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Global Funnel Control of Nonlinear Systems with Unknown and Time-Varying Fractional Powers

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Abstract

This paper is concerned with the global funnel control (FC) issue of the nonlinear systems with unknown dynamics and time-varying fractional powers. An FC strategy is proposed in this paper, which not only yields uniform performance insurance under any initial condition of the control system but also leads to a continuous and amplitude-reduced control input with respect to the existing solutions. Besides, it exhibits prominent simplicity, with no need for parametric details of time-varying fractional powers, adding a power integrator technique, parameter identification, function approximation or derivative calculation. A comparative simulation demonstrates the effectiveness and superiority of the developed method.

Keywords: global property; time-varying fractional powers; prescribed tracking performance; non-parametric uncertainties

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1. Introduction

Given the theoretical difficulties and practical demands, considerable research attention has been devoted to the control of the uncertain nonlinear systems involving fractional powers. In engineering contexts, such odd-power nonlinear systems can be found in various applications, with under-actuated mechanical systems [1] and dynamical boiler-turbine units [2] being typical instances. Notably, feedback linearization fails to be applied to this type of system, which stems from the uncontrollability of its Jacobian matrix. Additionally, the system exhibits a nonaffine relationship with the control input. As a result, formulating control strategies for odd-power systems presents considerable challenges. To deal with this problem, a variety of methods have been proposed, such as adaptive control [3–9], neural or fuzzy control [10–12], adding a power integrator technique [9,13–16], and funnel or prescribed performance control [11,12,17]. Nevertheless, the application scope of the aforementioned approaches is confined to integer powers. In contrast, some schemes for the control systems involving fractional powers have been developed recently [5,6,8,17–20]. In the literature [18,19], the fractional powers greater than zero and less than one are taken into consideration. It is important to highlight that a shared characteristic of the above-mentioned results is the requirement for known powers. Notably, the power integrator addition technique proves unsuitable for scenarios involving unknown powers, as it depends heavily on the homogeneous dominant component. To handle this situation, numerous approaches were put forward in the literature [9,14–16], but the bounds of powers need to be available for the control design. Additionally, the system nonlinearities of the aforementioned results are either constrained by known functions [1,4,7,13] or represented by a structure featuring both known functions and unknown parameters [9,16].

FC aims at fast and accurate reference tracking, with both the settling time and accuracy less than the respectively preselected values. This is accomplished by restricting the tracking error within a preselected performance boundary. In the conventional performance design, however, the choice of

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performance functions is contingent upon the initial condition of the control system [11,12,17,21–26]. Once the initial error is outside the performance envelop, the control signal becomes insolvable (e.g., $u = \ln(-1)$) or leads to a positive feedback loop. As a result, only semi-global performance of the control system is able to be ensured. For global FC, a predefined-time tuning function is designed to modify the tracking and intermediate errors within the FC framework [27]. By this means, the issue of global, fast and accurate convergence for all errors is converted into the problem of local constraints of the adjusted errors that can be addressed by using the barrier function-based control approach. Alternatively, employing a shifting function and a performance function where the initial value approaches infinity is also effective [28–33]. Nevertheless, the existing global FC methods work for the standard strict-feedback systems without fractional powers [27–31]. In addition to this, extensive simulation and experimental results show that the above global FC methods yield large control amplitude during the transient phase (e.g., control peak) when there is a large initial error. Due to actuator saturation of the practical control system, the performance requirement may not hold. Additionally, global stability can also be guaranteed by other control methods, e.g., the improved adaptive control [9,29–31,34–37], neural network control [32,38] and small-gain theorem [35]. Unfortunately, they are applicable to the standard strict-feedback systems [27–34,36]; assume the parametrically uncertain nonlinearities or their bounding functions [9,29–31,34,36]; involve sign functions in the control law [34,36].

The above discussion reveals that the global FC problem of the nonlinear systems with unknown, time-varying, and fractional powers remains open. To this end, a novel FC strategy is designed in this paper. Its superiority is enumerated as follows.

1. It is effective for the nonlinear systems with unknown, time-varying, fractional powers and totally unknown closed-loop dynamics, in contrast to the references [1,4,7–16,27–34].
2. It shows flexibility in the sense of the global performance insurance, the continuous control input, and the reduced control amplitude, with respect to the results [9,13,20–26].
3. The simplicity of FC is preserved, without needs for parameter identification [4–8], adding a power integrator technique [9,14–16], function approximation [10–12] or derivative calculation [31,33].

