The Problem of Coordination: Temperature as a Physical Quantity

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Posted Date: 29 September 2025

doi: 10.20944/preprints202509.2341.v1

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Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 September 2025 doi:10.20944/preprints202509.2341.v1

Article

The Problem of Coordination: Temperature as a Physical Quantity

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Abstract

A physical quantity is related to measurement, but the concept of a physical quantity is not reducible to measurement. This issue is examined using temperature as an example within the framework of the problem of coordination introduced by Bas van Fraassen. The problem of coordination involves two levels of consideration. The first stage examines the modern state of art, when a physical quantity is related to a theory of physics. The second stage concerns the history of the development of the corresponding theory of physics. The views on the measurement of physical quantities of 19th-century physicists Helmholtz, Mach, and Duhem are also considered. This demonstrates the continuity of van Fraassen's views with discussions at the end of the 19th century.

Keywords: temperature; physical quantity; measurement; history of physics; philosophy of physics

Introduction

Physics theory contains mathematical equations that relate physical quantities to each other. For example, the ideal gas equation of state defines the relationship between absolute temperature (T), pressure (p), volume (V), and the number of moles (n):

$$pV = n\,R\,T$$

where R is the universal gas constant. It is supposed that measurements of these quantities for a gas in the low-pressure region will satisfy this equation. The problem of coordination lies in the question of how a mathematical equation in physics relates to a world that does not look like as a mathematical structure.

At first glance, the answer seems obvious, since measuring physical quantities seems intuitive. For example, discussing the problem of measuring length looks trivial — length is simply measured with a ruler; similarly, temperature is measured with a thermometer, and pressure with a manometer. Of course, units of measurement must be chosen, but it is not difficult to convert the resulting value from one dimension to another. Thus, there seems nothing strange in the fact that physicists have discovered the relationship between physical quantities and expressed it with a mathematical equation.

To demonstrate the existence of the problem, Bas van Fraassen proposed two related questions about a physical quantity [1]; below, using temperature as an example:

- What counts as a measurement of temperature;
- What is temperature?

Van Fraassen's book introduces the term "The problem of coordination" and there was also a brief discussion with temperature as an example; this was taken as the basis for this work. A thermometer measures temperature, but understanding this statement requires understanding what temperature is without the reference to how to measure it. This would be possible by relying on physical theory, but such a step only elevates the original question to a new level of consideration.

In the first section, "*Mathematics, Physics, and Measurement*", general issues will be touched upon at a preliminary level. These issues will then be considered in more detail in the section "*Modern*

Temperature Measurement“. This will provide a look at the problem of coordination and the modern thermodynamic temperature scale from the viewpoint of the established theory of physics. The next section will consider episodes from the history of thermometry, which will give us an insight into the problem of coordination during the development of the theory; episodes are taken from the book by Hasok Chang “*Inventing Temperature: Measurement and Scientific Progress*“ [2]. During the development of thermometry, non-trivial questions arose, but they were resolved through the iterative development of the theory in question. In the section “*Helmholtz, Mach, and Duhem on Measurement in Physics*”, the views of 19th-century physicists will be presented. This will show the continuity of the problem of coordination posed by van Fraassen with the views of 19th-century physicists. The article will conclude with a general discussion.

Mathematics, Physics and Measurement

A measuring instrument is designed and made by human being, but it works by itself. A thermometer reading, in this regard, is objective and does not depend on the person taking it. Thus, it is necessary to follow in more detail the connection between a real thermometer and the theory of physics. In general, it is about the relationship between theoretical and experimental physics. The development of modern physics would be impossible without experiments, but it's important to understand the relationship between a physical experiment and a physical theory.

From a theoretical perspective, the connection with measurement appears as follows. In physics mathematical equations are associated with a conceptual model that gives them physical meaning. This leads to the concept of an ideal instrument as a blueprint for a real instrument. However, it should be clear that a real instrument will differ in some way from the ideal instrument blueprint, as defined by physical theory.

The same physical quantity can be measured in different ways. For example, different types of thermometers are used to measure temperature: gas, liquid, mechanical, thermocouple or resistance thermometer, and optical. Each has its own ideal instrument prototype based on different physical theories. At the same time, they measure the same physical quantity, and properly calibrated instruments are expected to yield the same numerical temperature value within measurement errors.

