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Review

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Article

Bell's Inequality: Theory, Quantum Entanglement, and Experimental Tests

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Abstract

This thesis presents a study of Bell's inequality and its experimental tests, with a focus on their implications for the concepts of locality and realism in quantum mechanics. The work reviews the theoretical framework underlying Bell's inequality, the assumptions of hidden variable theories, and the contrasting predictions of quantum mechanics. It further provides an overview of landmark experimental tests of Bell's inequality, emphasizing their role in shaping the modern understanding of quantum entanglement and nonlocality. The aim of the study is to offer a clear and pedagogical introduction to the subject, bridging the mathematical formalism with experimental verification of one of the most fundamental results in the foundations of quantum theory.

Keywords: Quantum Physics; Quantum Information; History of Science; Bell inequality; Fundamentals of Quantum Mechanics

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1. Abstract

1.1. History

From the twentieth century onward, a new concept began to be heard in the scientific environment; it was Max Planck who introduced the discipline of Quantum Mechanics, which explains the behavior of atoms and subatomic particles and examines the interactions of particles at the subatomic, i.e. microscopic level.

For events that classical mechanics could not fully explain, there was a need for quantum mechanics, which explains events that take place at the microscopic level. Quantum mechanics has so far failed to provide an adequate explanation for many different phenomena. For example, the measurement problem, the particle-wave duality, the wave function collapse when observations are made in the superposition principle of waves, quantum entanglement, and many events such as the so-called 'Spooky Action', also called EPR, do not have any information about exactly how and why they occur.

One of the great things about Quantum Physics during the formation of the science is that every time the founders stopped or could not complete a study, others came and continued from the same point, so that both the old and new quantum theory was close to being complete. The founders of quantum mechanics were Max Planck, Albert Einstein, Max Born, Max Born, Louis De-Broglie, Niels Bohr, P. Dirac, W. Pauli, W. Heisenberg and E. Schrödinger. One of the most important periods of quantum theory was in 1925, known as the birth of the new quantum.

After the birth of the new quantum theory, mathematical representations of the content of quantum theory began to be established. This is called the New Quantum Mechanics era and the scientists who contributed to it are W.Heisenberg, who worked on matrix mechanics and the uncertainty principle and reached serious results, E.schrödinger's wave mechanics, and after he produced the equation to represent the wave equation, Max Born said that the ψ presented by Schrödinger was not physical reality.

But his interpretation of $|\psi|^2$, the absolute value of its square $|\psi|^2$, as 'the probability of the existence of a certain quantum state' has now been seen as an important development, abandoning the

Bohr definition of the electrons in the atom moving in certain orbits and considering Born's idea of the possibility of electrons around the atom.

Starting with the quantum algebra developed by Paul Dirac and de Broglie's question why a particle should not have the character of a wave as a wave has the character of a particle, the electron $e-$ also proved that it has the character of a wave:

$$\lambda = \frac{h}{p} \tag{1}$$

In 1927, Bohr first proposed the definition of complementarity, which was presented by a new perspective using what he believed to be the essential point in understanding quantum mechanics (Wave-Particle duality).

According to the definition of complementarity, even if the wave and particle behavior of an object do not coexist, both are necessary to understand the properties of the object. This means that according to classical physics, if two descriptions of a state cannot coexist, at least one of them must be wrong. But not in quantum mechanics. To explain this, let us imagine that there is a set of experimental evidence, the first set can be interpreted only on the basis of wave properties and another set of evidence can be interpreted only on the basis of particle properties; these two sets of evidence are not contradictory. Because the evidence obtained under different experimental conditions does not converge, it must be viewed through the definition of complementarity.

According to Bohr, Heisenberg, Pauli and Born, a state "x" in an atomic system is undefined before it is measured. in other words , in other words, until the act of observation occurs, a superposition of wave probabilities arises, but when the observation is made, a certain probability appears to the observer and the terms of the remaining superposition disappear. this is called wave collapse. next section 1.2 describes the kopenhagen interpretation in detail.

