3,5]$	$[2,4]$	$[3,4]$	$[3,4]$	$[2,4]$	$[3,5]$
A_4	$[3,5]$	$[2,4]$	$[2,3]$	$[2,5]$	$[3,4]$	$[2,3]$
A_5	$[4,5]$	$[3,4]$	$[2,4]$	$[2,5]$	$[3,5]$	$[2,4]$

The known attribute weight information is given as:

Table 2. The initial attribute values of decision scheme.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	[0.15,0.18]	[0.16,0.27]	[0.17,0.28]	[0.14,0.19]	[0.13,0.26]	[0.15,0.22]
A_2	[0.13,0.28]	[0.16,0.17]	[0.12,0.18]	[0.14,0.29]	[0.12,0.16]	[0.16,0.20]
A_3	[0.15,0.18]	[0.16,0.27]	[0.17,0.18]	[0.14,0.19]	[0.13,0.26]	[0.15,0.20]
A_4	[0.15,0.18]	[0.16,0.27]	[0.17,0.28]	[0.15,0.29]	[0.12,0.16]	[0.16,0.22]
A_5	[0.15,0.18]	[0.16,0.27]	[0.17,0.18]	[0.14,0.19]	[0.13,0.26]	[0.16,0.20]

The normalized decision matrix is obtained according to Equation (4) as follows:

Table 3. Normalized decision-making information after transformation.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	[0.2822,0.889	[0.2535,0.520	[0.3563,0.827	[0.3381,0.889	[0.3381,0.859	[0.2081,0.414
A_2	[0.2822,0.592	[0.3381,0.564	[0.2138,0.414	[0.2535,0.592	[0.2535,0.859	[0.3468,0.827
A_3	[0.3763,0.889	[0.3381,0.564	[0.2138,0.414	[0.2535,0.592	[0.2535,0.859	[0.2081,0.551
A_4	[0.2258,0.592	[0.2535,0.564	[0.3563,0.827	[0.2028,0.889	[0.2535,0.572	[0.3468,0.827

A_5	[0.2258,0.444	[0.2535,0.376	[0.2673,0.827	[0.2028,0.889	[0.2028,0.572	[0.2601,0.827

2. From the decision matrix, the positive and negative ideal points are determined as:

Table 4. The grey positive and negative ideal point.

	A_1	A_2	A_3	A_4	A_5
$x_j^+(\otimes)$	[0.3563,0.8893]	[0.3468,0.8592]	[0.3763,0.8893]	[0.3563,0.8893]	[0.2673,0.8893]
$x_j^-(\otimes)$	[0.2081,0.4140]	[0.2138,0.4140]	[0.2081,0.4140]	[0.2028,0.5644]	[0.2028,0.3763]

The distances from each alternative to the positive and negative ideal points were calculated using Equations (11) and (12) respectively.

Table 5. The gain matrix.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	0.3401	0.0821	0.3109	0.3484	0.3280	0
A_2	0.1354	0.1380	0	0.1296	0.3161	0.3074
A_3	0.3565	0.1406	0.0040	0.1305	0.3164	0.0975
A_4	0.3565	0.0359	0.2156	0.2297	0.0363	0.2123
A_5	0.0510	0.0359	0.3226	0.3627	0.1389	0.3219

Table 6. The loss matrix.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	0.0524	0.2705	0.0434	0.0129	0.0249	0.3520
A_2	0.1938	0.2085	0.3286	0.1995	0.0660	0.0221
A_3	0	0.2313	0.3552	0.2269	0.0894	0.2666
A_4	0.2290	0.2410	0.0434	0.1085	0.2353	0.0439
A_5	0.3157	0.3629	0.0434	0.0456	0.2284	0.0437

The positive and negative prospect matrices for each alternative are calculated using Equation (14).