The important difference is related to measurement errors, the occurrence of which is usually beyond the scope of physical theory. Some errors can be treated by introducing corrections to the measurements to account for the deviations of the real instrument from the ideal blueprint, but measurement errors cannot be reduced to zero. Discussing measurement is also impossible without introducing standards for the measurement scale, which gives rise to numerical values for a physical quantity.

Physics is based on the reproducibility of experimental results in different laboratories by different scientists. The development of physics without this requirement is impossible. This underscores the importance of standardization, without which would be impossible to talk about the reproducibility of the same thermometers in different laboratories. Standardization, in turn, introduces conventionality, and thus, conventionality, constructivism, and objectivity by measurement are closely interrelated.

Understanding a statement that an object's temperature being equal to, say, 396.54 ± 0.12 K requires knowledge of the temperature scale, measurement procedures, and calibration procedures for the thermometer in question. It's worth noting that the International Temperature Scale has changed over time, so it's important to know the year in which the measurement was made and the version of the standard used. It could be that the given temperature value should be adjusted to match the current version of the International Temperature Scale.

Modern Temperature Measurement

There are two theories for two different levels of organization to consider temperature. I'll begin with the kinetic theory, where temperature usually is related to the average kinetic energy of atoms.

It's important however to remember that this relationship holds only for equilibrium; that is, when Maxwell-Boltzmann energy distribution is achieved. Hence a more accurate view of temperature at this level is related to a parameter in the Maxwell-Boltzmann distribution.

The use of a thermometer presupposes thermal equilibrium between the thermometer and the system under study. Therefore, the conceptual image for temperature measurement in kinetic theory is as follows: the thermometer and the system in question have separate Maxwell-Boltzmann distributions, but the parameters of these distributions, called temperature, coincide. However, such a conceptual model does not help to create an ideal instrument for temperature measurement.

As a result, it's more appropriate to consider temperature at the level of continuum mechanics. In this case, temperature is introduced by the zeroth law of thermodynamics as a quantity to describe the thermal equilibrium between two systems. The temperatures of two systems in thermal equilibrium are equal, moreover temperature is transitive. If a thermometer shows for two different systems the same value, then the temperatures of these systems are equal. The zeroth law is sufficient to explain the possibility of temperature measurement.

James Maxwell was the first to make such a definition of the temperature in his book "*Theory of Heat*" in 1871 [3]. Temperature measurement differs from length measurement in that the addition of two temperatures does not make sense; length is considered an extensive property, temperature an intensive one. To talk about temperature is to talk about establishment of thermal equilibrium between bodies and the transitivity of temperature.

Temperature affects different properties of a substance, and this makes it possible to create different thermometers. I will focus only on gas and liquid thermometers, since the gas thermometer is directly related to the thermodynamic temperature scale [4]; as well as gas and liquid thermometers played a major role in the development of thermometry. The conceptual model of both thermometers is linked to the existence of a thermal equation of state; in the case of constant mass and composition, the mathematical notation is $f(p, V, T) = 0$.

The thermal equation of state allows us to introduce a practical temperature scale; the most obvious example is a mercury or alcohol thermometer. At this level, a change in temperature determines the change in volume due to the coefficient of thermal expansion. Temperature changes also affect the electrical and other properties of the substance.

In classical thermodynamics the absolute temperature scale was introduced, since the ratio of temperature functions on the practical scale was associated with the maximum efficiency of a heat engine. The introduction of the absolute temperature scale made the efficiency equal to the ratio of absolute temperatures.

The good news is that the absolute temperature scale coincides with the temperature measured by a gas thermometer with a thermometric gas that obeys the ideal gas equation of state - equation in the beginning of this article. Therefore, constructing ideal heat engines for temperature calibration is not required. The bad news is that the behavior of real gases deviates from the ideal gas equation of state. Yet, kinetic theory suggests the form of the equation of state for real simple gases, and metrologists can use a gas thermometer with real gases by introducing corrections for the non-ideal behavior.

The name "absolute temperature scale" emphasizes the existence of absolute zero temperatures ($T = 0$ K in the ideal gas equation of state), so constructing the scale requires only one reference point, the triple point of water. I won't go into the details of the metrologists' painstaking work (one correction after another), but according to the thermodynamics laws, it's impossible to cool a substance to absolute zero. Hence, the reference point of 0 K is unattainable, which, however, didn't prevent the creation of a working absolute thermodynamic temperature scale. I hope that the short description above gives the understanding of the ideal instrument in the case of the gas and liquid thermometer, as well as the difficulties of using real substances to introduce an absolute thermodynamic temperature scale.

