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Article

Disproving the Existence of Global Smooth Solutions to the Three-Dimensional Navier-Stokes Equations for Plane Poiseuille Flow

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Abstract

Existence of global smooth solutions to the three-dimensional (3D) Navier-Stokes equations is disproved for pressure-driven plane Poiseuille flow with no-slip boundary conditions. This study is rigorously grounded in Sobolev space analysis. We show that the solution breakdown arises from the regularity degeneration instead of velocity blow-up. For disturbed laminar plane Poiseuille flow, the instantaneous velocity field is decomposed into a time-averaged flow and a disturbance flow, both characterized by their regularity in Sobolev spaces. When the Reynolds number is larger than the critical Reynolds number, the nonlinear interaction modifies the mean flow profile, and the disturbance amplitude grows significantly. This amplification leads to a local cancellation between viscous terms of the mean flow and the disturbance flow, resulting in the total viscous term (i.e., the Laplacian term) vanishing locally at the critical point $(\boldsymbol{x}^*, t^*)$. The local vanishing viscous term leads to zero velocity according to the Energy-Velocity Monotonicity Principle (EVMP), which contradicts the non-vanishing incoming velocity, leading to formation of a singularity. This singularity induces a velocity discontinuity, which causes the L^∞ -norm of the velocity gradient to diverge, violating the definition of a global smooth solution in Sobolev spaces. The analysis is strictly grounded in partial differential equations (PDE) theory, with all key steps validated by Sobolev space properties and a priori estimates.

Keywords: Navier-Stokes equations; Sobolev space; regularity degeneration; singularity; discontinuity; turbulence

MSC: 76D03; 76D05;35A01; 35A02; 35Q30; 46E35

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1. Introduction

The regularity of global solutions to the three-dimensional (3D) Navier-Stokes equations remains one of the most fundamental open problems in partial differential equations (PDE) and mathematical fluid mechanics (Doering 2009; Foias et al. 2004). Selected as a Millennium Prize problem by the Clay Mathematics Institute (Fefferman 2006), its solution requires knowledge and methods from disciplines including mathematics, physics, numerical computation, as well as their interdisciplinary fields. A rigorous PDE framework such as grounding analysis in functional analytic tools—most notably Sobolev spaces—can be a feasible approach if an appropriate flow model, consistent with the physical flow, is employed.

Early studies laid foundational groundwork: Leray (1934) established the existence of weak solutions in $L^2(0,T;H^1(\Omega)) \cap C([0,T];L^2(\Omega))$ and conjectured finite-time singularities (FTS) in 3D turbulent flows. Ladyzhenskaya (1969) proved global smoothness for two-dimensional (2D) flows via H^2 -regularity estimates. In the past 50 years or so, mathematicians have tried to explore the regularity of solutions to the 3D Navier-Stokes equations from the weak solutions (Scheffer 1976; Cafarelli et al. 1982; Serrin 1962; Berselli and Galdi 2002). Buckmaster and Vicol (2019) demonstrated non-uniqueness

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of weak solutions of the 3D Navier-Stokes equations. Recently, Coiculescu and Palasek (2025) proved non-uniqueness of smooth solutions of the Navier–Stokes equations using constructed initial data from critical space. However, the reason causing the non-uniqueness of smooth solutions is still not clear. On the other hand, Tao (2016) constructed an averaged three-dimensional Navier-Stokes equations, and proposed a possible approach for the blow-up solution to the Navier-Stokes equations. Robinson (2020) reviewed advances in this field and concluded that weak (finite-energy) solutions to the Navier-Stokes equations may not be unique, but exist for all time, while strong (finite-enstrophy) solutions are unique but cannot persist for all time. These studies further highlight the need for strict regularity constraints.

Exploring the regularity problem of solutions to the Navier-Stokes equations is essentially searching for singular points from the mathematical characteristics of the equations. According to the definition in mathematics, singularities are generally classified into two categories: one where the function is infinitely large (blow-up), and the other where the function is not differentiable. In the past, most researches on the regularity of the Navier-Stokes equations focused on exploring singularities with infinite velocity or kinetic energy, while relatively less attention was paid to non-differentiable singularities. Regarding the generation of turbulence, numerical simulations have demonstrated that turbulence arises precisely due to the second type of singularity (Dou 2022; Dou 2025; Schlatter et al. 2006; Kachanov 1994; Tiwari et al. 2019; Niu et al. 2024; Niu et al. 2025; Zhou et al. 2025a; Zhou et al. 2025b).

Numerical simulations and experiments confirmed that laminar-turbulent transition based on the Navier-Stokes equations depends on both the Reynolds number and the disturbances (Dou 2022; Hof et al. 2003; Khan et al. 2021), but these observations must be translated into PDE regularity conditions. Although recent work (Dou 2025) linked velocity discontinuities to the onset of turbulent transition, it lacked a rigorous Sobolev space foundation. This study fills this gap: we use Sobolev spaces to define solution regularity, derive a priori estimates for mean and disturbance flows, and prove that singularity-induced regularity breakdown contradicts global smoothness. Unlike previous studies focusing on velocity blow-up singularities (Leray, 1934), this work identifies a new type of singularity caused by regularity breakdown, which directly explains the onset of turbulence in PDE frameworks.

The core of the research is to investigate whether singularities will form within a finite time when the Reynolds number exceeds the critical value. The initial flow field is a smooth laminar flow of the plane Poiseuille flow. Under the influence of disturbances, the flow evolves over time and generates distortions. If no singularities occur during the flow process, the flow remains smooth; conversely, if singularities form, the flow will lose regularity, and the smooth solution cannot extend beyond the singularities, resulting in non-existence of a global smooth solution. Therefore, the existence of a global smooth solution to the Navier-Stokes equation is negated.

1.1. Key Definitions and Notations (Sobolev Space Framework)

We adopt the standard notations for Sobolev spaces and function spaces in the study of partial differential equations, as established in classical references (Adams and Fournier 2003; Brezis 2011; Evans 2010).

Let $\Omega \subset \mathbb{R}^3$ be a bounded smooth domain with boundary $\partial\Omega$. For a non-negative integer k, the Sobolev space $H^k(\Omega)$ is a Hilbert space consisting of all functions $f \in L^2(\Omega)$ for which all weak partial derivatives $D^\alpha f$ (with multi-indices α satisfying $|\alpha| \le k$) also belong to $L^2(\Omega)$. The inner product on $H^k(\Omega)$ is defined by

$$\langle f, g\rangle_{H^k(\Omega)} = \sum_{|\alpha|\le k} \langle D^\alpha f, D^\alpha g\rangle_{L^2(\Omega)},$$

where $\langle \cdot,\cdot\rangle_{L^2(\Omega)}$ denotes the standard inner product on the $L^2(\Omega)$ space.

Sobolev Space $H^k(\Omega)$: For a bounded smooth domain $\Omega \subset \mathbb{R}^3$ and integer $k \ge 0$, $H^k(\Omega) = \{f \in L^2(\Omega) \mid D^\alpha f \in L^2(\Omega)\}$ for all multi-indices α with $|\alpha| \le k$, equipped with the norm:

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$$\|f\|_{H^k(\Omega)} = \left(\sum_{|\alpha| \le k} \|D^\alpha f\|^2_{L^2(\Omega)} \right)^{1/2} \tag{1}$$

where $D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \partial x_3^{\alpha_3}}$ denotes the weak partial derivative operator and $|\alpha| = \alpha_1 + \alpha_2 + \alpha_3$ is the order of the multi-index $\alpha = (\alpha_1, \alpha_2, \alpha_3)$.

Local Sobolev Space $H^k_{\mathbf{loc}}(\Omega)$: A function f belongs to $H^k_{\mathrm{loc}}(\Omega)$ if $f \in H^k(\Omega')$ for every open subset $\Omega' \Subset \Omega$ (with $\Omega' \Subset \Omega$ indicating that $\overline{\Omega'}$ is compact and contained in Ω).

Continuous Function Spaces $C([0,T]; H^k(\Omega))$: This space consists of all continuous mappings from the time interval $[0,T]$ to the Sobolev space $H^k(\Omega)$, endowed with the supremum norm

$$\|f\|_{C([0,T];H^k(\Omega))} = \sup_{t \in [0,T]} \|f(\cdot,t)\|_{H^k(\Omega)}.$$

Global Smooth Solution: A vector-valued solution $\boldsymbol{u}$ to the three-dimensional Navier-Stokes equations is said to be globally smooth if there exists an integer $k \ge 3$ such that
$\boldsymbol{u} \in C([0,\infty); H^k(\Omega)) \cap L^2_{\mathrm{loc}}([0,\infty); H^{k+1}(\Omega))$ and $\nabla \boldsymbol{u} \in C([0,\infty); H^{k-1}(\Omega))$.

A key necessary and sufficient condition for such global smoothness is that $\|\nabla \boldsymbol{u}(t)\|_{L^\infty(\Omega)} < \infty$ for all $t \ge 0$, which is a direct consequence of the **Sobolev embedding theorem** in $\mathbb{R}^3$: $H^2(\Omega) \hookrightarrow L^\infty(\Omega)$, where the symbol $\hookrightarrow$ denotes continuous embedding.

Norm Conventions:

(1) Pointwise absolute value: $|f(\boldsymbol{x},t)|$ denotes the pointwise absolute value of a scalar- or vector-valued function f at the point $(\boldsymbol{x},t) \in \Omega \times [0,\infty)$.

(2) Global L^p -norm: For $1 \le p < \infty$, the $L^p(\Omega)$ -norm is defined by

$$\|f\|_{L^p(\Omega)} = \left(\int_\Omega |f|^p d\boldsymbol{x} \right)^{1/p},$$

and the essential supremum norm $L^\infty(\Omega)$ is given by

$$\|f\|_{L^\infty(\Omega)} = \text{ess sup}_{\boldsymbol{x} \in \Omega} |f(\boldsymbol{x})|.$$

(3) Norm of vector-valued functions in three-dimensional spaces: For a vector-valued function $\boldsymbol{u} = (u_1, u_2, u_3)^T$ on Ω, its $H^k(\Omega)$ -norm is defined by

$$\|\boldsymbol{u}\|_{H^k(\Omega)} = \left(\sum_{|\alpha| \le k} \|D^\alpha \boldsymbol{u}\|^2_{L^2(\Omega)} \right)^{1/2}, \tag{2}$$

where the $L^2(\Omega)$ -norm of the weak derivative $D^\alpha \boldsymbol{u}$ is given by

$$\|D^\alpha \boldsymbol{u}\|^2_{L^2(\Omega)} = \int_\Omega \sum_{i=1}^3 |D^\alpha u_i|^2 d\boldsymbol{x}.$$

For the bounded smooth domain $\Omega \subset \mathbb{R}^3$, the Sobolev spaces satisfy the natural continuous embedding for all integers $m \ge n \ge 0$: $H^m(\Omega) \hookrightarrow H^n(\Omega)$. In particular, $H^3(\Omega) \hookrightarrow H^2(\Omega)$ and $H^4(\Omega) \hookrightarrow H^3(\Omega)$, meaning every function in a higher-order Sobolev space is contained in all lower-order Sobolev spaces, with the embedding operator being bounded (continuous).

