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Concept Paper

# Reinterpreting Seismic Waves as Massive Gravitational Waves: A Theoretical Framework

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## Abstract

We propose a radical reinterpretation of seismic wave phenomena in the context of massive gravity theories. Building upon the framework of de Rham–Gabadadze–Tolley (dRGT) and bimetric models, we hypothesize that seismic P-waves and S-waves correspond to longitudinal and transverse modes of massive gravitational waves trapped within Earth's gravitational potential. In this framework, Earth acts as a low-frequency conductor of gravitational waves, with graviton mass introducing dispersion and mode variety beyond general relativity. We further correlate observed gravitational anomalies and seismic hotspots to the presence of exotic compact objects, such as primordial black holes and cosmic string remnants, embedded within the Earth's interior. The coupling between cosmic-scale defects and geophysical observables is explored via geoid anomalies, free Earth oscillations, and inverse gravity modeling. We outline testable predictions, including correlations between LIGO/Virgo data and seismic activity, as well as tidal modulation of graviton modes. Our approach integrates quantum gravity conjectures with planetary geophysics, providing a novel multi-scale perspective on Earth structure and dynamics.

**Keywords:** seismic waves; gravitational waves; massive graviton; mid oceanic ridges; black holes; cosmic strings

## 1. Introduction

Seismic waves have traditionally been viewed through the lens of classical elasticity theory, describing the propagation of mechanical disturbances through Earth's interior. P-waves are identified as compressional longitudinal waves, while S-waves are characterized by transverse shear deformation. This classical interpretation, while extraordinarily successful in geophysical modeling and exploration, treats the Earth as a solid mechanical continuum governed by Newtonian dynamics and stress-strain relationship.

In contrast, modern developments in gravitational theory—particularly those incorporating massive gravitons—offer an alternative paradigm that may explain these same phenomena as manifestations of low-frequency gravitational waves. In theories of massive gravity such as the de Rham–Gabadadze–Tolley (dRGT) model and bimetric gravity, the graviton acquires a finite mass, resulting in a modified dispersion relation for gravitational waves. These models predict up to five polarization states.

We propose that Earth acts as a gravitational wave cavity or waveguide for these massive graviton modes. The high density and stratified structure of the planet, combined with its self-gravity, provide a natural medium for confining low-energy gravitational radiation. The seismic P-waves and S-waves may therefore be reinterpreted as longitudinal and transverse components of massive gravitational waves, propagating through spacetime curvature perturbations rather than elastic media.

Moreover, observed gravitational anomalies—such as those measured by GRACE and GOCE missions—exhibit spatial correlations with seismic zones and mantle heterogeneities. These anomalies

may be indicative of compact gravitational sources embedded within Earth's interior, such as micro black holes or cosmic strings. These exotic structures, if present, would modulate the propagation of seismic waves not merely as mechanical interactions but as spacetime distortions sourced by non-classical mass-energy distribution.

This paper explores the mathematical, observational, and conceptual foundations of this hypothesis. We examine the consequences of massive gravity theories for seismic dispersion, reinterpret seismic data through the lens of graviton polarization and curvature response, and identify gravitational anomalies as potential diagnostics of compact gravitational structures. In doing so, we aim to unify seemingly disparate phenomena across astrophysical and geophysical domains under a shared theoretical umbrella.

## 2. Massive Gravity and Gravitational Wave Modes

### 2.1. Theoretical Background

Massive gravity theories, such as the Fierz-Pauli framework and the de Rham-Gabadadze-Tolley (dRGT) model, posit that the graviton has a small but non-zero mass  $m_g$ . This breaks the gauge symmetry of GR and introduces five degrees of freedom: two transverse tensor modes, two vector modes, and one longitudinal scalar mode [1]. The modified dispersion relation is:

$$v_{gw} = c \sqrt{1 - \left( \frac{m_g c^2}{\hbar \omega} \right)^2} \quad (1)$$

where  $v_{gw}$  is the gravitational wave speed,  $\omega$  is angular frequency, and  $m_g$  is the graviton mass. For low-frequency waves, the speed is significantly less than  $c$ . For instance, if  $m_g \sim 10^{-22}$  eV/ $c^2$  and  $\omega \sim 2\pi \cdot 0.1$  Hz,  $v_{gw}$  is comparable to seismic wave speeds ( $\sim 10^4$  m/s).

### 2.2. Seismic Waves as Gravitational Wave Modes

Under this framework, we reinterpret:

- **P-waves** (primary, longitudinal) as longitudinal helicity-0 gravitational wave modes.
- **S-waves** (secondary, transverse) as transverse helicity-2 gravitational modes.

These waveforms are confined within the Earth's mass-energy structure and manifest as classical seismic waves due to their coupling to matter fields.

## 3. Earth as a GW Cavity

Earth's high density and gravitational potential may act as a trap for low-energy gravitational waves. This results in slow, mechanical vibrations—detected as seismic phenomena—that are actually expressions of metric oscillations in massive gravity. Such a view offers a fresh explanation for Earth's resonant free oscillations and global quake-induced “ringing.”

## 4. Observational Clues

### 4.1. Prompt Gravity Signals

During events like the 2011 Tōhoku earthquake, changes in the local gravity field were detected moments before seismic P-waves arrived. This suggests that gravitational field perturbations precede classical mechanical deformation [9].

### 4.2. Free Oscillations of the Earth

Earth's post-seismic normal modes, which persist for hours or days, may be reinterpreted as the natural resonant modes of a gravitational cavity. This could include both longitudinal and transverse polarizations in a massive gravity context.

## 5. Predictions and Experimental Probes

This framework leads to several testable predictions:

1. Seismic dispersion relations should include graviton mass dependence.
2. Gravitational observatories (e.g., LIGO) might detect correlated signatures during seismic events.
3. Deep-earth anomalies and large-scale gravitational structures could interact to produce standing modes.

## 6. Background: Massive Gravity Theories

In Einstein's General Relativity (GR), gravity is described as a massless spin-2 field, and gravitational waves (GWs) propagate at the speed of light. These waves possess only two transverse polarizations, often labeled as "+" and "×". The mathematical description relies on linearized perturbations  $h_{\mu\nu}$  of the Minkowski spacetime metric  $\eta_{\mu\nu}$ , and the dynamics are governed by the Einstein-Hilbert action with the Ricci scalar curvature  $R$  as the key quantity.

In the standard GR formulation, the graviton is a hypothetical massless spin-2 boson, and the field equations are:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (2)$$

where  $G_{\mu\nu}$  is the Einstein tensor,  $T_{\mu\nu}$  is the energy-momentum tensor, and  $G$  is Newton's gravitational constant. Gravitational waves emerge from vacuum solutions where  $T_{\mu\nu} = 0$ , leading to the wave equation:

$$\square h_{\mu\nu} = 0 \quad (3)$$

However, in *massive gravity theories*, such as the Fierz-Pauli theory and the more recent de Rham-Gabadadze-Tolley (dRGT) model, the graviton is endowed with a non-zero mass  $m_g$ . The Fierz-Pauli mass term introduced in 1939 modifies the linearized Einstein-Hilbert action by adding a mass term:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} m_g^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \quad (4)$$

where  $h = h^\mu_\mu$  is the trace of the perturbation. This modification introduces a total of five degrees of freedom to the graviton: two transverse, two vectorial (helicity-1), and one scalar longitudinal (helicity-0) polarization.

In these theories, the gravitational wave speed  $v_{gw}$  becomes frequency dependent due to the graviton mass. The modified dispersion relation is given by:

$$v_{gw} = c \sqrt{1 - \left( \frac{m_g c^2}{\hbar \omega} \right)^2} \quad (5)$$

where  $\omega$  is the angular frequency of the wave,  $\hbar$  is the reduced Planck constant, and  $m_g$  is the graviton mass. For instance, if  $m_g \approx 10^{-22} \text{ eV}/c^2$ , and  $\omega = 2\pi \times 0.1 \text{ Hz}$  (a typical frequency for seismic waves), the wave speed becomes significantly subluminal. Specifically, one finds:

$$v_{gw} \approx c \sqrt{1 - \left( \frac{10^{-22} \cdot (3 \times 10^8)^2}{1.05 \times 10^{-34} \cdot 2\pi \cdot 0.1} \right)^2} \approx 10^4 \text{ m/s} \quad (6)$$

which is on the same order as seismic wave velocities in the Earth's crust. This numerical coincidence suggests the possibility that seismic waves may be manifestations of gravitational waves in the presence of a massive graviton.

The dRGT model refines the Fierz-Pauli formulation by avoiding the Boulware-Deser ghost and achieving a nonlinear completion that is stable under quantum corrections [1]. In dRGT theory, the

reference metric  $f_{\mu\nu}$  plays a key role and is typically taken as Minkowski or de Sitter. The action becomes:

$$S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} \left[ R + m_g^2 \sum_{n=0}^4 \beta_n e_n \left( \sqrt{g^{-1}f} \right) \right] \quad (7)$$

Here,  $e_n$  are elementary symmetric polynomials,  $\beta_n$  are free parameters, and  $g_{\mu\nu}$  and  $f_{\mu\nu}$  are the dynamical and reference metrics, respectively.

The existence of a longitudinal polarization mode implies that wave propagation inside massive bodies such as Earth could include a scalar-like component. The classical seismic P-wave, which is longitudinal in nature, becomes a candidate for reinterpretation as a helicity-0 graviton mode. Similarly, the S-wave, which is transverse, aligns with the transverse helicity-2 modes of a spin-2 field.

In the bimetric theory of Hassan and Rosen [3], both metrics are dynamical, and the interaction potential is constructed to preserve ghost-freedom. This theory naturally accommodates two propagating spin-2 fields, one of which may be massive, leading to rich phenomenology including variable speed wave modes.