2. Problem Formulation

2.1. System Description

Consider the following nonlinear system with time-varying fractional powers:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + [x_{i+1}]^{p_i(t)}, \ \ i = 1, \cdots, n-1, \\ \dot{x}_n = f_n(\bar{x}_n) + [u]^{p_n(t)}, \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in \mathfrak{R}^i$; $\bar{x}_n$ constitutes the system state; $p_i(t)$ is a continuous and time-varying scalar of which the scope covers both the integer and the fraction, $i = 1, \cdots, n$; $u \in \mathfrak{R}$ and $y \in \mathfrak{R}$ are the input and the output, respectively; the power sign function $[\cdot]^a$ is defined as $[\cdot]^a = \mathrm{sgn}(\cdot)|\cdot|^a$ for a real number $a > 0$; $f_i(\bar{x}_i) \in \mathfrak{R}$, $i = 1, \cdots, n$, are the nonlinear functions, each of which is continuous in their respective arguments.

Assumption 1. *There are constants, $\underline{p} > 0$ and $\overline{p} > 0$, for which*

$$\underline{p} \le p_i(t) \le \overline{p}, \ \ i = 1, \cdots, n, \ \ t \ge 0. \tag{2}$$

2.2. Control Objective

The control objective for (1) is to drive $y(t)$ to track $y_r(t)$, with

$$|y(t) - y_r(t)| < k_1(t) = (k_{10} - k_{1\infty})e^{-\mu_1 t} + k_{1\infty}, \quad t \geq \kappa, \tag{3}$$

where k_{10}, $k_{1\infty}$, μ_1 and κ are the preselected positive constants with $k_{10} > k_{1\infty}$; $k_{1\infty}$ denotes the largest acceptable deviation of the steady-state error; μ_1 denotes the expected rate at which the performance boundary $k_1(t)$ decreases from k_{10} to $k_{1\infty}$; κ denotes the expected settling time of reference tracking.

Assumption 2. *Both $y_r(t)$ and $\dot{y}_r(t)$ are bounded over $[0, \infty)$ [10,18,21–23,27].*

Remark 1. *This study is concentrated on the case where neither the specific knowledge of the the nonlinear dynamics in (1) nor the bound of time-varying fractional powers in Assumption 1 and the bound of the reference derivative in Assumption 2 are available for the control design below.*

To summarize, the problem investigated herein is summarized below.

Problem 1. *For the fractional-power nonlinear system in (1), design a controller off-line to ensure the boundedness of all closed-loop signals, and the control objective in (3) is achieved.*

3. Control Development

A predefined-time tuning function is employed:

$$\varepsilon(t) = \begin{cases} \sin\left(\frac{\pi t}{2\kappa}\right), & \text{if } t < \kappa, \\ 1, & \text{otherwise,} \end{cases} \tag{4}$$

where κ is given in (3). Adopt it to modify the tracking error:

$$e_1(t) = \varepsilon(t)(y(t) - y_r(t)). \tag{5}$$

For any $y(0)$ and $y_r(0)$, we know from (4) and (5) that

$$e_1(0) = \varepsilon(0) \cdot (y(0) - y_r(0)) = 0 \cdot (y(0) - y_r(0)) = 0, \tag{6}$$
$$e_1(t) = y(t) - y_r(t), \quad t \geq \kappa. \tag{7}$$

Based on (4), the tracking performance requirement in (3) is transformed to

$$|e_1(t)| < k_1(t), \quad t \geq 0. \tag{8}$$

On the purpose of (8), we employ a barrier function:

$$\eta_1(t) = \tan\left(\frac{\pi e_1(t)}{2k_1(t)}\right), \tag{9}$$

which yields the first intermediate control law:

$$\alpha_1(t) = -b_1\eta_1(t) - c_1 e_1(t), \tag{10}$$

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where $b_1 > 0$ and $c_1 > 0$ are free-design constants, standing for the virtual control gains. We now advance to

$$e_i(t) = \varepsilon(t)(x_i(t) - \alpha_{i-1}(t)), \tag{11}$$
$$k_i(t) = (k_{i0} - k_{i\infty})e^{-\mu_i t} + k_{i\infty}, \tag{12}$$
$$\eta_i(t) = \tan\left(\frac{\pi e_i(t)}{2k_i(t)}\right), \tag{13}$$
$$\alpha_i(t) = -b_i\eta_i(t) - c_i e_i(t), \tag{14}$$

for $i = 2, \cdots, n$, recursively, where $k_{i0} > k_{i\infty} > 0$, $\mu_i > 0$, $b_i > 0$ and $c_i > 0$ are the freely-chosen constants. Ultimately, the control law is acquired by

$$u(t) = \alpha_n(t). \tag{15}$$

Remark 2. *Different from the existing global FC laws [23–26], not only the barrier functions but also the tracking and intermediate errors are introduced to our control law in a proportional feedback way. By this means, the task of errors stabilization is divided into the barrier functions and errors together, i.e., the feedback errors share partial responsibility. The simulation results below show the significant advantage of the amended FC law in reduction of the control amplitude, especially during the transient phase of the control system.*