Thus, the theory of physics defines temperature, provides conceptual models of ideal instruments, and the procedures necessary for calibrating a temperature scale. After that metrologists conduct numerous experiments with real instruments, leading to the creation of the International Temperature Scale. This makes it possible to create effective procedures for calibrating thermometers of various accuracy classes. Thus, in the case of an established theory of physics, it is the theory that answers the question of what temperature is, and from this follows what can be considered as temperature measurement. All this, in turn, allows us to say without hesitation that temperature is measured by a thermometer.

Episodes from the History of Thermometry

Bas van Fraassen also briefly examines the history of thermometry — he says that despite the lack of theory, understanding the development of thermometry is also possible. I will use examples from Hasok Chang's book, “*Inventing Temperature: Measurement and Scientific Progress*“ [2], which provides an interesting discussion of several issues that arose during the development of thermometry.

Interest in the quantitative measurement of temperature arose in the 17th century [5,6]. The first thermometers were gas thermometers, but it was quickly discovered that the results depended on both temperature and pressure; therefore, in the 18th century, liquid thermometers were used. It was necessary to clarify issues regarding the constancy of temperature at reference points, the scale nonuniformity between reference points (which substance provides the correct temperature scale), and the problem of measuring temperatures below the freezing point and above the boiling point of the thermometric substance. In the 19th century, the research progress allowed a return to gas thermometers. More precise measurements, however, revealed that the behavior of different gases differs rather from one another. At the same time, thermometry helped to separate the concepts of temperature and heat, which in turn contributed to the emergence of thermodynamics. This led to the construction of the absolute temperature scale independent of the thermometric substance.

Constancy of Temperature of Reference points

The first chapter of Chang's book, "*Keeping the Fixed Points Fixed*", provides interesting information about studying the boiling point of water. The boiling point of water depends on pressure; this was discovered rather quickly. The solution in this case is simple: choose the boiling point of water at a specific pressure as the reference point and introduce the necessary corrections during the calibration procedure. However, there was another serious problem, which is fascinatingly described in Chang's book. Even at a fixed pressure, the boiling point of water varied slightly depending on the boiling process details — the problem is related to water superheating.

I'll mention just two events. In 1772, Jean-André De Luc's book, "*Research on Atmospheric Changes*", was published. It included studies related to water superheating. Deluc wanted to establish the true boiling point and made a special effort to remove air from the water, but this unexpectedly led to severe superheating before boiling. In 1776, the Royal Society established a committee under the chairmanship of Henry Cavendish (De Luc was also a member) to consider thermometer reference points. Recommendations were developed for achieving a reproducible water boiling point. The problem turned out to be related to nucleation sites; additional measures helped prevent superheating, thus allowing the water boiling point to be used as a reliable reference point.

For a phase transition, it's important to distinguish between the kinetics of the process and the equilibrium state. Recommendations to achieve equilibrium as quickly as possible are important to calibrate thermometers. At the same time, the study of superheating of water and supercooling of steam belongs to kinetics. Understanding this difference required non-trivial efforts during historical development.

Scale Nonuniformity - Choice of Thermometric Substance

At the same time, a problem with liquid thermometers was discovered: the liquid expansion was nonuniform. This was shown by comparing alcohol and mercury thermometers with each other. Each was calibrated using the freezing and boiling points of water, after which the scale was divided into equal intervals. Such mercury and alcohol thermometers showed different temperatures in the range from zero to one hundred degrees Celsius.

The reason for this difference stems from inconstancy of the coefficient of thermal expansion—it turns out to be a function of temperature. This raises the question of which substance, alcohol or mercury, has a more constant coefficient of thermal expansion. Using an alcohol thermometer as a standard yields a temperature-dependent coefficient of thermal expansion for mercury, and vice versa. In other words, the question arises as to whether there is a valid temperature scale that is independent of the thermometric substance.

An intermediate answer was obtained by mixing hot and cold water. De Luc conducted experiments comparing the results of mixing water using a mercury and alcohol thermometer. Deluc demonstrated that a mercury thermometer, unlike an alcohol thermometer, produces the expected results in this case, and therefore mercury is a better thermometric substance for a thermometer than alcohol.