Table 7. The positive prospect value matrix.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	0.3830	0.1080	0.3535	0.3913	0.3707	0
A_2	0.1687	0.1716	0	0.1622	0.3587	0.3500
A_3	0.3993	0.1744	0.0074	0.1633	0.3591	0.1260
A_4	0.0387	0.0517	0.2553	0.2701	0.0523	0.2518
A_5	0.0708	0.0517	0.4056	0.4056	0.1726	0.3646

Table 8. The negative prospect value matrix.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	−0.1493	−0.6758	−0.1256	−0.0410	−0.0752	−0.8611
A_2	−0.4971	−0.5319	−0.8081	−0.5107	−0.1845	−0.0675

A_3	0	−0.5851	−0.8682	−0.5748	−0.2440	−0.0675
A_4	−0.5797	−0.6075	−0.1256	−0.2917	−0.5944	−0.1269
A_5	−0.7791	−0.8854	−0.1256	−0.1314	−0.5783	−0.1263

3. With the objective of maximizing the comprehensive prospect value, an optimization model is established: $\max V_i = \sum_{j=1}^{n} v^+(x_{ij})\pi^+(w_j^L) + \sum_{j=1}^{n} v^-(x_{ij})\pi^-(w_j^U)$.

Table 9. The weights for the five attributes.

W^L	S_1	S_2	S_3	S_4	S_5	S_6	W^U	S_1	S_2	S_3	S_4	S_5	S_6
A_1	0.15	0.16	0.17	0.14	0.13	0.15	A_1	0.18	0.27	0.28	0.19	0.26	0.22
A_2	0.13	0.16	0.12	0.14	0.12	0.16	A_2	0.28	0.17	0.18	0.29	0.16	0.20
A_3	0.15	0.16	0.17	0.14	0.13	0.15	A_3	0.18	0.27	0.18	0.19	0.26	0.20
A_4	0.15	0.15	0.17	0.15	0.12	0.16	A_4	0.28	0.17	0.28	0.29	0.16	0.22
A_5	0.15	0.16	0.17	0.14	0.13	0.16	A_5	0.18	0.27	0.18	0.19	0.26	0.20

The optimal weights for the gains and losses are calculated using the information entropy method (Equations (17)–(20)) as follows:

$$\pi^+\left(w^L\right) = \left(w_1^+, w_2^+, \cdots, w_n^+\right) = \left(0.1299, 0.0273, 0.7044, 0.0329, 0.0639, 0.0416\right),$$

$$\pi^-\left(w^U\right) = \left(w_1^-, w_2^-, \cdots, w_n^-\right) = \left(0.2001, 0.1944, 0.2001, 0.1832, 0.2130, 0.0091\right),$$

where, $\sum_{j=1}^{6} w_j^+ = 1, \sum_{j=1}^{6} w_j^- = 1$.

4. By substituting the optimal weight vectors $\pi^+\left(w^U\right) = \left(w_1^+, w_2^+, \cdots, w_n^+\right)$, $\pi^-\left(w^L\right) = \left(w_1^-, w_2^-, \cdots, w_n^-\right)$ into the comprehensive prospect value function, we obtain the optimal integrated prospect values for each alternative:

$$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$$

Table 10. The optimal comprehensive prospect value.

	A_1	A_2	A_3	A_4	A_5
V_i	0.1205	−0.4287	−0.3555	−0.2315	−0.1941

The comprehensive prospect values reveal that only A_1 yields a positive value, whereas all the others are negative. Consequently, for complex venture capital investments, A_1 is the optimal choice for project investment.

6. Conclusion

This study addresses MADM problems in which attribute weights are inconsistent and entirely unknown and attribute values are expressed as interval grey numbers. The proposed framework integrates three key components.

1. The interval Grey Number Distance Model: Measures the disparity between alternatives and ideal benchmarks.
2. Prospect Value Theory: Quantifies gains/losses based on decision-makers' psychological preferences.

3. Information entropy-based weight optimization: Bijective weights are derived to balance the subjective biases.

Advantages Over Existing Models,

1. Behavioral Realism: Explicitly accounts for decision-makers' varying perceptions of attribute values.
2. Adaptability: Suitable for high-uncertainty scenarios with incomplete weight information.
3. Practical Utility: The entropy-driven weighting method enhances the reliability of outcomes in complex real-world decisions.

This framework provides a robust tool for venture capital firms and other high-stakes decision environments in which risk perception and data uncertainty play critical roles.

Author Contributions: Conceptualization, F.L.; methodology, F.L.; software, F.L. and J.G.; validation, F.L.; formal analysis, F.L. and J.G.; investigation, F.L.; resources, F.L.; data curation, F.L.; writing—original draft preparation, F.L.; writing—review and editing, F.L. and J.G.; visualization, F.L. and J.G.; supervision, F.L. and J.G.; project administration, F.L. and J.G.; funding acquisition, F.L. and J.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding authors.