4. Performance Analysis

Lemma 1. *For any $y_r(t)$ and $x_i(0)$, $i = 1, \cdots, n$, we obtain*

$$e_i(0) = 0, \ \ i = 1, \cdots, n, \tag{16}$$
$$u(0) = 0. \tag{17}$$

Proof. Substituting (6) into (9) with (3) yields

$$\eta_1(0) = \tan\left(\frac{\pi e_1(0)}{2k_1(0)}\right) = 0. \tag{18}$$

Putting (6) and (18) into (10), one has

$$\alpha_1(0) = -b_1\eta_1(0) - c_1 e_1(0) = 0. \tag{19}$$

By (4) and (11), it is obtained that

$$e_2(0) = \varepsilon(0) \cdot (x_2(0) - \alpha_1(0)) = 0 \cdot (x_2(0) - \alpha_1(0)) = 0 \cdot x_2(0) - 0 \cdot \alpha_1(0) = 0. \tag{20}$$

Substituting it into (13) with (12) yields

$$\eta_2(0) = \tan\left(\frac{\pi e_2(0)}{2k_2(0)}\right) = 0. \tag{21}$$

Putting (20) and (21) into (14), one has

$$\alpha_2(0) = -b_2\eta_2(0) - c_2 e_2(0) = 0. \tag{22}$$

From (4), (11) and (22), there holds

$$e_3(0) = \varepsilon(0) \cdot (x_3(0) - \alpha_2(0)) = 0 \cdot (x_3(0) - \alpha_2(0)) = 0 \cdot x_3(0) - 0 \cdot \alpha_2(0) = 0. \tag{23}$$

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Continue along the same path to examine $e_i(t)$, $i = 4, \cdots, n$, one by one. We are able to conclude that

$$e_i(0) = 0, \tag{24}$$
$$\eta_i(0) = 0, \tag{25}$$
$$\alpha_i(0) = 0, \tag{26}$$

for $i = 4, \cdots, n$. Based on (15), (17) holds. □

Lemma 2. *For any $t_2 > 0$ and each $i \in \{1, \cdots, n\}$, $|\dot{\alpha}_i| < \infty$ during $t \in [0, t_2)$, provided*

1. *$e_i(t)$ evolves inside $(-k_i(t), k_i(t))$ and keeps at a distance from $-k_i(t)$ and $k_i(t)$ during $t \in [0, t_2)$;*
2. *$\dot{e}_i$ and $\eta_i(t)$ are both bounded during $t \in [0, t_2)$.*

Proof. The derivatives of (9) and (13) are computed by

$$\dot{\eta}_i(t) = \frac{\pi \beta_i(t)}{2}\left(\dot{e}_i(t) - \frac{e_i(t)\dot{k}_i(t)}{k_i(t)}\right), \ \ i = 1, \cdots, n, \ \ t < t_2, \tag{27}$$

$$\beta_i(t) = \frac{1}{k_i(t)\cos^2\left(\frac{\pi e_i(t)}{2k_i(t)}\right)}, \ \ i = 1, \cdots, n, \ \ t < t_2. \tag{28}$$

Differentiating (10) and (14) gives

$$\dot{\alpha}_i(t) = -b_i\dot{\eta}_i(t) - c_i\dot{e}_i(t), \ \ i = 1, \cdots, n, \ \ t < t_2. \tag{29}$$

Substituting (27) into (29) yields

$$\dot{\alpha}_i(t) = -\frac{\pi b_i \beta_i(t)}{2}\left(\dot{e}_i(t) - \frac{e_i(t)\dot{k}_i(t)}{k_i(t)}\right) - c_i\dot{e}_i(t), \ \ i = 1, \cdots, n, \ \ t < t_2. \tag{30}$$

It follows from (3) and (12) that $\dot{k}_i(t)$ and $\frac{1}{k_i(t)}$ are bounded, $i = 1, \cdots, n$. The boundedness of $\eta_i(t)$ in (9) and (13) guarantees that of $\beta_i(t)$ in (28), $i \in \{1, \cdots, n\}$. Therefore, $|\dot{\alpha}_i(t)| < \infty$ on $[0, t_2)$ under the assumed conditions of Lemma 2, $i \in \{1, \cdots, n\}$. □