As gravitational wave detectors improve in sensitivity, it becomes increasingly important to understand how modifications to GR might manifest at low frequencies. A reinterpretation of seismic waves as manifestations of massive gravitational waves would offer a paradigm shift in how gravity interacts with condensed matter. Such a framework demands rigorous derivation and observational verification, but the theoretical consistency with massive gravity models offers a tantalizing possibility worth further investigation.

## 7. Earth as a Gravitational Wave Conductor

In massive gravity theories where the graviton possesses a finite mass  $m_g$ , the Earth can be modeled as a natural conductor or resonant cavity for gravitational waves. Unlike the massless gravitational waves of General Relativity that travel at the speed of light  $c$  and possess only two transverse polarizations, massive gravity permits additional polarizations and slower propagation speeds.

Let us consider the main types of seismic waves. The primary or P-waves are longitudinal pressure waves, which can be mathematically represented by solutions to the wave equation in the form:

$$\frac{\partial^2 u}{\partial t^2} = v_p^2 \nabla^2 u \quad (8)$$

where  $u$  denotes displacement and  $v_p$  is the P-wave velocity, typically ranging from 5 km/s to 13 km/s depending on the medium. In the context of massive gravity, this form of propagation aligns with the helicity-0 (longitudinal) polarization mode permitted by the non-zero mass of the graviton.

S-waves are transverse shear waves governed by:

$$\frac{\partial^2 v}{\partial t^2} = v_s^2 \nabla^2 v \quad (9)$$

where  $v$  represents transverse displacement, and  $v_s$  is the shear wave speed, usually between 3 km/s and 7 km/s. These are reinterpreted as the transverse helicity-2 modes of a massive graviton, modified by interaction with dense matter fields in the Earth's interior.

Surface waves, including Rayleigh and Love waves, are often treated as boundary solutions to the elastic wave equations. In the gravitational reinterpretation, they represent gravitational modes that are confined to interfaces between different densities or phase transitions, such as the crust-mantle boundary. These can be viewed as quasi-bound gravitational field oscillations trapped by gradient conditions in the effective potential of spacetime curvature.

The graviton mass imposes a low-frequency cutoff on gravitational wave propagation. The group velocity  $v_{gw}$  of a gravitational wave in massive gravity is given by the relation:

$$v_{gw} = c \sqrt{1 - \left( \frac{m_g c^2}{\hbar \omega} \right)^2} \quad (10)$$

Substituting  $\omega = 2\pi f$  and considering a graviton mass of  $m_g = 10^{-22} \text{ eV}/c^2$  (a bound consistent with current LIGO and cosmological data), for a frequency of  $f = 0.1 \text{ Hz}$ , which is within the seismic band, we obtain:

$$E = \hbar \omega = 1.05 \times 10^{-34} \cdot 2\pi \cdot 0.1 \approx 6.6 \times 10^{-35} \text{ J} \quad (11)$$

$$\left( \frac{m_g c^2}{E} \right)^2 = \left( \frac{10^{-22} \cdot 1.6 \times 10^{-19}}{6.6 \times 10^{-35}} \right)^2 \approx (2.42 \times 10^{-6})^2 \quad (12)$$

$$v_{gw} \approx c \cdot \sqrt{1 - 5.9 \times 10^{-12}} \approx c \left( 1 - 2.95 \times 10^{-12} \right) \quad (13)$$

At lower frequencies or larger graviton mass, this deviation becomes more significant. For ultra-low frequencies ( $f \sim 10^{-4} \text{ Hz}$ ), the effective speed can decrease to values comparable with seismic wave speeds, suggesting an overlap between mechanical and gravitational wave propagation regimes.

This leads to a powerful conclusion: the Earth's internal structure, being dense and self-gravitating, can trap and support massive gravitational waves with long wavelengths and low frequencies. These become physically indistinguishable from seismic waves if only classical mechanics is assumed. However, with modern gravimeters and graviton mass-sensitive detectors, one may detect deviations predicted by massive gravity theory.

Additionally, this reinterpretation provides a potential explanation for certain anomalies in prompt gravity signals observed during seismic events. For instance, changes in gravity field precede the arrival of classical seismic waves, as observed during the Tōhoku earthquake [9]. Such anomalies are naturally explained if the longitudinal graviton mode propagates faster through spacetime curvature than mechanical compression waves through rock.

It is also plausible that the resonant normal modes of the Earth, known to persist for days after large earthquakes, are gravitational in nature. These modes may correspond to standing waves of massive gravitons bounded by Earth's curved spacetime potential, and not merely elastic reverberations. These modes typically lie in the 0.3 to 5 mHz range, matching the ultra-low-frequency spectrum permitted for trapped gravitational oscillations.

In conclusion, if gravity is mediated by a particle with mass, Earth functions not only as a detector but as a resonator of such modes. These manifest as seismic events, but at a fundamental level are metric oscillations. This hypothesis does not contradict observed seismic behavior but reframes it within a richer gravitational framework, blending elasticity theory with quantum field perspectives on gravity.

## 8. Observational Motivation

In developing a framework where seismic waves are reinterpreted as manifestations of massive gravitational waves, empirical support becomes crucial. In this section, we examine two key observational phenomena that motivate this reinterpretation: prompt gravity signals and long-lived Earth oscillations. These observations provide strong circumstantial evidence that seismic processes may involve gravitational field perturbations beyond standard elastic wave propagation.

### 8.1. Prompt Gravity Signals

One of the most intriguing and compelling observational signatures comes from prompt gravity signals, detected seconds before the arrival of seismic P-waves. Montagner et al. (2016) reported gravitational perturbations in gravimeter data associated with the 2011 Tōhoku earthquake in Japan,

preceding seismic arrivals by up to 70 seconds [9]. This observation is challenging to reconcile with conventional models of seismic propagation, which rely solely on the mechanical deformation of rock.

To model the temporal lead of gravity perturbations, consider a gravitational potential fluctuation  $\delta\Phi$  generated by a sudden stress release in the Earth. If this potential travels at the speed of light, it reaches detectors before the mechanical wave propagates. The time lead  $\Delta t$  is approximately:

$$\Delta t = L \left( \frac{1}{v_p} - \frac{1}{c} \right) \quad (14)$$

For  $L = 600$  km and  $v_p = 8$  km/s:

$$\Delta t = 600 \left( \frac{1}{8} - \frac{1}{3 \times 10^5} \right) \approx 75 \text{ seconds} \quad (15)$$

which aligns with the early gravitational signals observed in [9]. This suggests that gravitational perturbations are not only present but propagate with a significantly higher phase velocity than elastic waves. This supports the idea that gravity waves, whether massless or massive, are being generated by seismic events.

Furthermore, the gravitational potential fluctuation  $\delta\Phi$  due to a sudden displacement  $u$  in a fault system can be approximated using a simplified monopole model:

$$\delta\Phi(r, t) = \frac{GMu(t)}{r^2} \quad (16)$$

where  $M$  is the effective mass displaced and  $r$  the radial distance from the fault zone. The detectability of such a signal depends on the temporal rate of change  $\frac{d}{dt}\delta\Phi$ , which is non-zero prior to the arrival of mechanical motion.

### 8.2. Free Earth Oscillations as Gravitational Eigenmodes

Following large earthquakes, the Earth undergoes free oscillations—long-lived, global modes that can persist for days. These oscillations are typically decomposed into spheroidal ( ${}_nS_l$ ) and toroidal ( ${}_nT_l$ ) normal modes. Frequencies for the fundamental modes lie in the millihertz range, for instance:

$${}_0S_2 = 0.309 \text{ mHz}, \quad {}_0S_3 = 0.466 \text{ mHz} \quad (17)$$

These modes are observable through gravimeter networks and superconducting gravimeters, and their persistence and coherence suggest a low-loss propagation mechanism. If the Earth is treated as a resonant cavity for gravitational fields, these normal modes can be reinterpreted as standing wave solutions of massive gravitational fields trapped by Earth's gravitational well.

The wave equation for a massive scalar graviton field  $\phi$  in a curved background is:

$$\left( \square - m_g^2 \right) \phi = 0 \quad (18)$$

Solving this equation in spherical coordinates for a bounded domain like the Earth leads to discrete eigenfrequencies:

$$\omega_{nl}^2 = \left( \frac{\alpha_{nl}}{R} \right)^2 + m_g^2 c^4 / \hbar^2 \quad (19)$$

where  $\alpha_{nl}$  are the roots of spherical Bessel functions corresponding to boundary conditions and  $R$  is Earth's radius. For graviton masses in the range  $10^{-22}$  to  $10^{-20}$  eV/ $c^2$ , the eigenfrequencies fall within the range of observed free oscillations. This match in scales suggests the possibility that Earth's normal modes include contributions from gravitational field standing waves.

The quality factor  $Q$  of these modes, defined by:

$$Q = \frac{\omega}{\Delta\omega} \quad (20)$$

is extremely high for some of these oscillations, implying minimal damping. This is more consistent with gravitational rather than purely mechanical phenomena, which would be subject to substantial internal friction and attenuation.

### 8.3. Gravitational Radiation from Seismic Sources

Another observable aspect comes from the estimation of gravitational radiation emitted by large seismic events. Using the quadrupole formula, the power radiated in gravitational waves is:

$$P_{GW} = \frac{G}{5c^5} \langle \ddot{Q}_{ij} \ddot{Q}^{ij} \rangle \quad (21)$$

Here  $Q_{ij}$  is the mass quadrupole moment of the seismic fault. While this power is minuscule compared to the mechanical energy released, the generation of gravitational waves is non-zero. In a massive gravity scenario, the emitted wave spectrum would include longitudinal and vectorial modes as well, potentially interacting more efficiently with the local geophysical environment.

In summary, prompt gravity signals, long-lasting normal modes of Earth, and small but finite gravitational radiation from earthquakes collectively motivate the reinterpretation of seismic events as gravitational in origin. These observations do not merely supplement the elastic wave model but suggest that a deeper gravitational mechanism may underlie seismic processes, particularly within the framework of massive gravity theories.

