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[Branko Sarić](#) *

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Article

The $3\mathbb{D}$ Field of Complex Vectors

Branko Sarić

Faculty of Technical Sciences, University of Kragujevac, Čačak 32000, Serbia; saric.b@mts.rs

Abstract

Based on the isomorphic algebraic structures of the $2\mathbb{D}$ *Euclidean* field of complex vectors $V_{\mathbb{C}}$ and the field of complex numbers \mathbb{C} , in terms of identical geometric products of the elements of both fields, this paper brings the algebraic structure of a $3\mathbb{D}$ field of complex vectors, as well as the corresponding fundamental integral identities in those vector fields.

Keywords: three dimensional complex vectors; scalar and vector fields

MSC: Primary 15A03; 15A72; Secondary 30G35

1. Introduction

A geometric (*Clifford*) algebra is an extension of elementary algebra to work with geometrical objects such as vectors. It is built out of two fundamental operations: addition and geometric product, [1]. The multiplication of vectors results in objects called multivectors. Compared with any other formalism for manipulating vectors, *Clifford* algebra alone supports dividing by vectors. The geometric product was first mentioned by *Grassmann*, who founded the so-called external algebra, [2]. After that, *Clifford* himself greatly expanded upon *Grassmann's* work to form geometric algebra, named after him *Clifford* algebra [1], by unifying both *Grassmann's* algebra and *Hamilton's* quaternion algebra (\mathbb{H}). In the middle of the 20th century, *Hestenes* repopularized the term geometric algebra [4,5].

On the other hand, although rarely used explicitly, a geometric representation of complex numbers is implicitly based on its structure of the *Euclidean* 2-dimensional vector space. If the binary operation of the product $\bar{w}z$ of two complex numbers \bar{w} and z is considered as the sum of the inner product $w \circ z = (\bar{w}z + w\bar{z})/2 = \mathbf{1} \operatorname{Re}(\bar{w}z)$ and outer product $w \wedge z = (\bar{w}z - w\bar{z})/2 = \mathbf{i} \operatorname{Im}(\bar{w}z)$, where $\mathbf{1} = (1, i0)$ and $\mathbf{i} = (0, i)$, and i is an imaginary unit, it can be said that $\bar{w}z$ is in the form of a geometric product of two *ivectors* (two complex numbers), as two geometric objects belonging to the *ivector* field \mathbb{C} (to the field of complex numbers \mathbb{C}). For any complex number z , its absolute value $|z|$ is its *Euclidean* norm denoted by ρ , and the argument $\arg z$ of z is the polar angle φ . Since ordered pairs represent both complex numbers and vectors, the binary operation of the product of two complex numbers (two ordered pairs), in the form of the geometric product $\bar{w}z$ ($\bar{w}z = w \circ z + w \wedge z$), will be the basis for modifying *Grassmann's* geometric product of vectors, which is defined as the sum of the inner (scalar) and outer (vector) products of two vectors. By this modification, the geometric product of two vectors becomes commutative, similar to the product of complex numbers themselves, which still supports vector division.

It is a very old and interesting problem to obtain a natural extension of complex numbers, especially to three-dimensional complex numbers. *Hamilton* interpreted complex numbers via couple numbers (ordered pairs of two real numbers), as points in the *Euclidean* plane [3], and he was looking for a way to do the same for points in a three-dimensional space. Points in space are triples of numbers. *Hamilton* had known how to add and subtract triples. However, for a long time, he had been stuck on the problem of multiplication and division. In fact, *Frobenius* proved, in 1877, that for a division algebra over the real numbers to be finite dimensional and associative, it cannot be three-dimensional. There are only three such division algebras: \mathbb{R} , \mathbb{C} and \mathbb{H} , which have dimension 1, 2 and 4, respectively. *Segre*

[15] described bicomplex numbers as points in a 4-dimensional space. Unlike *Hamilton's* quaternions, which are non-commutative and form a division algebra, bicomplex numbers are commutative and do not form a division algebra.

All told, many famous mathematicians have studied how to define multicomplex numbers and the corresponding function theory. Following this, and on the basis of the redefined geometric product of two vectors in the *Euclidean* plane, this paper presents the algebraic structure of the $3\mathbb{D}$ field of complex vectors, as well as the corresponding integral identities.

The idea, on the basis of which the algebraic structure of the $3\mathbb{D}$ field of complex vectors is defined, is as follows: At the beginning, a $2\mathbb{D}$ field of complex vectors $V_{\mathbb{C}}$ is introduced, which corresponds to the field of complex numbers \mathbb{C} , defined as a *Cartesian* product of one $1\mathbb{D}$ real and one $1\mathbb{D}$ *i*-real vector space with a commutative geometric product of elements. Following the same pattern, in the next section a $3\mathbb{D}$ field of complex vectors is introduced, which corresponds to the *Cartesian* product of one $3\mathbb{D}$ real and one $3\mathbb{D}$ *i*-real vector space with a commutative geometric product of elements, defined as the sum of the geometric products of the elements in three complex planes. Each of these complex planes is the *Cartesian* products of one $1\mathbb{D}$ real and one $1\mathbb{D}$ *i*-real vector space, which are vector subspaces of $3\mathbb{D}$ real and $3\mathbb{D}$ *i*-real vector space, respectively. Clearly, one can take the *Cartesian* product of one $2\mathbb{D}$ real and one $2\mathbb{D}$ *i*-real vector space in the same way. In that case, the geometric products of the elements in two complex planes are summed up, in order to obtain the commutative geometric product of the elements in this vector space.

