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Maximal Rotational Velocity of Fluid Elements (Ω_{FluidMAX}): A New Perspective on Turbulent Singularities

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Abstract:

The classical Navier-Stokes (NS) equations, rooted in the continuum hypothesis, essentially extend Newton's second law to describe fluid motion. However, in their standard derivation, these equations rely solely on mass and momentum conservation and do not incorporate angular momentum conservation or dynamical equations for the rotation of fluid elements. As a result, they cannot explicitly describe the inertial torque arising from the fluid element's own moment of inertia and angular acceleration. This simplification is reasonable and has been widely validated in translation-dominated flows. Nevertheless, in turbulent flows, local angular accelerations can become significant, and inertial torques may dominate the dynamics, rendering the NS equations physically incomplete in modeling strongly rotational, nonequilibrium flows. Consequently, turbulence models based on the NS equations—such as Reynolds-Averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES)—often rely on empirical or semi-empirical closure assumptions to compensate for the missing physical mechanisms, thereby limiting their ability to reveal the intrinsic dynamical processes of turbulence. This paper argues that the rotational degree of freedom of fluid elements should be treated independently from translational motion. We propose a testable hypothesis: **the Maximum Rotational Angular Velocity of Fluid Elements** (**Ω_{FluidMAX}**). We posit that the angular velocity of fluid elements in a flow field is subject to a physical upper bound, the magnitude of which is determined by the fluid's speed of sound a and the characteristic length scale ℓ of the fluid element. A preliminary expression is given by: $\Omega_{\text{FluidMAX}} = K_i \cdot \frac{a}{2\pi\ell}$, where K_i is a dimensionless constant to be calibrated experimentally. When external energy input drives the angular velocity of a fluid element toward this limit, excess energy can no longer be stored as rotational kinetic energy and is instead forced to convert into thermal energy, pressure fluctuations, or wave energy (e.g., acoustic or shock waves). This leads to a sharp rise in local energy density and strong nonlinear effects, resulting in what we define as a **physical singularity**. This mechanism offers a new physical interpretation for singularity formation in turbulence. It not only addresses the **mathematical singularity** problem arising from unbounded rotation in the NS framework but also provides a clear physical picture of energy dissipation and extreme events. The core value of this hypothesis lies in transforming the concept of **physical singularity** from an abstract mathematical notion into an observable physical state. Future validation requires high-resolution experimental techniques—such as ultra-high-speed particle image velocimetry (PIV), quantum sensing, or femtosecond laser spectroscopy—to verify the existence of **Ω_{FluidMAX}**. If confirmed, this hypothesis could revolutionize turbulence modeling by enabling the reformulation of subgrid-scale dissipation mechanisms in RANS and LES. Theoretically, it may also provide a unified framework for understanding extreme phenomena such as plasma turbulence in fusion devices and angular momentum transport in black hole accretion disks, thereby advancing turbulence modeling from an empirically fitted paradigm toward a physically driven one.

Keywords: Navier-Stokes (NS) equations; inertial torque; singularities in fluid turbulence; the maximum rotational angular velocity of fluid elements (Ω_{FluidMAX})

1. Introduction

Turbulence is a ubiquitous and complex flow phenomenon in both natural and engineering systems. Its multiscale structure, strong nonlinearity, and stochastic nature make it one of the most challenging problems in fluid mechanics. Since Reynolds' systematic investigation of turbulence in the late 19th century, despite over a century of development, fundamental mechanisms—such as energy cascading, vortex dynamics, and singularity formation—remain incompletely understood. Turbulence is thus often regarded as "the last unsolved problem in classical physics" [1, 2].

The theoretical foundation of modern fluid mechanics rests upon the **Navier-Stokes (NS) equations**, which, under the continuum assumption, represent an application of Newton's second law to fluid motion. These equations describe the translational acceleration of fluid elements (via the material derivative) through the conservation of mass and momentum. In laminar or weakly perturbed flows, rotational effects of fluid elements are typically negligible. The moment of inertia, being small at microscopic scales, can be treated as a higher-order infinitesimal, and neglecting inertial torques is a reasonable simplification. This approach has been extensively validated by experiments and engineering practice, forming the cornerstone of classical fluid dynamics [3].