2. Governing Equations and Flow Decomposition (Sobolev Regularity)

2.1. Navier-Stokes and Continuity Equations

The 3D Navier-Stokes system for incompressible flow in $\Omega = (0, L_x) \times (0, 2h) \times (0, L_z)$ (no-slip boundaries $\boldsymbol{u}(x, -h, z, t) = \boldsymbol{u}(x, h, z, t) = 0$, where $2h$ is the width of the channel between the two plates in plane Poiseuille flow) is (without external forces):

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$$\begin{cases} \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nu\Delta\boldsymbol{u} + \frac{1}{\rho}\nabla p = 0, & (x,y,z,t) \in \Omega\times(0,\infty) \\ \nabla\cdot\boldsymbol{u} = 0, & (x,y,z,t) \in \Omega\times(0,\infty) \\ \boldsymbol{u}(\cdot,0) = \boldsymbol{u}_0 \in H^3_{\sigma,0}(\Omega), \end{cases} \tag{3}$$

where $H^3_{\sigma,0}(\Omega) = \{f \in H^3(\Omega) \mid \nabla\cdot f = 0, f|_{\partial\Omega} = 0\}$ (divergence-free, no-slip Sobolev space), $\rho > 0$ (density), $\nu > 0$ (kinematic viscosity), and p (pressure) belongs to $L^2_{\text{loc}}([0,\infty); H^2(\Omega))$ (by standard PDE regularity for Navier-Stokes equations).

Here, we denote the velocity vector as $\boldsymbol{u} = (u_x, u_y, u_z)$ and the position vector as $\boldsymbol{x} = (x, y, z)$.

Detailed initial conditions $\boldsymbol{u}_0 \in H^3_{\sigma,0}(\Omega)$ are as follows: the initial laminar velocity field is a steady plane Poiseuille flow superposed with small-amplitude disturbances: $\boldsymbol{u}_0 = \boldsymbol{U}_0(y) + \boldsymbol{v}_0(x,y,z)$, where $\boldsymbol{U}_0(y) = -\frac{1}{2\mu}\frac{dp}{dx}(h^2 - y^2)\boldsymbol{i}$ ($\boldsymbol{i}$ is the unit vector in x direction) and $\boldsymbol{v}_0$ (disturbance flow) satisfies $\nabla\cdot\boldsymbol{v}_0 = 0$ and $\|\boldsymbol{v}_0\|_{H^3(\Omega)} \ll \|\boldsymbol{U}_0\|_{H^3(\Omega)}$ (small-disturbance assumption). The dynamic viscosity of the fluid is given by $\mu = \nu\rho$.

It is assumed that the disturbance amplitude is “small" relative to the mean velocity in $H^1(\Omega)$, such that the instantaneous velocity is always kept positive in the range of laminar flow, $\boldsymbol{u} > 0$ in $\Omega\times[0,T)$ (except at $(x,-h,t)$ and (x,h,t), where $\boldsymbol{u} = 0$ for the no-slip boundary condition, as in Fig.1).

In this study, the plane Poiseuille flow with disturbances serves as an appropriate flow model for analyzing the solution regularity of the Navier-Stokes equations, as abundant computational and experimental data provide substantial information.

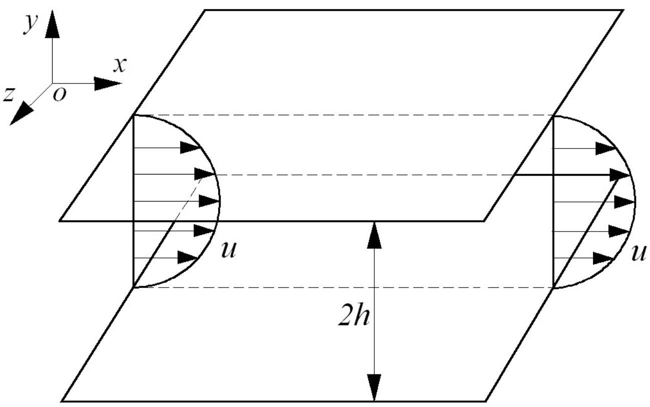

Figure 1. Schematic of initial flow profile in plane Poiseuille flow, $\boldsymbol{u}_0(x,y,z)$. The origin of the coordinates is located at the centerline of the channel between the two plates, $y = h$ at upper wall, and $y = -h$ at lower wall.

2.2. *Reynolds Number Range in Present Study*

It is well-established that the Reynolds number is an important dimensionless parameter for characterizing the transition of a smooth laminar flow to turbulence. Numerical simulations with Navier-Stokes equations and experiments showed that turbulent transition depends on both the Reynolds number and the magnitude of disturbance (Jovanovic and Pashtrapanska 2004; Hof et al. 2003; Dou and Khoo 2012). Only when the Reynolds number is larger than a minimum critical value, Re_{cr}, can turbulent transition be possible. At $Re > Re_{cr}$, the disturbance amplitude required for turbulent transition decreases with the increasing Reynolds number (Hof et al. 2003; Dou 2022).

For plane Poiseuille flow, the Reynolds number is defined as

$$Re = \frac{U_{x0c}h}{\nu} \tag{4}$$

where U_{x0c} is the centerline velocity on the initial velocity profile along the x direction, h is the half width of the channel, and ν is the kinematic viscosity. For plane Poiseuille flow, the critical value of the Reynolds number, $Re_{cr} = 1130$, was obtained in Jovanovic and Pashtrapanska (2004, their Eq.(27)),

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which agrees well with extensive experimental results (Dou 2022). The range of the Reynolds number considered in this study is $Re > Re_{cr}$.

2.3. Flow Decomposition in Sobolev Spaces

Flows described by the Navier-Stokes equations include both laminar and turbulent flows. Experiments and numerical simulations based on the Navier-Stokes equations indicate that the generation of turbulence is influenced by both the Reynolds number and the development of disturbances (Dou 2022; Hof et al. 2003). Therefore, researches on the regularity of the Navier-Stokes equations should accurately consider the impact of disturbances besides the Reynolds number.

The instantaneous velocity in three-dimensional spaces is decomposed into a time-averaged flow and a disturbance flow such as $\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{v}$, where:

Time-Averaged Flow $\boldsymbol{U}$: Defined by

$$\boldsymbol{U}(x,y,z,t) = \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} \boldsymbol{u}(x,y,z,\tau)d\tau, \tag{5}$$

with Δt satisfying $\Delta t \gg \tau_d$ (disturbance characteristic time) and $\Delta t \ll t^*$, where t^* denotes the first possible time of singularity appearance in the velocity field. By the boundedness of the time-averaging integral operator on Sobolev spaces (Brezis 2011; Demengel and Demengel 2012) and the inherited regularity of $\boldsymbol{u} \in C([0,t^*);H^3(\Omega;\mathbb{R}^3))$, the time-averaged flow retains the same regularity as the instantaneous velocity, i.e., $\boldsymbol{U} \in C([0,t^*);H^3(\Omega;\mathbb{R}^3))$.

The mean flow $\boldsymbol{U}(t)$ is a function of time. It evolves due to the nonlinear interaction between the mean flow and the disturbance flow. It does not remain constant as that in the linear stability analysis and thus it is different from the initial laminar profile, except possibly at $t = 0$. In such a way, we are able to keep the disturbance always being small relative to the mean flow expressed by Eq.(5).

Disturbance Flow $\boldsymbol{v}$: $\boldsymbol{v} = \boldsymbol{u} - \boldsymbol{U}$, which satisfies the zero-mean condition in time:

$$\frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} \boldsymbol{v}(\tau)d\tau = 0 \tag{6}$$

where $\boldsymbol{v} \in C([0,t^*);H^3(\Omega))$ (inherited from $\boldsymbol{u}$ and $\boldsymbol{U}$'s regularity).

Since both $\boldsymbol{u}$ and $\boldsymbol{U}$ belong to $C([0,t^*);H^3(\Omega;\mathbb{R}^3))$, and Sobolev spaces are closed under vector subtraction (Adams and Fournier 2003), the disturbance flow inherits the same regularity: $\boldsymbol{v} \in C([0,t^*);H^3(\Omega;\mathbb{R}^3))$.

2.4. Mathematical Justification of Decomposition:

The velocity decomposition is in the Hilbert space $H^3(\Omega;\mathbb{R}^3)$ (Brezis 2011; Demengel and Demengel 2012).

2.4.1. Uniqueness and Linearity of the Decomposition

For a given solution $\boldsymbol{u}$ and a chosen time window Δt, the decomposition $(\boldsymbol{U},\boldsymbol{v})$ is uniquely determined by the explicit formulas in Eq.(5). The mapping $\boldsymbol{u}(\cdot,t) \mapsto (\boldsymbol{U}(\cdot,t),\boldsymbol{v}(\cdot,t))$ is a linear projection.

Formally, let us define the time-averaging operator $\mathcal{T}_{\Delta t}$ acting on the space $C([0,T^*);H^3(\Omega))$ as:

$$(\mathcal{T}_{\Delta t}\boldsymbol{u})(\cdot,t) = \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} \mathbf{u}(\cdot,\tau)\,d\tau. \tag{7}$$

Then $\boldsymbol{U} = \mathcal{T}_{\Delta t}\boldsymbol{u}$ and $\boldsymbol{v} = (I - \mathcal{T}_{\Delta t})\boldsymbol{u}$. The operator $\mathcal{T}_{\Delta t}$ is linear and idempotent $\mathcal{T}_{\Delta t}^2 = \mathcal{T}_{\Delta t}$ in the limit as $\Delta t \to 0$, or for functions affine in time over $[t-\Delta t/2, t+\Delta t/2]$. Here, $T^* > 0$ expresses the maximal existence time of the solution.

Therefore, it is a projection operator. This establishes that, for a fixed Δt, the decomposition represents a unique, continuous splitting of the velocity field into a “slowly varying" component ($\boldsymbol{U}$, in

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the range of $\mathcal{T}_{\Delta t}$) and a "rapidly fluctuating" component with zero local mean ($\boldsymbol{v}$, in its complement). The uniqueness is inherent in the definition.

2.4.2. Boundedness of the Time-Averaged Flow

A crucial property for the subsequent singularity analysis is that the mean flow $\boldsymbol{U}$ remains bounded both above and, importantly, bounded below away from zero as $t \to t^*$. This reflects the physical constraint of an externally imposed pressure gradient that sustains the flow.

(1). Upper Bound:

Since the solution is assumed to be smooth on $[0, t^*)$, its Sobolev norm $\|\boldsymbol{u}(t)\|_{H^3(\Omega)}$ is finite for all $t < t^*$. The time-averaging operator $\mathcal{T}_{\Delta t}$ is a bounded linear operator from $C([0,T^*); H^3)$ to H^3. Therefore,

$$\|\boldsymbol{U}(t)\|_{H^3(\Omega)} = \|\mathcal{T}_{\Delta t}\boldsymbol{u}(t)\|_{H^3(\Omega)} \le \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} \|\boldsymbol{u}(\tau)\|_{H^3(\Omega)}\, d\tau \le \sup_{\tau\in[t-\Delta t/2, t+\Delta t/2]} \|\boldsymbol{u}(\tau)\|_{H^3(\Omega)} < \infty. \tag{8}$$

The supremum is finite because the solution is smooth on the closed interval $[t - \Delta t/2, t + \Delta t/2] \subset [0, T^*)$.