1.2. Kopenhagen Interpretation

The Copenhagen Interpretation can be defined as a set of different views and principles of quantum mechanics by Niels Bohr, W. Heisenberg and Max Born at the Institute of Theoretical Physics at the University of Copenhagen. It is an interpretation created to discuss answers to questions related to some undiscussed but mysterious and highly important issues in quantum mechanics.

Niels Bohr believed that Quantum Physics was a generalization of classical physics, in the 1930s N.Bohr tried to answer some questions from these questions:

(1) Does quantum mechanics have an objective reality?
(2) How accurate is our knowledge about physical objects?
(3) What is the role of the experiment (does it affect the results of the experiment?)
(4) What is the linking relationship between quantum mechanics and classical mechanics?

Classical physics is deterministic, which means that by determining the initial values of a system, we can learn all the components of a given system that describe both the position and motion of a given system at any given moment as required by a given law. For example, let's take the motion of an object along the x-axis, if this object starts at zero and its velocity is 1 unit per second, its position is easily determinable after two seconds. In addition, it is possible to obtain information about the motion of the object both in the past and in the future by using the information describing the motion of the object even before the observation is made in a way that is independent of the act of observation.

In classical physics, the act of observation (observation instrument) The act of observation, which does not interact with the system, does not change the result of the observation, and classical physics says that it is objective.

Niels Bohr also took a very practical approach in trying to answer the questions addressed (quantum physics is a natural generalization of classical physics). he believed that in this way the classical concepts did not change their meaning but only limited their application. In 1928, N Bohr

introduced the principle of complementarity, the de Borglie wave-particle duality, which states that the electron sometimes behaves like a particle and sometimes like a wave, so in fact sometimes electrons have both types of behavior and it is possible to observe both types at the same time, for example in the double slit experiment. Bohr thought that the complementarity of the kinematic and dynamical properties of quantum particles could only be observed in mutually exclusive experiments.

According to scientists such as W. Heisenberg, Max Born and Dirac Pauli, they thought that the description of a state in an atomic system before measuring it is undefined. it only has the potential for certain values at certain probabilities.

The state of a certain system before observing it is considered to be the sum of these certain probabilities, and when any observation action is performed on this system, the system behaves differently and can be explained in this way "in any system, no information about the system is said until the observation action is performed. its state before the measurement can be considered as the sum of the probabilities of the outcome of the entire observation. As soon as the act of observation is performed, the wave function undergoes a wave collapse and only one outcome emerges from this probability distribution." This is one of the most famous explanations of the kopenhagen interpretation, the measurment problem. The Copenhagen interpretation claimed that the act of observation can change the outcome of the experiment, contrary to classical physics. To explain this, see In Schrödünger's thought experiment, before measuring or observing a licked system, only two possibilities of the expected probabilities of the system in this situation are realizable, and according to this experiment, the object (the cat) can be observed as licked or dead.

$$| \psi >= \frac{1}{\sqrt{2}}|Alive> + \frac{1}{\sqrt{2}}|Dead> \tag{2}$$

Einstein faced the biggest challenge at the solvey meeting in 1930, he tried to disprove Heisenberg's uncertainty principle with different experiments and ideas to the uncertainty principle based on the copenhagen interpretation. he first started with the light box thought experiment "let there be a box full of light. he argued that both the energy and the time of emission of a photon from this box can be determined exactly". in this way he believed that he had found a situation that violated Heisenberg's uncertainty. but later Bohr proved Einstein wrong.

$$(\Delta E \Delta T \geq \frac{h}{4\pi} \tag{3}$$

'Energy-Time version of the uncertainty principle'

In conclusion, the crux of the Copenhagen interpretation can be seen as the measurement problem and wave collapse, and it aims to explain that quantum mechanics violates fundamental principles on which classical physics is based. of these principles, The principle of causality refers to the idea that an event must have a cause that brings about another event or state. The principle of space and time (the principle of locality) is the idea that physical events first affect the nearest point or surroundings where the event occurs. determinism, continuity, it has been argued that the Copenhagen interpretation violates fundamental principles of classical physics.