However, in turbulent flows, fluid elements undergo intense deformation and rotation. Vortex structures cascade from large scales down to microscales—approaching even the molecular mean free path—leading to significant local angular accelerations. Under such conditions, the inertial torque, determined jointly by the moment of inertia and angular acceleration, may become a dominant factor in fluid motion. Since the **NS equations** do not explicitly include an angular momentum conservation equation or terms accounting for inertial torques associated with rotational degrees of freedom, they exhibit a systematic deficiency in the physical modeling of rotational dynamics. This modeling limitation prevents the **NS equations** from accurately capturing key processes such as vortex stretching, fragmentation, and energy transfer within rotational modes, thereby constraining a deeper understanding of the essence of turbulence.

Contemporary turbulence research largely remains within the **NS equation** framework, relying on empirical parameters (e.g., turbulent viscosity in k-ε models) or numerical techniques (e.g., Large Eddy Simulation (LES), Direct Numerical Simulation (DNS)) to approximate statistical properties of turbulence. While these methods are practically useful in specific engineering contexts, their theoretical basis inherently neglects the dynamical response of rotational degrees of freedom. As such, they amount to "mathematical compensation" for the physical incompleteness of the **NS equations**, rather than a restoration of the underlying physical mechanisms. This has led to a long-standing dilemma in turbulence research: "phenomenological fitting prevails over mechanistic revelation, " hindering the development of universal, cross-scale, and cross-regime predictive capabilities.

The motivation of this work stems from a critical re-examination of this modeling limitation. Drawing inspiration from rigid body dynamics—where motion is decomposed into center-of-mass translation (governed by the theorem of motion of the center of mass) and rotation about the center of mass (governed by the angular momentum theorem)—we argue that in turbulence research, the rotational degree of freedom of fluid elements should be treated as an independent physical variable and explicitly incorporated into the mechanical modeling framework. Although the vorticity transport equation implicitly contains some rotational dynamics, it is itself derived from the **NS equations** and thus cannot compensate for the absence of inertial torque terms in the original formulation.

Based on this perspective, this paper systematically analyzes the physical consequences of neglecting inertial torques in the **NS equations**, demonstrating their modeling insufficiency in strongly rotational, nonequilibrium flows. We further propose a testable hypothesis: **the Maximum**

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Rotational Angular Velocity of Fluid Elements ($\Omega_{\mathrm{FluidMAX}}$). This hypothesis posits that the angular velocity of fluid elements in a flow field is subject to a universal physical upper bound. When continuous external energy input drives the angular velocity toward this limit, excess energy can no longer be accommodated as rotational kinetic energy and is instead forced to convert into thermal energy, pressure fluctuations, or other forms of wave energy. This results in a sharp rise in local energy density and strong nonlinear effects—what we define as a **physical singularity**. This hypothesis offers a new physical pathway to explain singularity formation in turbulence, potentially filling a critical theoretical gap in the NS framework regarding rotational constraints. It may thereby catalyze a paradigm shift in turbulence research—from "mathematical fitting" to "physics-driven modeling."

2. Historical Lag in Rotational Physics and the Decomposition Paradigm from Rigid Body Dynamics

2.1. Historical Lag in Rotational Physical Laws

Rotational motion, despite being ubiquitous in nature, has historically seen a significant delay in the development of its associated physical laws compared to translational dynamics. This historical asymmetry reflects a systematic delay in human understanding of rotational degrees of freedom. Key examples include:

1. **The late establishment of the angular momentum theorem: Although Kepler discovered the three laws of planetary motion in the 17th century—implicitly containing the concept of angular momentum conservation—the principle of angular momentum as a fundamental conserved quantity was not formally established as a cornerstone of physics until the early 20th century, approximately 200 years after the formulation of linear momentum conservation. Had Kepler possessed the concept of angular momentum conservation, he might have deduced the inverse-square nature of gravitational force earlier. While this counterfactual remains unverifiable, it highlights the potential impact of delayed rotational theory on the trajectory of scientific discovery.**
2. **Quantum entanglement and fluid turbulence:** The former involves nonlocal correlations of spin states in microscopic particles, while the latter manifests as nonlinear cascading and fragmentation of macroscopic vortex structures. Despite their vastly different scales, both phenomena are fundamentally governed by rotation or angular momentum as a primary degree of freedom. This suggests that rotational dynamics may harbor universal principles that transcend scale, linking quantum and classical systems through a common physical foundation.

This historical and phenomenological parallel raises two critical questions:

(1), Is the nonlinear and geometrically complex nature of rotation inherently more difficult to abstract into fundamental laws than translation?

(2), Could the neglect or oversimplification of rotational degrees of freedom be a deep-rooted obstacle to breakthroughs in fields ranging from turbulence to quantum many-body physics?

Re-evaluating the role of rotation in physical modeling may not only deepen our understanding of classical mechanics but also open new theoretical pathways for explaining complex systems such as turbulence and quantum entanglement.

2.2. Insights from Rigid Body Dynamics

In classical mechanics, the planar motion of a rigid body is successfully decomposed into two independent dynamical components: translation of the center of mass and rotation about the center of mass. This decomposition is described by two fundamental equations:

1, The **theorem of motion of the center of mass**, which governs translational dynamics;

2, The angular momentum theorem relative to the center of mass, which governs rotational dynamics.

These two equations are dynamically independent and mutually irreplaceable; both must be solved simultaneously to fully describe the motion of a rigid body.

This principle of "**separated modeling**" offers profound insight for fluid mechanics. Although fluid elements are not rigid bodies, their motion similarly comprises translational and rotational degrees of freedom. However, the classical **NS equations** are derived solely from mass and momentum conservation, effectively describing only the translational acceleration (material derivative) of fluid elements. No independent equation for rotational motion is introduced. While the vorticity transport equation can be derived from the **NS equations** and provides indirect information about rotation, it originates from the same incomplete framework. Crucially, the original **NS equations** do not explicitly include the inertial torque term arising from the moment of inertia and angular acceleration. As a result, the full dynamical response of the rotational degree of freedom remains unmodeled.

2.3. Complexity of Fluid Element Motion and Modeling Limitations

The motion of a fluid element is far more complex than that of a rigid body in planar motion: its shape continuously deforms, internal shear, stretching, and vortical structures emerge, and rotational behavior is highly nonlinear and localized. To accurately describe such motion, both translational and rotational degrees of freedom must, in principle, be considered simultaneously.

However, during the derivation of the **NS equations**, the moment of inertia of fluid elements is treated as a higher-order infinitesimal and neglected. Rotational effects are only indirectly captured through viscous stress terms. This simplification is valid in laminar or weakly rotational flows, but becomes problematic in turbulent regimes, where local angular accelerations can be extremely large and inertial torques cannot be neglected. In such conditions, the absence of an explicit angular momentum conservation equation in the NS framework prevents an accurate description of energy transfer, concentration, and dissipation within rotational degrees of freedom. This leads to a **physical modeling deficiency** in capturing core mechanisms such as vortex stretching, fragmentation, and singularity formation.

This paper argues that the rotation of fluid elements should be treated as an independent dynamical degree of freedom, requiring the explicit inclusion of inertial torque terms in the mechanical framework. Such an extension is not merely a correction to the **NS equations**, but a necessary step toward building next-generation, physics-driven turbulence models. Future research in fluid mechanics should aim to develop a "**translation-rotation coupled**" modeling framework capable of simultaneously describing both translational and rotational dynamics. High-resolution experimental techniques—such as three-dimensional particle tracking velocimetry (3D-PTV), quantum sensing, or ultrafast imaging—should be employed to validate the physical reality and limiting behavior (e.g., maximum angular velocity) of fluid element rotation.