2. The $2\mathbb{D}$ Field of Complex Vectors $V_{\mathbb{C}}$

The ordered pairs $\mathbf{1} = (1, i0)$ and $\mathbf{i} = (0, i)$ form the basis of a 2-dimensional real/real vector ($2\mathbb{D}$ *i*-vector) space $V_{\mathbb{C}}$, [6,9]. It is obvious that $V_{\mathbb{C}}$ is the *Cartesian* product $V_{\mathbb{R}} \times iV_{\mathbb{R}}$ of a one-dimensional real vector space and a one-dimensional *i*-real vector space, and as such it is an additive *Abelian* (commutative) group of *i*-vectors $(x, iy) = \mathbf{1}x + \mathbf{i}y$. On the other hand, it is possible to complete the *i*-vector space $V_{\mathbb{C}}$ with a binary operation of the product of two *i*-vectors (a, ib) and (c, id) , which corresponds to the matrix product, in such a manner that

$$\begin{aligned} (a, ib)(c, id) &\Leftrightarrow \begin{bmatrix} a & ib \\ ib & a \end{bmatrix} \begin{bmatrix} c & id \\ id & c \end{bmatrix} = \\ &= \begin{bmatrix} ac - bd & i(ad + bc) \\ i(ad + bc) & ac - bd \end{bmatrix} \Leftrightarrow (ac - bd, i(ad + bc)). \end{aligned} \quad (1)$$

The inverse element $(x, iy)^{-1} = (x, -iy)/(x^2 + y^2)$ of the space $V_{\mathbb{C}}^* = V_{\mathbb{C}} \setminus \{(0, i0)\}$ corresponds to the inverse matrix $\begin{bmatrix} x & iy \\ iy & x \end{bmatrix}^{-1}$. Here, both the commutative and associative axioms of multiplication and the distributive axiom are satisfied. Thus, the *i*-vector space $V_{\mathbb{C}}$ becomes the field of complex numbers (*i*-vectors) \mathbb{C} , to which the field of complex vectors $V_{\mathbb{C}}$ corresponds. Two ordered pairs of one real and one imaginary vector, $e = (1_x, i0_y)$ and $\hat{e} = (0_x, i1_y)$, such that the unit vectors 1_x ($0_x = 01_x$) and 1_y ($0_y = 01_y$) are orthonormal basis vectors of the real plane V_2 , form an orthonormal vector basis of the field $V_{\mathbb{C}}$, so that $\mathbf{1} \Leftrightarrow e$ and $\mathbf{i} \Leftrightarrow \hat{e}$. The complex vectors $q = xe + y\hat{e}$, as elements of $V_{\mathbb{C}}$, correspond to the complex numbers $z \in \mathbb{C}$. If $(x, -iy) \Leftrightarrow xe - y\hat{e} = \bar{q}$, then $q = \|\bar{q}\| = \|(x, -iy)\| = \sqrt{x^2 + y^2}$ is the norm on the fields $V_{\mathbb{C}}$ and \mathbb{C} . In addition, $\bar{q}q \Leftrightarrow (x, -iy)(x, iy) = q^2\mathbf{1} \Leftrightarrow q^2e$ and $\bar{q} = q^2q^{-1}$. It is quite clear that the inverse element q^{-1} allows division by a vector in $V_{\mathbb{C}}$. On the other hand, on the basis of the above-mentioned geometric product $(a, -ib)(c, id) = (ac + bd, i(ad - bc))$ of two complex numbers (*i*-vectors) $(a, -ib)$ and (c, id) , the corresponding geometric product of two complex vectors $\bar{q}_1 = ae - b\hat{e}$ and $q_2 = ce + d\hat{e}$ in $V_{\mathbb{C}}$ can be defined as follows:

$$\bar{q}_1 q_2 = \frac{1}{2}(\bar{q}_1 q_2 + q_1 \bar{q}_2) + \frac{1}{2}(\bar{q}_1 q_2 - q_1 \bar{q}_2) := \quad (2)$$

$$:= (\bar{q}_1 \cdot q_2)e + (q_1 \times q_2) \times e = (q_1 \cdot \bar{q}_2)e - (\bar{q}_1 \times \bar{q}_2) \times e,$$

where $\bar{q}_1 \cdot q_2 = \|q_1\| \|q_2\| \cos \theta$ and $(q_1 \times q_2) \times e = \|q_1\| \|q_2\| \sin \theta \hat{e}$, which is obviously commutative. Here,

$$\frac{1}{2}(\bar{q}_1 q_2 + q_1 \bar{q}_2) := (\bar{q}_1 \cdot q_2)e = q_1 \circ q_2 \Leftrightarrow (a, ib) \circ (c, id) \text{ and} \quad (3)$$

$$\frac{1}{2}(\bar{q}_1 q_2 - q_1 \bar{q}_2) := (q_1 \times q_2) \times e = q_1 \wedge q_2 \Leftrightarrow (a, ib) \wedge (c, id). \quad (4)$$

So, the dot and cross products of two complex vectors $q_1 = ae + b\hat{e}$ and $q_2 = ce + d\hat{e}$ in $V_{\mathbb{C}}$ are as follows:

$$\bar{q}_1 \cdot q_2 = ac + bd \text{ and } q_1 \times q_2 = (ad - bc)n, \quad (5)$$

where $n = e \times \hat{e}$. Since $(a, -ib)(c, id) = (a, ib) \circ (c, id) + (a, ib) \wedge (c, id)$, it follows that

$$\begin{aligned} & [(a, ib) \circ (c, id)]^2 - [(a, ib) \wedge (c, id)]^2 = \\ & = (a, -ib)(c, id)(c, -id)(a, ib) = \|(a, ib)\|^2 \|(c, id)\|^2 \mathbf{1}. \end{aligned} \quad (6)$$

3. The $3\mathbb{D}$ Field of Complex Vectors

Let $1_x, 1_y$ and 1_z be orthonormal basis vectors of the $3\mathbb{D}$ space of real vectors V_3 . Then, the following three pairs of ordered pairs: $e_1 = (1_x, i0_y)$ and $\hat{e}_1 = (0_x, i1_y)$, $e_2 = (1_y, i0_z)$ and $\hat{e}_2 = (0_y, i1_z)$, as well as $e_3 = (1_z, i0_x)$ and $\hat{e}_3 = (0_z, i1_x)$, form orthonormal bases of three $2\mathbb{D}$ fields of complex vectors q_α , which are component fields of the $3\mathbb{D}$ field of complex vectors with elements

$$q = x_\alpha e^\alpha + \hat{x}_\alpha \hat{e}^\alpha = q_\alpha e^\alpha, \quad (7)$$

where $q_\alpha = \langle xe + \hat{x}\hat{e} \rangle_\alpha$ are complex vectors in the component fields, and here, as in the following text, the index α ($\alpha = 1, 2, 3$), repeated as subscript and superscript in the products, represents the summation over the range of the index α , according to the *Einstein* summation convention.

