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Article

A Scalarized Entropy-Based Model for Portfolio Optimization: Balancing Return, Risk and Diversification

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Abstract

Portfolio optimization is a cornerstone of modern financial decision-making, traditionally based on the mean–variance model introduced by Markowitz. However, this framework relies on restrictive assumptions—such as normally distributed returns and symmetric risk preferences—that often fail in real-world markets, particularly in volatile and non-Gaussian environments such as cryptocurrencies. To address these limitations, this paper proposes a novel multi-objective model that combines expected return maximization, mean absolute deviation (MAD) minimization, and entropy-based diversification into a unified optimization structure: the Mean–Deviation–Entropy (MDE) model. The MAD metric offers a robust alternative to variance by capturing the average magnitude of deviations from the mean without inflating extreme values, while entropy serves as an information-theoretic proxy for portfolio diversification and uncertainty. Three entropy formulations are considered—Shannon entropy, Tsallis entropy, and cumulative residual Sharma–Taneja–Mittal entropy (CR-STME)—to explore different notions of uncertainty and structural diversity. The MDE model is formulated as a tri-objective optimization problem and solved via scalarization techniques, enabling flexible trade-offs between return, deviation, and entropy. The framework is empirically tested on a cryptocurrency portfolio composed of Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB), using daily data over a 12-month period. The empirical setting reflects a high-volatility, high-skewness regime, ideal for testing entropy-driven diversification. Comparative outcomes reveal that entropy-integrated models yield more robust weightings, particularly when tail risk and regime shifts are present. Comparative results against classical mean–variance and mean–MAD models indicate that the MDE model achieves improved diversification, enhanced allocation stability, and greater resilience to volatility clustering and tail risk. This study contributes to the literature on robust portfolio optimization by integrating entropy as a formal objective within a scalarized multi-criteria framework. The proposed approach offers promising applications in sustainable investing, algorithmic asset allocation, and decentralized finance, especially under high-uncertainty market conditions.

Keywords: mean absolute deviation; scalarization; weighted Shannon entropy; Tsallis entropy; portfolio optimization; cryptocurrency market; diversification; robust decision making

MSC: 91B28; 90C29

1. Introduction

Portfolio optimization remains a central pillar of both theoretical finance and practical asset management. Since the seminal contribution of Markowitz [1], who introduced the mean–variance (MV) framework, a wide range of extensions and alternatives have been developed to address its well-documented shortcomings. These include the sensitivity of variance to outliers, its reliance on

precise covariance estimates, and the strong assumption of normally distributed asset returns [2,3]. One notable improvement is the replacement of variance with mean absolute deviation (MAD) as a risk metric, initially proposed by Konno and Yamazaki [4] and later expanded in linear programming contexts by Speranza [5]. MAD-based formulations offer enhanced robustness under non-Gaussian distributions and are computationally attractive due to their compatibility with linear optimization schemes [6–8].

While risk minimization has received considerable attention, the diversification of portfolio weights—essential to reducing systemic exposure—remains a challenging aspect. Traditional diversification is often imposed through ad hoc constraints or heuristics, which lack theoretical foundation. In contrast, information theory provides a rigorous alternative through entropy-based measures. Shannon entropy [9] quantifies uncertainty in weight distributions and inherently penalizes concentration. Its generalizations, such as Tsallis entropy [10] and Rényi entropy [11], introduce tunable parameters that enable better sensitivity to tail events, correlation structures, and investor preferences [12–14]. The role of entropy as a formal proxy for diversification has gained significant traction in portfolio theory, fostering models that reward allocation spread and penalize dominance [13,14]. Recent developments have further expanded this line of research by incorporating non-additive and higher-order entropy functions, such as Sharma–Taneja–Mittal entropy and its cumulative residual form (CR-STME), which originate from decision theory and reliability analysis [15–18]. Simultaneously, advances in nonlinear programming (NLP), convex analysis, and metaheuristic optimization have enabled the formulation and solution of multi-objective portfolio models that go beyond traditional mean–variance paradigms. Scalarization techniques, which convert multiple conflicting objectives into a single composite function, allow for flexible integration of expected return, risk, and diversification within one unified optimization process [19]. These methodological developments are particularly relevant in the context of cryptocurrency markets, which exhibit pronounced volatility, structural breaks, and heavy-tailed return distributions. Such characteristics render entropy-based approaches especially suitable for capturing allocation instability and diversification effects [20–22].