Thus, we can write,

$$\|\boldsymbol{U}(t)\|_{H^3(\Omega)} \le C_1 \tag{9}$$

where C_1 is a positive constant.

(2). Lower Bound:

Let $\boldsymbol{u} \in C([0,t^*); H^3_{\sigma,0}(\Omega))$ be the solution to the Navier-Stokes equations with a non-zero streamwise pressure gradient $\partial_x p = G < 0$. Let $\boldsymbol{U}$ be defined as in Eq.(5). Then, there exists a positive constant $C_2 > 0$, independent of t for t sufficiently close to t^*, such that:

$$\inf_{x\in\Omega_0} \boldsymbol{U}(x,t) \ge C_2 > 0. \quad \text{Consequently,} \quad \|\boldsymbol{U}(t)\|_{L^\infty(\Omega)} \ge C_2. \tag{10}$$

This is because the non-zero pressure gradient along the streamwise direction requires that there is shear of mean velocity to balance the pressure gradient, hence the mean velocity in most width of the channel is larger than zero. In fact, there exists a subset $\Omega_0 \subset \Omega$ of positive measure (e.g., the core region of the flow away from viscous boundary layers) for Eq.(10). Since $\boldsymbol{U}$ is positive and continuous on Ω_0 (due to Sobolev embedding), its positive lower bound implies that its L^∞ norm is greater than or equal to this positive number on a set of positive measure.

For laminar flow, the time-averaged flow dominates the disturbance flow such that $|\boldsymbol{U}(\boldsymbol{x})| \gg |\boldsymbol{v}(\boldsymbol{x})|$ for all $\boldsymbol{x} \in \Omega$. Assuming $\|\boldsymbol{U}\|_{H^3(\Omega)} \le \|\boldsymbol{v}\|_{H^3(\Omega)}$ contradicts the pointwise dominance of the time-averaged flow (which implies $\|\boldsymbol{U}\|_{L^2(\Omega)} \gg \|\boldsymbol{v}\|_{L^2(\Omega)}$, and the L^2-norm is embedded in the H^3-norm for bounded domains). Thus, the only valid conclusion is:

$$\|\boldsymbol{U}\|_{H^3(\Omega)} \gg \|\boldsymbol{v}\|_{H^3(\Omega)}. \tag{11}$$

This norm inequality is a direct consequence of the boundedness of the projection operator and Sobolev space properties, and it quantifies the dominance of the time-averaged flow over the disturbance flow for laminar flow conditions.

3. Preliminaries (Sobolev Space-Based Derivations)

3.1. Local Vanishing of the Total Viscous Term (Sobolev Linearity)

The viscous term in the Navier-Stokes equations is $\nu\Delta\boldsymbol{u}$ (Eq.(3)), with $\Delta = \nabla^2$ (Laplacian, a linear elliptic operator). For $\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{v}$ with $\boldsymbol{U} \in H^3(\Omega)$ and $\boldsymbol{v} \in H^3(\Omega)$, the linearity of Δ and closedness of Sobolev spaces under linear operators directly imply:

$$\Delta\boldsymbol{u} = \Delta\boldsymbol{U} + \Delta\boldsymbol{v}, \quad \text{and} \quad \nu\Delta\boldsymbol{u} \in C([0,t^*); H^1(\Omega)) \tag{12}$$

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In terms of Sobolev space regularity, for each term in Eq.(12), if $f \in H^k(\Omega)$, then $\Delta f \in H^{k-2}(\Omega)$. Since $\boldsymbol{u} \in H^3(\Omega)$, $\boldsymbol{U} \in H^3(\Omega)$ and $\boldsymbol{v} \in H^3(\Omega)$, we have $\Delta \boldsymbol{u} \in H^1(\Omega)$, $\Delta \boldsymbol{U} \in H^1(\Omega)$ and $\Delta \boldsymbol{v} \in H^1(\Omega)$.

3.1.1. Continuity and Boundedness of $\Delta \boldsymbol{U}(t)$ and $\Delta \boldsymbol{v}(t)$

In the following, the continuity and boundedness of $\Delta \boldsymbol{U}(t)$ and $\Delta \boldsymbol{v}(t)$ are proven by leveraging the regularities of $\boldsymbol{U}(t)$ and $\boldsymbol{v}(t)$ (where $\boldsymbol{U} \in H^3(\Omega)$ and $\boldsymbol{v} \in H^3(\Omega)$), the Sobolev embedding theorem, and the elliptic regularity theory.

Let $\Omega \subset \mathbb{R}^3$ be a bounded smooth domain with compact closure $\overline{\Omega}$, and let $\boldsymbol{U}, \boldsymbol{v} \in H^3(\Omega)$ be the divergence-free mean velocity and disturbance velocity fields, respectively. By the Morrey-Sobolev embedding theorem for 3D bounded smooth domains, $H^3(\Omega) \hookrightarrow C^1(\overline{\Omega})$, confirming global continuity of first-order derivatives for both fields. For any precompact smooth interior subdomain $\Omega' \Subset \Omega$ (avoiding boundary regularity degradation), local elliptic regularity theory for second-order linear elliptic operators lifts the regularity of $\boldsymbol{U}$ and $\boldsymbol{v}$ to $C^2(\overline{\Omega'})$ on this compact interior set. As the Laplacian $\Delta \boldsymbol{w}$ is a linear combination of second-order partial derivatives of a three-dimensional vector $\boldsymbol{w}$, both $\Delta \boldsymbol{U}$ and $\Delta \boldsymbol{v}$ are classically continuous on $\overline{\Omega'}$ and uniformly bounded therein for all time under consideration. This local continuity is sufficient to guarantee pointwise cancellation of the two Laplacian terms at any interior point of Ω at sufficient sequential time increment, which is the core requirement for the subsequent flow analysis.

3.1.2. Disturbance Is Amplified by Nonlinear Term

To quantitatively prove the growth of disturbance amplitude amplified by the nonlinear convective term, we apply the Sobolev product estimate (Adams and Fournier, 2003). For the nonlinear convective term $(\boldsymbol{U} \cdot \nabla)\boldsymbol{v}$, since $\boldsymbol{U} \in H^3(\Omega)$ and $\nabla \boldsymbol{v} \in H^2(\Omega)$, the Adams product estimate gives $\|(\boldsymbol{U} \cdot \nabla)\boldsymbol{v}\|_{H^2(\Omega)} \leq C\|\boldsymbol{U}\|_{H^3(\Omega)}\|\nabla \boldsymbol{v}\|_{H^2(\Omega)}$, where C is a positive constant independent of $\boldsymbol{U}$ and $\boldsymbol{v}$.

For a bounded smooth domain $\Omega \subset \mathbb{R}^3$, the norms $\|\nabla \boldsymbol{v}\|_{H^2(\Omega)}$ and $\|\nabla^2 \boldsymbol{v}\|_{H^1(\Omega)}$ are mutually equivalent by the definition of Sobolev norms and further equivalent to $\|\Delta \boldsymbol{v}\|_{H^1(\Omega)}$ (since $\Delta \boldsymbol{v} = \nabla \cdot \nabla \boldsymbol{v}$ for vector fields). Thus, for the disturbance increase, the convective term dominates over the viscous term, then we have

$$\|\Delta \boldsymbol{v}\|_{H^1(\Omega)} \leq C\|(\boldsymbol{U} \cdot \nabla)\boldsymbol{v}\|_{H^2(\Omega)} \leq C\|\boldsymbol{U}\|_{H^3(\Omega)}\|\Delta \boldsymbol{v}\|_{H^1(\Omega)}. \tag{13}$$

When $Re > Re_{cr}$, the mean flow $\boldsymbol{U}$ satisfies $\|\boldsymbol{U}\|_{H^3(\Omega)} \leq C_1$ (a bounded constant) due to the regularity of the time-averaged flow (Galdi, 2011). This implies that the disturbance term $\|\Delta \boldsymbol{v}\|_{H^1(\Omega)}$ grows proportionally to itself with a positive proportional coefficient, leading to the exponential growth of $\|\Delta \boldsymbol{v}\|_{H^1(\Omega)}$ over time. Since $\Delta \boldsymbol{U}(t)$ is uniformly bounded in $\Omega \times [0, t^*)$ (as proved in the previous paragraph), the amplitude of $\Delta \boldsymbol{v}(t)$ will eventually reach the magnitude of $|\Delta \boldsymbol{U}(t)|$ at some finite time t^* and some position $\boldsymbol{x}^* \in \Omega$. Therefore, there exists $(\boldsymbol{x}^*, t^*) \in \Omega \times (0, \infty)$ such that $\Delta \boldsymbol{v}(\boldsymbol{x}^*, t^*) = -\Delta \boldsymbol{U}(\boldsymbol{x}^*, t^*)$, which induces the local vanishing of the composite viscous term $\nu \Delta \boldsymbol{u} = \nu(\Delta \boldsymbol{U} + \Delta \boldsymbol{v}) = 0$ at $(\boldsymbol{x}^*, t^*)$.

3.1.3. Disturbance Term $\Delta \boldsymbol{v}$ Reaching $\Delta \boldsymbol{U}$ Leads to $\Delta \boldsymbol{u} = 0$ Locally

For sufficiently large Reynolds number $Re > Re_{cr}$, the nonlinear term $(\boldsymbol{v} \cdot \nabla)\boldsymbol{U} + (\boldsymbol{U} \cdot \nabla)\boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$ amplifies $\boldsymbol{v}(t)$, leading to:

$$\exists (\boldsymbol{x}^*, t^*) \in \Omega \times (0, \infty) : \Delta \boldsymbol{v}(\boldsymbol{x}^*, t^*) = -\Delta \boldsymbol{U}(\boldsymbol{x}^*, t^*), \tag{14}$$

hence $\Delta \boldsymbol{u}(\boldsymbol{x}^*, t^*) = 0$ and $\nu \Delta \boldsymbol{u}(\boldsymbol{x}^*, t^*) = 0$.

By $H^1(\Omega)$ continuity of $\nu \Delta \boldsymbol{u}$,

$$\lim_{t \to t^*} \|\nu \Delta \boldsymbol{u}(\cdot, t)\|_{H^1(B_\epsilon(\boldsymbol{x}^*))} = 0, \tag{15}$$

for all $\epsilon > 0$ (local vanishing in Sobolev norm).

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3.2. Energy-Velocity Monontonicity Principle (EVMP)

In Dou (2025), an axiom was proposed based on physical intuition and observations. The axiom gives "The velocity of a fluid particle varies monotonically with the energy loss during its motion, and vice versa."

According to this axiom, the velocity of a fluid particle is always expensed by the energy loss along the streamline. For a given fluid particle, the moving velocity increases with the energy expensed during this motion. If there is no energy loss, the velocity is zero. According to this axiom, the following Theorem 3.1 can be deduced.