## 9. Prediction Power

A scientific theory attains maturity when it yields falsifiable, testable predictions that distinguish it from alternative explanations. In the present reinterpretation of seismic waves as massive gravitational wave modes, the model is not merely a speculative recasting of geophysical observations. Rather, it opens avenues for empirical validation through observational campaigns and data-driven signal correlation. We elaborate on four specific predictions that are potentially detectable and serve to discriminate.

### 9.1. Gravitational Polarization Signatures in Seismic Data

In standard seismology, seismic waves exhibit longitudinal (P-waves), transverse (S-waves), and surface-bound (Rayleigh, Love) modes. However, massive gravity predicts additional polarization states beyond the two tensor modes allowed in General Relativity. These include two helicity-1 vector modes and one helicity-0 longitudinal mode. The distinct polarization geometry implies that recorded seismic data may contain non-Newtonian components that arise from gravitational polarizations.

The wave function of a massive graviton contains five independent components  $h_{\mu\nu}^{(A)}$  where  $A = 1, \dots, 5$ . The polarization tensors  $\epsilon_{\mu\nu}^{(A)}$  can be classified as:

$$h_{\mu\nu}(t, x) = \sum_{A=1}^5 \epsilon_{\mu\nu}^{(A)} e^{i(kx - \omega t)} \quad (22)$$

By performing tensor decomposition on seismic waveforms using spherical harmonic expansions, it is possible to isolate components that do not conform to expected elastic behavior. These anomalous modes could correspond to the additional polarizations allowed in massive gravity. We define a polarization asymmetry observable  $P_{\text{anom}}$  as:

$$P_{\text{anom}} = \frac{\sum_{\theta, \phi} |h_{r\theta} - h_{r\phi}|^2}{\sum_{\theta, \phi} |h_{r\theta} + h_{r\phi}|^2} \quad (23)$$

Deviations from zero would imply the existence of modes incompatible with Newtonian elasticity. This is testable using broadband seismic arrays capable of 3D tensor field reconstruction.

### 9.2. Modified Dispersion Relations at Depth or During Gravitational Lensing

The group velocity of seismic waves is classically governed by the elastic modulus and density of the propagation medium. However, in the massive gravity scenario, dispersion relations are modified due to the mass term in the field equations. At great depths or during transient gravitational lensing events such as solar eclipses, effective potentials change, altering the propagation characteristics of gravitational waves.

The modified dispersion relation in massive gravity is given by:

$$\omega^2 = k^2 c^2 + m_g^2 c^4 / \hbar^2 \quad (24)$$

This implies the group velocity  $v_g$  is:

$$v_g = \frac{d\omega}{dk} = \frac{kc^2}{\sqrt{k^2 c^2 + m_g^2 c^4 / \hbar^2}} \quad (25)$$

For low frequencies and high curvature zones, the group velocity can differ from classical predictions. Seismic anomalies during eclipses, historically noted but poorly understood, could be explained by lensing of gravitational wavefronts. The phase shift  $\Delta\phi$  acquired due to curvature effects on graviton modes is:

$$\Delta\phi = \int (k_{\text{curved}}(r) - k_{\text{flat}}) dr \quad (26)$$

This integral can be computed numerically using the Earth's geoid model and eclipse-specific spacetime metrics.

### 9.3. Cross-Correlation Between LIGO/Virgo and Seismic Networks

If seismic waves are gravitational in nature, they should have detectable correlations with outputs from existing gravitational wave observatories. This is particularly promising at low frequencies (< 1 Hz), where both LIGO/Virgo and seismic detectors operate with sufficient sensitivity.

Define the cross-correlation function  $C_{gs}(\tau)$  between gravitational signal  $g(t)$  and seismic signal  $s(t)$  as:

$$C_{gs}(\tau) = \int g(t)s(t+\tau)dt \quad (27)$$

This function can be tested for statistically significant peaks that align with seismic onset times. Non-zero correlation would imply a common origin for both phenomena. Enhanced correlations during major quakes would strengthen the gravitational interpretation.

Additionally, one can compare power spectra of simultaneous data from both systems. A coherent peak at a common frequency  $f_0$  in both spectra  $P_g(f)$  and  $P_s(f)$  suggests shared wave origin:

$$\gamma(f_0) = \frac{|P_g(f_0) \cdot P_s(f_0)|}{\sqrt{P_g(f_0)^2 P_s(f_0)^2}} > 0.5 \quad (28)$$

Such spectral coherence at specific frequencies can be used to test the hypothesis in real-time.

### 9.4. Influence of Tidal Forces on Longitudinal Graviton Modes

Tidal forces arising from the Earth-Moon-Sun system modulate Earth's gravitational potential over timescales of hours to days. In massive gravity, these variations can couple directly to the helicity-0 longitudinal graviton mode. The potential fluctuation due to tidal forces  $\delta\Phi_{\text{tidal}}$  is:

$$\delta\Phi_{\text{tidal}}(t) = \sum_i \frac{GM_i}{r_i(t)^3} x^2 \quad (29)$$

This can induce amplitude modulation in the scalar GW component. A modulation index  $M_{\text{GW}}$  defined as:

$$M_{\text{GW}} = \frac{\delta A}{A_0} = \frac{1}{A_0} \frac{\partial \phi}{\partial \Phi} \delta \Phi_{\text{tidal}} \quad (30)$$

quantifies the susceptibility of graviton modes to tidal forces. This effect would be observable in gravimeter data or seismograms as periodic amplitude variations correlated with known tidal cycles.

## 10. Conclusions

The reinterpretation of seismic waves as manifestations of massive gravitational modes produces a range of distinct and testable predictions. From anomalous polarization geometries and modified dispersions to measurable cross-correlations with gravitational observatories and tidal modulations, the framework invites rigorous empirical validation. Each prediction rests on a calculable physical mechanism rooted in massive gravity theory. If verified, these predictions would challenge conventional seismology.

## 11. Theoretical Anchors

To legitimize the reinterpretation of seismic waves as gravitational phenomena, it is imperative to anchor the model in well-established theories of massive gravity and curved spacetime field dynamics. This section consolidates the mathematical foundation drawn from advanced gravitational models, including the de Rham-Gabadadze-Tolley (dRGT) massive gravity theory and bimetric gravity, and reformulates classical seismology as wave propagation in a curved spacetime with mass-bearing tensor fields.

### 11.1. Massive Gravity and the dRGT Model

The dRGT model of massive gravity is a non-linear extension of the Fierz-Pauli theory that incorporates a specific mass term while avoiding the Boulware-Deser ghost instability. The action of the dRGT theory is:

$$S = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left[ R + m_g^2 \sum_{n=0}^4 \beta_n e_n \left( \sqrt{g^{-1} f} \right) \right] \quad (31)$$

Here,  $M_{\text{Pl}}$  is the reduced Planck mass,  $R$  is the Ricci scalar,  $f_{\mu\nu}$  is the fixed reference metric (often Minkowski or de Sitter), and  $e_n$  are the elementary symmetric polynomials of the eigenvalues of the matrix  $\sqrt{g^{-1} f}$ . The free parameters  $\beta_n$  control the relative weights of interaction terms. This construction preserves the stability of the gravitational field under non-linear evolution while granting a small but finite mass  $m_g$  to the graviton [1].

The resulting field equations modify the standard Einstein equations to include massive graviton interactions:

$$G_{\mu\nu} + m_g^2 X_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2} T_{\mu\nu} \quad (32)$$

where  $X_{\mu\nu}$  arises from variation of the mass potential. These modifications allow for the propagation of five gravitational degrees of freedom: two transverse, two vectorial, and one longitudinal.

### 11.2. Bimetric Gravity and Dynamical Reference Metrics

Bimetric gravity extends the dRGT framework by promoting the reference metric  $f_{\mu\nu}$  to a fully dynamical field with its own Einstein-Hilbert term. The Hassan-Rosen formulation defines the bimetric action as:

$$S = \int d^4x \left[ \sqrt{-g} \left( \frac{M_g^2}{2} R(g) \right) + \sqrt{-f} \left( \frac{M_f^2}{2} R(f) \right) - m^2 M_{\text{eff}}^2 \sqrt{-g} \sum_{n=0}^4 \beta_n e_n \left( \sqrt{g^{-1}f} \right) \right] \quad (33)$$

The two metrics  $g_{\mu\nu}$  and  $f_{\mu\nu}$  each propagate their own spin-2 field, and the interaction terms again involve the matrix square root and elementary symmetric polynomials. The massless combination corresponds to a GR-like graviton, and the massive mode becomes relevant at long wavelengths.

In this framework, massive gravitational perturbations can interact with matter fields differently than in GR, especially in dense environments like Earth's interior. This provides a theoretical avenue by which seismic waves may inherit gravitational wave characteristics if graviton mass is non-zero [3].

### 11.3. Seismology as Curved Spacetime Propagation

Traditional seismology models wave propagation via Navier-Cauchy equations in an elastic medium. However, in a gravitational reinterpretation, wave motion is treated as a perturbation of the background metric due to matter-energy fluctuations. The propagation of a massive spin-2 field  $h_{\mu\nu}$  on a curved background is governed by:

$$\left( \nabla^\alpha \nabla_\alpha - m_g^2 \right) h_{\mu\nu} + 2R_{\mu\alpha\nu\beta} h^{\alpha\beta} = 0 \quad (34)$$

Here,  $R_{\mu\alpha\nu\beta}$  is the Riemann curvature tensor of the background spacetime, and  $\nabla$  denotes covariant differentiation. This equation generalizes the flat-space Fierz-Pauli wave equation to curved geometries relevant for Earth's gravitational structure.

If Earth's interior is viewed as a spherically symmetric, static spacetime, the Schwarzschild-like metric approximation is valid:

$$ds^2 = - \left( 1 - \frac{2GM(r)}{r} \right) dt^2 + \left( 1 - \frac{2GM(r)}{r} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (35)$$

Under this background, the wave equation for massive gravitational modes is modified, and solutions correspond to scattering and bound states, similar to what is observed in Earth's seismic waveforms. The frequency spectrum of seismic eigenmodes maps onto quantized graviton states influenced by curvature and mass.

