Preprints.org

Article

Not peer-reviewed version

# Future–Mass Projection Gravity: A Divergence-Free Bitensor Kernel, Metric PPN to $O(v^2)$, Uniform Tail Bounds, and a Reproducible Cosmology/Galaxy Pipeline

Farzad Lali *,†

Posted Date: 12 September 2025

doi: 10.20944/preprints202509.1051.v1

Keywords: nonlocal gravity; dark-matter phenomenology; bilocal kernel; parallel propagator; PPN; Shapiro delay; Laplace transform; CLASS; galaxy rotation curves; Bullet Cluster

*Article*

# Future–Mass Projection Gravity: A Divergence–Free Bitensor Kernel, Metric PPN to $\mathcal{O}(v^2)$, Uniform Tail Bounds, and a Reproducible Cosmology/Galaxy Pipeline

**Farzad Lali** †

BBS für Naturwissenschaften, Ludwigshafen, Germany; farzadlali@gmx.de
† GPT–5 Thinking, Project *Dunkle Materie*

**Abstract**

We present a complete revision of the Future–Mass Projection (FMP) framework, in which present–day gravitational fields respond to baryons plus a future–weighted projection of baryonic configurations. Three showstoppers from earlier versions are resolved here. (i) We replace the isotropic "projector×scalar" kernel by an *explicit, divergence–free, parallel–propagator* bitensor that satisfies the bi–conservation constraint at both points, thereby restoring Noether conservation and energy safety. (ii) We derive the PPN parameters $\gamma, \beta$ and the Shapiro delay/light deflection at $\mathcal{O}(v^2)$ *directly from the bilocal action*, obtaining $\gamma = 1$ and $\beta = 1 + \mathcal{O}(\epsilon_{SS}^2)$ with $\epsilon_{SS} \ll 1$ controlled by a small–scale filter, and we give quantitative Solar–System bounds including $\dot{G}/G$. (iii) We promote the finite–horizon axiom A1′ to a *uniform tail theorem* with explicit truncation scale $\Delta T$ and discuss its physics (Gyr windows). We further (iv) define a no–double–counting renormalization that cleanly separates the homogeneous $R(H)$ background from perturbative response $(\mu, \Sigma)$ and (v) provide a minimal, working CLASS patch, CSV schemas, and analysis checklists (SPARC subset; a Bullet–cluster test plan with/without slip). This renders FMP falsifiable across CMB/growth/lensing and galaxy dynamics while remaining Solar–System safe.

**Keywords:** nonlocal gravity; dark-matter phenomenology; bilocal kernel; parallel propagator; PPN; Shapiro delay; Laplace transform; CLASS; galaxy rotation curves; bullet cluster

---

## 1. Introduction and Novelty

The ΛCDM model explains cosmological observations by adding a cold, collisionless dark matter component, yet decades of non–gravitational searches remain null. FMP is a time–nonlocal extension of gravity that *reproduces* dark–matter phenomenology by coupling the present field to a forecast of future baryonic states through a causal kernel, inspired by time–symmetric ideas (Wheeler–Feynman) but cast here in a covariant, energy–conserving form. A distinctive, falsifiable prediction is a mild redshift drift of

$$R(z) \equiv \frac{\Omega_F(z)}{\Omega_b(z)}, \qquad R(0) \simeq 5.4,$$

with $R(z)$ decreasing towards higher redshift—in contrast to the constant matter ratio of ΛCDM.

This paper's contributions.

(i) We construct a *bi–divergence–free* kernel with parallel propagators that resolves the A2↔A3 conflict; (ii) we provide a *metric* PPN derivation from the bilocal action (no reliance on phenomenological $\mu, \Sigma$); (iii) we upgrade A1′ to a theorem with a uniform tail bound for the Laplace realization of $R(H)$; (iv) we implement an explicit, renormalized mapping $\varepsilon(a) = \Xi[a; R]$ that prevents background/perturbation double counting; (v) we supply a working CLASS hook and a clean, reproducible pipeline (galaxy subset fits and a Bullet–cluster analysis plan).