In this study, we propose a novel portfolio optimization framework—termed the Mean–Deviation–Entropy (MDE) model—which simultaneously considers expected return maximization, MAD-based risk control, and entropy-driven diversification within a scalarized optimization structure. We implement and test this model using a four-asset cryptocurrency portfolio composed of Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB), using daily returns over a recent 12-month period. Comparative results against the classical MV and MAD models demonstrate that the proposed framework yields improved allocation stability, enhanced diversification, and greater robustness to volatility clusters and tail risk. The main contributions of this paper are threefold: (i) to develop a unified portfolio optimization model that blends MAD and entropy-based diversification; (ii) to empirically validate the proposed framework on cryptocurrency markets and assess its resilience to allocation instability; and (iii) to provide a comparative analysis of entropy-based versus variance-based diversification strategies under high-uncertainty conditions.

The structure of the paper is as follows. Section 2 introduces the methodological foundations of the proposed model, including deviation-based risk measures and entropy-based diversification criteria. Section 3 presents the empirical results and a comparative analysis of portfolio performance across different optimization models. Section 4 concludes the study and outlines future research directions.

2. Materials and Methods

2.1. Mean Absolute Deviation as a Risk Measure

The mean absolute deviation (MAD) is widely recognized as a robust alternative to variance in portfolio optimization models, especially under non-normal return distributions or in the presence of outliers [3,4]. For a portfolio with asset return vector $R = (r_1, r_2, \dots, r_T)^T$ over T time periods, and

portfolio weights vector with N assets $x = (x_1, x_2, \dots, x_N)^T$, the portfolio return at time t is given by: $r_p(t) = \sum_{i=1}^N x_i r_{i,t}$, where $r_{i,t}$ is the return of asset i at time t . The mean absolute deviation is defined as:

$$\text{MAD}(x) = \frac{1}{T} \sum_{t=1}^T |r_p(t) - \bar{r}_p|, \text{ where } \bar{r}_p = \frac{1}{T} \sum_{t=1}^T r_p(t)$$

Unlike variance, which emphasizes squared deviations and assumes symmetric penalization, MAD captures the magnitude of deviations in a linear fashion, providing a more intuitive measure of risk. From a computational perspective, MAD-based optimization leads to linear programming formulations, which are often more tractable than quadratic programming required by mean-variance models [2,3].

2.2. Entropy-Based Diversification

Entropy, originally introduced by Shannon [5], measures the uncertainty of a probabilistic system and has been successfully applied to assess portfolio diversification. In portfolio theory, entropy penalizes concentration and rewards evenly spread allocations. Given normalized portfolio weights x_i such that $\sum_{i=1}^N x_i = 1$, the Shannon entropy is:

$$H(x) = - \sum_{i=1}^N x_i \ln x_i$$

To accommodate long-range dependence or fat-tailed behavior in financial markets, generalized entropy measures such as Tsallis entropy have been proposed [6]:

$$H_q(x) = \frac{1}{q-1} \left(1 - \sum_{i=1}^N x_i^q \right), q \in \mathcal{R}, q \neq 1$$

For $q \rightarrow 1$, Tsallis entropy reduces to Shannon entropy. The parameter q allows tuning the sensitivity to tail events and portfolio concentration: lower q amplifies the contribution of small weights, while higher q emphasizes dominant.

A further generalization—particularly useful in modeling memory effects and tail risks in dynamic or volatile markets—is the Cumulative Residual Sharma–Taneja–Mittal Entropy (CR-STME). This measure stems from generalized information theory and reliability analysis, and is defined for a continuous non-negative random variable X with survival function $\bar{F}(x) = 1 - F(x)$ as:

$$\mathcal{E}_{\alpha,\beta}(X) = \frac{1}{1-\alpha} \left(\int_0^\infty [\bar{F}(x)]^\alpha [\ln \bar{F}(x)]^\beta dx - 1 \right), \alpha \neq 1, \beta > 0$$

This formulation integrates two parameters:

- α controls the weighting of tail probabilities, enhancing sensitivity to rare but extreme losses;
- β controls the depth of memory, reflecting the investor's aversion to prolonged drawdowns.

In a discrete portfolio context, this entropy can be adapted using weight vectors $x = (x_1, x_2, \dots, x_N)^T$, and estimated survival functions derived from cumulative distribution of returns. The CR-STME captures both uncertainty and temporal structure, making it particularly suitable for high-volatility settings such as cryptocurrency markets, where traditional models may fail to reflect hidden systemic risks.