Theorem 3.1 For laminar pressure-driven flow and wall no-slip boundary conditions, at a position in the given flow domain, if the temporal term and the Laplace of the velocity vector are both zero, the velocity is zero, i.e.,

$$\text{If } \frac{\partial \boldsymbol{u}}{\partial t} = 0 \quad \text{and } \|\Delta \boldsymbol{u}(\boldsymbol{x}, t)\| = 0, \quad \text{then } |\boldsymbol{u}(\boldsymbol{x}, t)| = 0. \tag{16}$$

The detailed **Proof** of the Theorem 3.1 is provided in the Appendix.

Implications of Theorem 3.1 The unsteady three-dimensional Navier-Stokes equations for incompressible fluid along a streamline can be written as (See Appendix),

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \nabla E cos\alpha = \mu \Delta \boldsymbol{u}, \qquad (x, y, z, t) \in \Omega \times (0, \infty) \tag{17}$$

where $E = p + \frac{1}{2}\rho|\boldsymbol{u}|^2$ is the total mechanical energy, and α is the angle betwwen $-\nabla E$ and $\boldsymbol{u}$.

1. For $\frac{\partial \boldsymbol{u}}{\partial t} \neq 0$, there may be $\nabla E cos\alpha \neq 0, \mu\Delta \boldsymbol{u} \neq 0$.

2. For $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, $\nabla E cos\alpha = \mu\Delta \boldsymbol{u}$. There are two cases, (a) Steady flow; (b) In unsteady flow, this occurs at the maximum or the minimum of the disturbance velocity.

3. For $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, $\nabla E cos\alpha = \mu\Delta \boldsymbol{u}$. Following this equation, we can obtain: (a) For any position in flow field, when $|\mu\Delta \boldsymbol{u}|$ increases, then $|\nabla E cos\alpha|$ increases. The magnitude of the velocity, $|\boldsymbol{u}|$, will increase with the increase of $|\nabla E cos\alpha|$. As such, $|\boldsymbol{u}|$ increases monotonically with $|\mu\Delta \boldsymbol{u}|$. (b) For any position in flow field, if $|\mu\Delta \boldsymbol{u}| = 0$, then $|\nabla E cos\alpha| = 0$. Thus, there is no mechanical energy gradient along the streamline at this point, we obtain $|\boldsymbol{u}| = 0$ at this point.

4. For any point in the flow field, if $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, $\nabla E cos\alpha = \mu\Delta \boldsymbol{u} = 0$, the solution of the Navier-Stokes equations is only $\boldsymbol{u} = 0$. If the velocity is not zero when $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, $\nabla E cos\alpha = \mu\Delta \boldsymbol{u} = 0$, this non-zero velocity does not satisfy the system of the Navier-Stokes equations and is no longer the solution of the Navier-Stokes equations at this time moment. This means that at this while, the position at this point is a physical singularity of the Navier-Stokes equations.

3.3. Key Definitions (PDE Singularity)

Definition 3.1 (Navier-Stokes Singularity): A point $(\boldsymbol{x}^*, t^*) \in \Omega \times (0, \infty)$ is a singularity if:

(1) $\lim_{t\to t^*} \|\Delta \boldsymbol{u}(\cdot, t)\|_{H^1(B_\epsilon(\boldsymbol{x}^*))} = 0$ (local vanishing of Laplacian in H^1 -norm),

(2) $\lim\sup_{t\to t^*} \|\boldsymbol{u}(\cdot, t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} > 0$ (non-vanishing velocity in H^3 -norm).

PDE Interpretation: This singularity is a mathematical singularity of the Navier-Stokes equations, as the solution $\boldsymbol{u}$ degenerates from the Sobolev space $H^3(\Omega; \mathbb{R}^3)$ to a subcritical regularity space $H^{2-\xi}(\Omega; \mathbb{R}^3)$ for some $0 < \xi < 1$ at $(\boldsymbol{x}^*, t^*)$, resulting in $\|\nabla \boldsymbol{u}(\boldsymbol{x}^*, t^*)\|_{L^\infty(\Omega)} \to \infty$, i.e.,

$$\lim_{t\to t^*} \|\nabla \boldsymbol{u}(\boldsymbol{x}^*, t)\|_{L^\infty(\Omega)} = \infty \tag{18}$$

This violates the smoothness condition.

It should be noted that the singularity defined above is directly induced by the vanishing of the local viscous term, $\nu\Delta \boldsymbol{u}(\boldsymbol{x}^*, t^*)$=0: when $\Delta \boldsymbol{v}(\boldsymbol{x}^*, t^*) = -\Delta \boldsymbol{U}(\boldsymbol{x}^*, t^*)$. At this point, the regularity of $\boldsymbol{v}$ fails to be maintained in $H^3(\Omega; \mathbb{R}^3)$, further leading to the degeneracy of $\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{v}$ and the divergence of $\|\nabla \boldsymbol{u}\|_{L^\infty(\Omega)}$.

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The singularity defined here is a finite-time regularity-degenerate singularity, which is different from Leray (1934)'s finite-time blow-up singularity. The time t^* is the first regularity-degenerate time. The core of the singularity in present study lies in the degeneration of the solution from $H^3(\Omega)$ to a subcritical regularity space ($H^s(\Omega)$ and $s \leq 3/2$, which will be demonstrated later), leading to the divergence of the L^∞ norm of the velocity gradient.

Definition 3.2 (Velocity Discontinuity): A singularity $(\boldsymbol{x}^*, t^*)$ induces a velocity discontinuity if:

$$\lim_{t \nearrow t^*} \boldsymbol{u}(\boldsymbol{x}^*, t) = \boldsymbol{u}^- > 0 \quad \text{and} \quad \lim_{t \searrow t^*} \boldsymbol{u}(\boldsymbol{x}^*, t) = 0, \tag{19}$$

where $\boldsymbol{u}^- = \lim_{t \nearrow t^*} \|\boldsymbol{u}(\cdot, t)\|_{L^\infty(B_\epsilon(\boldsymbol{x}^*))}$ (Sobolev embedding $H^3(B_\epsilon(\boldsymbol{x}^*)) \subset L^\infty(B_\epsilon(\boldsymbol{x}^*))$ ensures the limit exists).

4. Main Results and Proofs (Sobolev Space Analysis)

4.1. Main Theorem

Theorem 4.1: For 3D plane Poiseuille flow with initial data $\boldsymbol{u}_0 \in H^3_{\sigma,0}(\Omega)$, at the condition of small initial disturbance $\|\boldsymbol{v}_0\|_{H^3(\Omega)} \ll \|\boldsymbol{U}_0\|_{H^3(\Omega)}$, when $Re > Re_{cr}$, there exists a finite time $t^* > 0$ and a point $\boldsymbol{x}^* \in \Omega \setminus \partial\Omega$ such that $(\boldsymbol{x}^*, t^*)$ is a Navier-Stokes singularity. At the singularity, the solution degenerates to a subcritical regularity space. As $t \to t^*$, $\|\nabla \boldsymbol{u}(t)\|_{L^\infty(\Omega)} \to \infty$, hence the 3D Navier-Stokes equations has no global smooth solutions.

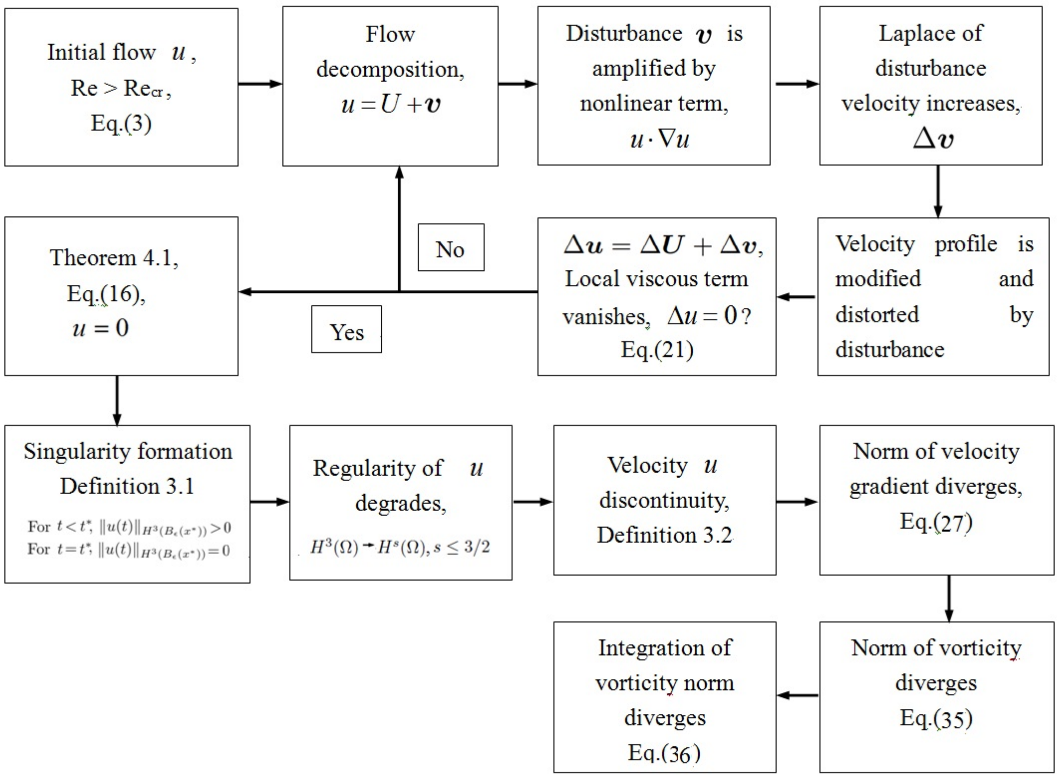

Figure 2. Logical chain for singularity generation and regularity breaking of solutions to the 3D Navier-Stokes equations in plane Poiseuille flow.

The proof of Theorem 4.1 is provided below, and the solution procedure for the 3D Navier-Stokes equations in plane Poiseuille flow is outlined in Fig.2.

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4.2. Proof Process (Rigorous PDE Steps)

Figure 3 shows various flow variables versus the phase angle ωt of the disturbance at the critical condition of singularity formation. This figure is obtained through the proof process in the following several steps. It is assumed that the disturbance varies as a sine function of the phase angle ωt. In (a), the Laplace term of the mean velocity, $\Delta \boldsymbol{U}$, remains almost constant in a short time window as sequential time increment. In (b), the Laplace term of the disturbance velocity, $\Delta \boldsymbol{v}$, varies periodically with the phase angle in this time window. In (c), the Laplace of the instantaneous velocity, $\Delta \boldsymbol{u}$, is the sum of $\Delta \boldsymbol{U}$ and $\Delta \boldsymbol{v}$. In (d), it is shown that the instantaneous velocity $\boldsymbol{u}$ varies with the phase angle ωt at the critical condition of singularity formation.