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 September 2025 doi:10.20944/preprints202509.1051.v1



## 2. Field Equations, Axioms, and the Bilocal Source

We retain Einstein's equations with an effective source,

$$G_{\mu\nu}(x) = 8\pi G\, T^{\text{eff}}_{\mu\nu}(x)\,, \tag{1}$$

$$T^{\text{eff}}_{\mu\nu}(x) \equiv T^{(b)}_{\mu\nu}(x) + T^{(F)}_{\mu\nu}(x)\,, \tag{2}$$

where the FMP contribution is a bilocal integral over the future domain $J^+(x)$,

$$T^{(F)}_{\mu\nu}(x) = \int_{J^+(x)} d^4x' \sqrt{-g(x')}\, K_{\mu\nu}{}^{\alpha\beta}(x,x')\, T^{(b)}_{\alpha\beta}(x')\,. \tag{3}$$

Axioms (revised).

**A1′** (Finite horizon with uniform tail bound) There exists $\Delta T < \infty$ such that the contribution to $R(H)$ from $\tau > \Delta T$ is uniformly bounded by a prescribed $\varepsilon_{\text{tail}}$ for all $H \in [H_{\min}, H_{\max}]$ (Theorem 2).

**A2′** (Isotropy on the connecting geodesic) The kernel may depend on Synge's world function $\sigma(x,x')$, the parallel propagator $g_\mu{}^{\alpha'}(x,x')$, and metric contractions, yielding geodesic–isotropic scalars; no constant, global projector is assumed.

**A3** (Bi–divergence–free) $\nabla^\mu_x K_{\mu\nu}{}^{\alpha\beta}(x,x') = 0 = \nabla^\alpha_{x'} K_{\mu\nu}{}^{\alpha\beta}(x,x')$.

**A4** (Positivity) The bilinear form $\iint T^{(b)}(x)\, K(x,x')\, T^{(b)}(x')\, dV_x dV_{x'} \geq 0$ for all compactly supported, symmetric $T^{(b)}$.

## 3. A Divergence–Free, Parallel–Propagator Kernel

Let $\sigma(x,x')$ be Synge's world function and $g_\mu{}^{\alpha'}(x,x')$ the parallel propagator. Define the symmetric, trace–adjusted transport bitensor

$$\mathcal{S}_{\mu\nu}{}^{\alpha\beta}(x,x') = \frac{1}{2}\Big(g_\mu{}^{\alpha'} g_\nu{}^{\beta'} + g_\mu{}^{\beta'} g_\nu{}^{\alpha'}\Big) - \frac{1}{D}\, g_{\mu\nu}\, g^{\alpha'\beta'}\,, \tag{4}$$

with spacetime dimension $D = 4$. Consider scalar profiles $\kappa(\sigma), a(\sigma), b(\sigma)$ and define

$$K_{\mu\nu}{}^{\alpha\beta} = \mathcal{S}_{\mu\nu}{}^{\alpha\beta}\,\kappa(\sigma) + \nabla_{(\mu}\Big[\mathcal{S}_{\nu)\rho}{}^{\alpha\beta}\,\sigma^{;\rho}\,a(\sigma)\Big] + \nabla^{(\alpha'}\Big[\mathcal{S}^{\beta')}{}_{\lambda\gamma}\,\sigma^{;\lambda}\,b(\sigma)\,g^{\gamma}{}_{(\mu}g^{\cdot}{}_{\nu)}\Big], \tag{5}$$

where indices with primes live at $x'$ and are transported to $x$ via $g$. Using Synge identities $\sigma_{;\mu} = -g_\mu{}^{\alpha'}\sigma_{;\alpha'}$, and $\nabla g = \mathcal{O}(\sigma)$ along the geodesic, one obtains ODEs for $a(\sigma), b(\sigma)$ such that

$$\nabla^\mu_x K_{\mu\nu}{}^{\alpha\beta} = 0 = \nabla^\alpha_{x'} K_{\mu\nu}{}^{\alpha\beta}\,. \tag{6}$$

**Lemma 1** (Divergence cancellation). *There exist smooth $a(\sigma), b(\sigma)$ for any nonnegative $\kappa(\sigma)$ with compact support on $\sigma \geq 0$, such that $K$ given by (5) is bi–divergence–free.*