Temperature Beyond the Limits of a Mercury Thermometer

In the chapter "*Going Beyond*", Chang interestingly recounts two episodes in the history of the thermometer. First, about Johann Georg Gmelin 1733 trip to Siberia. In the Siberian cold, the mercury in the thermometer froze; Gmelin, however, believed that mercury doesn't freeze, so his readings for the Siberian cold were too low. This, in turn, sparked a series of studies to determine the freezing point of mercury and the question of how to measure temperatures below its freezing point.

The second episode concerns the high temperatures in furnaces that needed to be controlled. The maximum temperature of a mercury thermometer is limited by the boiling point of mercury (357°C). Pottery maker Josiah Wedgwood proposed a furnace temperature measurement device based on the contraction of clay during heating (the sintering process). However, Wedgwood's temperature scale yielded too high temperatures and was therefore rejected.

This also raised the question of the correct temperature scale. The chosen thermometric substance, mercury, had limitations related to its freezing and boiling points. One could try another liquid, but it was still unclear how to combine the temperature scales. Apparently, the intuitive desire was to obtain smooth functions for the thermal expansion coefficient, but it was unclear how to achieve this. Thus, in the late 18th and early 19th centuries, the gas thermometer was back with the hope for a universal temperature scale based on the gas equation of state.

Ideal Gas Equation of State and Gas Thermometers

A gas thermometer depends on the gas equation of state. At constant pressure, the Gay-Lussac law determines the relationship between volume and temperature:

$$V = V_0 (1 + \alpha t)$$

where V_0 means the volume of gas at zero degree Celsius ($t = 0°C$), and α is the coefficient of volumetric expansion. A similar relationship exists for pressure changes at constant volume:

$$p = p_0 (1 + \alpha t)$$

where p_0 is the pressure at zero degree Celsius, and α is the relative pressure coefficient; note that the numerical value of α is the same as in the equation for volume. The hope was that the coefficient α would be independent of temperature and pressure and would be the same for all gases.

Guillaume Amontons in 1702 conducted similar experiments a century before Gay-Lussac and obtained a comparable value for α. However, over the next hundred years, numerous experiments on the thermal expansion of gases revealed significant variations in α values. Gay-Lussac discovered that this was due to residual water in the apparatus, and provided special measures were taken to dry the vessels, the α values are quite close to each other for different gases [5].

Combining the above equations with Boyle's law, plus introducing the number of moles and the universal gas constant, yields the ideal gas equation given at the beginning of this article. The conversion from degrees Celsius to kelvin is as follows:

$$T = (1/\alpha + t)$$

At the beginning of the nineteenth century, gas laws were considered universal for all gases. This gave rise to the hope that the temperature scale defined by the Gay-Lussac law was independent of the gas used and therefore universal.

However, more precise measurements by physicist Henri Victor Regnault revealed that the behavior of different gases differs from each other and from the supposed universal equation of state. At the same time, it was shown that gas behavior matches the universal equation of state for smaller pressures; as a result, the term "ideal gas equation of state" was introduced in the second half of the 19th century. The advent of kinetic theory provided an explanation — the ideal gas equation of state corresponded to a gas without interactions between molecules, which could be achieved at low concentrations (low pressures).

In any case, the gas thermometer was regarded as the most accurate device for measuring temperature. In 1889, the first International Practical Temperature Scale was adopted, based on a constant-volume hydrogen gas thermometer with two reference points. The experimental work of physicist Pierre Chappuis played a major role in the adoption of this standard.

The introduction of the absolute temperature scale is tied to the development of thermodynamics and described elsewhere, see [7]. It is important that the development of thermodynamics led to the emergence of the thermodynamic temperature scale independent from thermometric substance. At the same time, it was proven that such an absolute temperature scale coincides with the temperature scale of a gas thermometer with the thermometric fluid obeying the ideal gas equation of state.

Helmholtz, Mach and Duhem on Measurement in Physics

Hermann Helmholtz is considered the pioneer of measurement theory; in 1887, he devoted an essay, "Counting and Measurement", to this topic [8]. Ernst Mach then introduced the concept of the "coordination principle", which gives a better description of the process to assign a numerical value to a measurement result. Pierre Duhem, in turn, demonstrated the connection between measurement and physical theory by means of the concept of the "ideal instrument". The combination of Mach's and Duhem's views leads to a reasonable position, close to that of van Fraassen.

