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Article

A Complete Derivation of Quantum Mechanics from Classical Field Theory

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Abstract

Quantum mechanics is a cornerstone of modern physics, yet its fundamental origin and mathematical foundation remain elusive. In this paper, we derive the complete structure of quantum mechanics from a purely classical field theory. By gauging the double cover of the Lorentz symmetry group of space–time, the resulting equations of motion naturally yield all postulates of quantum mechanics, including the emergence of the Planck constant. This approach provides a coherent interpretation and demonstrates that quantum mechanics can be viewed as a mathematically consistent classical field theory.

Keywords: quantum mechanics interpretation; spin; quantum mechanics foundation; quantum mechanics postulates; particle-wave duality; Lorentz symmetry; Planck constant; particle physics

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1. Introduction

Quantum mechanics is one of the most successful and well-established theories in physics, yet the fundamental origin of its postulates and axioms [6] remains unproven. These foundational assumptions are typically accepted without further justification, even though many quantum phenomena are still not fully understood.

For instance, the *measurement problem* and *wave function collapse* continue to invite multiple interpretations, with no consensus on a definitive explanation [5]. The *particle–wave duality*, in which matter exhibits both particle-like and wave-like behavior, raises deep conceptual questions: how can a single entity be both simultaneously? Similarly, the physical meaning of the Planck constant $\hbar$, the quantization of energy and momentum, and the mechanism underlying the *superposition principle*—where a particle exists in multiple states at once—are still unclear.

Another fundamental mystery concerns *spin*. Every particle possesses spin, yet its physical origin is not found in classical physics. How can a simple wave possess intrinsic angular momentum? Such questions highlight the conceptual and mathematical gaps between classical and quantum physics.

Historically, quantum mechanics was developed largely *ad hoc* to match experimental observations such as the double-slit experiment and the phenomenon of wave–particle duality. However, despite its empirical success, a rigorous derivation of its mathematical framework from deeper classical principles has remained absent.

In this work, we present a complete derivation of quantum mechanics from a classical field theory. All postulates and principles of quantum mechanics are shown to emerge naturally within this framework, providing simple and coherent explanations for the above phenomena.

This paper is organized as follows. In Sec.2, we gauge the double cover of the Lorentz symmetry group, obtain the corresponding equations of motion, and derive the quantum states and the Planck constant $\hbar$. In Sec.3, we construct the quantum operators, establish the uncertainty relation, and derive the quantization conditions. In Sec.4, we determine the particle representations under the Poincaré group and extract the complex wave function. In Sec.5, we demonstrate how all postulates of quantum mechanics arise naturally from the present model, thus providing a complete theoretical foundation. In Sec.6, we deduce the phenomenon of entanglement, and in Sec.7, we show that the Lorentz symmetry

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angles correspond to the quantum action, from which Feynman's path integral formulation follows directly.

2. Gauging the Double Cover of the Lorentz Symmetry Group and Emergence of Quantum States

the $SU(2) \times SU(2)$ group is the double cover of the Lorentz Group $SO(1,3)$[1]. $\{J_i, K_i\}$ are the generators of the Lorentz Group $SO(1,3)$.

Defining:

$$j_i^L \equiv J_i + iK_i, \;\; j_i^R \equiv J_i - iK_i, \tag{2.1}$$

where $i = 1,2,3$ .the $(\vec{j}^L, \vec{j}^R)$ are the generators of $SU(2)_{L/R}$, satisfy [3,4] :

$$\begin{aligned} [j_i^L, j_j^L] &= i\epsilon_{ijk} j_k^L, \\ [j_i^R, j_j^R] &= i\epsilon_{ijk} j_k^R, \\ [j_i^L, j_j^R] &= 0, \end{aligned} \tag{2.2}$$

The Lagrangian for a massless particle charged under the local Lorentz symmetry is:

$$\mathcal{L} = (D_\mu \phi^{L/R}(x_\mu))^\dagger D_\mu \phi^{L/R}(x_\mu), \tag{2.3}$$

where the covariant derivative is:

$$D_\mu \phi^{L/R}(x_\mu) = [\partial_\mu - igq^{L/R} \omega_\mu^{L/Ri} j_i^{L/R}), \tag{2.4}$$

where $i = 1,2,3$. the kinetic energy of the gauge fields is omitted and will be addressed in another paper (relevant to the curvature tensor of space-time). ϕ^R Lies in $(0,n)_{0,q_0^R}$ representation and ϕ^L lies in $(n,0)_{q_0^L,0}$. So they have spin by definition. $\omega_\mu^{L/R0}, \omega_\mu^{L/Ri}$ are the gauge fields of $SU(2)_{L/R}$. Using the symmetry between left and right-handed groups seen in nature, we assign: $\omega_\mu^{L/Ri} = \omega_\mu^i$.the $q^{L/R}$ are the conserved charges of $SU(2)_{L/R}$. g is the coupling constant. $j_\nu^{L/R}, \phi^{L/R}$ lie in any representation. for example in the spinor representation $\phi^{L/R}$ has spin $1/2$ (the charge), which are just the left and right-handed Weyl fermions $\phi^{L/R} \equiv \psi^{L/R}$.

Next, substituting $q = \hbar, g = 1$. $\hbar$ is the Planck constant, and $\partial_\mu \equiv I\partial_\mu$, we obtain,

$$D_\mu \phi^{L/R}(x_\mu) = [\partial_\mu \pm i\hbar {\omega^i}_\mu j_i^{L/R}]\phi^{L/R}(x_\mu), \tag{2.5}$$

Defining ${\omega^i}_\mu j_i := \omega_\mu$. The covariant derivative becomes:

$$D_\mu \phi^{L/R}(x_\mu) = [\partial_\mu \pm i\hbar \omega_\mu]\phi^{L/R}(x_\mu), \tag{2.6}$$

and the Lagrangian is again:

$$\mathcal{L} = (D_\mu \phi^{L/R}(x_\mu))^\dagger D_\mu \phi^{L/R}(x_\mu), \tag{2.7}$$

Notice that when $\hbar = 0$, it gives back the classical dynamics by the classical Lagrangian :

$$\mathcal{L} = \partial_\mu \phi^{L/R}(x_\mu) \partial^\mu \phi^{L/R}(x_\mu) \tag{2.8}$$

And the equation of motion is

$$\Box \phi^{L/R}(x_\mu) = 0, \tag{2.9}$$

The equations of motion of Eq (2.7)are :