Lemma 3. *Under (4), consider a continuous scalar function $q(t)$ with bounded $q(0)$. Firstly, for any $t_3 > 0$, if*

$$\lim_{t \to t_3} |q(t)| = \infty, \tag{31}$$

then

$$\lim_{t \to t_3} \varepsilon(t)|q(t)| = \infty. \tag{32}$$

Secondly, for any $t_4 > 0$, if

$$|\varepsilon(t)q(t)| < \infty, \ \ t < t_4, \tag{33}$$

then

$$|q(t)| < \infty, \ \ t < t_4. \tag{34}$$

Proof. We show (32) and (34) by contradiction. At the outset, suppose (31) but

$$\varepsilon(t)|q(t)| < \infty. \tag{35}$$

This means that $\lim_{t\to t_3} \varepsilon(t) = 0$, which in turn indicates from (4) that $t_3 = 0$. As a result, (31) is rephrased by

$$\lim_{t\to 0}|q(t)| = |q(0)| = \infty. \tag{36}$$

However, $q(0)$ is bounded, which contradicts (36). Hence, (35) is invalid, and instead, (32) is established. Suppose (33) but there is $t^* \in [0, t_4)$ for which

$$\lim_{t\to t^*}|q(t)| = \infty. \tag{37}$$

By (33) and (37), we further have

$$\lim_{t\to t^*} \varepsilon(t) = 0, \tag{38}$$

which in turn implies from (4) that $t^* = 0$. Thus, (37) is rewritten by

$$\lim_{t\to 0}|q(t)| = |q(0)| = \infty, \tag{39}$$

which however contradicts the fact that $|q(0)| < \infty$. Thereby, (37) is invalid, and instead, (34) is true. □

Theorem 1. *Problem 1 is tackled by the control strategy composed of (4), (5) and (9)–(15), under Assumptions 1 and 2.*

Proof. We commence with the argument that:

$$|e_i(t)| < k_i(t), \;\; \forall\, t \geq 0, \tag{40}$$

for $i = 1, \cdots, n$. This assertion is validated using the proof by contradiction method. From (3) and (12), we have $k_i(0) > 0$, $i = 1, \cdots, n$, which in conjunction with Lemma 1 give (40) at $t = 0$. Note that $x_i(t)$, $i = 1, \cdots, n$, in (1) and $k_i(t)$, $i = 1, \cdots, n$, in (3) and (12) are all uniformly continuous. The uniform continuity of $e_1(t)$ follows from (4) and Assumption 2. Thus, the continuity of $\eta_1(t)$ in (9) and $\alpha_1(t)$ in (10) is guaranteed, if $|e_1(t)| < k_1(t)$. Further, the continuity of $e_2(t)$ in (11) holds under the identical condition. Continuing along the same path to examine $e_i(t)$, $i = 3, \cdots, n$, one by one, we are able to conclude that $e_i(t)$ is continuous in the case of $|e_j(t)| < k_j(t)$, $j = 1, \cdots, i-1$. These findings indicate that a violation of (40) implies the existence of $\tau > 0$ for which

$$\lim_{t\to \tau^-} |e_j(t)| = \lim_{t\to \tau^-} k_j(t), \;\; \exists j \in \{1, \cdots, n\}, \tag{41}$$

with

$$|e_i(t)| < k_i(t), \;\; i = 1, \cdots, n, \;\; t < \tau. \tag{42}$$

Next, (41) with (42) is supposed, and each case in (41) is to be enumerated for verification. For brevity, the arguments of some functions may not be shown.

Case 1: Initially, we examine

$$\lim_{t\to \tau^-} |e_1(t)| = \lim_{t\to \tau^-} k_1(t). \tag{43}$$

Under (42), a precondition for (43) is

$$\lim_{t\to \tau^-} \frac{\mathrm{d}|e_1(t)|}{\mathrm{d}t} \geq \lim_{t\to \tau^-} \dot{k}_1(t). \tag{44}$$

Differentiating (5) by (1) yields

$$\dot{e}_1 = \dot{\varepsilon}(x_1 - y_r) + \varepsilon(\dot{x}_1 - \dot{y}_r) = \omega_1 + \varepsilon[x_2]^{p_1}, \tag{45}$$

where

$$\omega_1 = \dot{\varepsilon}(x_1 - y_r) + \varepsilon(f_1 - \dot{y}_r). \tag{46}$$

There further holds

$$\begin{aligned} \lim_{t\to\tau^-} \frac{\mathrm{d}|e_1(t)|}{\mathrm{d}t} &= \lim_{t\to\tau^-} \operatorname{sgn}(e_1)\omega_1 + \lim_{t\to\tau^-} \operatorname{sgn}(e_1)\varepsilon[x_2]^{p_1} \\ &= \lim_{t\to\tau^-} \operatorname{sgn}(e_1)\omega_1 + \lim_{t\to\tau^-} \operatorname{sgn}(e_1)\operatorname{sgn}(x_2)\varepsilon|x_2|^{p_1} \end{aligned} \tag{47}$$