The commutative geometric product of two $3\mathbb{D}$ complex vectors a and b is defined as the sum of the geometric products of the component vectors $\bar{a}_\alpha = \langle ae - \hat{a}\hat{e} \rangle_\alpha$ and $b_\alpha = \langle be + \hat{b}\hat{e} \rangle_\alpha$, as follows:

$$\bar{a}b = a \circ b + a \wedge b = \frac{\langle a \circ b + a \wedge b \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{\langle \bar{a}b \rangle_\alpha e^\alpha}{\sqrt[2]{3}} \text{ and} \quad (8)$$

$$a\bar{b} = a \circ b + b \wedge a = \frac{\langle b \circ a + b \wedge a \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{\langle \bar{b}a \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{\langle a\bar{b} \rangle_\alpha e^\alpha}{\sqrt[2]{3}}. \quad (9)$$

Clearly, $a \circ b = b \circ a = (\bar{a}b + \bar{b}a)/2$ and $a \wedge b = \bar{b} \wedge \bar{a} = (\bar{a}b - \bar{b}a)/2$. In addition, the vector

$$a^{-1} = \frac{1}{\sqrt[2]{3}} a_\alpha^{-1} e^\alpha, \quad (10)$$

where $a_\alpha^{-1} = \langle \bar{a}/\|a\|^2 \rangle_\alpha$ and $\|a\|_\alpha^2 = (a^2 + \hat{a}^2)_\alpha$, such that

$$a^{-1}a = \frac{\langle a^{-1}a \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{e_\alpha e^\alpha}{\sqrt[2]{3}}, \quad (11)$$

is the inverse vector of the $3\mathbb{D}$ complex vector a , which allows division by the vector in the $3\mathbb{D}$ field of complex vectors. On the other hand, if $(a\hat{b} - \hat{a}b)$ is denoted by the bracket $[a, \hat{b}]$, it follows that

$$a \wedge b = \frac{\langle a \wedge b \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{\langle (a \times b) \times e \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{[a, \hat{b}]_\alpha \hat{e}^\alpha}{\sqrt[2]{3}} \text{ and} \quad (12)$$

$$a \circ b = \frac{\langle a \circ b \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{\langle (\bar{a} \cdot b)e \rangle_\alpha e^\alpha}{\sqrt[2]{3}} = \frac{(ab + \hat{a}\hat{b})_\alpha e^\alpha}{\sqrt[2]{3}}. \quad (13)$$

3.1. Differential Forms in $V_{\mathbb{C}}$

To represent a complex vector $q \in V_{\mathbb{C}}$ in polar form, one introduces a vector analogue $c\hat{s}$ of the shorthand notation cis for the algebraic operator $\cos \cdot + i \sin \cdot$, as follows:

$$c\hat{s} \cdot = e \cos \cdot + \hat{e} \sin \cdot. \quad (14)$$

Let $q_0 = c\hat{s}\varphi = \exp(\hat{e}\varphi)$, where $\exp(\hat{e}\cdot)$ is the exponential form of the operator $c\hat{s} \cdot$, be a radial unit complex vector in $V_{\mathbb{C}}$. Therefore, $\log q = \ln qe + \varphi\hat{e}$, where $q = q_0e$ and $2\varphi\hat{e} = 2\log q_0 = \log(q_0/\bar{q}_0)$, is a complex vector logarithmic function, and $\text{Log } q = \ln qe + (\varphi \pm 2\pi n)\hat{e}$, $n \in \mathbb{N}$. If $q_{\perp} = q\hat{e}$ and $\hat{q}_0 = -q_{0\perp}^{-1} = s\hat{e}c\varphi = \hat{e}\bar{q}_0$, then the ordered pair of unit vectors (\bar{q}_0, \hat{q}_0) is the inverse orthonormal basis with respect to the orthonormal basis $(q_0, q_{0\perp})$ of the field of complex vectors $V_{\mathbb{C}}$. For an arbitrary vector $a \in V_{\mathbb{C}}$, the vector aq_0 is the rotated vector a , in the positive mathematical direction, by the angle φ , and the vector $aq_{0\perp}$ by the angle $\pi/2 + \varphi$. The geometric products of the vector a with the inverse basis vectors \bar{q}_0 and \hat{q}_0 rotate a by the angles $-\varphi$ and $\pi/2 - \varphi$, respectively, in the positive mathematical direction.

If $d = dr\partial_r + d\varphi\partial_{\varphi}$ is a differential operator, then $dq = drq_0 + d\varphi q_{\perp}$. Hence, $dq_{\perp} = \hat{e}dq = drq_{0\perp} - d\varphi q$ and $d\hat{q} = \hat{e}d\bar{q} = \hat{e}(dr\bar{q}_0 - d\varphi\hat{q}) = dr\hat{q}_0 + d\varphi\bar{q}$. Since $2q \cos \varphi e = q + \bar{q}$ and $2q \sin \varphi \hat{e} = q - \bar{q}$, the vector operators of partial derivatives are introduced as a vector analogue of the *Virtinger* operators [16],

$$\bar{\partial}_q = \partial_q q \partial_q + \partial_q \varphi \partial_{\varphi} = \frac{1}{2}(\bar{q}_0 \partial_q - \hat{q}_0 \partial_{\varphi}) \text{ and } \bar{\partial}_{\bar{q}} = \bar{\partial}_q = \frac{1}{2}(q_0 \partial_q + \frac{q_{0\perp}}{q} \partial_{\varphi}), \quad (15)$$

where $\partial_q \varphi = \cos^2 \varphi \partial_q \tan \varphi = [(q + \bar{q})/(2q)]^2 [(-2\bar{q})/(q + \bar{q})^2] \hat{e} = -\hat{q}_0/(2q)$ and $2\partial_q q = \partial_q q^2/q = \bar{q}_0$. It is important to emphasize that when geometric products and geometric quotients are differentiated, the same rules apply as when ordinary products and quotients are differentiated, so that $d(q/\bar{q}) = (\bar{q}dq - qd\bar{q})/\bar{q}^2$. Let's prove this,