2. Theory

2.1. EPR Theory

2.1.1. Einstein-Podolsky-Rosen (EPR)

In 1935, five years after Einstein challenged the uncertainty principle on which quantum physics is based with the light box thought experiment, Albert Einstein shared a paper with his Princeton colleagues, Boris Podolsky and Nathan Rosen, titled (Can a Quantum Mechanical Description of Physical Reality be Considered Complete?), known as the EPR paper. In this paper, Einstein and his colleagues argued that quantum mechanics is not a complete theory, or more precisely, that quantum mechanics is not a complete description of physical reality, and that the $|\psi|$ wave function does not provide sufficient information about the system, implying that it has no physical meaning. "For any

theory to be complete, every element of physical reality must have a counterpart in the physical theory," EPR emphasized in the original paper.

EIt was in this paper that Einstein first introduced the definition of 'Reality' and interpreted it from a realistic point of view. From a realist point of view, quantum theory is an incomplete theory, because even if all the information that quantum mechanics can tell us about a system of quantum mechanics is clearly known through the wave function, we cannot determine all the properties of the system. the meaning of the expression reality is that each quantity has a certain value, which is due to our lack of knowledge. since this value is not known before the measurement, and only at the end of the measurement can we learn this value, the result appears to be random.

The EPR paradox assumes two particles in the original thought experiment in a state of quantum entanglement (Quantum Entaglement) (correlated). According to the Copenhagen interpretation, the state of these two particles is indeterminate unless a measurement is made, and it is only when the act of measurement is performed that the state of the particle becomes definite and clear.

In this experiment, it is assumed that the particles are in a special quantum state, i.e. the two particles are entangled. It is a designed thought experiment that examines situations in which the particles are separated and measurements are made on both particles separately. When position and velocity are measured on one of these particles, the position and velocity of the other particle are also determined, whereas according to the Copenhagen interpretation, both quantities (q,p) of both particles must be indeterminate before the act of measurement is performed. But in this case, the fact that after a measurement is made on one of the particles, the same measured quantity is determined for the other particle, moreover, he put forward the EPR paradox, which occurs independently of the distance between the particles. In this system, it was thought that when a measurement is made on one particle, the other particle can instantly change its quanta state. This event coincides with Einestein's theory that there is no speed faster than the speed of light, which is restricted by the theory of relativity, so it creates a situation where relativity is violated, and for this reason he called this effect 'Spooky Action at the distance'.

2.1.2. EPR-Bohm Spin Version

In 1952, David Bohm simplified the EPR thought experiment into an EPR-spin version. In this experiment, the uncharged pi meson particle decays into an electron and a positron.

$$\pi^0 \rightarrow e^- + e^+$$
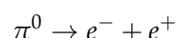
The electron and positron emerging from the decaying pi meson are in a state of entanglement since they originate from the same source. The experiment assumes that the pion is at rest and the particles are exiting in opposite directions. Let us assume that this system is in a singlet state, that is, in a special state such that the sum of the spins of the particles is 0. The total angular momentum of particles (fermions) with spin 1/2 has two possibilities, either spin 0 or spin 1. The reason for the first case (spin0) is that one of the particles has spin +1/2 and the other has spin -1/2, which means that the normal sum rule applies in that case.

Let 'S' represent the Total angular momentum of this system, the result of the total angular momentum of particles with spin 1/2 gives four possibilities. The notation corresponding to these cases is (↑) corresponding to the spin value (+1/2) and (↓) corresponding to the spin value (-1/2), the four probability outcomes are written like this:

(↑↑) , (↓↓) (↑↓) , (↓↑)

J: For the total angular momentum, j1 corresponds to the spin of the first particle and j2 corresponds to the spin of the second particle ($j = \pm 1/2$).

$$J = j1 + j2 \rightarrow J = 1, J = 0.$$

In the case where J = 0, the magnetic quantum number 'M' will also be equal to zero, The state where J = 0 and M = 0 is called the Singlet state, and if one particle is known to have a downward spin, it can be predicted that the other particle will have the same value, but in the opposite direction. The ψ entanglement state describing the state of particles can be expressed in this way:

$$|\psi> = |0,0> = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

In the expression quantum entanglement, it can be thought that it covers all the possibilities that may arise, that is, it shows that the measurements are coherent with each other, not which combination will arise. Assuming that the decaying electron and positron particles are separated by a certain distance (10 light years), if one measures the spin of the electron and finds that it has an upward spin, one can simultaneously say that the spin of the positron will be downward. If spin is measured on both particles separately, regardless of the distance between the two particles, the spins of the two particles will be different.