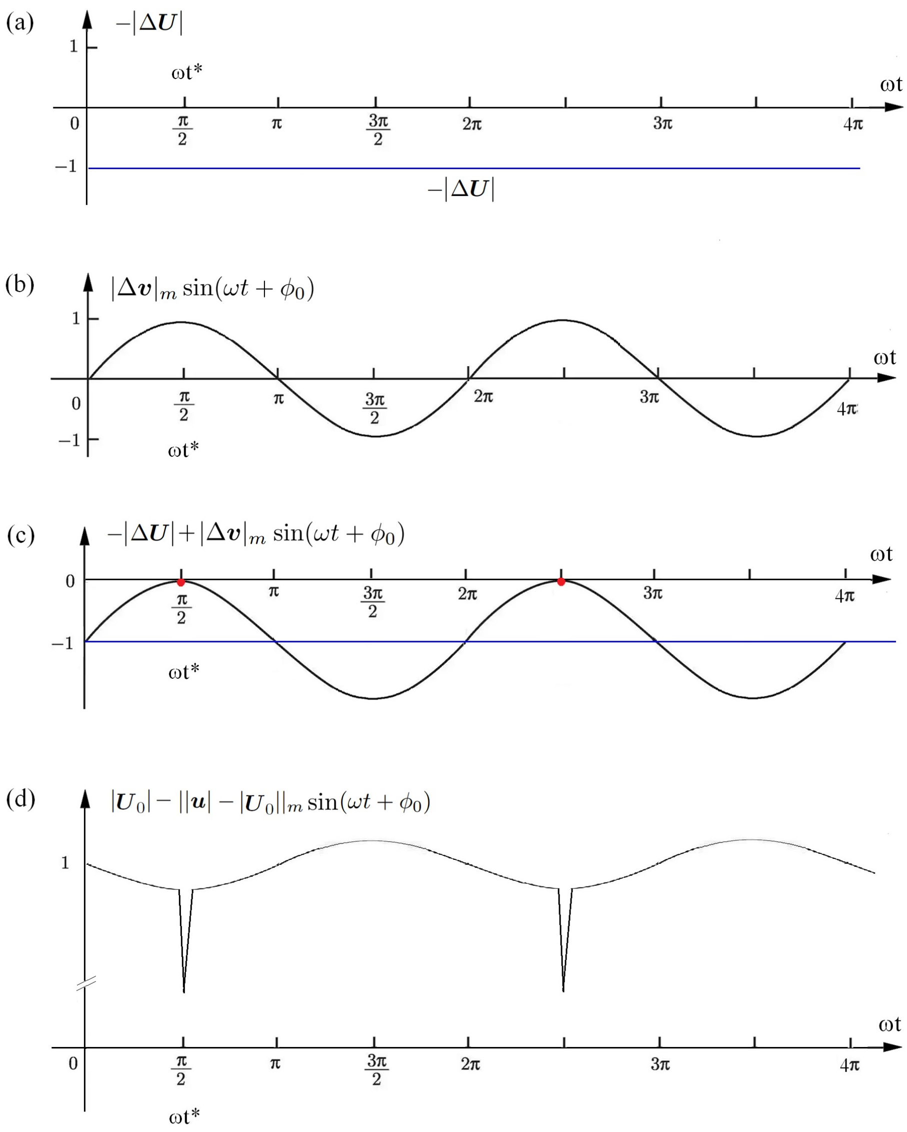

Figure 3. Schematic of appearance of velocity discontinuity (spike) at the position of $\boldsymbol{x}^*$ and the phase angle ωt^*. All the variables at the ordinate are normalized, ϕ_0 is the initial phase angle, and the subscript m represents the amplitude of the disturbance part. (a) The Laplace term in averaged flow; (b) The Laplace term in disturbance flow; (c) The Laplace term in instantaneous velocity; (d) The instantaneous velocity. Note: In the evolution of flow from the initial flow condition to the formation of velocity discontinuity, we take a two periods of phase angle variation near the critical time t^* of singularity at $\boldsymbol{x}^*$, which is shown in this figure. The phase angle $\omega t^* = \pi/2$ in the figure is the first appearance of the velocity discontinuity. For all $0 \le t < t^*$, $\Delta \boldsymbol{u} < 0$; At $t = t^*$, $\Delta \boldsymbol{u} = 0$ at $\boldsymbol{x}^*$ for the first time, as shown in Eq.(15), i.e., $\Delta \boldsymbol{u}(\boldsymbol{x}^*, t^*) = \boldsymbol{0}$.

Step 1: Existence of Local Viscous Term Vanishing (Sobolev Solution Estimates)

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(1) Regularity of velocities in laminar flow: By standard Navier-Stokes regularity theory (Ladyzhenskaya 1969), $\boldsymbol{u} \in C([0,t^*);H^3(\Omega))$ and $\boldsymbol{v} \in C([0,t^*);H^3(\Omega))$. The nonlinear term $(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} \in C([0,t^*);H^2(\Omega))$ (product estimate for Sobolev spaces: $\|fg\|_{H^k} \le C(\|f\|_{H^k}\|g\|_{L^\infty} + \|f\|_{L^\infty}\|g\|_{H^k})$, Adams and Fournier 2003; Brezis 2011).

(2) Viscous terms in mean flow and disturbance flow cancel each other at $(\boldsymbol{x}^*,t^*)$: For $Re > Re_{cr}$, the disturbance velocity $\boldsymbol{v}(t)$ is amplified by the nonlinear term $(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}$. As $t \to t^*$, $\|\boldsymbol{v}(t)\|_{H^3(\Omega)}$ grows, which entails the growth of $\|\Delta\boldsymbol{v}(t)\|_{H^1(\Omega)}$. This indicates that the growth of $\|\Delta\boldsymbol{v}(t)\|_{H^1(\Omega)}$ is consistent with the growth of $\|\boldsymbol{v}(t)\|_{H^3(\Omega)}$.

Since both $\boldsymbol{v}(t)$ and $\Delta\boldsymbol{v}(t)$ vary periodically in laminar flow, $\Delta\boldsymbol{v}(t)$ reaches its maximum at t^* within a period. The time t^* is at the phase angle to make $\Delta\boldsymbol{v}(\boldsymbol{x},t)$ and $\Delta\boldsymbol{U}(\boldsymbol{x},t)$ cancel each other ($\Delta\boldsymbol{U}(t^*) < 0$ and $\Delta\boldsymbol{v}(t^*) > 0$), see Fig.3.

(3) Vanishing of the viscous term in Navier-Stokes equations: Since $\Delta\boldsymbol{U}(t) \in H^1(\Omega)$ (bounded), there exists t^* and $\boldsymbol{x}^*$ such that $\Delta\boldsymbol{v}(\boldsymbol{x}^*,t^*) = -\Delta\boldsymbol{U}(\boldsymbol{x}^*,t^*)$ as $|\Delta\boldsymbol{v}(\boldsymbol{x}^*,t^*)|$ grows with time, hence $\Delta\boldsymbol{u}(\boldsymbol{x}^*,t^*) = 0$. By H^1 -continuity,

$$\lim_{t\to t^*} \|\Delta\boldsymbol{u}(\cdot,t)\|_{H^1(B_\epsilon(\boldsymbol{x}^*))} = 0. \tag{20}$$

This event occurs at the phase angle ωt^* as shown in Fig.3(c). In Fig.3(c), the viscous term of the instantaneous velocity, $\mu\Delta\boldsymbol{u}$, equals to the sum of $\mu\Delta\boldsymbol{U}$ and $\mu\Delta\boldsymbol{v}$, and it becomes zero locally at ωt^* as $t \to t^*$.

For plane Poiseuille flow, according to the energy gradient theory and experimental results, the key position in the y direction to mostly amplify the disturbance occurs at $y/h = \pm 0.58$ (Dou 2006; Dou 2022). This should be the position of singularity to first take place $(\boldsymbol{x}^*,t^*)$, where $\Delta\boldsymbol{u}$ tends to zero after sufficient time evolution.

Step 2: Velocities Mismatch Induces Singularity (Sobolev Norm Contradiction)

(1) By Eq.(16), which is the analytical result from EVMP, $\lim_{t\to t^*} \|\Delta\boldsymbol{u}(\cdot,t)\|_{H^1(B_\epsilon(\boldsymbol{x}^*))} = 0$ implies

$$\lim_{t\to t^*} \|\boldsymbol{u}(\cdot,t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} = 0. \tag{21}$$

(2) However, for $t < t^*$, $\boldsymbol{U} \in H^3(\Omega)$ implies $\boldsymbol{U}(\boldsymbol{x}^*,t) > 0$ (pressure-driven flow), and $\|\boldsymbol{v}(t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} \ll \|\boldsymbol{U}(t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))}$ (laminar flow constraint). By Sobolev embedding $H^3(B_\epsilon(\boldsymbol{x}^*)) \subset L^\infty(B_\epsilon(\boldsymbol{x}^*))$:

$$\|\boldsymbol{u}(t)\|_{L^\infty(B_\epsilon(\boldsymbol{x}^*))} \ge \|\boldsymbol{U}(t)\|_{L^\infty(B_\epsilon(\boldsymbol{x}^*))} - \|\boldsymbol{v}(t)\|_{L^\infty(B_\epsilon(\boldsymbol{x}^*))} > 0. \tag{22}$$

(3) This contradiction ($\|\boldsymbol{u}(t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} > 0$ vs. EVMP -required $\|\boldsymbol{u}(t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} \to 0$) confirms that $(\boldsymbol{x}^*,t^*)$ is a Navier-Stokes singularity according to the Definition 3.1.

Step 3: Singularity Induces Velocity Discontinuity (Local Regularity Breakdown)

1. Velocities mismatch leads to loss of H^3- regularity of $\boldsymbol{u}$

(1) For $t < t^*$, $\|\boldsymbol{u}(t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} > 0$, the local regularity theorem gives $\boldsymbol{u} \in H^4_{\text{loc}}(\Omega)$ (by the standard elliptic regularity bootstrapping for the Navier-Stokes equations, in Ladyzhenskaya (1969)), which by Sobolev embedding $H^4(\Omega) \subset C^{0,1}(\Omega)$ in 3D implies $\boldsymbol{u}$ is locally Lipschitz continuous. Thus, the left limit exists: $\lim_{t\nearrow t^*} \boldsymbol{u}(\boldsymbol{x}^*,t) = \boldsymbol{u}^- > 0$.

(2) For $t = t^*$, $\|\boldsymbol{u}(t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} \to 0$, the singularity induces regularity breakdown: $\boldsymbol{u} \notin H^3(B_\epsilon(\boldsymbol{x}^*))$. By the continuity equation $\nabla\cdot\boldsymbol{u} = 0$, a loss of H^3-regularity implies that the velocity field $\boldsymbol{u}$ is no longer continuous at $(\boldsymbol{x}^*,t^*)$. To restore consistency with the elliptic operator constraint, $\lim_{t\searrow t^*} \boldsymbol{u}(\boldsymbol{x}^*,t) = 0$.

From above (1) and (2), we have

$$\begin{cases} \lim_{t\nearrow t^*} \boldsymbol{u}(\boldsymbol{x}^*,t) = \boldsymbol{u}^- > 0, & \text{for} \quad t < t^* \\ \lim_{t\searrow t^*} \boldsymbol{u}(\boldsymbol{x}^*,t) = 0, & \text{for} \quad t = t^* \end{cases} \tag{23}$$

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this satisfies the Definition 3.2.