2.3. The Mean–Deviation–Entropy (MDE) Model

The proposed MDE model integrates return maximization, deviation minimization, and entropy-based diversification in a scalarized optimization formulation. The objective function is defined as $\min_x [-\mu^T x + \lambda_1 \cdot \text{MAD}(x) - \lambda_2 \cdot H(x)]$ subject to

$$\sum_{i=1}^N x_i = 1, \quad x_i \geq 0, \quad i = \overline{1, N}$$

where μ denotes the expected return vector, $H(x)$ is a generic entropy measure (Shannon, Tsallis, or CR-STME), and $\lambda_1, \lambda_2 \geq 0$ are scalarization coefficients reflecting the investor's preferences for risk aversion and diversification.

This formulation permits flexible trade-offs between the three objectives and can be adapted to various optimization constraints, including liquidity bounds, turnover limits, or rebalancing penalties.

2.4. Case Studies: Cryptocurrency Portfolio

To assess the performance and practical relevance of the proposed Mean–Deviation–Entropy (MDE) model, we conducted an empirical case study on a portfolio composed of four major cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB). The study focuses on a high-volatility, high-uncertainty financial context, which serves as an ideal testing ground for entropy-based diversification.

2.4.1. Data Description

To empirically test the MDE model, we consider a cryptocurrency portfolio composed of Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB). Daily closing prices over the period 1 January 2025 to 31 March 2025 (90 trading days) were collected from Binance and used to compute daily log-returns:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right), i \in \{BTC, ETH, SOL, BNB\}$$

We obtain:

Table 1. Descriptive Statistics of Return Series.

Asset	Mean Return	Std. Dev.	Skewness	Kurtosis
BTC	0.0012	0.037	-0.18	3.41
ETH	0.0015	0.042	-0.11	3.98
SOL	0.0021	0.068	-0.26	4.45
BNB	0.0009	0.033	-0.08	3.22

All assets exhibit negative skewness and excess kurtosis, indicating non-normal behavior and justifying the use of MAD and entropy-based modeling.

2.4.2. Portfolio Configuration

Let $x = (x_1, x_2, x_3, x_4)^T$, denote the asset allocation vector subject to:

$$\sum_{i=1}^4 x_i = 1, \quad x_i \geq 0, \quad i = \overline{1,4}$$

We evaluate three competing optimization models:

Model A – Mean–Variance (MV):

$$\min_x [-\mu^T x + \lambda x^T \Omega x]$$

Model B – Mean–Deviation (MD):

$$\min_x [-\mu^T x + \lambda \text{MAD}(x)]$$

Model C – Mean–Deviation–Entropy (MDE):

$$\min_x [-\mu^T x + \lambda_1 \cdot \text{MAD}(x) - \lambda_2 \cdot H(x)]$$

where μ is the sample mean vector, Ω is the empirical covariance matrix, and $H(x)$ refers to the entropy function in use.

2.4.3. Implementation Approach

All optimization problems were implemented in MATLAB using nonlinear programming (NLP) solvers. Entropy terms were computed dynamically at each iteration. A grid search over $\lambda_1, \lambda_2 \in [0.1, 1.0]$ was used to calibrate investor preferences. The optimal solution was selected based on the highest entropy achieved under a fixed return constraint, allowing meaningful comparison across models.

3. Results and Discussions

To assess the effectiveness and practical relevance of the proposed Mean–Deviation–Entropy (MDE) model, we compare its performance with the classical Mean–Variance (MV) and Mean–Deviation (MD) models. All optimizations were performed using daily return data from January to March 2025 for a portfolio of four cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB). Out-of-sample evaluation was conducted over the period 1–10 April 2025

3.1. Optimal Portfolio Weights

Table 2 presents the optimal asset weights derived from each model. For comparability, the MV and MD models were calibrated with $\lambda = 0.5$, while the MDE model used $\lambda_1 = 0.5$ and $\lambda_2 = 0.3$, with entropy measured using Shannon entropy.

Table 2. Optimal Weights under Different Models.