From a realistic point of view, the scientists who came up with the EPR thought experiment thought that 'the electron and positron had a predetermined state (spin) for each of them since they were first separated, but it was revealed at the moment of observation due to lack of knowledge or incomplete knowledge of the observer'. However, according to the Copenhagen interpretation, as long as no observation is made on a particle, the drums of the system are in superposition and nothing is said about the system, but as soon as the act of observation takes place, the wave function collapses and a certain state of the system emerges, and in this experiment, if the realistic point of view were correct as it claims, there would be the possibility that both particles would be found with down spin or both particles would be found with up spin.

The (hidden variable) mentioned in the EPR paper actually arose from the point where the definition of the physical reality of the wave function was thought to be incomplete, that is, it was thought to come in addition to the wave equation ψ as any constant. This hidden variable theory was shown by John S. Bell in 1964 not to exist in quantum mechanics.

3. Bell and Beyond

3.1. Hidden Variable Theory and Locality

It was John S. Bell in 1964, about 30 years after the EPR paper appeared, who became interested in the hidden variables theory and the locality problem in quantum mechanics. Bell modified the EPR thought experiment and started to investigate how two particles traveling in different directions are connected to each other by measuring spin on different axes and how changes on one axis affect the other. Although it is a hidden variable theory, it actually says that the two particles, which we called coherent in the previous section, have their quantum state determined even before the act of measurement is performed on them. Einstein and EPR generally believed in this principle, which can be explained with a simple thought experiment:

In the macroscopic world, consider two balls of different colors: one black and one white. Assume that one of these balls remains on Earth while the other is sent to another planet, and initially there is no information about which ball is where. In this case, the probability of finding either ball (black or white) at a given location is equal, i.e. 50%. If, at the moment of observation on Earth, the ball is found to be white, then it can immediately be concluded that the ball on the other planet must be black.

But when one enters the quantum micro world, two entangled quantum particles (electron and positron) will be in superposition and the wave function will be in this form as discussed earlier ;

$$|\psi> = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

In this case, let the measurement be applied as a specific quantum property (spin) of these two particles, and the spin of both particles before the observation is made has both spin states simultaneously, which was the claim of the Copenhagen Interpretation (Copenhagen Interpretation). Since the particles are in the singlet state when the up spin is found when the observation is made, the spin of the other must be down to make its total angular momentum zero. Therefore, if the hidden variable theory is true and in reality there are no hidden variables in the universe, this would mean that the uncertainty and superposition statement before the observation would be false and that both particles have such a hidden property that even before the observation the spin of both particles should be in the set state, but that value cannot be known due to lack of information.

It was also said earlier that it is independent of the distance between the particles, which would be contradictory to the principle of locality because the speed of light limits this situation and when observations are made on the particles, that information would be transferred to the other particle simultaneously or instantaneously at a speed greater than the speed of light. Bell said that "If the hidden variable theory exists and is local, it is incompatible with quantum mechanics. if it is compatible with quantum mechanics, it will not be local. " he said.

3.2. Bell Inequality

In 1964, John S. Bell obtained an inequality that provided the hidden variable theory. Bell used a method of logic called "Reductio ad absurdum", which is the process of accepting a claim as true and concluding that the claim is false. In this case (hidden variable theory), assuming that it was true, the result of the operations was to prove that the theorem was false.

EPR-Bohm version of the thought experiment $\frac{1}{2}$ In the case of a-spin entanglement singlet, two particles are generated and separated in opposite directions. Let t=0 be realized by measuring the projections of the spins of the particles, with A and B (measuring devices) at equal distances from both left and right.

$$|0,0> = \frac{1}{\sqrt{2}}(|\frac{1}{2}, -\frac{1}{2} > - | -\frac{1}{2}, \frac{1}{2} >)$$

"entangled spin represents the quantum state of $\frac{1}{2}$ particles."