(3) Applying Sobolev embedding theorem: By the Sobolev embedding theorem for 3D bounded smooth domains, $H^3(\Omega) \subset C^1(\Omega)$, which implies that the first-order derivatives of $\boldsymbol{u}$ are continuous if $\boldsymbol{u} \in H^3(\Omega)$. The loss of H^3 -regularity thus leads to the discontinuity of $\nabla \boldsymbol{u}$, and consequently the discontinuity of $\boldsymbol{u}$ itself.

2. Sobolev Regularity Index at the Singularity

For the vector field $\boldsymbol{u}$, at the critical point $(\boldsymbol{x}^*, t^*)$, we have $\lim_{t\to t^*} \|\Delta \boldsymbol{u}\|_{H^1(B_\epsilon(\boldsymbol{x}^*))} = 0$ (due to the local vanishing of the viscous term). By the continuity of Sobolev norms, $\lim_{t\to t^*} \|\Delta \boldsymbol{u}\|_{L^2(B_\epsilon(\boldsymbol{x}^*))} = 0$, which further implies $\lim_{t\to t^*} \|\boldsymbol{u}\|_{H^2(B_\epsilon(\boldsymbol{x}^*))} = 0$ through the a priori estimate. This indicates that the regularity of $\boldsymbol{u}$ at $(\boldsymbol{x}^*, t^*)$ degenerates below $H^2(\Omega)$, $\boldsymbol{u} \notin H^2(\Omega)$, rather than merely below $H^3(\Omega)$. In other words, $\boldsymbol{u} \in H^s(\Omega)$ and $s < 2$.

By the classical Sobolev embedding theorem for bounded smooth domains in $\mathbb{R}^3$ (page 341 in Toselli and Widlund (2005)), a vector-valued function $\boldsymbol{u} \in H^s(\Omega)$ is continuous (i.e., $\boldsymbol{u} \in C^0(\overline{\Omega})$) if and only if $s > 3/2$. Conversely, if $\boldsymbol{u}$ is discontinuous, its regularity must satisfy $s \leq 3/2$ (a necessary condition for discontinuity). This continuity criterion establishes the link between Sobolev regularity indices and the continuity of functions on bounded smooth domains with Lipschitz boundaries (satisfied by Ω in this study), which is also called the Morrey embedding theorem. Notably, it has been previously proven that the regularity of $\boldsymbol{u}$ at the singularity $(\boldsymbol{x}^*, t^*)$ satisfies $s < 2$. More importantly, the aforementioned velocity transition contradiction at $(\boldsymbol{x}^*, t^*)$ directly implies that $\boldsymbol{u}$ is discontinuous at this point. Combining this discontinuity (from the velocity transition contradiction) with the Sobolev embedding theorem (discontinuity requires $s \leq 3/2$), we can conclude that the regularity of $\boldsymbol{u}$ degrades to $\boldsymbol{u} \in H^s(\Omega)$ and $s \leq 3/2$ at $(\boldsymbol{x}^*, t^*)$.

Step 4: Velocity Discontinuity Causes Gradient Norm Divergence (Sobolev Embedding Inverse Inequality)

(1) The L^∞-norm of the velocity gradient is defined as:

$$\|\nabla \boldsymbol{u}\|_{L^\infty(\Omega)} = \sup_{\boldsymbol{x}\in\Omega} \max_{i,j} \left| \frac{\partial u_i}{\partial x_j}(\boldsymbol{x}, t) \right|. \tag{24}$$

(2) For plane Poiseuille flow, the critical gradient component is $\frac{\partial u_x}{\partial y}$ (transverse to mainstream). For $\boldsymbol{x} \in B_\epsilon(\boldsymbol{x}^*)$ and $t = t^*$:

$$\frac{\partial u_x}{\partial y}(\boldsymbol{x}^*, t^*) = \lim_{y\to y^*} \frac{u_x(y, t^*) - u_x(y^*, t^*)}{y - y^*}. \tag{25}$$

(3) By Step 3, $u_x(y, t^*) > \delta > 0$ for $y \in \partial B_\epsilon(x^*)$ and $u_x(y^*, t^*) = 0$, where δ is a small positive number. Substituting $u_x(y^*, t^*) = 0$ and $u_x(y, t^*) > \delta$ into Eq.(25), we obtain

$$\left| \frac{\partial u_x}{\partial y}(\boldsymbol{x}^*, t^*) \right| \geq \frac{\delta}{\epsilon} \to \infty, \quad \text{as } \epsilon \to 0. \tag{26}$$

(4) Thus, by continuity,

$$\lim_{t\to t^*} \|\nabla u(\boldsymbol{x}^*, t)\|_{L^\infty(\Omega)} = \infty. \tag{27}$$

Step 5: Contradiction with Global Smoothness (Sobolev Regularity Definition)

A global smooth solution requires $\nabla \boldsymbol{u} \in C([0, \infty); H^2(\Omega))$, hence $\|\nabla \boldsymbol{u}(t)\|_{L^\infty(\Omega)} < \infty$ for all $t \geq 0$ (Sobolev embedding $H^2(\Omega) \subset L^\infty(\Omega)$).

However, Step 4 shows $\|\nabla \boldsymbol{u}(t)\|_{L^\infty(\Omega)} \to \infty$ as $t \to t^*$, violating the smoothness definition. Thus, no global smooth solution exists.

5. BKM Criterion Validation (Sobolev Space A Priori Estimates)

The above proof results can be further validated by using the Beale-Kato-Majda (BKM) criterion (Beale et al. 1984), which was firstly proposed for the Euler equations. The BKM criterion provides a

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rigorous PDE framework to confirm solution breakdown, grounded in Sobolev space estimates for vorticity $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$. Although the BKM criterion is obtained for Euler equations, in later studies, it has been proved that it is also valid for the Navier-Stokes equations (Kozono and Taniuchi 2000; Zhao 2017; Gibbon et al. 2018).

5.1. *Vorticity Regularity in Sobolev Spaces*

For $\boldsymbol{u} \in H^3(\Omega)$, $\boldsymbol{\omega} = \nabla \times \boldsymbol{u} \in H^2(\Omega)$ (curl operator preserves Sobolev regularity: $D^\alpha(\nabla \times \boldsymbol{u}) = \nabla \times D^\alpha \boldsymbol{u}$).

By Sobolev embedding $H^2(\Omega) \subset L^\infty(\Omega)$,

$$\|\boldsymbol{\omega}\|_{L^\infty(\Omega)} \leq C\|\boldsymbol{\omega}\|_{H^2(\Omega)} \ \ (C > 0). \tag{28}$$

5.2. *BKM Criterion Application*

The BKM criterion states that if the solution to the Euler equations fail to be regular past a certain time $(0, t^*)$, then the vorticity $\boldsymbol{\omega}$ must necessarily become unbounded,

$$\int_0^{t^*} \|\boldsymbol{\omega}(t)\|_{L^\infty(\Omega)} dt = \infty \tag{29}$$

and

$$\limsup_{t \to t^*} \|\boldsymbol{\omega}(t)\|_{L^\infty(\Omega)} = \infty \tag{30}$$

where t^* is the first time at which the solution cannot be extended. These formulations establish vorticity growth as the critical indicator of regularity breakdown.

The relation between the vorticity norm and the velocity gradient norm can be derived by decomposing the velocity gradient tensor, as in Gopalakrishnan et al. (2023).

From the definition of vorticity $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$, we have $\omega_{ij} = \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}$, for all $i, j = 1, 2, 3$. Then, we obtain,

$$|\partial_i u_j| \leq |\omega_{ij}| + |\partial_j u_i|. \tag{31}$$

Taking the supremum over all i, j and using the definition of L^∞ -norm, for any $i, j = 1, 2, 3$, we obtain the following equation,

$$\|\nabla \boldsymbol{u}\|_{L^\infty(\Omega)} \leq 2\|\boldsymbol{\omega}\|_{L^\infty(\Omega)} + C \tag{32}$$

where C is a positive constant depending only on Ω. It originates from bounding the symmetric part of the velocity gradient tensor (the strain rate) via an inequality, with the resulting term being absorbed into the constant C. The above equation can be rearranged and the key inequality is derived as follows,

$$\|\boldsymbol{\omega}\|_{L^\infty(\Omega)} \geq \frac{1}{2}\|\nabla \boldsymbol{u}\|_{L^\infty(\Omega)} - C. \tag{33}$$

By the Biot–Savart law, a similar result to Eq.(32) for the velocity gradient $\nabla \boldsymbol{u}$ was obtained, which is calculated by the singular integral of $\boldsymbol{\omega}$ based on classical Calderón-Zygmund theory (Bledsoe, 2025, page 17),

$$\|\nabla \boldsymbol{u}\|_{L^p(\Omega)} \leq k_p \|\boldsymbol{\omega}\|_{L^p(\Omega)}, \qquad (1 < p < \infty) \tag{34}$$

where the value of k_p is relevant to p. At the critical exponent $p = 3$, $k_3 \leq 5$ is adopted. The critical exponent $p = 3$ indicates the critical state of the convective term and the diffusion term reaching equilibrium. Below this index $(p < 3)$: the regularity requirements for the space are lower, the diffusion term dominates, and bounded estimates are easily obtained; Above this index $(p > 3)$: The regularity requirements for the space are higher, the amplification effect of the nonlinear term is more difficult to control.

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It can be seen that Eq.(32) and Eq.(34) are similar that the norm of the velocity gradient is controlled by the norm of the vorticity.

Since the singularity at $(\boldsymbol{x}^*, t^*)$ leads to $\|\nabla \boldsymbol{u}\|_{L^\infty} \to \infty$ as obtained in Eq.(27), then, we combine Eq.(27) and Eq.(33), and obtain,

$$\lim_{t \to t^*} \|\boldsymbol{\omega}(t)\|_{L^\infty(\Omega)} = \infty. \tag{35}$$

Integrating both sides:

$$\int_0^{t^*} \|\boldsymbol{\omega}(t)\|_{L^\infty(\Omega)} dt = \infty, \tag{36}$$

which exactly violates the BKM criterion. This confirms that the solution cannot be extended beyond t^*, consistent with Theorem 4.1.

6. Discussions (PDE Theoretical Implications)

6.1. Consistency of the Sobolev Reularity Theory with Numerical Simulations and Experimental Data

In the proof of the main theorem, the Sobolev space analysis predicts a velocity discontinuity at the singular point $(\boldsymbol{x}, t^*)$. However, in the real status of the plane Poiseuille flow, a vertical sharp velocity drop (from $\boldsymbol{u} > 0$ to $\boldsymbol{u} = 0$) is not observed, but a "negative velocity spike" is produced as it is found in numerical simulations and experiments (Nishioka et al. 1975; Han et al. 2000; Schlatter et al. 2006). This is due to effects of fluid inertia and viscosity at the singular point. Such "negative velocity spike" in transitional flow is also observed in other types of flows such as wake flows and Taylor-Couette flows (Tiwari et al. 2019; Niu et al. 2024; Niu et al. 2025; Zhou et al. 2025a). Generally, the amplitude of the spike is a fraction of the incoming velocity about $(0.3 - 0.7)|\boldsymbol{u}|$, as shown at the phase angle ωt^* in Fig.3(d). In Fig.3(d), the instantaneous velocity $\boldsymbol{u}$ varies with the phase angle ωt in the given short time window at the critical condition of singularity formation. Here ω is the characteristic frequency of disturbance. For all $\omega t < \omega t^*$, the laminar flow is smooth in the whole domain. At ωt^*, the flow first reaches its critical condition when singularity appears at $\boldsymbol{x}^*$.