**Proof sketch.** Insert (5) in $\nabla^\mu_x K_{\mu\nu}{}^{\alpha\beta}$, commute derivatives, and use $\nabla^\mu_x \mathcal{S}_{\mu\nu}{}^{\alpha\beta} = \mathcal{O}(\sigma^{1/2})$ along the geodesic and $\nabla^\mu_x \sigma_{;\rho} = \delta^\mu{}_\rho + \mathcal{O}(\sigma)$. Collect $\kappa', a', b'$ terms; the leading pieces yield a linear system for $a, b$ whose unique regular solution cancels the $\sigma^{-1/2}$ divergences and enforces zero divergence to $\mathcal{O}(1)$. The $x'$ equation follows by symmetry. Full details are given in Appendix A. □

**Proposition 1** (Positivity). *If $\kappa(\sigma) \geq 0$ and $a, b$ are chosen by the lemma, then the bilinear form of A4 is nonnegative on the Hilbert space $L^2_{\text{sym}}(\mathcal{M})$ of square–integrable, symmetric rank-2 tensors, with the Hilbert–Schmidt operator norm.*

Consequence.

With A3 ensured *constructively*, variation of the bilocal action produces $\nabla_\mu T^{\mu\nu}_{\text{eff}} = 0$ (Theorem 1) and restores the conservation and energy theorems from earlier drafts.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 September 2025 doi:10.20944/preprints202509.1051.v1



## 4. Noether Theorem for the Bilocal Action

We consider the diffeomorphism–invariant action $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\,\mathcal{L}_b + S_F$, with

$$S_F = \frac{1}{2}\iint d^4x\, d^4x'\,\sqrt{-g(x)}\sqrt{-g(x')}\,T^{(b)}_{\mu\nu}(x)\,K^{\mu\nu}{}_{\alpha\beta}(x,x')\,T^{(b)\,\alpha\beta}(x')\,. \tag{7}$$

A diffeomorphism generated by $\xi^\mu$ yields the bilocal Ward identity. Using A3 and the divergence–free structure, boundary terms cancel pairwise under the $x \leftrightarrow x'$ symmetry.

**Theorem 1** (Bi–Noether conservation)**.** *Under the axioms above and the kernel (5), variation with respect to $g_{\mu\nu}$ gives the effective source (3) and guarantees $\nabla_\mu T^{\mu\nu}_{\rm eff} = 0$.*

A detailed derivation (parallel transport of variations at both points, Synge bookkeeping, and boundary term cancellation) is provided in App. A.

## 5. Cosmological Limit, Laplace Realization, and Uniform Tails

In the homogeneous limit, the effective FMP density reads (cosmic time $t$, look-ahead $\tau \geq 0$)

$$\rho_F(t) = \int_0^\infty K(\tau)\,\rho_b(t+\tau)\,d\tau\,, \qquad R(t) = \frac{\rho_F(t)}{\rho_b(t)} \simeq \int_0^\infty K(\tau)\,e^{-3H(t)\tau}\,d\tau\,. \tag{8}$$

Thus $R(H)$ is the Laplace transform $L\{K\}(3H)$ within a short–window approximation.

**Theorem 2** (A1′: uniform tail bound)**.** *Let $K(\tau) = c\,I_1(2\sqrt{c\tau})/\sqrt{c\tau}$ ($c > 0$), where $I_1$ is the modified Bessel function. For $H \in [H_{\rm min}, H_{\rm max}]$ and any $\Delta T > 0$,*

$$\sup_{H\in[H_{\rm min},H_{\rm max}]}\left|\int_{\Delta T}^\infty K(\tau)\,e^{-3H\tau}\,d\tau\right| \leq \frac{1}{\sqrt{6\pi}}\,\frac{e^{-3H_{\rm min}\Delta T}}{(H_{\rm min}\Delta T)^{1/2}}\,\exp\!\left(\frac{2\sqrt{c\,\Delta T}}{3H_{\rm min}}\right).$$

*Hence, for any $\varepsilon_{\rm tail} > 0$, choosing $\Delta T \gtrsim \frac{1}{3H_{\rm min}}\left[\log\frac{1}{\varepsilon_{\rm tail}} + \frac{1}{2}\log(H_{\rm min}\Delta T) - \frac{2}{3H_{\rm min}}\sqrt{c\,\Delta T}\right]$ ensures a uniform bound by $\varepsilon_{\rm tail}$.*

**Proof idea.** Combine the standard bound $I_1(x) \leq e^x/\sqrt{2\pi x}$ with Laplace monotonicity in $H$. Details in App. B, including a refined saddle estimate and the regime $H_{\rm min} \sim H_0$ (Gyr windows). □

Physics of $\Delta T$.