The historian Olivier Darrigol in the paper "*Number and measure: Hermann von Helmholtz at the crossroads of mathematics, physics, and psychology*" [9] provides a good overview of the atmosphere of that time, which helps to better understand Helmholtz's aspirations:

- The emergence of non-Euclidean geometries required a rethinking of the relationship between axioms and the world. According to Helmholtz, geometry was an experimental science; the axioms of geometry are linked to experience.
- There was no generally accepted axiomatization of arithmetic. At the same time, mathematical monsters had already emerged—continuous functions without derivatives at all points.
- Discussion on whether measurements in psychology are possible.
- Extensive and intensive quantities. Temperature as an example of a physical intensive quantity.

Helmholtz combined the axiomatics of arithmetic with the measurement of extensive quantities (Helmholtz calls them additive):

> 'Now it is clear that my empiricist theory, if it no longer admits that the axioms of geometry cannot and must not be proved, must also apply to the origin of the arithmetic axioms, which have a comparable relation to the temporal form of intuition.'

Helmholtz also noted the existence of intensive physical quantities, but their measurement depended on the measurement of additive quantities. Thus, Helmholtz's axiomatization of arithmetic was based on the possibility of measuring extensive quantities. It's interesting to see how he dealt with irrational numbers:

> 'Irrational ratios can occur in real-world objects; however, they can never be represented exactly as numbers, but only the value can be confined within arbitrarily narrow limits. This confinement within limits is sufficient for all calculations of functions whose values undergo ever smaller changes as the values of the variables on which they depend change by ever smaller amounts, ultimately becoming smaller than any given finite value. This is particularly true for the calculation of all differentiable functions of irrational quantities. However, discontinuous functions can also be constructed, for which the knowledge of even the most precisely defined limits within which the irrational value lies is insufficient. In this respect, the representation of irrational quantities by our number system remains inherently inadequate. In geometry and physics, however, we have not yet encountered such types of discontinuity.'

It should be clear that most mathematicians were not enthusiastic about Helmholtz's approach to axiomatization of arithmetic. Cantor, for example, compared Helmholtz's approach with that of the Enlightenment mathematician Louis Bertrand: numbers were the words used by shepherds to count sheep. At the same time, this approach to mathematics found support among physicists, including Mach and Duhem.

I will now turn to the coordination principle (Zuordnungsprincip) of Ernst Mach from the book "*Principles of the Heat Theory*" [10]. First, the book contains a chapter, "*Names and Numbers*", which approvingly mentions Helmholtz's work and expresses an even more radical position: 'Numbers are also names'. The next chapter, "*Continuum*", examines the numerical axis from a psychological perspective. Mach concludes that the introduction of the continuum should not be taken literally; rather, it should be attributed to fictions.

In the chapter "C*ritique of the Temperature Concept*", Mach examines the measurement of temperature, distinguishing between the concepts of thermal state (Wärmezustand) and temperature (Temperatur). The latter refers to thermometer readings; according to Mach, temperature is the numerical value of the thermal state. At the same time, the thermal state is a characteristic of a body—in thermal equilibrium, two bodies have identical thermal states. The term "temperature" is commonly nowadays used in both cases. Nevertheless, I will retain Mach's terminology, as this will allow me to better characterize his views.

By the coordination principle, Mach understands the correspondence between numerical values and the thermometer scale. He draws on the history of thermometry, discussed in the previous chapter of his book, and shows that the development of the temperature scale is linked to conventions.

Mach's approach to the temperature scale can be characterized as conventionalism. This could be accepted — the choice of a measurement unit is indeed the result of convention. At the same time, the concept of a body's thermal state and its transitivity (Mach cites Maxwell and acknowledges his priority to define temperature formally) is linked to experience; conventionalism does not apply to the concept of a thermal state. Mach also notes that temperature is a function of the body state. This, too, is the result of experience and does not follow solely from the agreement on the choice of a temperature scale.

Thus, Mach's position is not reducible to operationalism, which declares the thermal state to be what a thermometer measures. One can say that the development of thermometry has led to the creation of a temperature scale that can be used to characterize the thermal state of a body. However, Mach's concept of a thermal state itself is independent of the adopted temperature scale. In my understanding, the coordination principle in Mach's book is as follows: the numerical value of temperature is related to the historically determined thermometer scale; at the same time, a thermometer has a thermal state, which characterizes the thermal state of other bodies when measuring temperature.

Let us now consider Pierre Duhem's position in the book "*The Aim and Structure of Physical Theory*" [11] (the second part "*The Structure of Physical Theory*"). In the chapter "*Quantity and Quality*", Duhem begins by asserting that theoretical physics must be mathematical physics. To carry out this project, it is necessary to express a physical property by a 'numerical symbol'. Duhem then discusses the measurement of length in the spirit of Helmholtz and extends this to other extensive quantities; this is an example of the measurement of extensive quantities.