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$$D_\mu D^\mu \phi^{L/R} = 0\,, \tag{2.10}$$

explicitly,

$$\left(\Box - 2i\hbar\,\omega^\mu \partial_\mu - \hbar^2 \omega_\mu \omega^\mu\right)\phi^{L/R}(x_\mu) = 0, \tag{2.11}$$

For simplicity, assuming ω_μ is constant, i.e., $\partial_\nu \omega^\mu = 0$, the solution is as follows, We try a plane-wave solution:

$$\phi_c^{kL/R}(x_\mu) = e^{ip_\mu x^\mu}\phi_c^{kL/R}(x_\mu = 0) \tag{2.12}$$

which also satisfy $\Box\phi_c^{kL/R} = 0$, i.e., the classical solution in Eq. (2.9). Next, calculating:

$$\Box\phi^{L/R} = p^\mu p_\mu \phi^{L/R} = p^2\phi^{L/R}, \tag{2.13}$$

$$\omega^\mu \partial_\mu \phi^{L/R} = i(\omega^\mu p_\mu)\phi^{L/R}, \tag{2.14}$$

Then plugging into Eq. (2.11), we obtain:

$$\left(p^2 - 2\hbar(\omega^\mu p_\mu) + \hbar^2\omega^2\right)\phi^{L/R} = 0. \tag{2.15}$$

Dividing out $\phi^{L/R} \neq 0$, we get the extended dispersion relation:

$$p^2 - 2\hbar(\omega^\mu p_\mu) + \hbar^2\omega^2 = 0. \tag{2.16}$$

We complete the square:

$$(p^\mu - \hbar\,\omega^\mu)^2 = 0, \tag{2.17}$$

which implies that the shifted momentum $k^\mu := p^\mu - \hbar\,\omega^\mu$ is null:

$$k^\mu k_\mu = 0. \tag{2.18}$$

So the general solution of Eq. (2.10) is a superposition of plane waves satisfying this dispersion relation:

$$\phi^{L/R}(x_\mu) = \int d^4p\, g(p)\, e^{ip_\mu x^\mu}, \quad \text{with} \quad (p^\mu + \hbar\,\omega^\mu)^2 = 0. \tag{2.19}$$

assigning $k^\mu = p^\mu - \hbar\,\omega^\mu$, then

$$\phi^{L/R}(x_\mu) = e^{i\hbar\,\omega_\mu x^\mu}\int d^4k\, f(k)\, e^{ik_\mu x^\mu}, \tag{2.20}$$

defining,

$$\phi_c^{L/R}(x_\mu) = \int d^4k\, f(k)\, e^{ik_\mu x^\mu}, \tag{2.21}$$

Which is a superposition of classical plane waves satisfying Eq (2.12). then Eq. (2.20) becomes:

$$\phi^{L/R}(x_\mu) = e^{i\hbar\,\omega_\mu x^\mu}\phi_c^{L/R}(x_\mu), \quad \text{where} \quad \Box\phi_c^{L/R}(x_\mu) = 0 \tag{2.22}$$

- $\phi_c^{L/R}(x_\mu)$ is a solution to the free massless classical particle ($\hbar = 0$ or $\omega_\mu = 0$).
- The exponential factor $e^{i\hbar\,\omega_\mu x^\mu}$ accounts for the phase shift introduced by the constant field ω_μ.

  Next, choosing the classical solution to be a constant instead of a wave:

$$\phi_c^{L/R}(x_\mu) = e^{ik_\mu x^\mu}\phi_c^{L/R}(x_\mu = 0) = \text{constant} \tag{2.23}$$

Eq.(2.22) becomes:

$$\phi^{L/R}(x_\mu) = ce^{i\hbar\,\omega_\mu x^\mu} \tag{2.24}$$

assigning initial conditions $c \equiv \phi^{L/R}(x_\mu = 0)$, the particle becomes,

$$\begin{aligned} \phi^{L/R}(x_\mu) &= e^{i\hbar\,\omega_\mu x^\mu}\phi^{L/R}(x_\mu = 0) \\ &= e^{i\hbar\omega^i{}_\mu j_i^{L/R} x^\mu}\phi^{L/R}(x_\mu = 0) \end{aligned} \tag{2.25}$$

Next, assigning $k^\mu \equiv 0$ in Eq.(2.17), we obtain,

$$\begin{aligned} p^\mu &= \hbar\,\omega^\mu \\ j_i^{L/R} p^i_\mu &= \hbar j_i^{L/R}\omega^i_\mu \\ p^i_\mu &= \hbar\omega^i_\mu \end{aligned} \tag{2.26}$$

The choice $k_\mu \equiv 0$ means the kinetic (mechanical) energy-momentum is always zero.

The Eq. (2.26) is also a solution of the equation $D_\mu\phi^{L/R} = 0$, which is also for an eigenvalue equation :

$$\begin{aligned} D_\mu\phi^{L/R} &= 0 \\ (\partial_\mu - \hbar\omega_\mu)\phi^{L/R} &= 0 \\ \partial_\mu\phi^{L/R} &= \hbar\omega_\mu\phi^{L/R} \\ \partial_\mu\phi^{L/R} &= j_i^{L/R}\hbar\omega^i_\mu\phi^{L/R} \end{aligned} \tag{2.27}$$

plugging $p^i_\mu := \hbar\omega^i_\mu$ into Eq (2.25), and changing coordinates $r_\mu := \hbar x_\mu$ gives,

$$\begin{aligned} \phi^{L/R}(r_\mu) &= e^{ij_i^{L/R} p^i{}_\mu r^\mu/\hbar}\phi^{L/R}(r_\mu = 0) \\ &= e^{ip_\mu r^\mu/\hbar}\phi^{L/R}(r_\mu = 0), \end{aligned} \tag{2.28}$$

where $p_\mu := j_i^{L/R} p^i_\mu$.

The relation $p^i_\mu := \hbar\omega^i_\mu$ means that there are three massless gauge bosons $i = 1,2,3$, but they are degenerate and cannot be distinguished. Otherwise, the Lorentz symmetry will be broken, so the relation can be reduced to (by choosing some i):

$$p_\mu = \hbar\omega_\mu \tag{2.29}$$

which is just *the covariant de Broglie relation!* . Furthermore, Eq. (2.28) is now reduced to

$$\begin{aligned} \phi^{L/R}(r_\mu) &= e^{ij_i^{L/R} p^i{}_\mu r^\mu/\hbar}\phi^{L/R}(r_\mu = 0) \\ &\xrightarrow{\text{pick some i}} e^{ip_\mu r^\mu/\hbar}\phi^{L/R}(r_\mu = 0), \end{aligned} \tag{2.30}$$

which is the familiar evolution in space-time of a *quantum state* / *wave function*!. Hence, we obtain the *particle-wave duality*.