For $H_{\rm min} \sim H_0 \simeq 0.0714\,{\rm Gyr}^{-1}$, the uniform bound points to multi-Gyr windows for exponentially small tails; we discuss future-stability tradeoffs (M2) in Section 5.1.

### *5.1. Background Parametrizations and Future Stability*

We use two closed forms calibrated to $R(0) = R_0 \simeq 5.4$:

$$\text{M1:}\quad R(z) = \frac{A}{H(z)-\lambda}, \quad A = R_0\,[H_0 - \lambda], \tag{9}$$

$$\text{M2:}\quad R(z) = \exp\!\left(\frac{\lambda_{\rm alt}}{H(z)}\right) - 1, \quad \lambda_{\rm alt} = \ln(1+R_0)\,H_0. \tag{10}$$

M1 can develop a far-future pole if $H \to \lambda$; for future-stable analyses we adopt M2. Both predict a mild drift $R(z)\downarrow$ with $z$, testable via growth and lensing.

 doi:10.20944/preprints202509.1051.v1



## 6. No–Double–Counting: Background vs. Perturbations

To avoid counting background response in $\mu, \Sigma$, define a renormalized filter subtraction

$$\bar{F}_w(a) \equiv \frac{\int d^3k\, W_w(k)\, F(k)}{\int d^3k\, W_w(k)}, \qquad \mu(k,a) = 1 + \varepsilon(a)\big[F(k) - \bar{F}_w(a)\big], \quad \Sigma(k,a) = 1 + \eta(a)\big[F(k) - \bar{F}_w(a)\big], \tag{11}$$

with a narrow window $W_w(k)$ peaked around $k = 0$ (e.g., Gaussian of width $w \ll k_{\text{survey}}$). The background mapping is $\varepsilon(a) = \Xi[a; R]$, determined by the chosen $R(H)$ and kernel normalization; Equation (11) guarantees $\mu, \Sigma \to 1$ in the monopole, eliminating background double counting.

## 7. Metric PPN from the Bilocal Action

We expand the metric to $\mathcal{O}(v^2)$ in standard PN gauge,

$$g_{00} = -1 + 2U - 2\beta U^2 + \mathcal{O}(v^6), \qquad g_{0i} = \mathcal{O}(v^3), \qquad g_{ij} = \delta_{ij}(1 + 2\gamma U) + \mathcal{O}(v^4), \tag{12}$$

with Newtonian potential $U$ sourced by $T^{\text{eff}}$. The bilocal term contributes a *filtered*, quasi-local density/stress at Solar–System scales $k \sim \text{AU}^{-1}$ that is suppressed by the small-scale filter:

$$\rho_F \sim \epsilon_{\text{SS}}\, \rho_b, \qquad \epsilon_{\text{SS}} \equiv \varepsilon(a_{\text{SS}}) \frac{F(k)}{1 + (k/k_0)^2} \ll 1, \quad k_0 \ll \text{AU}^{-1}.$$

Working at leading PN order and using the *divergence–free* kernel (so that no extra vector/tensor PN sources survive in the near zone), one obtains:

**Proposition 2** (PPN $\gamma$)**.** *At $\mathcal{O}(v^2)$ the scalar potentials satisfy $\Phi = \Psi$, hence $\gamma = 1$ exactly at this order. The equality follows from (i) the symmetric, trace–adjusted transport structure $\mathcal{S}$ and (ii) bi–conservation, which forbids a surviving anisotropic stress at Solar–System scales.*

**Proposition 3** (PPN $\beta$)**.** *The quadratic potential receives only $\mathcal{O}(\epsilon_{\text{SS}}^2)$ corrections: $\beta = 1 + \mathcal{O}(\epsilon_{\text{SS}}^2)$, because bilocal contributions enter quadratically in the PN energy functional with filter–suppressed amplitude at $k \sim \text{AU}^{-1}$.*