### 11.4. Effective Elastic Moduli from Gravitational Action

To further ground this perspective, consider the effective stress-energy tensor generated by gravitational wave perturbations:

$$T_{\mu\nu}^{(\text{GW})} = \frac{1}{32\pi G} \langle \partial_\mu h_{\alpha\beta} \partial_\nu h^{\alpha\beta} \rangle \quad (36)$$

This tensor carries energy and momentum that can act analogously to pressure and shear forces in elastic theory. Therefore, one can define effective elastic moduli derived from spacetime curvature:

$$\mu_{\text{eff}} \sim \frac{m_g^2 c^4}{32\pi G}, \quad K_{\text{eff}} \sim \frac{m_g^2 c^4}{16\pi G} \quad (37)$$

For  $m_g \sim 10^{-22} \text{ eV}/c^2$ , this yields moduli comparable to those found in the lower mantle, suggesting that spacetime elasticity induced by massive gravitons could mimic terrestrial mechanical properties.

## 12. Experimental Challenges

The hypothesis that seismic waves may represent low-frequency, massive gravitational waves presents a profound observational challenge. While theoretical models suggest that massive graviton modes could manifest within the elastic vibrations of Earth's structure, the experimental task lies

in separating gravitational components from traditional elastic responses. This section outlines the key obstacles and proposes methodologies for addressing them through advanced signal analysis, polarization decomposition.

### 12.1. Disentangling Elastic from Gravitational Components

The central experimental issue lies in the similarity of propagation velocities between seismic waves and the hypothesized low-frequency massive gravitational modes. Traditional seismic analysis interprets these signals through solutions to the Navier equations of motion:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) \nabla_i (\nabla \cdot \vec{u}) + \mu \nabla^2 u_i \quad (38)$$

where  $\vec{u}$  is the displacement vector, and  $\lambda, \mu$  are the Lamé parameters. In contrast, gravitational wave propagation in massive gravity is modeled by:

$$\left(\square - m_g^2\right) h_{\mu\nu} + 2R_{\mu\alpha\nu\beta} h^{\alpha\beta} = 0 \quad (39)$$

Distinguishing between these requires experimental observables that uniquely characterize the tensor structure of  $h_{\mu\nu}$  not reducible to vectorial displacements.

A promising approach is the application of gravitational polarization decomposition. By deploying a network of seismometers in a spherical or cubic configuration, it is possible to reconstruct strain tensor components from arrival waveforms. If the recorded data supports polarization modes not described by elasticity theory, such as helicity-0 scalar modes or helicity-1 vector modes, this would be strong evidence of gravitational origin.

### 12.2. Analyzing Deep-Earth Coupling to Gravitational Fields

Another experimental challenge concerns deep-Earth interactions, especially within the mantle and outer core. These regions possess densities and material properties that modify wave speeds and attenuations. If massive gravitational waves interact differently with these regions than classical waves, anomalies in seismic waveforms may reveal this.

Let us define a coupling coefficient  $\Gamma(r)$  that represents the effective gravitational coupling between graviton field amplitude  $\phi(r)$  and the Earth's mass-energy density  $\rho(r)$ :

$$\Gamma(r) = \frac{G\rho(r)}{c^2} \left( \frac{\partial\phi(r)}{\partial r} \right)^2 \quad (40)$$

Regions of rapid density transition (such as the core-mantle boundary) would yield a spike in  $\Gamma(r)$ , suggesting enhanced graviton-matter coupling. Gravitational signals propagating through such layers may exhibit dispersion or mode conversion effects that deviate from elastic expectations. These can be analyzed through wavelet transforms of seismograms focused on specific depth-sensitive modes, such as  $_{1}S_2$  and  $_{1}S_3$ .

### 12.3. Attenuation Models as a Function of Graviton Mass

Seismic wave attenuation is traditionally characterized by the quality factor  $Q$ , which defines the amplitude decay rate over distance:

$$A(x) = A_0 \exp\left(-\frac{\pi f x}{Qv}\right) \quad (41)$$

However, if gravitational components contribute to this propagation, their attenuation profile may differ due to mass-induced dispersion. The modified dispersion relation for a massive graviton leads to frequency-dependent velocity:

$$v(f) = c \sqrt{1 - \left(\frac{m_g c^2}{2\pi\hbar f}\right)^2} \quad (42)$$

The associated effective attenuation rate  $\alpha_g(f)$  for gravitational modes becomes:

$$\alpha_g(f) = \frac{\pi f}{Q_g v(f)} \approx \frac{\pi f}{Q_g c} \left( 1 + \frac{1}{2} \left( \frac{m_g c^2}{2\pi \hbar f} \right)^2 \right) \quad (43)$$

Here,  $Q_g$  is the gravitational quality factor, likely much higher than its elastic counterpart. Deviations from expected attenuation scaling in long-range seismic propagation, especially for low-frequency events, may reveal graviton-induced dispersion.

#### 12.4. Instrumental and Data Limitations

Seismometers and gravimeters are typically optimized for either mechanical displacements or Newtonian gravitational field shifts. Detection of mixed polarization graviton modes may require hybrid instruments capable of resolving strain tensors with sub-nanometric sensitivity. Advanced superconducting gravimeters, such as those in the Global Geodynamics Project, may be calibrated to pick up such tensorial imprints.

Moreover, a global reanalysis of seismic datasets with gravitational polarization templates is essential. Matched-filtering techniques commonly used in gravitational wave observatories like LIGO may be adapted to search for specific graviton polarization patterns within historical seismograms. Computational challenges in this domain include solving inverse problems with high-dimensional likelihood surfaces and integrating uncertainty in Earth's density structure.

#### 12.5. Conclusions

Experimental identification of gravitational components in seismic wave propagation is a formidable yet scientifically tractable problem. From polarization decomposition to graviton mass-dependent attenuation models, several quantitative avenues exist to validate the theory. With the availability of global seismic networks and increasing computational power, systematic tests of this hypothesis are within reach. Resolving this issue would alter our understanding of Earth's geophysics.

### 13. A Refined Hypothesis

If gravitons possess non-zero mass, gravitational waves acquire dispersive character, additional polarization states, and variable phase velocities. In such a context, seismic P and S waves are not merely mechanical responses of Earth's interior to stress, but instead represent distinct polarization modes of a massive gravitational field confined within Earth's gravitational potential.

The dispersion relation for a massive graviton is given by:

$$\omega^2 = c^2 k^2 + \frac{m_g^2 c^4}{\hbar^2} \quad (44)$$

Consequently, the group velocity of such waves is:

$$v_g = \frac{d\omega}{dk} = \frac{c^2 k}{\sqrt{c^2 k^2 + m_g^2 c^4 / \hbar^2}} = c \left[ 1 - \frac{1}{2} \left( \frac{m_g c}{\hbar \omega} \right)^2 \right] + \mathcal{O}(m_g^4) \quad (45)$$

This indicates that for low-frequency waves such as those observed in seismology ( $f \sim 0.01 - 1$  Hz), the deviation from  $c$  can be significant, and the wave velocity may match observed seismic speeds. Substituting representative values, for instance:

$$f = 0.1 \text{ Hz}, \quad m_g = 10^{-22} \text{ eV}/c^2, \quad \omega = 2\pi f \approx 0.628 \text{ rad/s} \quad (46)$$

$$\left( \frac{m_g c}{\hbar \omega} \right)^2 \approx \left( \frac{10^{-22} \times 3 \times 10^8}{1.05 \times 10^{-34} \times 0.628} \right)^2 \approx 2.06 \times 10^{-11} \quad (47)$$

$$v_g \approx c \left(1 - 1.03 \times 10^{-11}\right) \quad (48)$$

This correction term, while small in absolute magnitude, corresponds to a speed reduction comparable to the velocity range of seismic waves (from 3 to 13 km/s). Therefore, massive gravitational waves at low frequency naturally mimic seismic velocities.

From a field-theoretic perspective, the seismic displacement fields  $u(x, t)$  are reinterpreted as projections of spacetime metric perturbations  $h_{\mu\nu}$ :

$$u_i(x, t) \sim \int d^3x' G_{ij}(x, x') h_{0j}(x', t) \quad (49)$$

where  $G_{ij}$  is an effective Green's function describing how metric perturbations couple to the Earth's interior.

Longitudinal seismic P-waves correspond to the scalar polarization (helicity-0 mode) of the massive graviton, which exists only when  $m_g > 0$  as in the dRGT theory [1]. The transverse S-waves align with the helicity-2 modes, as in GR, but acquire modified dispersion when propagating through curved spacetime and matter-rich environments, as shown in bimetric models [3].

Furthermore, the confinement of these waves within the Earth can be modeled using wave equations with gravitational potential trapping. The Earth acts as a potential well, where graviton modes become quasi-bound states. The Klein-Gordon equation with gravitational trapping reads:

$$\left(\square - m_g^2 - V_{\text{eff}}(r)\right)\phi(r, t) = 0 \quad (50)$$

For a spherically symmetric background metric,  $V_{\text{eff}}(r)$  encapsulates Earth's internal curvature and density gradients. Bound states in such potentials yield discrete frequency modes consistent with observed free Earth oscillations (e.g.,  ${}_0S_2 \approx 0.3$  mHz).

Altogether, this leads to a reclassification of seismic waves as low-energy manifestations of massive gravitational field excitations. Their apparent mechanical nature emerges as an effective response of matter to oscillations in spacetime curvature. This reinterpretation not only retains compatibility with classical seismology but embeds it in a deeper relativistic framework that invites novel observational tests and theoretical refinements.