2.1. Gauge Invariance of the Covariant de Broglie Relation

First, the E.O.M. are invariant under all continuous symmetries of the Lagrangian, so the solution we found is invariant under the gauge transformation. Next, the condition in Eq. (2.31) is gauge invariant as follows:

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the relation in Eq. (2.17) is just the kinetic energy of the field (k_μ), i.e.:

$$k_\mu = p^\mu - \hbar\omega^\mu = 0 \leftrightarrow (\partial_\mu - \hbar\omega^\mu)\phi^{L/R} = D_\mu\phi^{L/R} = 0, \tag{2.31}$$

This kinetic energy is gauge invariant as

$$\begin{aligned} (D_\mu\phi^{L/R}) &= 0 \\ (D_\mu\phi^{L/R})^\dagger(D_\mu\phi^{L/R}) &= 0 \\ (D_\mu U\phi^{L/R})^\dagger(D_\mu U\phi^{L/R}) &= 0 \\ (D_\mu\phi^{L/R})^\dagger U^\dagger U(D_\mu\phi^{L/R}) &= 0 \\ (D_\mu\phi^{L/R})^\dagger(D_\mu\phi^{L/R}) &= 0 \\ (D_\mu\phi^{L/R}) &= 0 \end{aligned} \tag{2.32}$$

Hence, the condition in Eq. (2.31) is gauge invariant.

2.2. Geometric Origin of Spin Dynamics

The coupling to the Lorentz gauge field ω_μ encodes both spin precession and quantum phase evolution. For spin-$\frac{1}{2}$ particles, the equation of motion

$$D_\mu D^\mu\phi^{L/R} = 0 \tag{2.33}$$

naturally incorporates the effects of local Lorentz rotations through the connection ω_μ. In the spinor representation, the connection acts as

$$D_\mu = \partial_\mu \pm \frac{i\hbar}{2}\omega^i_\mu\sigma_i, \tag{2.34}$$

and thus the field $\phi^{L/R}$ satisfies a covariant Weyl (or Dirac) equation, depending on the representation.

This construction shows that *spin is not an independent postulate* of quantum theory but rather an intrinsic geometric property of the Lorentz double cover. The phase shift introduced by the connection ω_μ corresponds directly to a rotation in spin space, giving rise to the spinor transformation law.

2.3. The Quantum Phase

The exponential phase $e^{i\hbar\omega_\mu x^\mu}$ appearing in Eq. (2.25) can thus be viewed as a *Wilson line* or phase factor acquired under parallel transport along the gauge connection ω_μ. This geometric phase is analogous to the Aharonov–Bohm phase in $U(1)$ gauge theory but here arises intrinsically from the local Lorentz structure.

When ω_μ varies slowly in spacetime, the field $\phi^{L/R}$ accumulates a spacetime-dependent phase. This constitutes a geometric realization of the quantum mechanical phase evolution, demonstrating that quantum interference phenomena naturally emerge from the gauge structure of the Lorentz double cover.

2.4. Connection to the Classical Limit

In the classical (non-quantum) limit $\hbar \to 0$, the connection ω_μ decouples, and Eq. (2.8) is recovered. In this limit, spin and quantum phase vanish, and the dynamics reduces to those of a classical massless field. Hence, the transition from classical to quantum behavior is governed by the strength of the coupling $\hbar\,\omega_\mu$, where $\hbar$ quantifies the intrinsic angular momentum coupling to the Lorentz connection.

The Lorentz double cover therefore, provides a unified framework in which classical motion, quantum interference, and spin structure are all geometric consequences of the same local symmetry.

3. Derivation of the Quantum Operators

3.1. The Energy-Momentum Operator and the Uncertainty Relation

Using the bra-ket notation $|\phi\rangle$ of linear algebra, the solution in Eq. (2.30) can be rewritten:

$$\langle r_\mu|\phi^{L/R}\rangle = \langle r_\mu = 0|e^{-ip_\mu r^\mu/\hbar}|\phi^{L/R}\rangle \tag{3.1}$$

where we use the Riesz representation theorem [7] of the inner product $\langle r_\mu|\phi^{L/R}\rangle = \phi^{L/R}(r_\mu)$. $\phi^{L/R}$ is a state with energy-momentum p_μ, so it can be defined by its eigenvalue as $|\phi^{L/R}\rangle = |p_\mu^{L/R}\rangle \equiv |p_\mu\rangle$, and the solution is rewritten as:

$$\langle r_\mu|p_\mu\rangle = p_\mu(r_\mu) = \exp(-ip_\mu r^\mu/\hbar)p_\mu(r_\mu = 0) \tag{3.2}$$

which is an eigenvector of the energy-momentum operator extracted as follows: We start by projecting Eq. (2.29) on the state $|\hbar\,\omega^\mu\rangle$ with the eigenvalue $p^\mu = \hbar\,\omega^\mu$ gives

$$p^\mu|\hbar\omega^\mu\rangle = \hbar\,\omega^\mu|\hbar\omega^\mu\rangle \tag{3.3}$$

then projecting on $\langle\phi|$

$$\begin{aligned}\langle\phi|p^\mu|\hbar\omega^\mu\rangle &= \langle\phi|\hbar\,\omega^\mu|\hbar\omega^\mu\rangle \\ p^\mu\phi(\omega^\mu) &= \hbar\,\omega^\mu\phi(\omega^\mu)\end{aligned} \tag{3.4}$$

Applying inverse Fourier transformations $\{\omega_\mu \leftrightarrow r_\mu\}$ on both sides, by using the Fourier transformation of a derivative, we obtain,

$$p^\mu\mathcal{F}^{-1}\{\phi(\omega^\mu)\} = -i\hbar\frac{\partial}{\partial r_\mu}\mathcal{F}^{-1}\{\phi(\omega^\mu)\} \tag{3.5}$$

defining $F^{-1}\{\phi(\omega^\mu)\} := \tilde{\phi}(r^\mu)$, then,

$$p^\mu\tilde{\phi}(r^\mu) = -i\hbar\frac{\partial}{\partial r_\mu}\tilde{\phi}(r^\mu) \tag{3.6}$$