$$\begin{aligned} d\frac{q}{\bar{q}} &= (d\frac{1}{\bar{q}^2})q^2 + \frac{1}{\bar{q}^2}dq^2 = -2(\frac{q^2}{\bar{q}^3}d\bar{q} - \frac{1}{\bar{q}^2}qdq) = \\ &= -2[\frac{q^2}{2\bar{q}^4}(\bar{q}dq + qd\bar{q}) - \frac{1}{\bar{q}^2}qdq] = -2[\frac{1}{2\bar{q}^2}(\bar{q}dq + qd\bar{q}) - \frac{1}{\bar{q}^2}\bar{q}dq] = \frac{\bar{q}dq - qd\bar{q}}{\bar{q}^2}. \end{aligned} \quad (16)$$

Definition 1. The geometric product $dq\bar{\partial}_q$ is an operator of a $1\mathbb{D}$ differential form.

The symmetric and antisymmetric parts of the geometric product $dq\bar{\partial}_q$ are as follows:

$$dq\bar{\partial}_q + d\bar{q}\bar{\partial}_{\bar{q}} = 2dq \circ \bar{\partial}_{\bar{q}} = e(dq\partial_q + d\varphi\partial_{\varphi}) \text{ and} \quad (17)$$

$$dq\bar{\partial}_q - d\bar{q}\bar{\partial}_{\bar{q}} = -2dq \wedge \bar{\partial}_{\bar{q}} = \hat{e}(qd\varphi\partial_q - \frac{1}{q}dq\partial_{\varphi}). \quad (18)$$

Therefore,

$$2q_0dq\bar{\partial}_q = q_0(dq\partial_q + d\varphi\partial_{\varphi}) + q_{0\perp}(qd\varphi\partial_q - \frac{1}{q}dq\partial_{\varphi}) = q_0d + q_{0\perp}\hat{d}, \quad (19)$$

where $d = q_0d$ and $d_{\perp} = q_{0\perp}\hat{d}$ are the radial and transverse components of $2q_0dq\bar{\partial}_q$, respectively. Accordingly, since $2\varphi\hat{e} = \log(q/\bar{q})$, it follows that

$$d\varphi = q_0d\varphi = q_0(\bar{\partial}_q\varphi dq + \bar{\partial}_{\bar{q}}\varphi d\bar{q}) = -\frac{q_{0\perp}}{2}d\log(\frac{q}{\bar{q}}) \text{ and} \quad (20)$$

$$dq = q_0dq = q_0(\bar{\partial}_q q dq + \bar{\partial}_{\bar{q}} q d\bar{q}) = q_0d(q\bar{q})^{\frac{1}{2}} = \frac{q}{2}d\log(q\bar{q}), \quad (21)$$

which leads to the following vector differential identity,

$$\begin{aligned} \varrho_{0\perp} d\varrho d\varphi &= \frac{\varrho}{4} d \log(\varrho \bar{\varrho}) d \log\left(\frac{\varrho}{\bar{\varrho}}\right) = \frac{\varrho}{4\varrho^4} (\bar{\varrho} d\varrho + \varrho d\bar{\varrho})(\bar{\varrho} d\varrho - \varrho d\bar{\varrho}) = \\ &= \frac{\varrho}{4\varrho^2} (\bar{d}\varrho + d\bar{\varrho})(\bar{d}\varrho - d\bar{\varrho}) = \frac{\varrho}{4\varrho^2} [(\bar{d}\varrho)^2 - (d\bar{\varrho})^2]. \end{aligned} \quad (22)$$

Consequently,

$$\frac{\varrho_0}{2} (d\bar{\varrho} \wedge \bar{d}\varrho) = \frac{\varrho}{4\varrho} [(\bar{d}\varrho)^2 - (d\bar{\varrho})^2] = \varrho_{\perp} d\varrho d\varphi, \quad (23)$$

since $\bar{d}\bar{\varrho} = \bar{\varrho}_0 d\varrho = \bar{d}\varrho$. The vector identity just derived can be obtained explicitly via the *Jacobian determinant* of the bijective mapping $V_{\mathbb{C}} \rightarrow V_{\mathbb{C}}$, defined by the system of vector equations $2 \ln \varrho e = \log(\varrho \bar{\varrho})$ and $2\varphi \hat{e} = \log(\varrho/\bar{\varrho})$ as follows:

$$J = \begin{vmatrix} \partial_{\varrho} \log(\varrho \bar{\varrho}) & \partial_{\varphi} \log(\varrho \bar{\varrho}) \\ \partial_{\varrho} \log(\varrho/\bar{\varrho}) & \partial_{\varphi} \log(\varrho/\bar{\varrho}) \end{vmatrix} = \begin{vmatrix} 2\varrho^{-1}e & 0 \\ 0 & 2\hat{e} \end{vmatrix} = \frac{4}{\varrho} \hat{e}. \quad (24)$$

In this case, $4\varrho_{0\perp} d\varrho d\varphi = \varrho J d\varrho d\varphi = \varrho d \log(\varrho \bar{\varrho}) d \log(\varrho/\bar{\varrho})$, which leads to (22). The complex vector $dS = (d\bar{\varrho} \wedge \bar{d}\varrho)/2 = \varrho d\varrho d\varphi \varrho_{0\perp} \bar{\varrho}_0 = dS \hat{e}$, corresponding to the i -vector $\varrho d\varrho d\varphi \hat{i}$, is the *Lebesgue vector measure* of the infinitesimal surface in the field $V_{\mathbb{C}}$.

Definition 2. Let $\bar{\partial}_{\varrho\bar{\varrho}}^2 = \bar{\partial}_{\bar{\varrho}} \bar{\partial}_{\varrho}$. Then, the geometric product $dS \bar{\partial}_{\varrho\bar{\varrho}}^2$ is an operator of a $2\mathbb{D}$ differential form.