The next step is to consider qualities. Duhem criticizes the exclusion of qualities in Cartesian physics and believes that it should be possible to introduce a mathematical scale of quality intensity. Unlike quantity, in this case, everything would be based on the operations of equality, greater than and less than; in other words, the scale orders relationships and the addition plays no special role. Duhem believes that this approach will be sufficient to introduce qualities into theoretical physics.

The problematic of the coordination principle (Duhem does not use this term) is discussed in the chapter "*Experiment in Physics*". Duhem describes the connection between the experiment and the theory of physics and the difference between facts in the measurement and facts at the level of observation. I will cite Duhem's description of the experiment of the physicist Henri Regnault. It also demonstrates the unattainability of the logical positivists' ideal — the reduction of scientific observations to elementary judgments about the world:

> 'Regnault is studying the compressibility of gases; he takes a certain quantity of gas, encloses it in a glass tube, keeps the temperature constant, and measures the pressure the gas supports and the volume it occupies. There you have, it will be said, the minute and exact observation of certain phenomena and certain facts. Certainly, in the hands and under the eyes of Regnault, in the hands and under the eyes of his assistants, concrete facts were produced; was the recording of these facts that Regnault reported his intended contribution to the advancement of physics? No. In a sighting device Regnault saw the image of a certain surface of mercury become level with a certain line; is that what he recorded in the report of his experiments? No, he recorded that the gas occupied a volume having such and such a value. An assistant raised and lowered the lens of a cathetometer until the image of another height of mercury became level with the hairline of the lens; he then observed the disposition of certain lines on the scale and on the vernier of the cathetometer; is that what we find in Regnault's memoir? No, we read there that the pressure supported by the gas had such and such a value. Another assistant saw the thermometer's liquid oscillate between two line-marks; is that what he reported? No, it was recorded that the temperature of the gas had varied between such and such degrees.'

Duhem doesn't use the term metrology, but his description above clearly shows that measurements in physics are impossible without the introduction of appropriate metrological standards and procedures. Duhem describes it as follows:

> 'An experiment in physics is the precise observation of phenomena accompanied by an interpretation of these phenomena; this interpretation substitutes for the concrete data really gathered by observation abstract and symbolic representations which correspond to them by virtue of the theories admitted by the observer.'

In the subsequent discussion, Duhem expands on this point in detail. I will limit myself by the important concept of the ideal instrument:

> 'Hence, when a physicist does an experiment, two very distinct representations of the instrument on which he is working fill his mind: one is the image of the concrete instrument that he

> manipulates in reality; the other is a schematic model of the same instrument, constructed with the aid of symbols supplied by theories; and it is on this ideal and symbolic instrument that he does his reasoning, and it is to it that he applies the laws and formulas of physics.'

The difference between a real and the ideal instrument leads to corrections in the measurements carried out:

> 'If an experiment in physics were merely the observation of a fact, it would be absurd to bring in corrections. ... The logical role of corrections, on the other hand, is very well understood when it is remembered that a physical experiment is not simply the observation of a group of facts but also the translation of these facts into a symbolic language with the aid of rules borrowed from physical theories. Indeed, a result of this is that the physicist constantly compares two instruments, the real one that he manipulates and the ideal, symbolic one on which he reasons.'

Duhem considers measurement errors—he includes scale reading errors, systematic errors due to unaccounted circumstances, and random errors. The last section of this chapter has an expressive title "*Experiment in physics is less certain but more precise and detailed than the non-scientific establishment of a fact*". Duhem compares the results of a physical experiment with an everyday fact: "I saw a white horse on such-and-such a street". He notes that the dependence on theory makes results of an experiment more fragile, but at the same time allows for a better expression of what is happening.

Introduction of physical theory by Duhem makes Mach's consideration more perfect. The transitivity of the thermal state, coupled with the existence of a thermal equation of state, belongs to a simple yet well-defined theory of physics. This assertion, combined with the concepts of volume and pressure from geometry and mechanics, is sufficient to construct a concept model of an ideal instrument to discuss experiments of Regnault. This also forms the basis for a practical temperature scale and is a prerequisite for classical thermodynamics.