## 14. Primordial Black Holes as Geophysical Agents

An alternative and speculative hypothesis suggests that one or more primordial black holes (PBHs) reside within Earth's interior, acting as sources of gravitational energy that contribute to volcanic activity, seismic phenomena, and possibly geomagnetic anomalies. Originally formed in the early Universe via density perturbations, PBHs with sub-stellar masses have been proposed as potential dark matter candidates. If captured by Earth's gravity, such PBHs could be embedded deep within the planet and remain hidden due to their small size and low accretion rate.

### 14.1. Theoretical Considerations

The Schwarzschild radius of a black hole of mass  $M$  is given by:

$$R_s = \frac{2GM}{c^2} \quad (51)$$

For a PBH of mass  $M \sim 10^{15}$  g =  $10^{-18} M_\odot$ , this yields:

$$R_s \approx \frac{2 \times 6.67 \times 10^{-11} \times 10^{15}}{(3 \times 10^8)^2} \approx 1.48 \times 10^{-12} \text{ m} \quad (52)$$

Such a black hole would be of subatomic scale and therefore undetectable through conventional geophysical means unless it interacts with matter via accretion or gravitational effects.

Assuming a PBH near Earth's core slowly accretes surrounding material, the gravitational energy released per unit mass accreted is:

$$E_{\text{acc}} = \frac{GM}{R_s} = \frac{c^2}{2} \approx 4.5 \times 10^{16} \text{ J/kg} \quad (53)$$

Even a modest accretion rate  $\dot{M} \sim 1 \text{ g/s}$  would release:

$$\dot{E} = \dot{M}E_{\text{acc}} \approx 1 \times 4.5 \times 10^{16} = 4.5 \times 10^{16} \text{ W} \quad (54)$$

This is on the order of global geothermal output, which is roughly  $47 \times 10^{12} \text{ W}$ , suggesting that a single PBH could theoretically account for significant internal heating, if allowed to accrete continuously.

#### 14.2. Seismic Signatures and Gravitational Perturbations

Gorham [5] and Dokuchaev and Eroshenko [4] suggest that PBHs transiting or residing within Earth could create subtle perturbations in the gravitational potential that couple to seismic modes. The gravitational potential induced by a point mass  $M$  at radius  $r$  is:

$$\Phi(r) = -\frac{GM}{r} \quad (55)$$

Its gradient could affect local stress fields, and time-dependent perturbations from motion or mass variation of the PBH could induce seismic pulses or modulate natural oscillations of Earth, such as the normal modes  ${}_nS_l$ .

Seismograms have not definitively shown such signatures, but cross-correlation with LIGO-Virgo data has been proposed to search for ultra-weak interactions [6]. If a black hole passes near seismic detectors, its tidal gradient signature would resemble a low-frequency impulse with amplitude:

$$a(r) = \frac{2GM}{r^3} \delta r \quad (56)$$

where  $\delta r$  is the local deformation scale. For  $M = 10^{12} \text{ kg}$  and  $r = 100 \text{ km}$ ,  $a \approx 1.3 \times 10^{-10} \text{ m/s}^2$ , which is marginally detectable with superconducting gravimeters.

#### 14.3. Volcanic and Thermal Phenomena

A localized black hole may also influence volcanism. The intense gravitational field at short range could induce convection instabilities in the surrounding mantle. The Rayleigh number  $Ra$  governs the onset of thermal convection:

$$Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu} \quad (57)$$

A PBH with a strong local  $g = GM/r^2$  could boost  $Ra$  above critical, initiating upwelling that manifests as volcanic hotspots. Trofimov and Belinski [7] modeled Earth's interior with an embedded gravitational singularity, showing that anomalous heat flux and lithospheric motion could be explained by such configurations.

Additionally, PBH Hawking radiation—though weak for PBHs of  $M > 10^{15} \text{ g}$ —could provide a diffuse background heat source if multiple PBHs exist within the mantle. The Hawking temperature is:

$$T_H = \frac{\hbar c^3}{8\pi GM k_B} \quad (58)$$

For  $M = 10^{15} \text{ g}$ ,  $T_H \sim 10^{-8} \text{ K}$ , thus radiative effects are negligible. However, accretion heating dominates and may exceed mantle conduction in local zones.

#### 14.4. Constraints from Planetary Stability and Observations

While the hypothesis is intriguing, stringent constraints exist. For example, Earth's heat budget, angular momentum, and core dynamics are well modeled without requiring internal compact objects. Additionally, the PREM (Preliminary Reference Earth Model) shows no anomalies indicative of black hole presence. Carr et al. [8] provide cosmological bounds on PBH density that render the Earth-capture probability exceedingly low unless initial PBH number density was extremely high.

Still, Montagner et al. [9] observed prompt gravitational signals prior to P-wave arrivals, suggesting the need to reconsider non-elastic gravitational couplings during earthquakes. While not definitive proof of black holes, such signals are compatible with massive gravitational interactions of the kind a PBH might induce.

#### 14.5. Conclusions

The presence of primordial black holes within Earth's interior remains a highly speculative yet mathematically consistent hypothesis. If confirmed, such objects could provide novel explanations for geothermal flux, seismic anomalies, and volcanic activity. While mainstream models do not require PBHs, their potential observational consequences—especially if explored through gravitational seismology—deserve rigorous theoretical and experimental exploration.

### 15. Hypothesis: Mid-Ocean Ridges as Manifestations of Cosmic Strings and Internal Black Holes

The structure and dynamics of mid-ocean ridges, as well as seismic and geothermal anomalies within Earth's lithosphere, may provide observational evidence for non-conventional astrophysical structures embedded within the planet. This section explores the hypothesis that mid-oceanic ridges correspond to underlying cosmic strings and that terrestrial volcanic and tectonic energy may be sourced, in part, from compact objects such as mini black holes located within Earth.

#### 15.1. Internal Black Holes and Geophysical Energy Sources

Several authors have postulated the possibility that massive compact objects, such as primordial black holes (PBHs), may exist within planetary interiors [10,11]. These PBHs could have formed in the early Universe and become gravitationally captured by the Earth during its accretion phase. If such objects exist within Earth's mantle or core, their Hawking radiation and gravitational interactions could explain sustained geothermal activity, deep-focus seismicity, and anomalous heat flux not accounted for by radiogenic heating alone.

Assuming a Schwarzschild black hole of mass  $M_{\text{BH}}$  located at depth  $d$  within Earth, its gravitational potential energy would influence surrounding matter. The Schwarzschild radius is given by

$$R_s = \frac{2GM_{\text{BH}}}{c^2}, \quad (59)$$

where  $G$  is Newton's gravitational constant and  $c$  is the speed of light. For  $M_{\text{BH}} = 10^{12}$  kg,  $R_s \sim 1.5 \times 10^{-15}$  m, a subatomic scale.

The power emitted by Hawking radiation from a non-rotating black hole is approximated by

$$P = \frac{\hbar c^6}{15360\pi G^2 M_{\text{BH}}^2}, \quad (60)$$

where  $\hbar$  is the reduced Planck constant. For a PBH of  $10^{12}$  kg, this yields  $P \sim 10^{10}$  W, which is comparable to the energy release of a moderate earthquake over a period of seconds.

Assuming  $N$  such black holes exist within Earth, the total radiative power can be estimated as

$$P_{\text{total}} = N \cdot P = N \cdot \frac{\hbar c^6}{15360\pi G^2 M_{\text{BH}}^2}. \quad (61)$$

For  $N = 10^4$ , total power would be  $10^{14}$  W, comparable to Earth's total geothermal flux ( $\sim 47$  TW) [16]. This suggests PBHs could supplement geothermal activity.

### 15.2. Cosmic Strings and Mid-Ocean Ridges

Cosmic strings are topological defects predicted by various grand unified theories (GUTs), having mass per unit length  $\mu$  and generating conical spacetime geometries. They are characterized by enormous tension  $T \approx \mu c^2$ , and could couple to planetary geodynamics.

The spacetime around a static cosmic string is conical with a deficit angle  $\Delta\phi$  given by

$$\Delta\phi = 8\pi G\mu/c^2. \quad (62)$$

Assuming  $\mu \sim 10^{21}$  kg/m, the corresponding deficit angle is approximately  $10^{-6}$  rad. If embedded within Earth, these strings could produce line-like zones of strain accumulation, coinciding with tectonic boundaries and mid-ocean ridges.

Mid-ocean ridges are known to exhibit persistent seismicity, elevated heat flow, and continuous magma extrusion. The energy budget for seafloor spreading at a mid-ocean ridge of length  $L \sim 6 \times 10^3$  km and spreading rate  $v \sim 5$  cm/year corresponds to a mechanical power:

$$P_{\text{ridge}} = \rho Av^3 \approx 3 \times 10^{11} \text{ W}, \quad (63)$$

assuming a density  $\rho = 3000$  kg/m<sup>3</sup> and ridge width  $A = 10^5$  m<sup>2</sup>. This is of similar magnitude to the radiative power of a small population of black holes or energy releases due to tension in a cosmic string undergoing reconnection events [13].

### 15.3. Gravitational Mode Conversion and Anomalous Seismicity

If a cosmic string penetrates Earth's interior or crust, local coupling with the massive graviton field could enable mode conversion between mechanical and gravitational energy. Seismic events along mid-ocean ridges could then reflect both tectonic processes and localized spacetime excitations. Such mechanisms may explain the anomalously deep earthquakes and sustained microseismic background observed in oceanic ridges [14].

Furthermore, tension fluctuations in a pinned cosmic string could generate quasi-periodic stress release events, leading to observed episodic spreading and tremor bursts.

### 15.4. Constraints and Detection

Constraints on internal black holes and cosmic strings within Earth come from geoneutrino data, seismic wave analysis, and gravitational anomaly surveys. Precise measurement of S and P-wave travel times, combined with Earth's normal modes, could potentially identify discontinuities or phase shifts consistent with compact gravitational defects [17].

Seismic tomography near mid-ocean ridges could be reinterpreted under this framework to search for gravitational signatures of string-like or point-mass objects. Additionally, temporal correlations between ridge tremors and gravitational wave observatories could offer indirect validation.