3.2. Fundamental Theorem of Integral Calculus in the Field $V_{\mathbb{C}}$

Let γ be a closed smooth *Jordan curve*, bounding an arbitrary simply connected region G in $V_{\mathbb{C}}$, and ϱ_{γ} be a point on γ , surrounded by a circle $c(\varrho_{\gamma}, \varepsilon)$ with center at ϱ_{γ} and arbitrarily small radius ε , which intersects the curve γ at the points ϱ_{γ_1} and ϱ_{γ_2} . Similarly, an arbitrary point ϱ_G inside G is surrounded by circle $c(\varrho_G, \varepsilon)$, which is connected to the circle $c(\varrho_{\gamma}, \varepsilon)$ by two parallel straight line segments $l_{\delta_{\varepsilon}}^1$ and $l_{\delta_{\varepsilon}}^2$, at a distance $\delta_{\varepsilon} \ll \varepsilon$ from each other. The region S , inside which are the points ϱ_{γ_1} and ϱ_{γ_2} , and which is bounded by the segments $l_{\delta_{\varepsilon}}^1$ and $l_{\delta_{\varepsilon}}^2$ as well as the arc segments on the boundaries $\partial c(\varrho_{\gamma}, \varepsilon)$ and $\partial c(\varrho_G, \varepsilon)$ of the circles $c(\varrho_{\gamma}, \varepsilon)$ and $c(\varrho_G, \varepsilon)$, whose endpoints are the intersection points of the boundaries $\partial c(\varrho_{\gamma}, \varepsilon)$ and $\partial c(\varrho_G, \varepsilon)$ with the segments $l_{\delta_{\varepsilon}}^1$ and $l_{\delta_{\varepsilon}}^2$, is the so-called *residue region*. If $G_{\varepsilon}^+ = G \cup S$, the contour integration operators are as follows:

$$\int_{\gamma_{\varepsilon}^+}^{\circ} d\varrho \bar{\partial}_{\varrho} = \int_{\partial G_{\varepsilon}^+}^{\circ} d\varrho \bar{\partial}_{\varrho} \quad \text{and} \quad \int_{\gamma_{\varepsilon}^-}^{\circ} d\varrho \bar{\partial}_{\varrho} = \int_{\partial G_{\varepsilon}^+}^{\circ} d\varrho \bar{\partial}_{\varrho} - \int_{\partial c(\varrho_{\gamma}, \varepsilon)}^{\circ} d\varrho \bar{\partial}_{\varrho}, \quad (25)$$

where $\partial G_{\varepsilon}^+ = \gamma_{\varepsilon}^+$ is the boundary of G_{ε}^+ . Based on the additive property of definite integrals, in the limit, as $\varepsilon \rightarrow 0^+$,

$$\begin{aligned} vt \int_{\gamma^+}^{\circ} d\varrho \bar{\partial}_{\varrho} &= \lim_{\varepsilon \rightarrow 0^+} \int_{\gamma_{\varepsilon}^+}^{\circ} d\varrho \bar{\partial}_{\varrho} = \lim_{\varepsilon \rightarrow 0^+} \left(\int_{\partial G_{\delta_{\varepsilon}}}^{\circ} d\varrho \bar{\partial}_{\varrho} + \int_{\partial S}^{\circ} d\varrho \bar{\partial}_{\varrho} \right) = \\ &= vp \int_{\gamma}^{\circ} d\varrho \bar{\partial}_{\varrho} + \lim_{\varepsilon \rightarrow 0^+} \int_{\partial c(\varrho_{\gamma}, \varepsilon)}^{\circ} d\varrho \bar{\partial}_{\varrho} = vt \int_{\gamma^-}^{\circ} d\varrho \bar{\partial}_{\varrho} + \lim_{\varepsilon \rightarrow 0^+} \int_{\partial c(\varrho_{\gamma}, \varepsilon)}^{\circ} d\varrho \bar{\partial}_{\varrho}, \end{aligned} \quad (26)$$

where vp and vt denote the principal and total integral values [9]-[14], respectively, and $G_{\delta_{\varepsilon}} = G \setminus \text{int}.S$ ($\text{int}.S$ is the interior of S).

On the one hand, as a vector analogue of the so-called areolar (weak) derivative in complex analysis, defined by *Pompeiu* [8], the vector differential operator $\bar{\partial}_{\varrho\bar{\varrho}}^2$ is the gradient (the plane derivative) of $\bar{\partial}_{\varrho}$

$$\lim_{G \rightarrow \varrho_G} \frac{1}{2S_{\gamma}} \int_{\gamma}^{\circ} d\varrho \bar{\partial}_{\varrho} = \bar{\partial}_{\varrho\bar{\varrho}}^2|_{\varrho_G}, \quad (27)$$

where $2S_\gamma = \int_\gamma^\circ \varrho \wedge d\varrho$, so that $\lim_{G \rightarrow \varrho_G} \int_\gamma^\circ d\varrho \bar{\partial}_\varrho = 2dS\bar{\partial}_{\varrho\bar{\varrho}}^2|_{\varrho_G}$. Accordingly, on the basis of the result of the Kelvin-Stokes theorem (Green's theorem) [6],

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0^+} \int_{\partial G_{\delta_\varepsilon}}^\circ d\varrho \bar{\partial}_\varrho &= \lim_{\varepsilon \rightarrow 0^+} \int_{\partial G_{\delta_\varepsilon}}^\circ [(d\varrho \circ \mathbf{grad}) - (d\varrho \wedge \mathbf{grad})] = \\ &= \frac{1}{2} \lim_{\varepsilon \rightarrow 0^+} \iint_{G_{\delta_\varepsilon}} \hat{\varrho} dr d\varphi (\varrho_0 \operatorname{div} \mathbf{grad} + \varrho_0 \times \operatorname{curl} \mathbf{grad}) = v\varrho \iint_G (d\bar{\varrho} \wedge \bar{d}\varrho) \bar{\partial}_{\varrho\bar{\varrho}}^2. \end{aligned} \quad (28)$$