3. Physical Modeling Limitations of Existing Turbulence Models

Classical turbulence models—such as Reynolds-Averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES)—are all fundamentally built upon the **NS equations**. However, since the **NS equations** neglect the moment of inertia and inertial torque of fluid elements during their derivation, they exhibit a systematic deficiency in modeling rotational degrees of freedom. This incompleteness in physical modeling leads to a common modeling limitation shared by all NS-based turbulence models: the inability to fully describe nonlinear dynamical processes dominated by rotation, forcing reliance on empirical parameters or numerical approximations for localized fitting.

3.1. Core Limitation of the NS Equations: Absence of Angular Momentum Conservation

The **NS equations** are based on the continuum assumption, and their momentum conservation equation describes only the relationship between translational acceleration of fluid elements and external forces (e.g., pressure gradients and viscous stresses). They do not explicitly include an angular momentum conservation equation or terms representing inertial torque arising from the moment of inertia and angular acceleration. This simplification implies the following physical consequences:

1, Rotational effects are indirectly represented only through gradients of viscous stress (e.g., the diffusion term in the vorticity transport equation), while inertial torque is entirely neglected.

2, In laminar or weakly rotational flows, where rotational effects are small, this approximation is physically reasonable.

1， In turbulent flows, however, fluid elements undergo intense deformation and vortex stretching, leading to large local angular accelerations. Under such conditions, inertial torque may become a dominant factor in the dynamics.

This limitation reflects not only a mathematical insufficiency but a fundamental omission in the physical modeling. As Xie Xuegang emphasized, angular momentum balance must be explicitly introduced into fluid dynamics; otherwise, non-conservation of angular momentum can compromise the accurate description of rotational structures [4, 5]. Joseph D. D. further demonstrated in multiphase flow studies that the classical **NS equations** fail to capture the rotational degrees of freedom of fluid elements and require augmentation with additional rotational dynamical equations [6].

3.2. Inherent Limitations of Mainstream Turbulence Models

Current mainstream turbulence models inherit this modeling deficiency from the NS framework. Their limitations manifest as follows:

3.2.1. Empirical Parameters as “Compensatory Fitting”

Reynolds-Averaged Navier-Stokes (RANS) models, such as the k-ε model, introduce a “turbulent viscosity coefficient” to represent fluctuation effects as an enhanced viscous stress. However, the value of this parameter is calibrated empirically, lacking a universal physical basis. It remains effective only in specific flow configurations (e.g., boundary layers, pipe flows) and fails to generalize to complex turbulence or shock-interacting flows. This strategy of “using empirical closure to compensate for missing physics” essentially avoids a deep mechanistic description of energy transfer within rotational degrees of freedom.

3.2.2. LES and DNS: “Computational Bottlenecks” and “Physical Blind Spots”

Large Eddy Simulation (LES) separates large- and small-scale motions via spatial filtering. However, subgrid-scale (SGS) models—such as the Smagorinsky model—still rely on empirical closures (e.g., eddy viscosity assumptions) and do not account for the inertial torque effects of small-scale vortex rotation. Direct Numerical Simulation (DNS), while aiming to “solve the **NS equations** directly, ” encounters physical breakdown at high Reynolds numbers when turbulent scales approach the molecular mean free path, where the continuum assumption itself fails. Moreover, the **NS equations**’ neglect of rotational degrees of freedom further amplifies the discrepancy between numerical solutions and physical reality. Lesieur and Métais pointed out that the **NS equations** must be supplemented with rotational terms to accurately describe vortex stretching and angular momentum transport in rotating turbulence [7]. This indicates that even with sufficient computational resolution, results may deviate from physical truth if the governing equations lack essential physical terms.