Acknowledgments: The authors thank the journal editor and anonymous reviewers for their guidance and constructive suggestions.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Abbreviations

The following abbreviation is used in this manuscript:

MADM multi-attribute decision-making

References

1. Xu, Z. Study on methods for multiple attribute decision making under some situations. *Southwest Jiao University* **2003**; pp. 6-31.
2. Wei G. A.; Study on methods for fuzzy multiple Attribute decision making under some situations. *Southwest Jiao University* **2009**; pp. 13-45.
3. Liu, S.; Dang, Y.; Fang, Z. Grey Systems Theory and Its Applications (10th Edition). *Beijing: Science Press* **2010;** pp. 18-31.
4. Wan, X.; Dang, Y. Multi-index grey target decision method based on adjustment coefficient. *Journal of Intelligent & Fuzzy Systems* **2015**, 29, 769-775.
5. Zhu, X.; Wen, H. Application of multi-index grey target decision method in prediction of coal temperature in goaf. *Combustion Science and Technology* **2024**, 196, 3527-3541.
6. Qiao, M. TODIM–VIKOR framework for development potential evaluation of forest health tourism based on the single-valued neutrosophic number multiple attribute group decision making. *Soft Computing* **2024**, 28, 8065-8076.
7. Tong, C.; Pei, S.; Xiao, L. Research on effectiveness evaluation of corporate culture construction based on the neutrosophic cubic number multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems* **2024**, 46, 2219-2231.

8. Jundan, H.; Qian, L.; Qi, D. A novel framework for comprehensive value evaluation of cultural tourism resources with 2-tuple linguistic neutrosophic numbers multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems* **2023**, 45, 7841-7858.
9. Ma, K.; Weng, H.; Chen, Y. A Dynamic Landslide Warning Model based on Grey System Theory. *IEEE Access* **2025**, 13, 22407-22419.
10. Luo, D.; Hao, H. Emergency management of agricultural drought disaster based on grey dynamic multi-attribute group decision method. *Water Saving Irrigation* **2022,** 6, 24-30.
11. Yang, L.; Xu, W.; Xiao, C.; Li, J. Multi-parameter Evaluation Method of Smart Community Grade Based on Multi-attribute Decision Making. *Computing Technology and Automation* **2022**, 41, 164-168.
12. Ci, T.; Liu, X. An Attribute Value Normalisation Method for Interval Number Multi-Attribute Decision Making Based on Decision Maker Preferences. *Statistics Decision* **2015**, 3, 36-38.
13. Ebrahimi, E.; Fathi, M.; Sobhani, S. A modification of technique for order preference by similarity to ideal solution (TOPSIS) through fuzzy similarity method (a numerical example of the personnel selection). *Journal of applied research on industrial engineering* **2023**, 10, 203-217.
14. Chen, X.; Liu, S. Grey multiple attribute group decision-making method with partial weight information. *Systems Engineering and Electronice* **2009**, 31, 843-846.
15. Pan, X.; He, S.; Wang, Y. A new decision analysis framework for multi-attribute decision-making under interval uncertainty. *Fuzzy Sets and Systems* **2024**, 480, 108867.
16. Kahneman, D.; Amos T. Prospect theory: An analysis of decision under risk. *Econometrica* **1979**, 47, 263-292.
17. Tversky, A.; Daniel, K. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty* **1992**, 5, 297-323.
18. Liu, Y.; Forrest, J.; Liu, S.; Liu, J. Multi-objective grey target decision-making based on prospect theory. *Control and Decision* **2013**, 28, 345-350.
19. Zhang, L.; Li, X. Entropy-weighted TOPSIS Multi-attribute Decision-making Model and Its Applications Based on Generalized Greyness. *Journal of Grey System* **2024**, 36, 15-26.
20. Li, P.; Wang, K.; Xu, Z. A two-sided matching decision-making method based on preference-approval structure and prospect theory. *Chinese Journal of Management Science* **2017**, 21, 592-606.
21. Chen, P. Effects of the entropy weight on TOPSIS. *Expert Systems with Applications* **2021**, 168, 114186.
22. Li, F.; Su, M.; Li, D. Combination evaluation model based on entropy weight method. *International Conference on Machine Learning and Computer Application* **2021**; pp. 1-5.