It follows from Assumption 2 and equation (4) that ε, $\dot{\varepsilon}$, y_r, and $\dot{y}_r$ are bounded. From (42), one has $|e_i| < \infty$, $i = 1, 2$, $t \in [0, \tau)$. By the second item of Lemma 3, there holds $|y - y_r| < \infty$ over $[0, \tau)$, which in turn warrants $|y| < \infty$ (i.e., $|x_1| < \infty$) on $[0, \tau)$. Due to the continuity of $f_1(x_1)$ in x_1, we have $|f_1| < \infty$, $t < \tau$. Inserting these findings into (46) gives

$$|\omega_1| < \infty, \quad t < \tau. \tag{48}$$

Note from (9) and (43) that

$$\lim_{t\to\tau^-} \operatorname{sgn}(e_1)\eta_1 = +\infty. \tag{49}$$

Under (10), there further holds

$$\lim_{t\to\tau^-} \operatorname{sgn}(e_1)\alpha_1 = -\infty. \tag{50}$$

Applying the first item of Lemma 3 to (50) yields

$$\lim_{t\to\tau^-} \operatorname{sgn}(e_1)\varepsilon\alpha_1 = -\infty. \tag{51}$$

By (11), we have

$$\varepsilon x_2 = e_2 + \varepsilon\alpha_1. \tag{52}$$

Further, there holds

$$\operatorname{sgn}(e_1)\varepsilon x_2 = \operatorname{sgn}(e_1)e_2 + \operatorname{sgn}(e_1)\varepsilon\alpha_1. \tag{53}$$

Putting (51) into (53) under (42) yields

$$\lim_{t\to\tau^-} \operatorname{sgn}(e_1)\varepsilon x_2 = -\infty. \tag{54}$$

By (4), there holds

$$\lim_{t\to\tau^-} \operatorname{sgn}(e_1)x_2 = -\infty. \tag{55}$$

It further follows that

$$\lim_{t\to\tau^-} \operatorname{sgn}(e_1)\operatorname{sgn}(x_2)|x_2|^{p_1} = -\infty. \tag{56}$$

Further, substituting (48) and (56) into (47) shows

$$\lim_{t\to\tau^-}\frac{\mathrm{d}|e_1(t)|}{\mathrm{d}t}=-\infty. \tag{57}$$

Note from (3) that $\dot{k}_1(t)$ is bounded. Apparently, (57) contradicts (44). Therefore, (43) is invalid. There instead exists a constant, $h_1>0$, for which

$$|e_1(t)|\leq k_1(t)-h_1<k_1(t),\ \ t<\tau. \tag{58}$$

Consequently, η_1 in (9) and α_1 in (10) remain bounded on $[0,\tau)$. Under (11) and (42), invoking the second item of Lemma 3 yields $(x_2-\alpha_1)$ is bounded on $[0,\tau)$, which implies that $|x_2|<\infty$, $t<\tau$. By (45), $|\dot{e}_1|<\infty$ during $t\in[0,\tau)$. This in company with (58) yields by Lemma 2 that $|\dot{\alpha}_1|<\infty$ over $[0,\tau)$.

Case 2: Consider

$$\lim_{t\to\tau^-}|e_2(t)|=\lim_{t\to\tau^-}k_2(t). \tag{59}$$

Under (42), a precondition for (59) is

$$\lim_{t\to\tau^-}\frac{\mathrm{d}|e_2(t)|}{\mathrm{d}t}\geq\lim_{t\to\tau^-}\dot{k}_2(t). \tag{60}$$

Taking the derivative of e_2 in (11) via (1), we have

$$\dot{e}_2=\dot{\varepsilon}(x_2-\alpha_1)+\varepsilon(\dot{x}_2-\dot{\alpha}_1)=\omega_2+\varepsilon[x_3]^{p_2}, \tag{61}$$

where

$$\omega_2=\dot{\varepsilon}(x_2-\alpha_1)+\varepsilon(f_2-\dot{\alpha}_1). \tag{62}$$

Further, there holds

$$\lim_{t\to\tau^-}\frac{\mathrm{d}|e_2(t)|}{\mathrm{d}t}=\lim_{t\to\tau^-}\mathrm{sgn}(e_2)\omega_2+\lim_{t\to\tau^-}\mathrm{sgn}(e_2)\varepsilon[x_3]^{p_2}. \tag{63}$$

Note that ε, $\dot{\varepsilon}$, x_1, x_2, α_1 and $\dot{\alpha}_1$ are all bounded during $t\in[0,\tau)$. Since $f_2(\bar{x}_2)$ is continuous with respect to $\bar{x}_2$, we further have $|f_2|<\infty$, $t<\tau$. Putting the above facts into (62) leads to

$$|\omega_2|<\infty,\ \ t<\tau. \tag{64}$$

One sees from (13) and (59) that

$$\lim_{t\to\tau^-}\mathrm{sgn}(e_2)\eta_2=+\infty. \tag{65}$$

By (14), there further holds

$$\lim_{t\to\tau^-}\mathrm{sgn}(e_2)\alpha_2=-\infty. \tag{66}$$

Applying the first item of Lemma 3 to (66) yields

$$\lim_{t\to\tau^-}\mathrm{sgn}(e_2)\varepsilon\alpha_2=-\infty. \tag{67}$$

From (11), we have

$$\varepsilon x_3=e_3+\varepsilon\alpha_2. \tag{68}$$

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Further, there holds

$$\mathrm{sgn}(e_2)\varepsilon x_3 = \mathrm{sgn}(e_2)e_3 + \mathrm{sgn}(e_2)\varepsilon\alpha_2. \tag{69}$$