Bell's method was not to perform measurements only on the x-z axes, but also to perform measurements at intermediate angles and calculate the correlation between the results to observe how the measurements are correlated and related to each other.

3.2.1. Bell Inequality Derivation

For the calculation of the correlation of the measurement results on the thought experiment it is necessary to introduce a few mathematical expressions; $\vec{nA}$ ve $\vec{nB}$ Let A and B be the measurement results (spin projection axes) of the measurement devices,

$$\vec{nA} = \hat{z}$$

$$\vec{nB} = \sin\theta\hat{x} + \cos\theta\hat{z}$$

spin up and down matrices when $\theta = 0$;

$$|+> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|-> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Let P(+,+) , P(+,-) , P(-,+) , P(-,+) and P(-,-) be the probability of each measurement between the outcomes on the axes, and the correlation operator $\hat{C}$,

$$\hat{C} = (\hat{\sigma} \cdot \vec{nA}) \otimes (\hat{\sigma} \cdot \vec{nB})$$

To find the correlation in this case, the expected value of the singlet state of C is taken;

$$C = < 0,0|\hat{C}|0,0 >$$

$$C = \langle 0,0| \, \hat{\sigma}_z \otimes (\sin\theta \, \hat{x} + \cos\theta \, \hat{z}) \, |0,0\rangle$$

Using Pauli spin matrices in the x-z plane;

$$\sigma x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The calculated result of the correlation predicted by quantum mechanics is $C = -\cos\theta$, which is the correlation or contact value between the measurement axes. Here, if $\theta = 0°$ between the measurement axes of devices A and B, then if an up spin is measured along the axis of device A, the correlation is 100% (i.e., $E(\theta) = -\cos 0 = -1$), which means that a down spin will be found at B.

Let's see if the result depends on this hidden variable when this experiment is restricted to a special hidden variable as Bell did; Here, if we choose θ as the hidden variable, which is defined as the angle between the axes where the particles are produced, α is the axes of measurement and the correlation is C;

$$C = \frac{1}{2\pi} \int_0^{2\pi} < \psi|\hat{C}|\psi > d\theta = -\frac{\cos\alpha}{2}$$

correlation result is independent of the selected latent variable. Bell-CHSH inequality (for hidden variable theory)

for this case two different axes are defined at points A and B, not one measurement axis; $(\vec{nA}, \vec{nb}, \vec{na'}, \vec{nb'})$ and let the spin measurement results A(λ, na) and B(λ, nb) be defined with λ being any hidden variable.

$$S = E(A, B) + E(A, B') + E(A', B') - E(A', B)$$

$$S \leq 2$$

The above inequality must also be satisfied for any quantum mechanical hidden variable experiment, but the Bell experiments "Bell test" was first performed in 1982 by the French physicist Alain Aspect in order to prove the prediction of the Bell inequality, and he came to these conclusions by performing the experiment based on the Bell inequality:

- It has been shown that quantum mechanics is incompatible with the predictions of classical mechanics, quantum mechanics violates the bell inequality, and hidden variable theory is incompatible with quantum physics.

A version of the Bell inequality was developed in the late 1960s by John Clauser and his advisor Shimony and together with PhD students developed the CHSH inequality, named after them. It was also in the 1970s that experimental testing of that inequality was first encountered with results.

In J. Clauser's experiment at Berkeley, two entangled photons were used instead of the decaying electron and positron described in this study, and the result obtained was consistent with the predictions of quantum mechanics. When Holt and Pipkin's experiment at Harvard yielded results

inconsistent with quantum mechanics, Clauser repeated their experiment and concluded that quantum mechanics violated Bell's inequality. However, no experiment without loopholes had been conducted until then.

4. Experiments

4.1. Generating Entangled Particles

There are many methods for producing entangled particles. In this study, the zero meson used in the EPR thought experiment decays into two particles, an electron and a positron, and it is assumed that the particles resulting from this decay share quantum properties. However, these processes may not be as straightforward as the thought experiment, so experiments using photons have been preferred.