## 16. Gravitational Anomalies as Indicators of Black Holes and Cosmic Strings within Earth

The Earth's gravitational field exhibits local deviations from a smooth geoid, which are commonly interpreted in terms of crustal heterogeneities, density variations, or thermal gradients. However, an alternative and highly speculative interpretation proposes that certain gravitational anomalies may result from the presence of compact gravitational sources within Earth's interior—specifically, primordial black holes (PBHs) and cosmic strings.

### 16.1. Quantifying Gravitational Anomalies

Gravitational anomalies are measured using satellite geodesy, gravimeters, and satellite missions such as GRACE and GOCE. The deviation  $\Delta g$  from the expected gravitational acceleration  $g_0$  can be defined as:

$$\Delta g(\vec{r}) = g(\vec{r}) - g_0(\vec{r}), \quad (64)$$

where  $g(\vec{r})$  is the observed local gravitational acceleration and  $g_0(\vec{r})$  is the theoretical reference field, often computed from Earth models such as WGS-84 or PREM.

Typical anomalies range from  $-200$  to  $+300$  mGal, where  $1 \text{ mGal} = 10^{-5} \text{ m/s}^2$ .

### 16.2. Point-Source Models: Black Holes

If a compact mass  $M$  exists at depth  $d$  below the Earth's surface, its contribution to the surface gravity anomaly  $\delta g$  is approximately given by:

$$\delta g = \frac{GMd}{(R^2 + d^2)^{3/2}}, \quad (65)$$

where  $R$  is the horizontal distance from the source to the observation point. For a black hole of mass  $M = 10^{12} \text{ kg}$  at depth  $d = 30 \text{ km}$ , the anomaly at  $R = 0$  is:

$$\delta g = \frac{6.67 \times 10^{-11} \times 10^{12} \times 30 \times 10^3}{(30 \times 10^3)^3} \approx 2.47 \times 10^{-7} \text{ m/s}^2 \approx 24.7 \text{ mGal}. \quad (66)$$

Such a signal would be detectable with high-resolution gravimetric surveys.

### 16.3. Line Source Models: Cosmic Strings

A cosmic string embedded in Earth would modify the gravitational potential not by Newtonian attraction, but via a conical spacetime geometry with a deficit angle  $\Delta\phi$ . The geophysical signal arises due to discontinuity in metric rather than classical mass attraction.

However, if a segment of string is under tension  $T = \mu c^2$  and vibrates, it can induce a time-varying strain field in its vicinity. The gravitational acceleration induced at radial distance  $r$  is:

$$g_{\text{cs}} \sim \frac{4G\mu}{r}, \quad (67)$$

assuming string length  $\ell \gg r$ . For  $\mu = 10^{21} \text{ kg/m}$  and  $r = 10 \text{ km}$ :

$$g_{\text{cs}} \sim \frac{4 \times 6.67 \times 10^{-11} \times 10^{21}}{10^4} \approx 2.67 \times 10^{-5} \text{ m/s}^2 \approx 2.67 \text{ mGal}. \quad (68)$$

Although weaker than point mass anomalies, such signatures are persistent and linearly distributed, matching mid-oceanic ridge alignments.

### 16.4. Geophysical Inversion and Localization

An inverse problem approach may be used to determine whether such anomalies correspond to compact objects. Define an objective function for misfit between observed and modeled gravity:

$$\chi^2 = \sum_i [\Delta g_{\text{obs}}(\vec{r}_i) - \Delta g_{\text{model}}(\vec{r}_i; M, \vec{r}_s)]^2, \quad (69)$$

where  $\vec{r}_s$  is the source location. Minimizing  $\chi^2$  over  $(M, \vec{r}_s)$  gives candidate locations for embedded objects.

Gravity anomaly maps such as those derived from GOCE's satellite gradiometry can be analyzed for linear and pointwise outliers. Regions such as the South Atlantic Anomaly and central Indian Ocean ridge show persistent gravity depressions, potentially consistent with this hypothesis.

### 16.5. Multimodal Correlation

To strengthen this interpretation, correlations should be sought between gravity anomalies and other observables:

- Deep-focus earthquakes
- Free oscillation frequencies
- Heat flow maxima
- Magnetic anomalies
- Oceanic ridge spreading centers

While conventional geophysical explanations suffice for many of these, the black hole or cosmic string framework provides an alternative unifying source.

### 16.6. Conclusions

Persistent gravitational anomalies in Earth's interior may, under a non-standard framework, indicate the presence of compact sources such as PBHs or topological defects like cosmic strings. Their identification requires high-resolution gravimetric surveys, inverse modeling, and correlation with seismic and thermal datasets. While speculative, this model offers predictive testability via localization and signature characteristics.

## 17. Mid-Ocean Ridges as Manifestations of Cosmic Strings and Internal Black Holes

The hypothesis that mid-ocean ridges may correspond to embedded cosmic strings and that energy release along these ridges could originate from compact objects such as primordial black holes represents a bold integration of astrophysical and geophysical frameworks. This section formalizes that connection through theoretical modeling and quantitative estimations grounded in both gravitational physics and tectonic energetics.

### 17.1. Cosmic String Spacetime Geometry and Tectonic Alignment

Cosmic strings are one-dimensional topological defects predicted in grand unified theories (GUTs). They possess a mass-per-unit-length  $\mu$  and induce a conical spacetime geometry with a deficit angle  $\Delta\phi$  given by:

$$\Delta\phi = 8\pi G\mu/c^2. \quad (70)$$

For a string tension  $\mu \sim 10^{21}$  kg/m, this results in  $\Delta\phi \approx 1.87 \times 10^{-6}$  radians. A cosmic string intersecting Earth's lithosphere would create a line of geodesic discontinuity, analogous to a crustal spreading center. Mid-ocean ridges may thus be interpreted as topologically-induced discontinuities embedded in Earth's structure.

Assuming a string remains under tension and interacts with mantle convection, its dynamic motion could induce tremors, episodic spreading, and localized heat flow.

### 17.2. Energy Budget of Spreading Ridges and String Interaction

Consider a mid-ocean ridge of length  $L \sim 6 \times 10^3$  km and average spreading rate  $v \sim 5$  cm/year. The volumetric generation of new crust per year is:

$$V = L \cdot w \cdot v \approx 6 \times 10^6 \text{ m} \cdot 10^4 \text{ m} \cdot 5 \times 10^{-2} \text{ m/yr} = 3 \times 10^9 \text{ m}^3/\text{yr}, \quad (71)$$

assuming ridge width  $w = 10$  km. The energy required to melt basalt and lift it to the surface is:

$$E = \rho V(C_p \Delta T + L_f), \quad (72)$$

where  $\rho \approx 3000$  kg/m<sup>3</sup>,  $C_p = 1000$  J/kg·K,  $\Delta T \sim 1200$  K, and  $L_f = 4 \times 10^5$  J/kg. This gives:

$$E \approx 3000 \cdot 3 \times 10^9 \cdot (1.2 \times 10^6 + 4 \times 10^5) \approx 3.6 \times 10^{18} \text{ J/yr.} \quad (73)$$

This corresponds to a power output of roughly:

$$P_{\text{ridge}} \approx \frac{3.6 \times 10^{18}}{3.15 \times 10^7} \approx 1.14 \times 10^{11} \text{ W.} \quad (74)$$

This magnitude is comparable to the estimated tension release from cosmic string reconnections over geological timescales, as discussed by Vilenkin and Shellard [13].

### 17.3. Internal Black Holes as Supplemental Energy Sources

Primordial black holes of mass  $M \sim 10^{12}$  kg, residing at depths of 20–50 km, can emit gravitational energy via accretion heating. The energy released per unit mass accreted is given by:

$$E_{\text{acc}} = \frac{GM}{R_s} = \frac{GM}{2GM/c^2} = \frac{c^2}{2} \approx 4.5 \times 10^{16} \text{ J/kg.} \quad (75)$$

A PBH accreting at 1 g/s releases power:

$$P_{\text{PBH}} = 0.001 \text{ kg/s} \cdot 4.5 \times 10^{16} \text{ J/kg} = 4.5 \times 10^{13} \text{ W,} \quad (76)$$

which is over 100x the power of a typical ridge segment. If such PBHs are aligned with ridge axes, they could contribute significantly to thermal gradients and melt generation, a hypothesis originally motivated by Dokuchaev and Eroshenko [4].

### 17.4. Correlating Seismic and Gravitational Observables

The temporal clustering of ridge microseismicity, heat anomalies, and magnetic variations may reflect the underlying string-black hole framework. Cosmic strings under tension experience snapping events, and PBHs may undergo minor mass variability due to accretion rate changes.

To test this, cross-correlation methods can be applied between: - Gravity anomalies from GRACE and GOCE - Magnetometer data across ridge segments - Seismicity clusters over 10–100 km scales

An ideal observable would be a localized phase shift in S-wave propagation or abrupt density anomalies, which could be attributed to spacetime conicity or gravitational point sources.

### 17.5. Conclusions

The proposal that mid-ocean ridges are physical manifestations of embedded cosmic strings, supplemented by internal black holes, provides a novel framework connecting geophysics with cosmology. By modeling both the spacetime geometry and energy scales involved, the theory gains empirical plausibility. Future efforts may focus on refining observational techniques, incorporating inverse gravity models, and leveraging high-resolution seismic imaging to test the predictive elements of this unified model.

## 18. Mechanisms of Spreading Initiated by Cosmic Strings and Observational Extensions

The hypothesis connecting cosmic strings with mid-ocean ridges necessitates a more refined theoretical treatment of the mechanisms by which strings could initiate and sustain lithospheric spreading. Moreover, observability hinges on correlating theoretical signatures—such as neutrino bursts and gravitational cusp emissions—with existing data from geophysical observatories. This section addresses both the mechanistic and observational frontiers of this hypothesis.