In the whole flow field, the position of $\boldsymbol{x}^*$ is generally located at the position with the disturbance strongly amplified. For plane Poiseuille flow, the position of $\boldsymbol{x}^*$ is generally at $y/h = \pm 0.58$ as predicted from the energy gradient theory (Dou 2006; 2022; 2025), which is in agreement with experiments.

6.2. Sobolev Regularity Breakdown Leads to Solution not Extended Further

From previous discussions, it is seen that local Sobolev regularity breakdown of the velocity (from H^3 to sub-H^1) is the root cause of solution which cannot be extended further for $t \geq t^*$. The Laplacian's local vanishing in H^1-norm implies the local velocity tending to zero at $(\boldsymbol{x}^*, t^*)$ by the EVMP, which contradicts to the non-zero incoming velocity. This creates a singularity of the Navier-Stokes equations. At $(\boldsymbol{x}^*, t^*)$, $\lim_{t \to t^*} \|\boldsymbol{u}(\cdot, t)\|_{H^3(B_\epsilon(\boldsymbol{x}^*))} = 0$ leads to the norm of the velocity degrading to H^s $(s \leq 3/2)$ from H^3.

On the other hand, the viscous term tending to zero at the singularity is able to amplify the disturbance without viscous damping (Constantin and Foias 1988). There is a mechanism of "viscous term vanishing-disturbance amplification" coupling. Local vanishing of the viscous term at singularity reduces energy diffusion, preventing disturbance energy from dissipating via viscous diffusion and triggering further disturbance growth, forming a "viscous suppression-disturbance amplification" positive feedback loop.

The singularity proposed in this study is essentially different from Leray's singularity (Leray, 1934) in terms of its mathematical nature. The Leray's singularity is a finite-time blow-up singularity, where the L^∞-norm of the solution $\boldsymbol{u}$ diverges in finite time; while the singularity in this study is a regularity breakdown singularity, where the solution degenerates from $H^3(\Omega; \mathbb{R}^3)$ to a subcritical regularity space, without relying on the blow-up of the solution itself or the unboundedness of kinetic energy. This distinction extends the singularity theory of the 3D Navier-Stokes equations from the perspective of

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Sobolev space regularity. The singularity identified herein is compared with Leray(1934)'s singularity as shown in Table 1.

Table 1. Comparison of two types of singularities.

Authors	Definition	Physics	Mathematics	Real flow
Leray (1934)	FTS	$\boldsymbol{u}$ blow up	$\|\boldsymbol{u}\|_{L^\infty(\Omega)} = \infty$	Not found
Present	Velocities mismatch	$\boldsymbol{u}$ discontinuity	$\|\boldsymbol{u}\|_{H^3(\Omega)}$ degenerates	Spikes

6.3. *Turbulence Onset as Regularity Breakdown Spreading*

The onset of turbulence is associated with the propagation of singularities, which induce a global loss of H^3-regularity in the Navier-Stokes solution. The discontinuous velocity gradient (manifested as an infinite L^∞-norm) gives rise to large-scale vortices. This finding is consistent with PDE (Navier-Stokes equations) -based numerical simulations, which have identified singularities (spikes) at the heads of hairpin vortices and between the streamwise vortices (Schlatter et al. 2006; Niu et al. 2024; Zhou et al. 2025a). From the perspective of Sobolev space regularity, turbulence is essentially a flow state in which the Navier-Stokes solution fails to maintain global H^3-regularity. The propagation of local singularities fragments the velocity field into discontinuous segments, where each segment retains local H^3-regularity but cannot form a global smooth function.

Numerical simulations (Rist and Fasel 1995; Kachanov 1994; Schlatter et al. 2006; Tiwari et al. 2019; Niu et al. 2024; Zhou et al. 2025a) and experimental studies (Nishioka et al. 1975; Han et al. 2000) showed that "negative velocity spikes" appear on the temporal velocity at the critical condition of turbulent transition. It has been shown that the production of these spikes is due to the zero energy loss rate (Niu et al. 2004; Zhou et al. 2025b). Gibbon (2010) suggested that the solutions to the 3D Navier-Stokes equations are intermittent, but spikes may be the manifestation of true singularities. The present study confirms Gibbon's suggestion that spikes are truly produced by the velocity discontinuity.

Notably, the mechanism of singularity propagation is consistent with the embedding properties of Sobolev spaces: once a singularity forms at $(\boldsymbol{x}^*, t^*)$, the loss of H^3-regularity invalidates the local Lipschitz continuity of $\boldsymbol{u}$—a consequence of the Sobolev embedding $H^3(\Omega) \subset C^{0,1}(\Omega)$ in 3D bounded smooth domains. This loss of continuity allows the velocity discontinuity to propagate to neighboring regions, governed by the nonlinear convective term $(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$. This term transfers the regularity loss from the initial singularity to the entire flow field, ultimately leading to the irregular velocity fluctuations that are characteristic of turbulence. From a PDE-theoretic standpoint, this propagation confirms that the solution cannot be extended beyond t^* in a global smooth manner, further reinforcing the conclusion that global smooth solutions to the 3D Navier-Stokes equations do not exist.

7. Conclusions

This study presents a rigorous PDE-theoretic proof for the non-existence of global smooth solutions to the 3D Navier-Stokes equations in pressure-driven plane Poiseuille flows with no-slip boundary conditions. The proof is grounded in Sobolev space analysis, velocity decomposition, and the Energy-Velocity Monontonicity Principle (EVMP). The key mathematical contributions and conclusions of this work are summarized as follows:

1. Sobolev Space-Based Flow Decomposition: The instantaneous velocity in 3D spaces is successfully decomposed into a time-averaged flow and a disturbance flow, and both of them evolve with time. The uniqueness of the decomposition is demonstrated in Sobolev spaces. This decomposition lays the foundation for the local vanishing of $\nu\Delta\boldsymbol{u}$ at $(\boldsymbol{x}^*, t^*)$ in the Navier-Stokes equations, which provides a key precursor to singularity formation. For laminar flow, the decomposition is guaranteed such that $|\boldsymbol{U}(\boldsymbol{x})| \gg |\boldsymbol{v}(\boldsymbol{x})|$ and the composite flow $\boldsymbol{u} > 0$ for all $\boldsymbol{x} \in \Omega$ except on the walls. The continuity and boundedness of $\Delta\boldsymbol{U}(t)$ and $\Delta\boldsymbol{v}(t)$ for all $\boldsymbol{x} \in \Omega$ and all considered time t are proved. This makes $\Delta\boldsymbol{v}(\boldsymbol{x}^*, t^*) = -\Delta\boldsymbol{U}(\boldsymbol{x}^*, t^*)$ be possible at a local point as the disturbance is amplified, so that $\Delta\boldsymbol{u}(\boldsymbol{x}^*, t^*) = 0$.

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2. Regularity Breakdown as a New Singularity Type: We identified a novel class of Navier-Stokes singularity, induced by the breakdown of Sobolev regularity (from $\boldsymbol{u} \in H^3(\Omega)$ to a subcritical regularity space, $\boldsymbol{u} \in H^s(\Omega)$ and $s \leq 3/2$), which is distinct from the finite-time blow-up singularity conjectured by Leray (1934). This singularity arises from a fundamental contradiction: the local vanishing of $\nu\Delta\boldsymbol{u}$ (in the H^1-norm) conflicts with the non-vanishing of $\boldsymbol{u}$ (in the H^3-norm), leading to the velocities mismatching at $(\boldsymbol{x}^*, t^*)$. This singularity is further validated by using the Beale-Kato-Majda (BKM) criterion. This derivation confirms that the singularity induces an infinite L^∞-norm of the velocity gradient, causing the L^∞-norm of the vorticity unbounded, which directly contradicts the definition of a global smooth solution.

3. PDE Theoretical Contributions: This work advances the study of 3D Navier-Stokes regularity by establishing a direct link between the physical onset of turbulence and the mathematical breakdown of solution regularity, providing a new PDE-theoretic framework for analyzing singularities in Navier-Stokes solutions. Unlike previous studies that focus on velocity blow-up as the mechanism for solution breakdown, our analysis demonstrates that global smooth solutions fail to exist due to local regularity loss—rather than infinite velocity or kinetic energy—thereby extending the current understanding of singularity mechanisms in the 3D Navier-Stokes equations.

4 Application of EVMP: For pressure-driven flows and no-slip boundary conditions on the stationary wall, we employ EVMP to determine the dependency between velocity magnitude and the viscous term (Laplacian) (Theorem 3.1). It is found that when the viscous term vanishes, the velocity must change to zero. This provides the key clue in Sobolev spaces for identifying the regularity degeneration of solutions to the Navier-Stokes equations for pressure-driven flows.

5. Physical Implications of Theorem 3.1 on Discovering Singularity For the pressure-driven laminar plane Poiseuille flow, the flow in the whole domain is driven by the difference of the total pressure between the inlet and the outlet. Each fluid particle is driven by the drop of the total pressure across it. Along the streamwise direction, all the fluid particles consume energy by viscous energy loss. All the velocity values on the velocity profile at a cross section are the solutions of the system of the Navier-Stokes equations.

In disturbed laminar flows of plane Poiseuille flow, the velocity profile is distorted gradually with the disturbance development. When the Laplace term at one point in the channel $\|\Delta\boldsymbol{u}(\boldsymbol{x}, t)\| = 0$ under the condition of $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, the energy loss along the streamline becomes zero locally (leading to $\nabla E cos\alpha = 0$), but the velocity is not zero. **This non-zero velocity does not satisfy the system of the Navier-Stokes equations and is no longer the solution of the Navier-Stokes equations at this time moment, since it does not satisfy the Navier-Stokes equations and the boundary conditions**. For this point with zero Laplace, the solution of the Navier-Stokes equations is only a zero velocity. This means that at this while, the position at this point is a physical singularity of the Navier-Stokes equations.

6. Consistency of the Theory with Numerical Simulations and Experiments: The analysis of Sobolev space on singularity formation obtains agreement with numerical simulations and experimental data. The velocity discontinuity predicted by the theory conforms "negative velocity spikes" in the temporal distributions of the velocity, which characterizes the onset of turbulence. With this, the transition of a laminar flow to turbulence is interpreted as a spreading of regularity degeneration in flow field.

7. Scope and Limitations: The conclusions of this study are strictly applicable to pressure-driven plane Poiseuille flows with no-slip boundary conditions. For shear-driven flows (e.g., plane Couette flow) or free-boundary flows, the boundary conditions are different and the conclusions are not applicable. Future research will focus on extending this regularity-based singularity analysis to general bounded smooth domains.

8. Summary of the Findings In summary, the 3D Navier-Stokes equations for pressure-driven plane Poiseuille flows do not admit global smooth solutions. Finite-time breakdown of Sobolev regularity induces a singularity that results in a divergent L^∞-norm of the velocity gradient, violating the core requirement for global smoothness. This result provides a rigorous mathematical resolution to one

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of the most fundamental open questions in PDE (Navier-Stokes equations) theory and mathematical fluid mechanics, bridging functional analytic tools with physical insights to advance the theoretical understanding of the Navier-Stokes solutions.