Putting (67) into (69) under (42) yields

$$\lim_{t\to\tau^-} \mathrm{sgn}(e_2)\varepsilon x_3 = -\infty. \tag{70}$$

By (4), there holds

$$\lim_{t\to\tau^-} \mathrm{sgn}(e_2)x_3 = -\infty. \tag{71}$$

It further follows that

$$\lim_{t\to\tau^-} \mathrm{sgn}(e_2)[x_3]^{p_2} = \lim_{t\to\tau^-} \mathrm{sgn}(e_2)\,\mathrm{sgn}(x_3)|x_3|^{p_2} = -\infty. \tag{72}$$

Further, substituting (64) and (72) into (63) shows

$$\lim_{t\to\tau^-} \frac{\mathrm{d}|e_2(t)|}{\mathrm{d}t} = -\infty. \tag{73}$$

Note from (12) that $\dot{k}_2(t)$ is bounded. Obviously, (73) contradicts (60). Hence, (59) is invalid. There instead is a constant, $h_2 > 0$, for which

$$|e_2(t)| \le k_2(t) - h_2 < k_2(t), \ \ t < \tau. \tag{74}$$

Further, η_2 in (13) and α_2 in (14) are bounded over $[0, \tau)$. Under (11) and (42), invoking the second item of Lemma 3 yields $(x_3 - \alpha_2)$ is bounded on $[0, \tau)$, which implies that $|x_3| < \infty$, $t < \tau$. By (61), $\dot{e}_2$ is bounded over $[0, \tau)$. This in conjunction with (74) yields by Lemma 2 that $|\dot{\alpha}_2| < \infty$ on $[0, \tau)$.

Case i $(i = 3, \cdots, n)$*:* Adopting the same analytical way from Case 2, we can conclude that there are a set of positive constants, $h_3, \cdots, h_n$, for which

$$|e_i(t)| \le k_i(t) - h_i < k_i(t), \ \ i = 3, \cdots, n, \ \ t < \tau. \tag{75}$$

Clearly, (58), (74) and (75) contradict (41). Therefore, (41) is invalid. There instead holds

$$|e_i(t)| \le k_i(t) - h_i < k_i(t), \ \ i = 1, \cdots, n, \ \ t \ge 0. \tag{76}$$

It is apparent that the claim in (40) is valid. This shows that the controller warrants the error constraints but evades the errors approaching the preselected boundaries. Further, it follows from (4), (5) and (40) for $i = 1$ that (3) is established.

It remains for us to verify that the rest of the signals in the closed loop are bounded. From (9), (13) and (76), there hold $\eta_i < \infty$ for $t \ge 0$, $i = 1, \cdots, n$. This in conjunction with (10), (14), (15) and (40) ensures the uniform boundedness of α_i and u, $i = 1, \cdots, n-1$. Based on (5), (11), (40) and the second item of Lemma 3, there hold $|x_1 - y_r| < \infty$ and $|x_i - \alpha_{i-1}| < \infty$ for $t \ge 0$, $i = 2, \cdots, n$. By Assumption 2, there further hold $x_i < \infty$, $i = 1, \cdots, n$, are bounded for $t \ge 0$. □

Remark 3. *The proof by contradiction reveals the inherent robustness of the developed control approach to the unknown nonlinearities and time-varying fractional powers. This phenomenon results from the infinity property of the barrier functions, as shown in (55) and (71). When extended to the nonlinear system with unknown powers in (1), the infinity property remains preserved, as indicated in (56) and (72). Therefore, only a bounded control input is needed, the techniques for approximation, identification and estimation are removed.*

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5. Simulation Study

To evaluate the proposed control strategy, a comparative simulation study is conducted. Consider the following second-order systems with time-varying fractional powers:

$$\begin{cases} \dot{x}_1 = -x_1^2 - x_1^3 + [x_2]^{1.5+\sin(t)}, \\ \dot{x}_2 = x_1^2 x_2^3 + [u]^{\frac{9}{5}+\cos(t)}, \\ y = x_1. \end{cases} \tag{77}$$