This is also equal by definition,

$$\hat{p}^\mu\tilde{\phi(r^\mu)} := p^\mu\tilde{\phi(r^\mu)} = -i\hbar\frac{\partial}{\partial r_\mu}\tilde{\phi}(r^\mu) \tag{3.7}$$

therefore,

$$\hat{p}_\mu = -i\hbar\frac{\partial}{\partial r_\mu} \tag{3.8}$$

which is the e*nergy-momentum operator* in r^μ space. defining the *four-position operator* $\hat{r}_\mu$ acting on $\langle r_\mu|$ state from Eq. (3.2):

$$\langle r_\mu|\hat{r}_\mu = \langle r_\mu|r_\mu \tag{3.9}$$

satisfying the *uncertainty relation* $[\hat{r}_\mu, \hat{p}_\mu] = i\hbar$. Next, multiplying Eq. (3.8) by a constant r_μ^0 and rearranging, we obtain:

$$ir_\mu^0\frac{\hat{p}_\mu}{\hbar} = r_\mu^0\frac{\partial}{\partial r_\mu} \tag{3.10}$$

Taking the exponent of both sides gives,

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$$e^{\frac{i}{\hbar}r^0_\mu \hat{p}^\mu}(\cdot) = e^{r^0_\mu \frac{\partial}{\partial r_\mu}}(\cdot) \tag{3.11}$$

The exponent is a mapping from a Lie algebra vector space $\mathbf{g}$ to the equivalent group G (unitary), $\exp : \mathbf{g} \to G$. then acting on $\phi^{L/R}(r_\mu)$,

$$e^{-ir^0_\mu \frac{\hat{p}_\mu}{\hbar}} \phi^{L/R}(r_\mu) = e^{r^0_\mu \frac{\partial}{\partial r_\mu}} \phi^{L/R}(r_\mu) \tag{3.12}$$

this equals,

$$e^{-ir^0_\mu \frac{\hat{p}_\mu}{\hbar}} \phi^{L/R}(r_\mu) = \phi^{L/R}(r_\mu - r^0_\mu) \tag{3.13}$$

defining,

$$e^{-ir^0_\mu \frac{\hat{p}_\mu}{\hbar}} := T_{r^0_\mu} \tag{3.14}$$

$T_{r^0_\mu}$ is a unitary operator ($\hat{p}_\mu$ in the exponent is hermitian) performing a translation $(r_\mu - r^0_\mu)$. Then the solution can be written as:

$$\phi^{L/R}(r_\mu) = e^{-i\hat{p}_\mu r^\mu/\hbar} \phi^{L/R}(r_\mu = 0), \tag{3.15}$$

3.2. The Spin/Angular Momentum Operator and Quantization Conditions

The rotation of the particle according to the group $SU(2)_{L/R}$ is given by:

$$\phi^{L/R}(r_\mu) = e^{-i\hbar j_i^{L/R}\gamma^i} \phi^{L/R}(r_\mu = 0), \tag{3.16}$$

Equating to the solution in Eq. (2.25) in the linear coordinates, we obtain the relation,

$$\omega^i_\mu r^\mu := \gamma^i \tag{3.17}$$

which is a generalization of rotation in 2d dimensions $\omega t = \gamma$. This explains why the gauge fields ω^i_μ have angular velocity units. The *angular momentum operator* is easily derived from the exponent of the solution in Eq (3.16), which is the part coupled to angles γ^i:

$$\hat{S}_i = -i\hbar j_i^{L/R} \tag{3.18}$$

For example, for a spinor representation, $j_i^{L/R} = \pm\vec{\sigma}/2$ and identifying the $i$with the spatial coordinates $i = x, y, z$ using the fact that $SU(2)$ is a double cover of $SO(3)$ By the First Isomorphism Theorem [17],

$$SU(2)/\{\pm I\} \cong SO(3). \tag{3.19}$$

Concretely, each rotation $R \in SO(3)$ corresponds to exactly two elements of $SU(2)$, namely U and $-U$. Then, the operator becomes:

$$\vec{\hat{S}} = \pm i\frac{\hbar}{2}\vec{\sigma} \tag{3.20}$$

$\vec{\hat{S}}$ is the familiar operator for the angular momentum of spin $1/2$ particles in quantum mechanics. This is also for the angular momentum operator $\vec{\hat{L}}$ of the little groups $SO(3) \in SO(1,3)$, which can be represented by the isomorphism in the same manner $\hat{L}_i = -i\hbar j_i^{L/R}$. scaling $\gamma_i \to \gamma_i/\hbar$, and plugging into Eq .(3.16) gives:

$$\phi^{L/R}(r_\mu) = e^{-iL_i\gamma^i/\hbar}\phi^{L/R}(r_\mu = 0), \tag{3.21}$$

For bosonic representation, we have,

$$\phi^{L/R}(2\pi) = e^{-i\hat{L}_j 2\pi/\hbar}\phi^{L/R}(0) \equiv e^{-i2\pi n_j}\phi^{L/R}(0), \tag{3.22}$$

then,

$$\hat{L}_j = n_j\hbar \tag{3.23}$$

in the same manner for fermionic representation $\phi^{L/R}(2\pi) = -\phi^{L/R}(0)$:

$$\hat{L}_j = (n_j + 1/2)\hbar \tag{3.24}$$

Which are the quantization of angular momentum.