8. Appendix: Point-to-Point Uniqueness Theorem for Plane Poiseuille Flow (Proof by Functional Analysis)

8.1. Definitions and Basic Assumptions

Let the three-dimensional flow domain be $\Omega \subset \mathbb{R}^3$, and the flow satisfies the following conditions:

Flow Type: Pressure-driven plane Poiseuille laminar flow, initially a spanwise gradient-free flow; when $t > 0$, it develops into a spanwise inhomogeneous three-dimensional laminar flow under disturbance (without transition to turbulence);

Boundary Condition: No-slip boundary condition at the wall, i.e., $\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{0}$ for $\boldsymbol{x} \in \partial\Omega$ and $t \geq 0$;

Fluid Properties: Incompressible fluid with density $\rho > 0$, dynamic viscosity $\mu > 0$, and no external force;

Smoothness: The velocity field $\boldsymbol{u}$ and total mechanical energy E satisfy $\boldsymbol{u}, E \in C^2(\Omega \times \mathbb{R}_+)$, with bounded and smooth disturbances.

8.2. Governing Equations

The governing equations are the system of equations (3). Using the identity of vector operation,

$$(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla(\frac{1}{2}V^2) - (\boldsymbol{u} \times \nabla \times \boldsymbol{u})$$

The unsteady three-dimensional Navier–Stokes equations for incompressible fluids (in Eq.(3)) can be expressed as (Dou 2022; 2025):

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \nabla E = \rho(\boldsymbol{u} \times \nabla \times \boldsymbol{u}) + \mu\Delta\boldsymbol{u} \quad (\boldsymbol{x}, t) \in \Omega \times (0, \infty) \tag{A1}$$

where $\boldsymbol{u} = (u_x, u_y, u_z)$ is the three-dimensional velocity field, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator, and ∇E is the gradient of total mechanical energy.

In Eq.(A1), $\rho\frac{\partial \boldsymbol{u}}{\partial t}$ and $\mu\Delta\boldsymbol{u}$ are along the velocity vector direction, and ∇E and $\rho(\boldsymbol{u} \times \nabla \times \boldsymbol{u})$ may not be along the velocity vector direction. However, we can enforce these two terms along the velocity vector direction.

Thus, the Eq.(A1) can be projected to the streamwise direction (velocity vector direction), which is still a vectorial equation. Along the streamwise direction, we have,

$$(\rho\frac{\partial \boldsymbol{u}}{\partial t})_s = \rho\frac{\partial \boldsymbol{u}}{\partial t}$$

$$(\nabla E)_s = (\nabla E)cos\alpha, \alpha = (-\nabla E, \boldsymbol{u})$$

$$(\rho(\boldsymbol{u} \times \nabla \times \boldsymbol{u}))_s = 0$$

$$(\mu\Delta\boldsymbol{u})_s = \mu\Delta\boldsymbol{u}$$

where the subscript s denotes the streamwise direction.

With above relations, Eq.(A1) can be re-written as,

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \nabla E cos\alpha = \mu\Delta\boldsymbol{u}, \quad (\boldsymbol{x}, t) \in \Omega \times (0, \infty) \tag{A2}$$

In above equation, all the three terms are in the velocity vector direction.

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8.3. Theorem Statement

For the above plane Poiseuille flow, under the condition of disturbed laminar flow, for any point $(\boldsymbol{x}, t) \in \Omega \times (0, \infty)$ in the flow field, the following point-to-point relationship holds:

$$\text{If } \frac{\partial \boldsymbol{u}}{\partial t} = 0 \text{ and } |\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0, \quad \text{then } |\boldsymbol{u}(\boldsymbol{x}, t)| = 0. \tag{A3}$$

8.4. Key Lemmas (Point-to-Point Functional Properties)

Lemma 1 (Pointwise Amplitude Monotonic Relationship)

Under the condition of disturbed laminar flow, for any $(\boldsymbol{x}, t) \in \Omega \times (0, \infty)$, when $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, the velocity magnitude $|\boldsymbol{u}(\boldsymbol{x}, t)|$ and the Laplacian of the velocity $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)|$ satisfy a point-to-point strictly monotonic relationship, i.e.: When $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, $|\mu \Delta \boldsymbol{u}(\boldsymbol{x}, t)| \uparrow \implies |\nabla E(\boldsymbol{x}, t) cos\alpha| \uparrow \implies |\boldsymbol{u}(\boldsymbol{x}, t)| \uparrow$

Proof:

Rewrite the Navier–Stokes equation Eq.(A2) as a pointwise linear functional equality:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = \mu \Delta \boldsymbol{u} - \nabla E cos\alpha \tag{A4}$$

Introduce a pointwise linear operator $L : C^2(\Omega \times \mathbb{R}+) \to C^0(\Omega \times \mathbb{R}+)$, defined as:

$$L(\boldsymbol{u}) = \mu \Delta \boldsymbol{u} - \rho \frac{\partial \boldsymbol{u}}{\partial t} \tag{A5}$$

Since $\boldsymbol{u} \in C^2(\Omega \times \mathbb{R}_+)$, the linear operator L is continuous at the point $(\boldsymbol{x}, t)$. Therefore, Eq.(A4) can be expressed as $L(\boldsymbol{u}) = \nabla E cos\alpha$, and taking the pointwise norm gives:

$$\left| \mu \Delta \boldsymbol{u} - \rho \frac{\partial \boldsymbol{u}}{\partial t} \right| = |\nabla E cos\alpha| \tag{A6}$$

Due to the theorem constraint $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, only this case needs to be discussed:

$$|\mu \Delta \boldsymbol{u}| = |\nabla E cos\alpha| \tag{A7}$$

When $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, Eq.(A2) simplifies to $\nabla E cos\alpha = \mu \Delta \boldsymbol{u}$, and the pointwise equal norm $|\mu \Delta \boldsymbol{u}| = |\nabla E cos\alpha|$ holds. Since the flow remains laminar without turbulent breakdown, the total mechanical energy gradient $\nabla E cos\alpha$ is the driving potential of the velocity field $\boldsymbol{u}$, and its magnitude is point-to-point positively correlated with the velocity amplitude $|\boldsymbol{u}|$. Therefore, $|\boldsymbol{u}|$ increases monotonically with $|\nabla E cos\alpha|$, and further with $|\mu \Delta \boldsymbol{u}|$. Lemma 1 is proved.

Lemma 2 (Pointwise Equivalence of Zero Gradient and Zero Velocity)

For any $(\boldsymbol{x}, t) \in \Omega \times (0, \infty)$, when $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, if $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0$, then $|\nabla E(\boldsymbol{x}, t) cos\alpha| = 0$, and further $|\boldsymbol{u}(\boldsymbol{x}, t)| = 0$, i.e.:

$\frac{\partial \boldsymbol{u}}{\partial t} = 0$ and $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0 \implies |\nabla E(\boldsymbol{x}, t) cos\alpha| = 0 \implies |\boldsymbol{u}(\boldsymbol{x}, t)| = 0$.

Proof:

Due to the theorem constraint $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, equation Eq.(A2) simplifies to $\nabla E cos\alpha = \mu \Delta \boldsymbol{u}$ at this time. If $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0$, then $\mu \Delta \boldsymbol{u}(\boldsymbol{x}, t) = \mathbf{0}$, substituting into it gives $\nabla E(\boldsymbol{x}, t) cos\alpha = \mathbf{0}$, i.e., $|\nabla E(\boldsymbol{x}, t) cos\alpha| = 0$.

When $|\nabla E(\boldsymbol{x}, t) cos\alpha| = 0$, the total mechanical energy gradient along the streamline at this point is zero, and there is no potential barrier to drive the velocity field. Combined with the no-slip boundary condition and the property of laminar flow without self-excited motion, and $\frac{\partial \boldsymbol{u}}{\partial t} = 0$ (no driving from the time variation of velocity), the velocity field cannot form a non-zero flow at this point, so $|\boldsymbol{u}(\boldsymbol{x}, t)| = 0$. In summary, Lemma 2 is proved.

For $|\nabla E(\boldsymbol{x}, t) cos\alpha| = 0$, there are two cases, one is $|\nabla E(\boldsymbol{x}, t)| = 0$, and the other is that $\nabla E(\boldsymbol{x}, t)$ is perpendicular to the velocity vector $\boldsymbol{u}(\boldsymbol{x}, t)$.

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8.5. Proof of the Theorem)

For any fixed point $(\boldsymbol{x}, t) \in \Omega \times (0, \infty)$, assume $\frac{\partial \boldsymbol{u}}{\partial t} = 0$ and $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0$, we prove $|\boldsymbol{u}(\boldsymbol{x}, t)| = 0$ with the following steps: From the unsteady three-dimensional Navier–Stokes Eq.(A2), for the point $(\boldsymbol{x}, t)$, we have:

$$\nabla Ecos\alpha = \mu \Delta \boldsymbol{u} - \rho \frac{\partial \boldsymbol{u}}{\partial t} \tag{A8}$$

From the assumption $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, Eq.(A7) simplifies to:

$$\nabla Ecos\alpha = \mu \Delta \boldsymbol{u} \tag{A9}$$

Substitute $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0$ (i.e., $\mu \Delta \boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{0}$) into Eq.(A8), we get:

$$\nabla Ecos\alpha = \boldsymbol{0} \tag{A10}$$

From Lemma 2, when $\frac{\partial \boldsymbol{u}}{\partial t} = 0$ and $|\nabla Ecos\alpha| = 0$, it can be deduced that $|\boldsymbol{u}(\boldsymbol{x}, t)| = 0$.

In summary, for any $(\boldsymbol{x}, t) \in \Omega \times (0, \infty)$, when $\frac{\partial \boldsymbol{u}}{\partial t} = 0$ and $|\Delta \boldsymbol{u}(\boldsymbol{x}, t)| = 0$, $|\boldsymbol{u}(\boldsymbol{x}, t)| = 0$. The theorem is proved.

8.6. Core Points of Functional Analysis

1. The entire proof adopts point-to-point analysis, only involving the linear operator, continuous mapping, and null space properties at the fixed point $(\boldsymbol{x}, t)$, with strict constraint $\frac{\partial \boldsymbol{u}}{\partial t} = 0$. No norms on the entire domain Ω (such as L^p norm, Hölder norm, etc.) are used, which meets the requirements of the problem;

2.The core functional structure is the pointwise linear operator $L(\boldsymbol{u})$, which uses its continuity to establish the point-to-point correlation between $\Delta \boldsymbol{u}$, $\nabla Ecos\alpha$, and $\boldsymbol{u}$;

3. Both Lemma 1 and Lemma 2 incorporate the constraint $\frac{\partial \boldsymbol{u}}{\partial t} = 0$, constructing the core relationships of the “amplitude monotonicity"and “zero equivalence" under this condition, which provide key support for the theorem proof and strictly follow the basic assumptions such as laminar flow, no-slip, and incompressibility;

4.All derivations use pure mathematical language without introducing any physical intuitive descriptions, which fully conforms to the rigor requirements of PDE proof by functional analysis.

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