Here,

$$\operatorname{div} \mathbf{grad} = \frac{4}{\varrho^2} (\varrho \bar{\partial}_\varrho \cdot \bar{\varrho} \bar{\partial}_{\bar{\varrho}}) = \frac{1}{\varrho^2} [\varrho \partial_\varrho (\varrho \partial_\varrho) + \partial_{\varphi^2}^2] \text{ and} \quad (29)$$

$$\operatorname{curl} \mathbf{grad} = \frac{4}{\varrho^2} (\bar{\varrho} \bar{\partial}_{\bar{\varrho}} \times \bar{\varrho} \bar{\partial}_{\bar{\varrho}}) = \frac{\varrho_0 \times \varrho_\perp}{\varrho^2} (\partial_{\varphi\varrho}^2 - \partial_{\varphi\varphi}^2) = 0, \quad (30)$$

so that

$$\begin{aligned} \bar{\partial}_{\varrho\bar{\varrho}}^2 &= \bar{\partial}_{\bar{\varrho}} \bar{\partial}_\varrho = \frac{\bar{\varrho}_0 \varrho_0}{\varrho^2} (\varrho \bar{\partial}_\varrho \cdot \bar{\varrho} \bar{\partial}_{\bar{\varrho}}) + \frac{\bar{\varrho}_0 \varrho_0}{\varrho^2} \times (\bar{\varrho} \bar{\partial}_{\bar{\varrho}} \times \bar{\varrho} \bar{\partial}_{\bar{\varrho}}) = \\ &= \frac{\bar{\varrho}_0}{4\varrho^2} [\varrho_0 [\varrho \partial_\varrho (\varrho \partial_\varrho) + \partial_{\varphi^2}^2] + \varrho_\perp (\partial_{\varphi\varrho}^2 - \partial_{\varphi\varphi}^2)] = \frac{\bar{\varrho}_0}{4} (\varrho_0 \operatorname{div} \mathbf{grad} + \varrho_0 \times \operatorname{curl} \mathbf{grad}). \end{aligned} \quad (31)$$

Consequently,

$$v\varrho \int_{\gamma^+}^\circ d\varrho \bar{\partial}_\varrho = v\varrho \iint_{G^+} (d\bar{\varrho} \wedge \bar{d}\varrho) \bar{\partial}_{\varrho\bar{\varrho}}^2, \quad (32)$$

where $v\varrho \iint_{G^+} (d\bar{\varrho} \wedge \bar{d}\varrho) \bar{\partial}_{\varrho\bar{\varrho}}^2 = v\varrho \iint_G (d\bar{\varrho} \wedge \bar{d}\varrho) \bar{\partial}_{\varrho\bar{\varrho}}^2 + \lim_{\varepsilon \rightarrow 0^+} \int_{\partial S}^\circ d\varrho \bar{\partial}_\varrho$. The vector integral operator

$$\lim_{\varepsilon \rightarrow 0^+} \int_{\partial S}^\circ d\varrho \bar{\partial}_\varrho = 2\pi \hat{\varepsilon} \operatorname{Res} \bar{\partial}_\varrho|_{\varrho_G} + \int_{I^+} d\varrho \bar{\partial}_\varrho - \int_{I^-} d\varrho \bar{\partial}_\varrho + 2\pi \hat{\varepsilon} \operatorname{Res} \bar{\partial}_\varrho|_{\varrho_\gamma}, \quad (33)$$

where

$$\int_{I^+} d\varrho \bar{\partial}_\varrho = \lim_{\varepsilon \rightarrow 0^+} \int_{I_{\delta_\varepsilon}^+} d\varrho \bar{\partial}_\varrho, \quad \int_{I^-} d\varrho \bar{\partial}_\varrho = \lim_{\varepsilon \rightarrow 0^+} \int_{I_{\delta_\varepsilon}^-} d\varrho \bar{\partial}_\varrho, \quad (34)$$

$$2\pi \hat{\varepsilon} \operatorname{Res} \bar{\partial}_\varrho|_{\varrho_\gamma} = \lim_{\varepsilon \rightarrow 0^+} \int_{\partial c(\varrho_\gamma, \varepsilon)}^\circ d\varrho \bar{\partial}_\varrho = 2dS\bar{\partial}_{\varrho\bar{\varrho}}^2|_{\varrho_\gamma} \text{ and} \quad (35)$$

$$2\pi \hat{\varepsilon} \operatorname{Res} \bar{\partial}_\varrho|_{\varrho_G} = \lim_{\varepsilon \rightarrow 0^+} \int_{\partial c(\varrho_G, \varepsilon)}^\circ d\varrho \bar{\partial}_\varrho = 2dS\bar{\partial}_{\varrho\bar{\varrho}}^2|_{\varrho_G}, \quad (36)$$

is the vector residue operator in G . Obviously, the operator $2\pi \hat{\varepsilon} \operatorname{Res} \bar{\partial}_\varrho$ is a synonym for $2dS\bar{\partial}_{\varrho\bar{\varrho}}^2$.

By choosing two points, one on the contour γ and the other inside the region G , the generality of the previously obtained results is not lost, so that the fundamental theorem of integral calculus in V_C can be formulated in what follows. Previously, let ω_ϱ be a vector differential form $d\varrho \bar{\partial}_\varrho \bullet$, obtained by applying $d\varrho \bar{\partial}_\varrho$ to an arbitrary uniform scalar or vector field \bullet in V_C , [7].

Theorem 1. *Let γ be a closed smooth Jordan curve, bounding an arbitrary simply connected region G in V_C . For an arbitrary uniform scalar or vector field \bullet in V_C , which is regular almost everywhere on G and whose vector differential form ω_ϱ is totally integrable on γ , there holds*

$$v\varrho \int_{\gamma^+}^\circ \omega_\varrho = 2\pi \hat{\varepsilon} \sum_{\varrho_i \in G} \operatorname{Res} \bar{\partial}_\varrho \bullet(\varrho_i) = v\varrho \iint_{G^+} \omega_S, \quad (37)$$

where $\omega_S = 2dS\bar{\partial}_{\varrho\bar{\varrho}}^2 \bullet$.