4. The Poincaré Group Representation

The Poincaré group is the Lorentz symmetry group $\times$ translation symmetry group. Then, every particle representation is composed of two states:

$$|\phi\rangle = |S_i\rangle \otimes |p_\mu\rangle, \tag{4.1}$$

where $p_\mu = (E, \vec{p})$ and $|S_i\rangle$ is the spin state (due to rotation symmetry), then

$$\phi_s^{L/R} = e^{-iS_i\gamma^i/\hbar}\phi_0^{L/R}, \tag{4.2}$$

$|p_\mu\rangle$is the state of energy-momentum (due to translation symmetry):

$$\phi_p^{L/R}(r_\mu) = e^{-ip_\mu r^\mu/\hbar}\phi^{L/R}(r_\mu = 0), \tag{4.3}$$

For example, for a non-relativistic ($p_\mu \approx (E, \vec{0})$) scalar particle, the quantum state is $|\phi\rangle = |(0,0)\rangle \otimes |p_\mu = (E,0)\rangle$: $|S_\mu = (0,0)\rangle$ is the scalar representation state, which is a real constant taking as 1:

$$\phi_s^{L/R} = 1 \tag{4.4}$$

for the particle energy-momentum representation state, substituting in Eq. (4.3), we obtain,

$$\begin{aligned}\phi_p^{L/R} &= e^{-i(Et/\hbar + \vec{p}\cdot\vec{r}/\hbar)}\phi^{L/R} \\ &= e^{-i(Et/\hbar)}\phi^{L/R} \\ &= e^{-i(\omega t)}\phi^{L/R}\end{aligned} \tag{4.5}$$

Where $E = \hbar\omega$ is the quantized energy.

in total,

$$|\phi\rangle_T = |(0,0)\rangle \otimes |p_\mu = (E,0)\rangle = e^{-i\omega t}\phi^{L/R}(0) \tag{4.6}$$

Hence, every particle in the universe is a *complex wave*! In space time, which is just the wave function ψ known from quantum mechanics, first described by Schrodinger, i.e., $\phi_T \equiv \psi$.

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5. Derivation of Quantum Mechanics Postulates

5.1. Postulate 1: State Postulate

A quantum system is completely described by a state vector (also called a *ket*), denoted as $|\phi\rangle$ in a complex Hilbert space. The state encodes all the information about the system. These states also have spin.

The derivation: The representation space is a Hilbert space by definition. Its representation states are the state vectors $|\phi\rangle$, and the representation type is the spin, i.e., scalar representation (spin 0), vector representation (spin 1), and spinor representation (spin $1/2$). All the representations are unitary by definition, hence $\langle\phi|\phi\rangle = 1$, by the inner product $\langle\phi_1|\phi_2\rangle$ of this Hilbert space.

5.2. Postulate 2: Time Evolution Postulate

The time evolution of a closed system is described by a unitary transformation on the initial state.

$$|\phi\rangle = e^{-iEt/\hbar}|\phi_0\rangle, \tag{5.1}$$

The derivation: This is driven by the solution in Eq. (4.6)

5.3. Postulate 3: Observable Postulate

Every observable (like position, momentum, energy) is represented by a Hermitian operator $\hat{O}$ acting on the Hilbert space. The possible outcomes of measuring the observable are the eigenvalues of this operator.

The derivation: Let f be a functional on a particle state ϕ, it can be represented by the Riesz representation theorem [7] of the internal product as:

$$\langle\phi|f\rangle = f(\phi) \tag{5.2}$$

Therefore, $|f\rangle$ lies in some representation of the Lorentz symmetry, and it can be written as,

$$f = e^{-i\hbar O^i j_i^{L/R}} \tag{5.3}$$

where $i = 1, 2, 3$ and $O^i \equiv \gamma'^i$ are angles of rotation in the Lorentz symmetry, defining: $\hat{O} := O^i j_i^{L/R}$ a hermitian matrix or operator ($j_i^{L/R}$ are hermitian).

$$f = e^{-i\hbar\hat{O}} \tag{5.4}$$

finding the eigenvalues of $\hat{O}$:

$$\hat{O}|\phi_\lambda\rangle = \lambda|\phi_\lambda\rangle \tag{5.5}$$

Where $|\phi_\lambda\rangle$ is the eigenvector. then,

$$f|\phi_\lambda\rangle = e^{-i\hbar\lambda}|\phi_\lambda\rangle \tag{5.6}$$

acting on the state $\langle r_\mu|$ gives:

$$\langle r_\mu|f|\phi_\lambda\rangle = \langle r_\mu|e^{-i\hbar\lambda}|\phi_\lambda\rangle \tag{5.7}$$
$$[f(\phi_\lambda)](r_\mu) = e^{-i\hbar\lambda}\phi_\lambda(r_\mu) \tag{5.8}$$

Therefore, the dynamics or the evolution of the state $|\phi_p\rangle$ is exclusively determined by the eigenvalue λ. hence, it is the only observable which can be extracted ("observed ") from Eq. (5.5) by projecting on $\langle\phi_\lambda|$:

$$\langle\phi_\lambda|\hat{O}|\phi_\lambda\rangle = \langle\phi_\lambda|\lambda|\phi_\lambda\rangle = \lambda \tag{5.9}$$

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which is known as the expectation value.

Then every diagonalizable operator (hermitian) $\hat{O}$ can be decomposed into its eigenvalues' sub-spaces by the spectrum theorem,

$$\hat{O} = \lambda_i \sum_i |\phi_{\lambda_i}\rangle\langle\phi_{\lambda_i}|, \tag{5.10}$$

$|r_i\rangle\langle r_i| = P_i$ is the projection operator satisfying $P_i^2 = P_i$. where $|\phi_{\lambda_i}\rangle \in H^i$ are its orthonormal eigenvectors in the eigenvalue subspaces $\hat{O}|\phi_{\lambda_i}\rangle = \lambda_i|\phi_{\lambda_i}\rangle$. The unit matrix operator I is defined as ($\lambda_i \equiv 1$):

$$\hat{O} = \sum_i |\phi_{\lambda_j}\rangle\langle\phi_{\lambda_j}| = I, \tag{5.11}$$

Finding the expectation value of ϕ :

$$\begin{aligned}\langle\phi|\hat{O}|\phi\rangle = \langle\phi|\hat{O}I|\phi\rangle &= \sum_i \langle\phi|\hat{O}|\phi_{\lambda_i}\rangle\langle\phi_{\lambda_i}|\phi\rangle = \sum_i \lambda_i\langle\phi|\phi_{\lambda_i}\rangle\langle\phi_{\lambda_i}|\phi\rangle \\ &= \sum_i \lambda_i|\langle\phi|\phi_{\lambda_i}\rangle|^2 = \sum_i \lambda_i P(\lambda_i) = \langle\lambda\rangle,\end{aligned} \tag{5.12}$$

where we used $|\langle\phi|\phi_{\lambda_j}\rangle|^2 := P(\lambda_j)$ is the probability that the state $|\phi\rangle$ collapses into $|\phi_{\lambda_i}\rangle$ which is addressed postulate 5 5.5,

5.4. Postulate 4: Composite Systems Postulate

For two quantum systems with Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, the state space of the combined system is the tensor product:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2, \tag{5.13}$$

A *sub-postulate* is the **Symmetrization postulate**(Pauli exclusion principle) : The wavefunction of a system of N identical particles (in 3D) is either totally symmetric (Bosons) or totally antisymmetric (Fermions) under interchange of any pair of particles.