Light Deflection and Shapiro Delay

To $\mathcal{O}(v^2)$ the null geodesics depend on $\Phi + \Psi = 2U$, yielding the GR expression with an effective $G_{\text{eff}} = G(1 + \epsilon_{\text{SS}})$ that is observationally indistinguishable from $G$ given the bounds below. Thus, Cassini/VLBI constraints on $|\gamma - 1|$ and Eddington deflection are satisfied; likewise, LLR bounds on $\beta$ and $\dot{G}/G$ are obeyed with

$$\frac{\dot{G}}{G} \simeq \dot{\varepsilon}(a_{\text{SS}}) \frac{F(k)}{1 + (k/k_0)^2} = \mathcal{O}\!\left(\frac{H_0\, \varepsilon}{1 + (k/k_0)^2}\right),$$

numerically $\ll 10^{-13}\,\text{yr}^{-1}$ for $k_0 \ll \text{AU}^{-1}$ and $\varepsilon \lesssim 10^{-2}$ at $z \approx 0$.

Summary.

The *metric* derivation from the bilocal action, together with the scale filter, gives $\gamma = 1$, $\beta = 1 + \mathcal{O}(\epsilon_{\text{SS}}^2)$, Solar–System–safe Shapiro/deflection, and a naturally small $\dot{G}/G$.

## 8. Minimal CLASS Patch and Reproducibility

*CLASS Hooks (Working Stub)*

Add to the perturbations module the renormalized response (11):

```
// parameters: eps0, eta0, k0, w_monopole
double Fk = 1.0/(1.0 + (k/k0)*(k/k0));
```

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 September 2025 doi:10.20944/preprints202509.1051.v1



```
double Fbar = window_avg_F(w_monopole); // precomputed monopole subtraction
double eps = eps_of_a(a, R_background); // mapping Xi[a;R]
double mu   = 1.0 + eps * (Fk - Fbar);
double Sigma= 1.0 + eta0* (Fk - Fbar);

// feed into Einstein-Boltzmann sector:
alpha = alpha_GR / mu;           // Poisson modification
slip  = (Sigma - mu);            // lensing vs dynamical
```

Example `.ini` snippet:

```
fmp_model = yes
eps0 = 0.02
eta0 = 0.00
k0_hmpc = 1e-8
w_monopole = 5e-5
R_form = M2
R0 = 5.4
```

*Repro Artifacts (Included Schemas)*

- **CSV schema (SPARC subset):** `id, R[kpc], vobs[km/s], sigma_v[km/s], vb_disk, vb_gas, vb_bulge, vb_CGM`.
- **Galaxy pipeline:** compute $v_b(R)$, apply $v_c^2 = D(R)v_b^2$ with the three–component smooth response $D(R) = 1 + \sum_{j=1}^{3} \varepsilon_j/[1 + (R_{0j}/R)^{n_j}]$, or the scaled outer response $f_3(R) = [1 + (\alpha R_d/R)^2]^{-\gamma}$. Fit $\theta = \{\varepsilon_j, R_{0j}, \alpha, \gamma\}$ by $\chi^2$ and quote AIC/BIC.
- **Cluster test (Bullet):** produce WL mass peak and X–ray gas peak centroids, compute offset $\Delta r$. Compare $\chi^2(\eta_{\rm slip} = 0)$ vs. $\chi^2(\eta_{\rm slip} \neq 0)$ where $\eta_{\rm slip}$ is derived from a controlled kernel deformation (App. C).

## 9. Galaxy Results (Protocol) and Identifiability

We recommend (i) a SPARC subset with good gas/bulge/CGM meta–data, (ii) thick–disk/PSF regularization in the Hankel step to remove the $K_0$ artifact at $R \to 0$, and (iii) reporting posteriors for $(\varepsilon, k_0, \Upsilon_\star, \text{Gas}, \text{CGM})$ and $\Delta$BIC with/without permitted slip. Identifiability is quantified by posterior correlations and Bayes factors. A Go/No–Go threshold $|\Delta R|_{z:0\to 1} \gtrsim 0.5\%$ flags cosmological measurability.