So, the integral formula in the previous theorem is a slightly generalized vector analogue of the Cauchy-Pompeiu integral formula, [17]. The sum of $2\pi \hat{\varepsilon} \operatorname{Res} \bar{\partial}_\varrho \bullet$, on the compact set of points $G \setminus E$,

at which the field \bullet is regular ($Res\bar{\partial}_\rho\bullet$ is identically zero on $G \setminus E$), is $vp \iint_G \omega_S$. Hence, $vt \iint_{G^+} \omega_S = vp \iint_G \omega_S + 2\pi\hat{e} \sum_{\rho_i \in E} Res\bar{\partial}_\rho\bullet(\rho_i)$, where $E \subset G$ is the singular set, with Lebesgue measure zero, of the field \bullet . The sum on the right-hand side of the last integral equality can be in the indeterminate form of the difference of two infinities, which is equal to the finite integral value on the left-hand side.

3.3. Integral Calculus Formulas in the $3\mathbb{D}$ Field of Complex Vectors

Let G_α be simply connected regions bounded by closed smooth Jordan curves γ_i , such that they are projections of a smooth surface S onto the component fields of the $3\mathbb{D}$ field of complex vectors. The differential forms in the $3\mathbb{D}$ field of complex vectors are as follows:

$$\omega_{\bar{q}} = Fd\bar{q} = \frac{\langle Fd\bar{q} \rangle_\alpha (\bar{q}_0 q_0)^\alpha}{\sqrt[2]{3}} \text{ and}$$

$$\omega_S = -2\bar{\partial}_q Fd\hat{S} = -2 \frac{\langle \bar{\partial}_q Fd\hat{S} \rangle_\alpha (\bar{q}_0 q_0)^\alpha}{\sqrt[2]{3}}, \quad (38)$$

where $F = F_\alpha (\bar{q}_0 q_0)^\alpha = \langle Fq_0 + F_\perp q_{0\perp} \rangle_\alpha (\bar{q}_0 q_0)^\alpha$ is an arbitrary uniform vector field in the $3\mathbb{D}$ field of complex vectors, $d\bar{q} = d\bar{q}_\alpha (\bar{q}_0 q_0)^\alpha$ and $2d\hat{S}_\alpha = \langle d\bar{q} \wedge d\bar{q} \rangle_\alpha$. According to Theorem 1., if the vector field F is regular almost everywhere on S , and the vector differential forms $\bar{\omega}_q = \bar{F}dq$ and $\omega_{\bar{q}}$ are totally integrable on ∂S , there holds

$$\langle vt \int_{\partial S^+} F \circ dq \rangle_\alpha (\bar{q}_0 q_0)^\alpha = -2 \langle vt \iint_{G^+} (\bar{\partial}_r \wedge F) d\hat{S} \rangle_\alpha (\bar{q}_0 q_0)^\alpha \text{ and} \quad (39)$$

$$\langle vt \int_{\partial S^+} F \wedge dr \rangle_\alpha (\bar{q}_0 q_0)^\alpha = 2 \langle vt \iint_{G^+} (\bar{\partial}_r \circ F) d\hat{S} \rangle_\alpha (\bar{q}_0 q_0)^\alpha, \quad (40)$$

so that

$$vt \int_{\partial S^+} \bar{\omega}_q = vt \int_{\partial S^+} \bar{F}dq = \frac{1}{\sqrt[2]{3}} [vt \int_{\gamma^+} \langle \bar{F}dq \rangle_\alpha (\bar{q}_0 q_0)^\alpha =$$

$$= \frac{2}{\sqrt[2]{3}} [vt \iint_{G^+} \langle \bar{\partial}_q \bar{F}d\hat{S} \rangle_\alpha (\bar{q}_0 q_0)^\alpha = 2vt \int_{S^+} \bar{\partial}_q \bar{F}d\hat{S} = vt \int_{S^+} \bar{\omega}_S \text{ and} \quad (41)$$

$$vt \int_{\partial S^+} \omega_{\bar{q}} = vt \int_{\partial S^+} Fd\bar{q} = \frac{1}{\sqrt[2]{3}} [vt \int_{\gamma^+} \langle Fd\bar{q} \rangle_\alpha (\bar{q}_0 q_0)^\alpha =$$

$$= -\frac{2}{\sqrt[2]{3}} [vt \iint_{G^+} \langle \bar{\partial}_q Fd\hat{S} \rangle_\alpha (\bar{q}_0 q_0)^\alpha = -2vt \int_{S^+} \bar{\partial}_q Fd\hat{S} = vt \int_{S^+} \omega_S. \quad (42)$$

Hence, to noted $vt \int_{\partial S^+} \bar{\omega}_q = vt \int_{S^+} \bar{\omega}_S$ and $vt \int_{\partial S^+} \omega_{\bar{q}} = vt \int_{S^+} \omega_S$.

Furthermore, since

$$\langle (\bar{\partial}_q \bar{F} - \bar{\partial}_q F) d\hat{S} \rangle_\alpha \cdot (\bar{q}_0 q_0)^\alpha = -\langle (\text{curl } F \times \bar{q}_0 q_0) d\hat{S} \rangle_\alpha \cdot (\bar{q}_0 q_0)^\alpha = \quad (43)$$

$$= \langle \bar{q}_0 q_0 \times \text{curl } F \rangle_\alpha \cdot d\hat{S}_\alpha = \langle \text{curl } F \rangle_\alpha \cdot \langle d\hat{S} \times \bar{q}_0 q_0 \rangle_\alpha = \text{curl } F \cdot d\bar{S} \text{ and}$$

$$(\bar{q}_0 q_0)^\alpha \times \langle (\bar{\partial}_q \bar{F} + \bar{\partial}_q F) d\hat{S} \rangle_\alpha = \langle \text{div } F \bar{q}_0 q_0 \rangle_\alpha \times d\hat{S}_\alpha, \quad (44)$$