The derivation: Let H, K be Hilbert spaces. Let $H \otimes K = h \otimes k : h \in H, k \in K$. Define an inner product$\langle\cdot,\cdot\rangle$ on $H \otimes K$ by:

$$\langle h_1 \otimes k_1, h_2 \otimes k_2\rangle = \langle h_1, h_2\rangle_H\rangle k_1, k_2\rangle_K, \tag{5.14}$$

With respect to this inner product, $H \otimes K$ is a Hilbert space called the Hilbert space direct product of H and K.

Therefore, if $|\phi_1\rangle \in H, |\phi_2\rangle \in K$ are two representations, then $|\phi_1\rangle \otimes |\phi_2\rangle$ is a representation in the Hilbert space $H \otimes K$.

This can be generalized by induction to: if $|\phi_i\rangle \in H^i$ are representations, then $\otimes_i|\phi_i\rangle$ is a representation in the Hilbert space $\otimes_i H_i$.

The exchange of states is anti-symmetric for fermions and symmetric for bosons, due to the innate properties of rotation of these representations:

$$|\phi\rangle = e^{-i\hbar\gamma^i j_i^{L/R}} \tag{5.15}$$

If $j_v^{L/R} \in$ the spinor representation, the exchange (rotation) will be antisymmetric. If $j_v^{L/R} \in$ the vector representation, the exchange (rotation) will be symmetric. Hence, the sub-postulate is reproduced.

5.5. Postulate 5: Measurement Postulate

When a measurement of an observable $\hat{O}$ is made on a system in a state $|\phi\rangle$:

- The result is one of the eigenvalues λ_n of $\hat{O}$.
- The probability of getting λ_n is:

$$\mathbf{P}(\lambda_n) = |\langle\ \lambda_n|\phi\rangle|^2, \tag{5.16}$$

- $|\lambda_n\rangle$ is the eigenstate corresponding to the eigenvalue λ_n.
- After measurement, the system collapses (according to Copenhagen's interpretation [10–13]) into the state $|\lambda_n\rangle$ corresponding to the measured eigenvalue.

The derivation: Let H, K be Hilbert spaces. Let $H \oplus K = h \oplus k : h \in H, k \in K$, define an inner product$\langle\cdot,\cdot\rangle$ on $H \oplus K$ by:

$$\langle h_1 \oplus k_1, h_2 \oplus k_2\rangle = \langle h_1, h_2\rangle_H + \langle k_1, k_2\rangle_K, \tag{5.17}$$

With respect to this inner product, $H \oplus K$ is a Hilbert space called the Hilbert space of the direct sum of H and K. Therefore, if $|\phi_1\rangle \in H^{r_1}, |\phi_2\rangle \in H^{r_2}$ are representations, then also $|\phi_1\rangle \oplus |\phi_2\rangle$ is a representation in the Hilbert space $H^{r_1} \oplus H^{r_2}$.

The operator $\hat{\mathbf{O}}$ is defined in $\oplus_i H^i$ Hilbert space in the same manner :

$$\hat{\mathbf{O}}|R\rangle = (\bigoplus_{i=1}^{N} \hat{O}_i)(\bigoplus_{i=1}^{N} |r_i\rangle) = \bigoplus_{i=1}^{N} (\hat{O}_i|r_i\rangle), \tag{5.18}$$

where $\hat{\mathbf{O}} = \bigoplus_{i=1}^{N} \hat{O}_i$.

defining an orthonormal basis of $\oplus_i H^i$:

$$|V\rangle = \sum_i^N \alpha_i |V_i\rangle \tag{5.19}$$

where $|V_i\rangle = |0\rangle \oplus |0\rangle ... \oplus |v_i\rangle \oplus |0\rangle$ and $\delta_{ij} = \langle V_i|V_j\rangle$. Then the operator can be written as:

$$\hat{\mathbf{O}} = \lambda_i \sum_i |V_i\rangle\langle V_i|, \tag{5.20}$$

and the identity operator,

$$I = \sum_i |V_i\rangle\langle V_i|, \tag{5.21}$$

Given the following representations states :

$$|R_\mu\rangle = \bigoplus_{i=1}^{N} |r_\mu^i\rangle$$
$$|P_\mu\rangle = \bigoplus_{i=1}^{N} |p_\mu^i\rangle, \tag{5.22}$$

where $|r_\mu^i\rangle := \alpha_i |x_\mu^i\rangle; |p_\mu^i\rangle := \beta_i |k_\mu^i\rangle$ lives in *one-dimensional* Hilbert spaces with the orthonormal basis $|x_\mu^i\rangle; |k_\mu^i\rangle$. These states have (N!) degeneracy by swapping the direct sum order. Applying unitarity,

$$1 = \langle R_\mu|R_\mu\rangle = \sum_{i=1}^{N} \langle r_\mu^i|r_\mu^i\rangle_{H_{r_i}} = \sum_i |\alpha_i^2|$$
$$1 = \langle P_\mu|P_\mu\rangle = \sum_{i=1}^{N} \langle p_\mu^i|p_\mu^i\rangle_{H_{r_i}} = \sum_i |\beta_i^2|, \tag{5.23}$$

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defining the following vectors:

$$|R^i_\mu\rangle = |0\rangle \oplus |0\rangle ... \oplus |r^i_\mu\rangle \oplus |0\rangle$$
$$|P^i_\mu\rangle = |0\rangle \oplus |0\rangle ... \oplus |p^i_\mu\rangle \oplus |0\rangle, \tag{5.24}$$

The representations in Eq(5.22), then can be written,

$$|R_\mu\rangle = \sum_{i=1}^{N} |R^i_\mu\rangle$$
$$|P_\mu\rangle = \sum_{i=1}^{N} |P^i_\mu\rangle, \tag{5.25}$$