3.2.3”. Passive Response” to Nonlinear Effects

Core features of turbulence—such as energy cascading, vortex fragmentation, and localized extreme gradients—are intrinsically linked to the nonlinear evolution of rotational degrees of freedom. Existing models can only passively capture the phenomenology of these effects (e.g., sudden increases in vorticity) through numerical means, but fail to explain mechanistically why energy cascades to smaller scales or why singular-like structures form locally. The root cause lies in the **NS equations**' lack of intrinsic constraints on rotational degrees of freedom, rendering current models more akin to "phenomenon recorders" than "mechanism revealers."

3.3. Mathematical Evidence: Divergence of Rotational Degrees of Freedom

Mathematical analysis further exposes the structural deficiency of the **NS equations** in rotational modeling. Tao Z. (Terence Tao) noted in his study of the regularity problem for the **NS equations** that, although total kinetic energy remains bounded, the nonlinear structure fails to effectively control the growth of vorticity (a proxy for rotational degrees of freedom), leaving open the possibility of "blowup in rotational degrees of freedom" [8, 9]. The physical interpretation of this mathematical phenomenon is clear: the **NS equations** lack a physical mechanism to suppress unbounded growth of angular velocity.

Frand M. White. also confirmed from a mathematical perspective that the **NS equations** "lack sufficient mathematical structure to describe angular momentum balance" in rotational flows [3]. Frisch et al. further observed that the nonlinear terms in the **NS equations** provide positive feedback during vortex stretching, enhancing vorticity, while the equations themselves contain no intrinsic mechanism to limit this process—potentially leading to theoretically infinite local vorticity [1]. This mathematical insight aligns with the physical argument: in the absence of a physical upper bound such as a maximum angular velocity, energy continues to be injected into rotational modes, ultimately leading to a **mathematical singularity**.

3.4. The Gap Between Mathematical Solutions and Physical Reality

It is important to emphasize that this paper does not dispute the validity of the NS equations in translation-dominated flows. Rather, it highlights that their *incomplete physical modeling* may lead to a *decoupling between mathematical solutions and physical reality*:

1. Even if smooth mathematical solutions to the **NS equations** exist (as in the Clay Mathematics Institute's "Millennium Prize Problem" on regularity), they may still fail to reflect the true physics of turbulence if rotational inertial effects are not properly accounted for.
2. Analogous to how Newtonian mechanics is superseded by relativity at high velocities, the **NS equations** may lose validity in highly turbulent regimes due to limitations in their underlying physical assumptions.

Therefore, the limitations of current turbulence models are, in essence, a continuation of the **NS equations**' failure to properly model rotational degrees of freedom. To overcome this bottleneck, the theoretical framework must incorporate an independent dynamical equation for rotational motion—following the same logic as the separation of translation and rotation in rigid body dynamics. This is not a rejection of the historical contributions of the **NS equations**, but a necessary *physical completion* of their framework, laying the foundation for a new generation of physics-driven turbulence models.

4. A Physical Framework for Turbulent Singularities: The Hypothesis of Maximum Rotational Angular Velocity ($\Omega_{\mathrm{FluidMAX}}$)

The "singularity problem" in turbulence has long been both a mathematical regularity challenge and a debate over the completeness of physical mechanisms. Building upon the preceding analysis of the **NS equations**' modeling deficiency in rotational degrees of freedom, this paper proposes a novel physical interpretation framework: the introduction of a **fluid element maximum rotational angular velocity**, denoted $\Omega_{\mathrm{FluidMAX}}$, as a fundamental physical constraint. This hypothesis aims to explain the formation of extreme nonlinear structures in turbulence and offers a contrast to traditional mathematical singularity paradigms.