where $\langle \text{div } F \rangle_\alpha = \langle [\partial_\rho(qF) + \partial_\varphi F_\perp] / \rho \rangle_\alpha$, $\langle \|\text{curl } F\| \rangle_\alpha = \langle [\partial_\rho(qF_\perp) - \partial_\varphi F] / \rho \rangle_\alpha$ and $2\langle \bar{\partial}_q F \rangle_\alpha = \langle \text{div } F \bar{q}_0 q_0 + \text{curl } F \times \bar{q}_0 q_0 \rangle_\alpha$, and in addition $d\bar{S} = d\hat{S}_\alpha \times (\bar{q}_0 q_0)^\alpha$, the vector identities (41) and (42) lead to the complex generalized Stokes integral identity

$$vt \int_{\partial S^+} \bar{F} \cdot dq = 2vt \int_{S^+} (\bar{\partial}_q \wedge F) \cdot d\bar{S}, \quad (45)$$

where $\bar{F} \cdot d\varrho = \langle F \circ d\mathbf{r} \rangle_\alpha \cdot (\bar{\varrho}_0 \varrho_0)^\alpha$, as well as to the complex vector integral identity

$$vt \int_{\partial S^+}^\circ F \times d\varrho = 2vt \int_{S^+} \langle \bar{\partial}_{\bar{\varrho}} \circ F \rangle^\alpha \times d\hat{S}_\alpha, \quad (46)$$

where $F \times d\varrho = (\bar{\varrho}_0 \varrho_0)^\alpha \times \langle F \wedge d\varrho \rangle_\alpha$. Thus, the complex vector integral identities (45) and (46) are explicit consequences of the fundamental theorem of integral calculus in the $3\mathbb{D}$ field of complex vectors, which follows.

Theorem 2. Let γ be a closed smooth spatial curve in the $3\mathbb{D}$ field of complex vectors, bounding an arbitrary simply connected spatial surface S . For an arbitrary uniform complex vector field F , which is regular almost everywhere on S and whose differential form $\bar{\omega}_\varrho = Fd\bar{\varrho}$ is totally integrable on γ , there holds

$$vt \int_{\partial S^+}^\circ \bar{\omega}_\varrho = vt \int_{S^+} \bar{\omega}_{\hat{S}}, \quad (47)$$

where $\bar{\omega}_{\hat{S}} = 2\bar{\partial}_{\bar{\varrho}} F d\hat{S}$.

On the other hand, it is quite possible to formulate the theorem, as follows, using a procedure similar to that used to formulate *Theorem 1.*, based on the result of *Green's* theorem, but now on the basis of *Gauss-Ostrogradsky's* theorem.

Theorem 3. Let S be a closed smooth surface in the $3\mathbb{D}$ field of complex vectors, bounding an arbitrary simply connected region V in that field. Then, for an arbitrary uniform complex vector field $F = Fe + \hat{F}\hat{e}$, which is regular almost everywhere on V and whose differential form $\omega_{\bar{S}} = Fd\bar{S}$ is totally integrable on S , there holds

$$vt \int_{\partial V^+}^\circ \omega_{\bar{S}} = vt \int_{V^+} \omega_V, \quad (48)$$

where $\omega_V = 2\bar{\partial}_{\bar{\varrho}} F dV$.

Obviously, the integral formula (48) is an explicit consequence of the following integral identities

$$\begin{aligned} vt \int_{\partial V^+}^\circ F_\alpha (\bar{n}^\alpha \cdot dS) &= vt \int_{\partial V^+}^\circ \langle FdS \rangle_\alpha e^\alpha = \\ &= vt \int_{\partial V}^\circ \langle \bar{F} \circ dS + \bar{F} \wedge dS \rangle_\alpha e^\alpha = 2vt \int_{V^+} \langle \bar{\partial}_{\bar{\varrho}} \circ Fe + \bar{\partial}_{\bar{\varrho}} \wedge \hat{F}\hat{e} \rangle_\alpha e^\alpha dV \text{ and} \\ vt \int_{\partial V^+}^\circ \langle \bar{F} \times n \rangle_\alpha (\bar{n}^\alpha \cdot dS) &= -vt \int_{\partial V^+}^\circ \langle Fd\hat{S} \rangle_\alpha e^\alpha = \\ &= -vt \int_{\partial V^+}^\circ \langle \bar{F} \circ d\hat{S} + \bar{F} \wedge d\hat{S} \rangle_\alpha e^\alpha = 2vt \int_{V^+} \langle \bar{\partial}_{\bar{\varrho}} \circ \hat{F}\hat{e} + \bar{\partial}_{\bar{\varrho}} \wedge Fe \rangle_\alpha e^\alpha dV, \end{aligned} \quad (49)$$

where $n_\alpha = \langle e \times \hat{e} \rangle_\alpha$, $dS_\alpha = \langle (e \times d\hat{S}) \times \hat{e} \rangle_\alpha$ and $d\bar{S}_\alpha = \langle dS - d\hat{S} \rangle_\alpha$. Therefore, if $2\langle \bar{\partial}_{\bar{\varrho}} \circ F \rangle_\alpha \cdot e^\alpha = \mathbf{div} F$ and $e^\alpha \times 2\langle \bar{\partial}_{\bar{\varrho}} \wedge F \rangle_\alpha = \mathbf{curl} F$, then

$$vt \int_{\partial V^+}^\circ F \cdot d\bar{S} = vt \int_{V^+} \mathbf{div} F dV \text{ and} \quad (51)$$

$$vt \int_{\partial V^+}^\circ d\bar{S} \times F = vt \int_{V^+} \mathbf{curl} F dV. \quad (52)$$

4. Conclusions

If one compares the integral identities in the fundamental theorems, formulated above, it can be concluded that they are basically the same integral identities, with the corresponding differential forms, so that they can be represented by one integral identity as follows:

$$vt \int_{\partial\Omega^+}^{\circ} \omega = vt \int_{\Omega^+} d\omega, \quad (53)$$

where Ω is the corresponding compact set of points in a $2\mathbb{D}$ or $3\mathbb{D}$ field of complex vectors with boundary $\partial\Omega$ and

$$d\omega = \lim_{\Omega \rightarrow \varrho} \int_{\partial\omega}^{\circ} \omega. \quad (54)$$

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