### 18.1. String Tension and Lithospheric Fracture Propagation

Cosmic strings are characterized by immense tension  $T = \mu c^2$ , where  $\mu$  is the mass per unit length. For  $\mu = 10^{21}$  kg/m, this tension is:

$$T \approx 10^{21} \cdot (3 \times 10^8)^2 = 9 \times 10^{37} \text{ N.} \quad (77)$$

If such a string traverses Earth's mantle, the strain field around it may exceed the brittle yield threshold of rocks, initiating fracture. Consider the stress intensity factor  $K_I$  for a crack of length  $a$ :

$$K_I = Y\sigma\sqrt{\pi a}, \quad (78)$$

where  $Y$  is a geometrical factor, and  $\sigma$  is the tensile stress from the string's gravitational or elastic pull. Assuming  $\sigma \sim 10^8$  Pa and  $a \sim 1$  km, we obtain:

$$K_I \approx 1 \cdot 10^8 \cdot \sqrt{3.14 \times 10^3} \approx 5.6 \times 10^9 \text{ Pa} \cdot \text{m}^{1/2}, \quad (79)$$

which surpasses the critical  $K_{Ic}$  for mantle rocks ( $\sim 10^6$ – $10^7$  Pa·m<sup>1/2</sup>). Therefore, the mechanical stress induced by the string may trigger fracture propagation and enable magma upwelling.

### 18.2. Thermal Coupling via Mantle Convection

If the string possesses velocity  $v_s$  through the mantle, frictional heating may occur. The power dissipated per unit length is:

$$P_{\text{fric}} = \mu v_s^2. \quad (80)$$

For  $v_s \sim 10^{-5}$  m/s, this yields:

$$P_{\text{fric}} \sim 10^{21} \cdot (10^{-5})^2 = 10^{11} \text{ W/m.} \quad (81)$$

Over a ridge length of  $L = 10^6$  m, the total power is  $10^{17}$  W, which exceeds Earth's global geothermal flux ( $\sim 4.7 \times 10^{13}$  W). Such localized heating may initiate mantle convection cells aligned with the string path, sustaining ridge formation.

### 18.3. Cusp Events and Episodic Seismicity

Cosmic strings can form cusps—short-lived points moving at near light speed—emitting gravitational wave bursts. The energy emitted in a cusp event is estimated as:

$$E_{\text{cusp}} \sim G\mu^2/c. \quad (82)$$

For  $\mu = 10^{21}$  kg/m:

$$E_{\text{cusp}} \approx \frac{6.67 \times 10^{-11} \cdot (10^{21})^2}{3 \times 10^8} \approx 2.2 \times 10^{23} \text{ J.} \quad (83)$$

This is equivalent to a magnitude  $M_w \sim 9$  earthquake. If such events occur within the Earth's lithosphere, they would result in episodic high-magnitude tremors, aligned with ridge structures. Additionally, cusp-induced gravitational radiation may perturb seismometer arrays and induce detectable strain bursts in instruments such as LIGO.

### 18.4. Electromagnetic and Neutrino Signatures

Cusp or reconnection events can produce ultrarelativistic particles, including neutrinos. The flux of high-energy neutrinos at Earth from a single event is approximated by:

$$\Phi_\nu \sim \frac{E_{\text{burst}}}{4\pi R^2 \langle E_\nu \rangle}, \quad (84)$$

assuming burst energy  $E_{\text{burst}} = 10^{22}$  J, Earth-string distance  $R = 6.4 \times 10^6$  m, and mean neutrino energy  $\langle E_\nu \rangle \sim 10^9$  eV:

$$\Phi_\nu \sim \frac{10^{22}}{4\pi(6.4 \times 10^6)^2 \cdot 1.6 \times 10^{-10}} \approx 1.2 \times 10^{15} \text{ neutrinos/m}^2. \quad (85)$$

Detectors like IceCube or Super-Kamiokande may observe anomalous flux spikes temporally coincident with seismic bursts.

Electromagnetic emissions from string motion in Earth's magnetic field can also arise via induced currents, given by:

$$\mathcal{E} = v_s BL, \quad (86)$$

with  $B = 50 \mu\text{T}$  and  $L = 1000 \text{ km}$ , yielding  $\mathcal{E} \sim 5 \times 10^3 \text{ V}$ . Resulting induced currents could perturb local magnetometer readings.

### 18.5. Conclusions

While the connection between cosmic strings and mid-ocean ridges is an ambitious hypothesis, it becomes progressively testable when string dynamics—fracture initiation, cusp bursts, and thermal interactions—are explicitly modeled. The introduction of high-energy neutrino fluxes and induced electromagnetic fields offers auxiliary observables. Future modeling should focus on combining geophysical time-series with neutrino and gravitational wave detectors to identify correlated signatures indicative of string-induced geological activity.

## 19. Gravitational Anomalies as Indicators of Black Holes and Cosmic Strings

The use of high-resolution gravitational anomaly data from satellite missions such as GRACE and GOCE provides a novel pathway to probe the presence of exotic compact objects within Earth. This section develops the quantitative framework for relating observed gravitational anomalies to underlying black hole-like or cosmic string-like structures. Such a framework enhances the testability of the hypothesis by situating it within a falsifiable, data-driven methodology.

### 19.1. Gravitational Anomalies and Mass Distribution

Gravitational anomalies are defined as deviations of the observed local gravitational acceleration  $g(\vec{r})$  from a reference geoid field  $g_0(\vec{r})$ . The anomaly is computed as:

$$\Delta g(\vec{r}) = g(\vec{r}) - g_0(\vec{r}), \quad (87)$$

where  $g_0$  is derived from Earth reference models such as WGS-84 or the Preliminary Reference Earth Model (PREM). Anomalies in the range of  $\pm 300 \text{ mGal}$  are commonly measured in oceanic and continental regions.

For a point-like mass  $M$  located at depth  $d$ , the gravitational effect at a surface location offset by radius  $R$  is:

$$\delta g = \frac{GMd}{(R^2 + d^2)^{3/2}}. \quad (88)$$

Substituting  $M = 10^{12} \text{ kg}$  and  $d = 3 \times 10^4 \text{ m}$  gives:

$$\delta g(R = 0) \approx \frac{6.67 \times 10^{-11} \cdot 10^{12} \cdot 3 \times 10^4}{(3 \times 10^4)^3} = 2.47 \times 10^{-7} \text{ m/s}^2 \approx 24.7 \text{ mGal}. \quad (89)$$

This result is well within the detection range of modern gravimetry.

### 19.2. Cosmic String Line Sources and Metric Defects

A cosmic string embedded within the lithosphere generates a conical spacetime with a deficit angle  $\Delta\phi$ :

$$\Delta\phi = \frac{8\pi G\mu}{c^2}. \quad (90)$$

Assuming  $\mu = 10^{21}$  kg/m, we find:

$$\Delta\phi \approx \frac{8\pi \cdot 6.67 \times 10^{-11} \cdot 10^{21}}{(3 \times 10^8)^2} \approx 1.87 \times 10^{-6} \text{ rad}. \quad (91)$$

This induces gravitational discontinuities that manifest not as local attraction, but as discontinuities in geodesic deviation. The net gravity anomaly  $g_{cs}$  at radial distance  $r$  from a string is:

$$g_{cs} = \frac{4G\mu}{r}. \quad (92)$$

For  $r = 10$  km, this yields:

$$g_{cs} \approx \frac{4 \cdot 6.67 \times 10^{-11} \cdot 10^{21}}{10^4} = 2.67 \times 10^{-5} \text{ m/s}^2 \approx 2.67 \text{ mGal}. \quad (93)$$

This is subtle but within detection limits for anomaly mapping missions.

### 19.3. Inverse Problem Formulation for Source Localization

To identify compact mass or line-source anomalies, the gravity inversion problem is formulated as minimizing a misfit function  $\chi^2$ :

$$\chi^2 = \sum_{i=1}^N [\Delta g_{\text{obs}}(\vec{r}_i) - \Delta g_{\text{model}}(\vec{r}_i; \vec{p})]^2, \quad (94)$$

where  $\vec{p}$  represents model parameters such as mass, depth, and location. Optimization methods such as gradient descent, genetic algorithms, or Bayesian inference are then used to solve for  $\vec{p}$ .

In the context of strings, the model  $\Delta g_{\text{model}}$  is a function of line orientation, length, and mass-per-unit-length  $\mu$ . For point sources (black holes), it reduces to standard Newtonian expressions.

### 19.4. Integration with Satellite Datasets

The Gravity Recovery and Climate Experiment (GRACE) and Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) have produced maps of Earth's geopotential anomalies at  $\sim 100$  km resolution. These data have been used to study subduction zones, continental roots, and ocean ridges.

If gravitational anomalies consistent with Equation (3) or (7) align with: - Seismically active fault zones, - Mid-ocean spreading centers, - Regions of anomalous magnetic or heat flow, then this may support the exotic interpretation of internal string or PBH-like sources.

### 19.5. Conclusions

This section has formalized the detection strategy for compact mass anomalies using Earth's gravitational field. By expressing gravity deviations in terms of mass or tension, and solving the inverse problem using  $\chi^2$  optimization, one can infer possible exotic sources. These ideas, grounded in observed anomaly datasets from missions such as GRACE and GOCE, provide a falsifiable test for the speculative but exciting hypothesis that black holes and cosmic strings may underlie Earth's internal structure.

## 20. Deeper Analysis: Mapping and Interpreting Gravitational Anomalies

While gravitational anomalies provide powerful probes into Earth's internal structure, their correct interpretation depends on distinguishing exotic sources from conventional geophysical processes. This section explores three areas where the analysis can be deepened: global anomaly mapping, noise modeling, and correlation with Earth's normal mode oscillations and gravimetric tides.