In the simulation, (77) is initialized at either $\bar{x}(0) = [3, -1]^{\mathrm{T}}$ or $\underline{x}(0) = [-0.5, 0.2]^{\mathrm{T}}$. The control task is to driver $y(t)$ to follow $y_r(t) = \sin(t)$ with

$$|y(t) - y_r(t)| < k_1(t) = (1 - 0.01)e^{-t} + 0.01, \quad t \geq 1. \tag{78}$$

The design of the controller follows from Theorem 1 with $\kappa = 1$, $b_1 = 4$, $c_1 = 3$, $b_2 = 2$, $c_2 = 2$ and $k_2(t) = k_1(t)$. Applying it to (77), Figures 1–5 illustrate the simulation results. As depicted in Figures 1 and 2, the output follows the reference under varying initial values, and the tracking error achieves convergence to the designated performance envelope within the predefined duration. Thereby, under different initial conditions, the prescribed performance specification in (78) is implemented, which implies the global attribute. Similarly, the preassigned performance specification for the intermediate error is also fulfilled, as displayed in Figure 3. Finally, one sees from Figures 4 and 5 that both the other state variable and the input are bounded under different initial conditions. Accordingly, the simulation findings confirm the effectiveness of the developed controller.

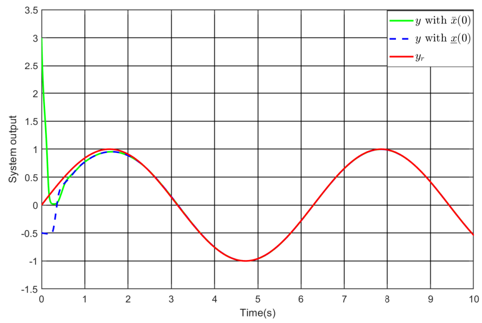

Figure 1. The system output under $\bar{x}(0)$ and $\underline{x}(0)$.

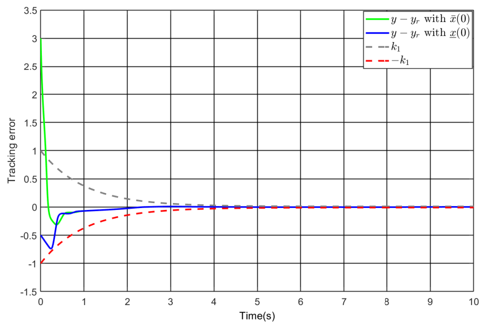

Figure 2. The tracking error under $\bar{x}(0)$ and $\underline{x}(0)$.

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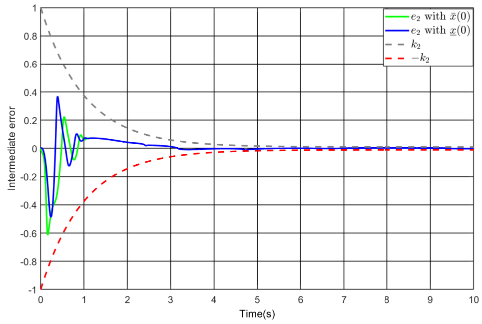

Figure 3. The intermediate error under $\bar{x}(0)$ and $\underline{x}(0)$.

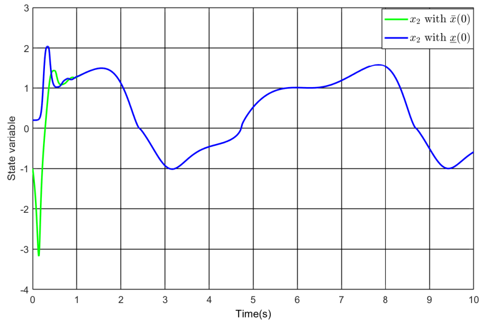

Figure 4. The state variable under $\bar{x}(0)$ and $\underline{x}(0)$.

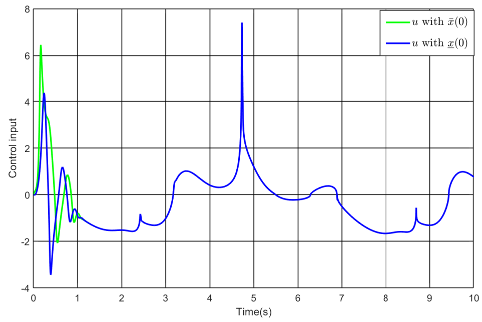

Figure 5. The control input under $\bar{x}(0)$ and $\underline{x}(0)$.

For comparison, an enhancing FC scheme is implemented for (77) under the identical control task. The controller is designed by [39]