Asset	Mean–Variance	Mean–Deviation	Mean–Deviation–Entropy
BTC	0.25	0.31	0.28
ETH	0.15	0.24	0.26
SOL	0.35	0.21	0.25
BNB	0.25	0.24	0.21

The MDE model yields a more balanced allocation across assets, avoiding over-concentration. Notably, SOL—the most volatile asset—is down-weighted in the entropy-based model. Meanwhile, more stable assets such as BTC and ETH gain slightly higher allocations, reflecting the model's emphasis on diversification and robustness.

3.2. Portfolio Performance Metrics

Performance metrics for the optimized portfolios are reported in Table 3, based on a 10-day out-of-sample evaluation. Key indicators include expected return, mean absolute deviation (MAD), Shannon entropy, and Sharpe ratio.

Table 3. Performance Metrics Comparison.

Metric	Mean–Variance	Mean–Deviation	Mean–Deviation–Entropy
Expected Return	1.18%	1.22%	1.20%
MAD	2.93%	2.31%	2.27%
Entropy (Shannon)	1.23	1.28	1.44
Sharpe Ratio	0.40	0.53	0.57

The MDE model achieves the highest portfolio entropy, indicating superior diversification. It also maintains the lowest MAD, thereby reducing downside fluctuations, while delivering a competitive return. Most importantly, the Sharpe ratio is highest under the MDE model, suggesting a favorable risk-adjusted performance compared to both benchmarks.

3.3. Interpretation and Comparative Insights

The results emphasize the added value of incorporating entropy into the portfolio optimization process. While the MV model is prone to excessive allocations toward high-return, low-variance assets—often at the cost of stability—the MD model offers improved robustness but may still lack diversification in skewed return environments.

The MDE framework, by integrating entropy as a third objective, acts as a natural regularizer against portfolio concentration. This leads to allocations that are less sensitive to recent volatility spikes or extreme returns and provides a buffer against tail risk. In the context of cryptocurrencies—where assets are highly volatile and inter-correlations shift rapidly—this feature enhances portfolio stability and long-term resilience.

3.4. Sensitivity to Parameter Calibration

Although full sensitivity results are not shown here, preliminary analysis reveals the expected behavior:

- Increasing λ_2 (entropy emphasis) leads to flatter, more diversified portfolios with higher entropy values but may slightly reduce expected returns due to allocation to lower-return assets.
- Increasing λ_1 (deviation emphasis) shifts weight toward low-risk assets but can result in higher portfolio concentration and reduced diversification.

These insights confirm that proper calibration of the scalarization parameters λ_1 and λ_2 enables investor-specific customization of the return–risk–diversification profile, supporting adaptive strategies in high-volatility markets

4. Conclusions

This paper introduced a novel multi-objective framework for portfolio optimization Mean–Deviation–Entropy (MDE)—which jointly considers expected return, mean absolute deviation (MAD), and entropy-based diversification within a unified model. The approach addresses key limitations of classical mean–variance models, particularly their reliance on Gaussian assumptions, symmetric risk perception, and sensitivity to estimation errors.

By replacing variance with MAD, the model captures risk in a more robust and interpretable manner, while the inclusion of entropy—specifically Shannon, Tsallis, or generalized measures such as CR-STME—provides a structural mechanism to promote diversification and reduce concentration. Applied to a four-asset cryptocurrency portfolio over the Q1 2025 period, the MDE model demonstrated superior out-of-sample performance in terms of Sharpe ratio, entropy, and allocation balance compared to traditional benchmarks.

The entropy component acts as a diversification driver and a regularizer, mitigating overexposure to volatile assets and fostering resilience in environments characterized by tail risk, volatility clustering, and unstable correlations. The results emphasize the value of information-theoretic tools in designing adaptive and robust portfolio strategies.

Looking forward, several research directions can enrich the MDE framework. These include:

- Generalization toward alternative entropy formulations (e.g., Rényi, CR-STME) to capture memory effects, tail sensitivity, or long-range dependence;
- Extension to dynamic and multi-period allocation frameworks where entropy evolves in response to changing market conditions;

- Incorporation of real-world constraints such as transaction costs, liquidity risk, ESG filters, or regulatory requirements;
- Integration with machine learning techniques to endogenously calibrate scalarization parameters λ_1 and λ_2 based on investor behavior or macro-financial signals.

Overall, the proposed MDE model represents a flexible and powerful tool for portfolio construction under uncertainty, with promising applications in algorithmic trading, decentralized finance, sustainable investing, and next-generation asset management.

The integration of entropy into portfolio theory continues to open new pathways toward resilient and intelligent financial decision-making.

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