There are also many different ways to produce entangled photons. Here, we will consider the approach followed in Aspect's experiment; In atomic physics, a cascade transition (or two-photon transition) can be used to excite electrons in a calcium atom with a laser. By following this method, two entangled photons can be obtained.

Here (Stimulated emission), when a photon of a specific energy and frequency is sent to an excited atom, it emits both the incident photon and a second photon with the same properties as the first incident photon, as it returns to its ground state. (The laser operating principle is also stimulated emission.)

The second method (spontaneous parametric downconversion) uses a nonlinear crystal, typically a beta barium borate crystal. When a high-energy photon hits this crystal, based on the logic of the polarizer, if the crystal has a vertical optical axis, it splits into two separate horizontal photons. Using the law of conservation of energy, it is clear that the energies of the outgoing photons are equal to the energy of the incoming photon. By placing two crystals with both vertical and horizontal optical axes side by side, the probability of the emitted photons being either vertical or horizontal arises, and they will be in a superposition state. This way, two entangled photons are produced.

$$|H>|V>+|V>|H>$$

The possibility that photons emitted from the BBO crystal may be either horizontally or vertically polarized can be represented by superposition.

4.2. Alain Aspect's Experiment

In 1982, French physicist A. Aspect conducted an experiment that yielded more precise results. He also used a strategy that had not been used in previous experiments; Again, in this experiment, he used two entangled photons with spin-1 and a polarizer that would allow the spin of the photon to be measured, and he adjusted the polarizer's response to be independent of the state on the other side and developed a method of changing the measurement axes of the polarizer during the experiment (when the photons were in the air) at different angles during the experiment. He considered that the speed of changing the polarizer would be less than the time between the two polarizers, which would help close the gaps, as Bell had suggested. The 'space-like interval' known in special relativity was also realized in this experiment, meaning that an event occurring at two different locations happened at the same time. In this case, both locality and closing the detector gaps were achieved, and quantum mechanics proved that Bell's inequality was violated.

In 2015, experiments were conducted that closed all loopholes in Bell inequality experiments, and in 2022, J. Clauser, A. Aspect, and A. Zeilinger were awarded the Nobel Prize for their experiments and work demonstrating the violation of Bell inequality in quantum mechanics.

5. Conclusions

5.1. Summary

Beginning in the 1920s and continuing through the 1930s with the EPR paradox and the Einstein–Bohr debates, the discussion seemed to have ended in 1964 with Bell's inequality. However, years of effort were required to determine whose view was correct and to gain greater certainty. This point was reached through the contributions of hundreds of scientists and researchers. The Copenhagen interpretation, as explained earlier, is now considered the strongest interpretation of quantum mechanics, in which probability forms the basis of the theory. According to this interpretation, the quantum state of particles only emerges upon observation; it is not meaningful to speak of a definite state before measurement, but rather a superposition of possibilities.

When an observation is made on one particle, the state of the other particle is also determined. Hidden-variable theories (HVTs) have been shown to be incompatible with quantum mechanics. In Bell's experiments, what Einstein called "spooky action at a distance" was demonstrated to hold true, showing that entangled particles instantly share the effect of observation, regardless of the distance between them. These experiments have deepened our understanding of the fundamental principles of quantum physics and laid the groundwork for major applications in quantum computing and quantum communication.

5.2. Future Work

Looking ahead, the implications of Bell's inequality extend beyond foundational physics into practical applications in **quantum information science**. The phenomenon of entanglement, once a philosophical debate, is now a resource for technologies such as quantum key distribution, quantum teleportation, and error-resistant quantum computation. Future research can explore:

- Developing more robust experimental setups to close remaining loopholes in Bell tests, ensuring stronger security guarantees in quantum cryptography.
- Utilizing multi-particle entanglement to enhance the scalability and stability of quantum computers.
- Investigating the intersection of Bell's nonlocality with quantum error correction codes to achieve fault-tolerant quantum computation.
- Exploring new protocols in quantum communication that leverage Bell-type correlations for ultra-secure global networks such as the BB84 protocol.

Thus, what began as a challenge to the foundations of quantum mechanics and quantum technologies has evolved into one of the most promising frontiers in modern quantum technology.

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