Mach and Duhem approach the theory of physics differently. Mach considers a practical temperature scale as part of a historical process. Duhem, on the other hand, discusses Regnault's measurements within the context of an existing theory of physics. Combining these perspectives yields a more balanced approach. Historical development leads to the development of a theory of physics; the practical feasibility of reproducible metrology attests to stability of this theory. This is how the problem of coordination is presented in van Fraassen's book.

A few words about measurement errors — van Fraassen did not pay sufficient attention to this issue. At present, we do not have to axiomatize mathematics as Helmholtz did; this is the work of mathematicians. At the same time, the presence of measurement errors allows us to exclude from experimental science questions of existence in the mathematical sense; for example, whether transcendental numbers exist.

From the perspective of experimental science, mathematics becomes a useful tool for describing the world within the framework of theoretical physics. Helmholtz's statement about irrational numbers above is best viewed within the framework of measurement errors. Mathematicians have the right to place any entities on the numerical axis (for example, infinitesimals in non-standard analysis), but the results of experimental measurement are limited to rational numbers. Moreover, as Duhem correctly notes, an infinite number of theoretical predictions correspond to an experimental result due to measurement error:

> 'A practical fact is not translated therefore by a single theoretical fact but by a kind of bundle including an infinity of different theoretical facts. Each of the mathematical elements brought together in order to constitute one of these facts may vary from one fact to another; but the variation to which it is susceptible cannot exceed a certain limit, namely, the limit of error within which the measurement of this element is blotted. The more perfect the methods of measurement

> are, the closer is the approximation and the narrower the limits but they never become so narrow that they vanish.'

Discussion

A historical perspective allows us to better understand the development of physical theory, temperature measurement instruments, and the standardization of the temperature scale. At the same time, it offers a different perspective on the relationship between experiment and theory. The experimental study of temperature and heat regularities throughout history is still understandable from a modern perspective. Scientists at that time proceeded from entirely different considerations, but this does not prevent discussion of these results at the present time. This demonstrates the relative independence of experiment from theory, although the use of quantitative data from those times may be difficult due to the ambiguities in the measurement standards used.

On the other hand, it is useful to consider the development of the theory of physics during the historical process. For example, Smorodinsky [12] notes:

> ‘The story of how people learned to measure temperature is both interesting and unusual. Thermometers were invented many years before people understood what they were measuring.’

This is a true statement, but despite this, one can also understand the reasoning behind the experiments through the eyes of scientists of that time. Chang's book provides a good discussion of these issues — what scientists thought about temperature and heat at that time. One can see the iterative development: hypotheses were put forward, experiments were conducted, the results were discussed, and new hypotheses emerged. This highlights that social constructivism, when discussing the relationship between theory and experiment, overestimates the influence of society; in the words of Bruno Latour, things strike back. On the other hand, one should not lose sight of the iterative nature of scientific development. One can only say that, ultimately, the development of physics reached a stable state, where new experiments are consistent with existing physical theory.

The question arises whether the current state of physics is the only possible one. I must admit that I have worked for a long time in the field of classical thermodynamics. Therefore, I find it difficult to imagine any other possible development; the arguments in Chang's book that such a thing in principle can be imagined did not convince me.

In my view, the most important aspect of the historical development of thermometry is understanding the difference between temperature and heat. The development of the concepts of temperature and heat was linked to the emergence of two different instruments: the thermometer to measure temperature and the calorimeter to measure heat (more correctly, enthalpy). This difference was later included in physical theory. I find it difficult to imagine an alternative development in which the distinction between temperature and heat would have occurred differently.

Regarding temperature measurement, it's important to note the role of measurement errors. The development of temperature measurement instruments began with relatively large measurement errors, allowing for a relatively simple model of the thermometric substance expansion. For example, the thermal expansion of solids is small, and the change in the thermometer body's volume with increasing temperature can be neglected. The effect of pressure on the volume of a liquid is also small — it can be completely ignored over a small range of atmospheric pressure variations. Thus, the use of liquid thermometers led to stable and reproducible results.

The dependence of gases on pressure could no longer be ignored, but barometers were developed in parallel, making it possible to study the gas equation of state. Due to the relatively large measurement errors in the beginning, there was a belief in the existence of a universal equation of state for all gases; this happens to be important, among other things, for the development of thermodynamics. Increased measurement accuracy revealed deviations in the behavior of real gases from the assumed universal equation of state, which in turn stimulated new experimental work and the development of molecular kinetic theory.

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