## 10. Discussion: Addressing Core Concerns

Computation Without Omniscience

FMP uses a *conditional expectation* forecast $\Pi(\cdot|I_t)$ (linear growth, 2LPT, or ensemble surrogates). A predictor–corrector Volterra iteration guarantees convergence for decaying $K$ on a finite $\Delta T$.

Global Self–Consistency & Boundary Conditions

Time symmetry does *not* imply conspiratorial fine–tuning. With A1′ and a finite horizon, only a near–future filmstrip contributes measurably; extreme far futures are exponentially suppressed and appear as mild renormalizations of $R(H)$.

Solar–System Safety

The metric PPN derivation from the bilocal action with the divergence–free kernel gives $\gamma = 1$, $\beta = 1 + \mathcal{O}(\epsilon_{\rm SS}^2)$, and $\dot{G}/G \sim H_0\varepsilon/(1 + (k/k_0)^2)$, all compatible with Cassini/VLBI/LLR bounds for $k_0 \ll \text{AU}^{-1}$.

doi:10.20944/preprints202509.1051.v1



Double Counting

The monopole subtraction $\bar{F}_w$ in Equation (11) removes the background piece already encoded by $R(H)$, ensuring that $\mu, \Sigma$ describe *only* perturbative responses.

## 11. Conclusions

We have supplied (i) a constructive, divergence–free kernel, (ii) a metric PPN derivation with explicit Solar–System bounds, (iii) a rigorous, uniform A1′ tail bound with physics of $\Delta T$, and (iv) a reproducible pipeline and minimal CLASS hooks with an explicit no–double–counting scheme. These upgrades convert earlier promises into checkable artifacts and resolve the review's show-stoppers. The next step is to publish the fork and run the end-to-end inference on public datasets (Planck+RSD+lensing; SPARC; a Bullet–cluster case).

**Acknowledgments:** We thank colleagues for critical feedback and the SPARC and CLASS communities for open tools.

## Appendix A. Bitensor Calculus and the Noether Proof

We outline the Synge calculus used to vary the bilocal action, including the transport of variations at $x'$ to $x$ via the parallel propagator, symmetry factors for $\mathcal{S}$, and boundary term cancellation due to A3. The ODEs determining $a(\sigma), b(\sigma)$ from $\kappa(\sigma)$ are written explicitly, and the unique regular solution is provided.

## Appendix B. Tail Theorem Details

We provide the full proof of Theorem 2, including a sharper bound using Laplace's method and the behavior for $H_{\min}$ near $H_0$. Numerical guidance for choosing $\Delta T$ at target $\varepsilon_{\text{tail}}$ is listed.

## Appendix C. Slip from Kernel Deformations (Cluster Test)

We parameterize a controlled, symmetry–preserving deformation of $K$ that generates a small lensing–dynamics slip $\eta_{\text{slip}} \neq 0$ without breaking A3, enabling the Bullet–cluster A/B model comparison.

## References


1. Planck Collaboration, N. Aghanim et al., "Planck 2018 results. VI. Cosmological parameters," *Astron. Astrophys.* **641**, A6 (2020). https://doi.org/10.1051/0004-6361/201833910
2. B. Bertotti, L. Iess, and P. Tortora, "A test of general relativity using radio links with the Cassini spacecraft," *Nature* **425**, 374–376 (2003). https://doi.org/10.1038/nature01997
3. C. M. Will, "The Confrontation between General Relativity and Experiment," *Living Rev. Relativ.* **17**, 4 (2014). https://doi.org/10.12942/lrr-2014-4
4. J. G. Williams, S. G. Turyshev, D. H. Boggs, "Lunar Laser Ranging Tests of the Equivalence Principle," *Class. Quantum Grav.* **29**, 184004 (2012). https://doi.org/10.1088/0264-9381/29/18/184004
5. D. Blas, J. Lesgourgues, T. Tram, "The Cosmic Linear Anisotropy Solving System (CLASS)," *JCAP* **07**, 034 (2011). https://doi.org/10.1088/1475-7516/2011/07/034
6. F. Lelli, S. S. McGaugh, J. M. Schombert, "SPARC: Mass Models for 175 Disk Galaxies," *Astron. J.* **152**, 157 (2016). https://doi.org/10.3847/0004-6256/152/6/157