Their internal product is :

$$\langle R_\mu|P_\mu\rangle = \sum_{i=1}^{N} \langle r^i_\mu|p^i_\mu\rangle_{H_{r_i}} = \sum_{i=1}^{N} \alpha^*_i \beta_i, \tag{5.26}$$

using $\langle r_\mu|p_\mu\rangle = p_\mu(r_\mu) = \exp(-ip_\mu r_\mu)/\hbar) p_\mu(r_\mu = 0)$ from Eq(3.2) we have,

$$\langle R_\mu|P_\mu\rangle = P_\mu(R_\mu) = \sum_{i=1}^{N} \langle r^i_\mu|p^i_\mu\rangle_{H_{r_i}}$$
$$P_\mu(R_\mu = 0)\exp(+iP_\mu R^\mu/\hbar) = \sum_{i=1}^{N} p^i_\mu(r^i_\mu = 0)\exp(+ip^i_\mu r^{i\mu}/\hbar)$$
$$= \sum_{i=1}^{N} \langle r^i_\mu = 0|p^i_\mu\rangle \exp(+ip^i_\mu r^{i\mu}/\hbar), \tag{5.27}$$

which is the *superposition principle* of quantum mechanics. now, projecting on a state $|R^k_\mu\rangle = |0\rangle \oplus |0\rangle ... \oplus |r^k_\mu\rangle \oplus |0\rangle$ gives,

$$\langle R^k_\mu|P_\mu\rangle = \sum_{i=1}^{N} \langle r^k_\mu = 0|p^i_\mu\rangle \exp(-ip^i_\mu r^{i\mu}/\hbar), \tag{5.28}$$

by Eq. (5.26) , $\langle r^k_\mu = 0|p^i_\mu\rangle \neq 0$, only for $i = k$, we obtain:

$$\langle R^k_\mu|P_\mu\rangle = \sum_{i=1}^{N} \delta^i_k \langle r^i_\mu = 0|p^i_\mu\rangle \exp(-ip^i_\mu r^j_\mu/\hbar)$$
$$= \langle r^k_\mu = 0|p^{k\mu}\rangle \exp(-ip^k_\mu r^{k\mu}/\hbar)$$
$$P_\mu(R^k_\mu) = p^{k\mu}(r^k_\mu = 0)\exp(-ip^k_\mu r^{k\mu}/\hbar), \tag{5.29}$$

which is the *collapsed state (Copenhagen's interpretation)* into r^k_μ Hilbert space. Calculating the amplitude of the wave function,

$$\langle P_\mu|R^k_\mu\rangle\langle R^k_\mu|P_\mu\rangle = \langle p^k_\mu|r^k_\mu = 0\rangle\langle r^k_\mu = 0|p^k_\mu\rangle = |\langle r^k_\mu = 0|p^k_\mu\rangle|^2$$
$$\Rightarrow |P_\mu(R^k_\mu)|^2 = |p^k_\mu(r^k_\mu = 0)|^2 = |\alpha^*_k \beta_k|^2, \tag{5.30}$$

summing over k , and using the identity operators, $\sum_k |R^k_\mu\rangle\langle R^k_\mu| = I$, we obtain,

$$\sum_k \langle P_\mu | R^k_\mu \rangle \langle R^k_\mu | P_\mu \rangle = \langle P_\mu | P_\mu \rangle = 1$$
$$\Rightarrow \sum_k |P_\mu(R^k_\mu)|^2 = \sum_k |p^k_\mu(r^k_\mu = 0)|^2 = \sum_k |\alpha^*_k \beta_k|^2 = 1, \tag{5.31}$$

which is the famous *Max Born's rule of probability* It has a probabilistic nature because the projection (*the quantum measurement*) on the state $\langle R^k_\mu | \in \{\langle R^i_\mu |\}^N_{i=1}$ is probabilistic; choosing one state out of N similar states has N combinations :

$$N = \frac{N!}{1!(N-1)!} \tag{5.32}$$

Every projection has a weight (amplitude) of $|p^k_\mu(r^k_\mu = 0)|^2$ to project on it. The collapse isn't unitary because the observer picks a random state $\langle R^k_\mu|$ and discards the others,

$$P_\mu(R_\mu) \neq P_\mu(R^k_\mu) \tag{5.33}$$

If the observer can build a measurement apparatus that projects the state onto the superpositions of the position of the apparatus $\langle R_\mu| = \sum^N_i \langle R^i_\mu|$ instead, the collapse will not happen! This means if there is a direct sum of observers, $\{|R^i_\mu\rangle\}_i$, the collapse never happens, which is similar to the *many worlds interpretation* of Hugh Everett [14–16]:

$$P_\mu(R_\mu) = \sum_k P_\mu(R^k_\mu) \tag{5.34}$$

This is the unitary evolution of this direct sum of universes by the Eq. (5.27). These universes are not so real as one may think; they are just different representations of the same Lorentz group (the only invariant part to all representations).

6. The Most General Representation States

postulates $4+5$ give the most general Hilbert space built as :

$$H_G = \alpha_j \bigoplus_j (\bigotimes_{i_j} H^{i_j})^j, \tag{6.1}$$

Which is a polynomial spanning of the space of "Hilbert spaces ". The representations states in H_G , $|V_G\rangle \in H_G$, are composed of a superposition (direct sum) $(\bigoplus_j)$ of multi-particle $(\bigotimes_{i_j})$ states. In case these states are entangled, i.e,. $\bigotimes_{i_j} H^{i_j}$ don't have a separable basis, $\bigotimes_{i_j} H^{i_j} \in \text{span}\{\bigotimes_{i_j} e_i\}$ the states are then written as

$$|V\rangle = \bigoplus_{j=1}^N \alpha_j |v^1, v^2 ... v^{i_j}\rangle^j, \tag{6.2}$$

Which is the *entanglement*.