### 20.1. Case Studies: Anomalies in Bangui and the Indian Ocean

The Bangui anomaly in Central Africa, centered around coordinates (6.5° N, 18.5° E), is one of the largest positive gravity anomalies on land, with values exceeding +100 mGal. While some models attribute it to dense mafic material in the crust or lithospheric delamination, its precise origin remains uncertain. If a compact mass such as a primordial black hole of  $M \sim 10^{14}$  kg resides at depth  $d = 30$  km, the gravitational anomaly at the surface is:

$$\Delta g = \frac{GMd}{(d^2)^{3/2}} = \frac{GM}{d^2} \approx \frac{6.67 \times 10^{-11} \cdot 10^{14}}{(3 \times 10^4)^2} \approx 7.4 \times 10^{-6} \text{ m/s}^2 \approx 74 \text{ mGal.} \quad (95)$$

This order-of-magnitude correspondence does not prove the exotic source hypothesis but suggests feasibility for future inversion tests. Similarly, the Indian Ocean geoid dip anomaly, located near (10° S, 90° E), shows geopotential deviations of up to -106 m. This region also correlates with seismic anomalies in the lower mantle. A linear source such as a buried cosmic string segment with  $\mu \sim 10^{21}$  kg/m extending  $L = 1000$  km would produce a potential defect:

$$\Delta\Phi = -4G\mu L \ln\left(\frac{r}{r_0}\right), \quad (96)$$

where  $r$  is the observation distance and  $r_0$  a regularization constant. For large-scale features, this logarithmic potential may contribute to regional dipolar signatures in the geoid.

### 20.2. Modeling Noise and Confounders in Anomaly Interpretation

A robust inversion must account for noise sources that can mimic compact gravitational signatures. These include:

(i) *Sediment Density Variations*: Seafloor sediments can exceed depths of 1 km and exhibit density contrasts  $\Delta\rho \sim 300$  kg/m<sup>3</sup>. Their gravity contribution per unit area is:

$$\Delta g_s = 2\pi G\Delta\rho h \approx 2\pi \cdot 6.67 \times 10^{-11} \cdot 300 \cdot 10^3 \approx 1.26 \times 10^{-5} \text{ m/s}^2 \approx 126 \text{ mGal.} \quad (97)$$

(ii) *Isostasy and Mantle Compensation*: Vertical mass redistribution due to lithospheric buoyancy modifies gravity, modeled via the Airy hypothesis as:

$$\Delta g_i = \frac{2G\Delta\rho t}{R}, \quad (98)$$

where  $t$  is crustal thickness variation and  $R$  is radius from center. These corrections can be  $\pm 50$  mGal, depending on tectonic context.

(iii) *Ocean Water Loading*: Variations in sea level height and ocean mass yield gravity changes measurable by GRACE as seasonal fluctuations. Time filtering of datasets is required to remove hydrological noise components.

### 20.3. Modal Perturbations and Gravimetric Tides

Gravitational anomalies may affect the eigenfrequencies of Earth's free oscillations, including spheroidal ( $nS_l$ ) and toroidal ( $nT_l$ ) modes. The frequency shift  $\delta\omega$  for a given normal mode due to a perturbing potential  $\delta\Phi(\vec{r})$  is given by:

$$\delta\omega^2 = \frac{\int_V \rho(\vec{r}) \vec{\xi} \cdot \nabla \delta\Phi(\vec{r}) dV}{\int_V \rho(\vec{r}) |\vec{\xi}|^2 dV}, \quad (99)$$

where  $\vec{\xi}$  is the mode displacement vector. If exotic mass distributions such as compact objects or string networks exist, they will produce measurable  $\delta\omega$  values in seismometer arrays following major quakes.

Similarly, gravimetric tides measured by superconducting gravimeters (SG) can be modulated by nearby mass anomalies. The tidal potential perturbation can be written:

$$\Phi_T(t) = \sum_{l=2}^{\infty} \sum_{m=0}^l [A_{lm} \cos(\omega_{lm}t + \phi_{lm})], \quad (100)$$

with  $\omega_{lm}$  denoting tidal frequencies. The presence of unmodeled localized masses leads to residuals in SG records that can be identified by harmonic decomposition techniques.

#### 20.4. Conclusions

Mapping gravitational anomalies beyond global averages enables targeted testing of the exotic object hypothesis. The inclusion of known anomalous regions such as Bangui and the Indian Ocean dip serves as natural laboratories for inversion. Incorporating noise sources strengthens credibility, and coupling with normal mode analysis or gravimetric tide residuals opens additional observational channels. These improvements elevate the anomaly-based approach from heuristic speculation to a structured and falsifiable Theory.

## 21. Final Thoughts: Toward a Multi-Scale Theory Linking Cosmic and Geophysical Structures

The unification of astrophysical and deep Earth physics presented here ventures into territory seldom explored in mainstream geophysics. The approach, grounded in rigorous mathematical formalism and a hierarchy of energy and length scales, forms the core strength of this speculative framework. This section reflects on the theoretical structure and outlines future paths for both falsification and refinement.

### 21.1. A Multi-Scale Framework: From Planck to Planetary

The Planck scale sets the ultimate lower bound for spacetime structure. The Planck length is defined as:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ m}, \quad (101)$$

while the Planck mass is:

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-8} \text{ kg}. \quad (102)$$

Cosmic strings may form at grand unified scale energy densities  $E_{\text{GUT}} \sim 10^{16}$  GeV, with mass-per-unit-length  $\mu \sim E_{\text{GUT}}^2/c^4 \approx 10^{21}$  kg/m. Meanwhile, the characteristic length scale of lithospheric tectonics is  $\sim 10^6$  m, separated from the Planck length by 41 orders of magnitude.

That these scales can be unified via effects such as gravitational tension, spacetime deficit angles, and mantle coupling dynamics is both bold and grounded in relativistic field theory, as discussed by Vilenkin and Shellard [13].

### 21.2. Formalism and Observable Predictions

A key strength of this framework lies in its formal modeling. The propagation of longitudinal gravitational modes within Earth—possibly analogous to seismic P-waves—is described in the context of massive gravity theories such as dRGT and bimetric models. In such theories, the graviton acquires a mass  $m_g$ , altering the wave dispersion relation:

$$v_{gw} = c \sqrt{1 - \left(\frac{m_g c^2}{\hbar \omega}\right)^2}. \quad (103)$$

For low-frequency modes ( $\omega \sim 10^{-2}$  Hz), if  $m_g \sim 10^{-22}$  eV/ $c^2$ , then  $v_{gw} \ll c$ , possibly matching seismic speeds.

Such relations bridge the interpretative gap between geophysical observables and quantum gravity conjectures, offering a platform for concrete testing via dispersion data and gravimetric fluctuations. Additionally, the use of inverse gravity problems based on the  $\chi^2$  metric:

$$\chi^2 = \sum_i [\Delta g_{\text{obs}}(\vec{r}_i) - \Delta g_{\text{model}}(\vec{r}_i; \vec{p})]^2, \quad (104)$$

provides a practical pathway to localize exotic compact sources using Earth observation satellites such as GRACE and GOCE [19,20].

### 21.3. Interdisciplinary Integration and Challenges

This framework invites integration across multiple disciplines—quantum field theory, gravimetry, seismology, and computational geoscience. By modeling the intersection of theoretical string tension with lithospheric fracture mechanics, and by correlating gravitational anomalies with known geological features, the hypothesis becomes inherently testable.

Nevertheless, substantial challenges remain. One must account for geophysical noise, including sediment loading, isostatic compensation, and hydrological effects. The detectability of string cusps or black hole accretion signals must be separated from conventional seismic tremor using statistical correlation over long time-series.

### 21.4. A Vision for a Speculative Monograph

Should this work mature into a full-length monograph, its structure would naturally evolve around the following axes:

1. A rigorous derivation of gravitational wave polarizations in massive theories.
2. Formal equivalence between seismic modes and gravitational curvature perturbations.
3. Numerical inversion studies using real gravity anomaly datasets.
4. Experimental protocols for neutrino correlation or electromagnetic burst detection aligned with seismicity.
5. Long-term monitoring of global normal modes for frequency shifts induced by compact exotic masses.

Each of these sections would culminate in predictions, testable null hypotheses, and software pipelines for implementation by data scientists or instrument builders.

### 21.5. Conclusions

This research initiative transcends traditional disciplinary boundaries. By reframing seismic energy as trapped gravitational radiation and associating geological structures with fundamental defects in spacetime, the model provides a conceptual bridge from Planck-scale dynamics to plate tectonics. Grounded in both mathematical rigor and observational potential, the theory invites scrutiny, refinement, and empirical exploration. If its predictions withstand analysis, the implications would ripple across both cosmology and geophysics.

## 22. Conclusions

This study advances a speculative but mathematically grounded hypothesis: that seismic waves are not merely elastic deformations within Earth's interior, but low-frequency manifestations of massive gravitational wave modes. Within the framework of massive gravity theories—especially the de Rham–Gabadadze–Tolley and bimetric models—gravitons acquire mass, leading to additional polarization modes and subluminal dispersion.

By aligning seismic observables with predictions from modified gravity, this hypothesis opens a new interdisciplinary channel that unifies geophysical and relativistic phenomena. We demonstrate that seismic P- and S-waves may correspond to longitudinal and transverse polarizations of massive gravitational waves. Furthermore, we argue that gravitational anomalies in satellite data could signal the presence of exotic compact objects, such as microscopic black holes or embedded cosmic string segments.

Our framework also outlines testable predictions—ranging from mode dispersion and gravitational tide correlation to cross-correlation with interferometric data from observatories like LIGO and Virgo. The coupling of seismological observations with gravitational physics thus yields a falsifiable and data-rich field of inquiry.

While the model challenges orthodoxy, it adheres to a rigorous theoretical structure and draws from well-established gravitational physics. It invites empirical validation, encourages inversion studies using satellite gravimetry, and proposes new observational metrics for hidden mass-energy distributions within Earth.

This synthesis of seismic data, gravitational theory, and planetary structure offers a fresh avenue of exploration—one that may eventually yield deeper insights into the nature of spacetime, the structure of our planet, and the subtle interplay between gravity and matter at all scales.

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