$$\begin{cases} e_1(t) = y(t) - y_r(t), \\ k(t) = \begin{cases} \operatorname{csch}\left(1.45 + \frac{0.4t}{1-t}\right) + 1, & \text{if } t < 1, \\ 1, & \text{otherwise}, \end{cases} \\ \bar{k}(t) = \operatorname{sgn}(e_1(0)) \cdot (k(t) - 1) + 0.01k(t), \\ \underline{k}(t) = \operatorname{sgn}(e_1(0)) \cdot (k(t) - 1) - 0.01k(t), \\ v(t) = \dfrac{e_1(t) - \underline{k}(t)}{\bar{k}(t) - \underline{k}(t)}, \\ \chi(t) = \dfrac{1}{v(t)(1 - v(t))\left(\bar{k}(t) - \underline{k}(t)\right)}, \\ \xi(t) = \ln\left(\dfrac{v(t)}{1 - v(t)}\right), \\ \phi(t) = \dfrac{\chi(t)}{\bar{k}(t) - \underline{k}(t)}\left(\underline{k}(t)\dot{\bar{k}}(t) - \dot{\underline{k}}(t)\bar{k}(t) - e_1(t)\left(\dot{\bar{k}}(t) - \dot{\underline{k}}(t)\right)\right), \\ \alpha_1(t) = -\dfrac{1}{\chi(t)}(4\xi(t) + \phi(t)) - (-x_1^2 - x_1^3) + \dot{y}_r, \\ e_2(t) = x_2(t) - \alpha_1(t), \\ u = -2e_2(t) + \dot{\alpha}_1(t) - \chi(t)\xi(t) - x_1^2 x_2^3. \end{cases} \tag{79}$$

Take $\underline{x}(0) = [-0.5, 0.2]^{\mathrm{T}}$ into consideration. Figures 6–9 exhibit the simulation findings. Despite ensuring that the control system achieves predefined performance and all signals remain bounded, the comparative controller demands that the nonlinearity is known and that the first and second-order derivatives of the reference are obtainable. Moreover, the choice of performance boundaries is contingent upon the initial condition of the control system. In contrast, the above requirements are eliminated by our approach. In addition, the comparative controller requires larger amplitude of the control input. To be specific, Figure 9 shows $\sup_{t\in[0,10]} |u(t)| > 20$ with the comparative controller and $\sup_{t\in[0,10]} |u(t)| < 10$ with our controller. Thus, the comparative simulation findings illustrate the advantage of the developed strategy.

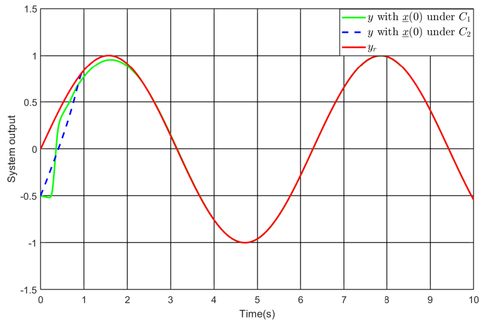

Figure 6. The system output under our controller C_1 and the comparative controller C_2.

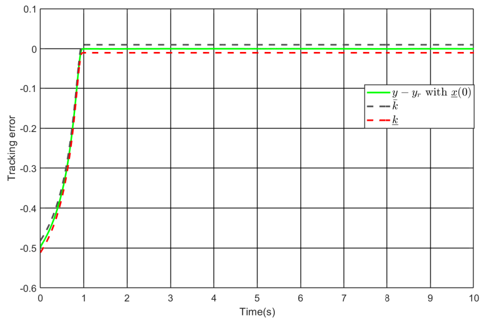

Figure 7. The tracking error under the comparative controller C_2.

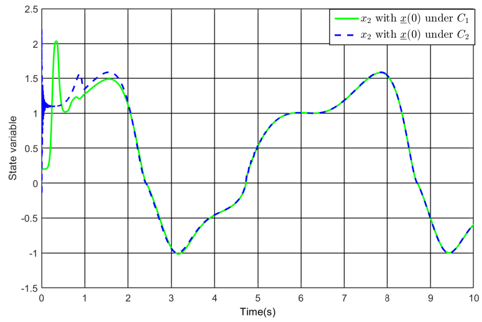

Figure 8. The state variable under our controller C_1 and the comparative controller C_2.

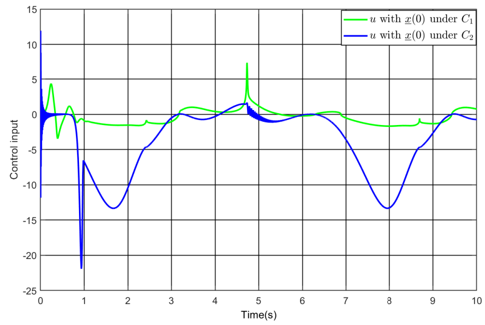

Figure 9. The control input under our controller C_1 and the comparative controller C_2.

6. Conclusion

We put forward a FC approach for reference tracking with prescribed performance in this paper. It is able to cope with unknown nonlinearities and unknown time-varying fractional powers. It achieves fast accurate reference tracking under arbitrary initial state of the control system and removes the requirements for specific information of the time-varying fractional powers, the parametrically uncertain form of system dynamics and the tools for approximation, identification and estimation. Moreover, the required control input is both continuous and shows lower amplitude than the conventional global FC laws. The simulation results validate our approach. Subsequent research will address extensions to MIMO systems.

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