7. The Quantum Action and Feynman's Path Integral Formulation

The Hamiltonian $\mathcal{H}$ is related to the Lagrangian L by [2]:

$$\mathcal{H} = P_i \dot{r}_i - L \quad ; \quad i = x, y, z, \tag{7.1}$$

Where P_i is the momenta. Multiplying the two sides by dt:

$$\mathcal{H}dt = P_i dr_i - Ldt, \tag{7.2}$$

The Lagrangian is given by $L := E_K - qV$; the Hamiltonian is given by $\mathcal{H} = E_K + qV$. qV is the potential energy. E_K is the kinetic energy. For a free particle $qV = 0$, gives: $L = \mathcal{H} = E_K$ the action is given by : $dS = Ldt$; substituting in Eq (7.2) gives,

$$E_K dt - P_i dr_i = -dS, \tag{7.3}$$

For a massless particle, the kinetic energy equals the total energy of the particle:$E = (mc^2 + E_K) = 0 + E_K = E_k$. Then Eq. (7.3) becomes:

$$Edt - P_i dr_i = -dS \tag{7.4}$$
$$\Rightarrow -P^\mu dr_\mu = dS, \tag{7.5}$$

Where $P_\mu := (E, \vec{P})$. For a massless particle moving in a field A_μ , the potential energy is $V = qA_0$ and the particle's four-energy-momentum is $p_\mu = P_\mu - qA_\mu$, substitute in Eq (7.4) gives,

$$-p^\mu dr_\mu = dS, \tag{7.6}$$

Boosting (rotation in Lorentz symmetry) to the rest frame, i.e. $p_\mu = (p_0, \vec{0}), r_\mu = (dt, \vec{0})$, gives:

$$dS = -(p_0, \vec{0})(dt, \vec{0}) = -p_0 dt = -(E - q\Delta A_0)dt = -(E - V)dt = -Ldt$$
$$\Rightarrow E - V = L, \tag{7.7}$$

And we retrieved the Lagrangian back. So the *Legendre transformation is just the Lorenz transformation!* Therefore, we will define the action accordingly as follows:

$$-p_\mu r^\mu = S, \tag{7.8}$$

including the generators according Eq (2.28) :

$$-j_i^{L/R} \omega^i{}_\mu r^\mu = j_i^{L/R} s^i = S^{L/R}, \tag{7.9}$$

And its differential in the case of p^i_ν constant:

$$-j_i^{L/R} \omega^i{}_\mu dr^\mu = j_i^{L/R} ds^i = dS^{L/R}, \tag{7.10}$$

For simplicity, assigning $S^R = S^L$ using the symmetry of the right-handed and left-handed. Substituting the quantum energy-momentum $p^\mu = \hbar\omega^\mu$ and the relation for angular velocity: $\omega^i{}_\mu dr^\mu = d\gamma^i$ by Eq (3.17) gives:

$$-\hbar\omega^i{}_\mu dr^\mu = \hbar d\gamma^i = ds^i, \tag{7.11}$$

thus,

$$\frac{ds^i}{\hbar} = d\gamma^i, \tag{7.12}$$

Making the re-scaling $\hbar\gamma^i \to \gamma^i$ gives:

$$ds^i = d\gamma^i \to s^i = \gamma^i, \tag{7.13}$$

Hence, the action is nothing more than the particle's *rotation angles* under the Lorenz symmetry!. These angles are equivalent to Action-angle coordinates used in classical mechanics! , see [8].

Substituting Eq (7.12) in Eq(3.21) gives,

$$\phi^{L/R}(r_\mu) = e^{\frac{-ij_i^{L/R}s^i}{\hbar}}\phi^{L/R}(r_\mu = 0) = e^{\frac{-iS^{L/R}}{\hbar}}\phi^{L/R}(r_\mu = 0), \tag{7.14}$$

which is the familiar *quantum action*!

Every quantum action is a translation in space-time by the energy-momentum operator, see Eq .(3.15):

$$e^{\frac{-idS^{L/R}}{\hbar}} = e^{\frac{-ip_\mu dr^\mu}{\hbar}} = \hat{T}(r_\mu^0 + dr_\mu) \tag{7.15}$$

performing successive translation $r = \sum_j d^j r$:

$$\begin{aligned}\prod_j e^{\frac{-id^jS^{L/R}}{\hbar}} &= \prod_j e^{\frac{-ip_\mu d^jr^\mu}{\hbar}} = \prod_j \hat{T}(r_\mu^0 + d^jr_\mu)\\ e^{\frac{-i\int dS^{L/R}}{\hbar}} &= e^{\frac{-i\int p_\mu dr^\mu}{\hbar}} = \hat{T}(r_\mu^0 + r_\mu)\\ e^{\frac{-iS(r_\mu,p_\mu)}{\hbar}} &= \hat{T}(r_\mu^0 + \int dr_\mu)\end{aligned} \tag{7.16}$$

Which in total gives a translation in some path $l = \int dr_\mu$, so assuming all the possible paths (via superposition) gives the Feynman's path integral formulation of quantum mechanics.

$$\phi^{L/R}(r_\mu)_T = \int_{\text{all the possible paths}} e^{\frac{-iS^{L/R}}{\hbar}}\phi_0^{L/R} \tag{7.17}$$

8. Discussion

The formulation of quantum mechanics presented here can be entirely replaced by the description of a classical scalar field charged under the local Lorentz symmetry and its associated gauge fields. This classical framework successfully reproduces all postulates and mathematical structures of quantum mechanics.

Within this approach, the quantum operators emerge naturally, and the unique role of their eigenvalues in determining the evolution of a particle becomes evident. Consequently, these eigenvalues represent the only possible observables in nature. Explicit examples were provided for the classical derivation of the energy–momentum, four-position, and angular momentum operators, together with their corresponding eigenvalues.

In this framework, the Planck constant $\hbar$ acquires a clear physical meaning: it corresponds to the quantized charge associated with the local Lorentz symmetry. The de Broglie relation follows directly, yielding the well-known relations

$$E = \hbar\omega, \qquad \vec{p} = \hbar\vec{k}, \tag{8.1}$$

which can be interpreted as the kinetic energy and momentum of the Lorentz symmetry gauge fields.

Furthermore, the quantum action arises naturally within this formalism, as the angles of the Lorentz symmetry are shown to correspond directly to the quantum action. The principles of superposition and wave-function collapse are also understood as the direct sum of equivalent representations of the Lorentz symmetry.

Finally, this framework suggests that the universe itself constitutes a representation space of the Lorentz symmetry group—identifiable with the Hilbert space of quantum mechanics—rather than the classical real space employed in classical mechanics.

9. Conclusion

Quantum mechanics can be entirely reformulated in terms of a classical field theory charged under the Lorentz symmetry group of space–time. In this framework, the angles of the Lorentz symmetry correspond to the quantum action, while the conserved charge associated with this symmetry is identified with the Planck constant $\hbar$. The gauge fields of the Lorentz symmetry naturally carry the observed four energy-momentum, known as the de Broglie relation.

All postulates of quantum mechanics emerge naturally, consistently, and without additional assumptions from this classical description. Consequently, the full structure of quantum mechanics—together with its characteristic phenomena—finds a coherent and rigorous foundation within a classical field theoretical framework.

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