4.1. The Principle of Maximum Angular Velocity and Rotational Constraints on Fluid Elements

In nature, the angular velocity of any physical structure with finite volume cannot increase indefinitely. It may be constrained by fundamental physical laws—akin to the speed of light c as a universal upper bound for linear velocity. This idea has been proposed in the study of rotational physical laws [10], termed the **Principle of Maximum Angular Velocity**: any physical structure with volume possesses a theoretical upper limit $\boldsymbol{\Omega}_{\mathbf{max}}$ on its angular velocity, which is related to the vacuum speed of light c and the mathematical constant π. The physical significance of this principle lies in the observation that a purely geometric point (zero volume) lacks rotational attributes (its angular velocity is zero), whereas any real physical entity—including fluid elements—must have its rotational behavior governed by some fundamental physical mechanism. Reference [10] establishes a quantitative framework for rotational motion by defining angular velocity as the first derivative of angular displacement ϕ with respect to proper time τ: $\omega = d\phi/d\tau$. This provides a theoretical foundation for analyzing the rotational dynamics of fluid elements.

Inspired by this principle, we propose the following hypothesis: a fluid element, as a "quasi-structured" entity with physical volume, also possesses an upper bound on its angular velocity, which we define as the **fluid element maximum rotational angular velocity**, $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ **(in rad/s)**. This upper limit is expected to depend on the fluid's intrinsic properties. We preliminarily assume that the dominant parameter is the speed of sound a, leading to the expression:

$$\Omega_{\text{FluidMAX}} = K_i \cdot \frac{a}{2\pi\ell} \quad (*)$$

where Ki is a dimensionless constant dependent on molecular structure, thermodynamic state, and other fluid-specific factors (to be calibrated experimentally), and ℓ is the characteristic length scale of the fluid element (e.g., the Kolmogorov scale). Furthermore, we impose the conservative constraint:

$$\boldsymbol{\Omega}_{\mathbf{FluidMAX}} \leq \boldsymbol{\Omega}_{\text{max}}$$

This reflects the physical saturation of rotational degrees of freedom under extreme conditions and may act as a limiting mechanism for the energy cascade in turbulence.

The physical origin of $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ may stem from fundamental limits such as:

1. **Maximal molecular collision frequency:** At extreme angular velocities, fluid elements rotate faster than molecular collisions can redistribute momentum. This mismatch between acceleration timescale ($\Delta t \approx \ell/\Omega$) and collision timescale creates a "kinetic bottleneck, " where rotational energy saturates due to insufficient angular momentum transfer at the molecular scale.

2. **Acoustic wave propagation limit:** Rotational stresses cannot exceed the finite speed of sound (a), as excess energy must dissipate via thermalization or shock formation to maintain local mechanical equilibrium. This aligns with second-law thermodynamics, where entropy production scales with angular velocity gradients across acoustic wavelengths.

3. **Quantum/relativistic constraints:** While negligible in classical flows, quantum effects constrain angular momentum resolution ($\Delta L \geq \hbar$) at molecular scales ($\ell \lesssim \lambda_{\text{mfp}}$), potentially linking Ω_{FluidMAX} to the classical-quantum transition. Relativistic considerations may further impose spacetime-based limits on rotational dynamics.

Collectively, these limits operate across scales—from molecular collisions (microscale) to acoustic propagation (mesoscale) and quantum/relativistic bounds (fundamental scale)—to constrain rotational angular velocity. These interdependent mechanisms jointly establish the physical upper bound of fluid rotation, rendering Ω_{FluidMAX} a universal regulator of turbulent energy dissipation pathways.

Note: The inequality $\boldsymbol{\Omega}_{\mathbf{FluidMAX}} \leq \boldsymbol{\Omega}_{\text{max}}$ is a conservative assumption. Even if the universal $\boldsymbol{\Omega}_{\text{max}}$ from [10] is not valid, $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ may still exist independently as a fluid-specific limit.

4.2. Formation Mechanism of Physical Singularities in Turbulence

Under the proposed hypothesis, the formation of physical singularities in turbulence can be described as follows:

During the energy cascade, large-scale vortices transfer energy to smaller scales through stretching and shearing, causing the angular velocity of fluid elements to increase. As this angular velocity approaches $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$, the system enters a **rotational saturation** state. If additional energy continues to be injected (e.g., via pressure gradient work or gravitational potential release), the excess energy cannot be stored as rotational kinetic energy due to the physical limit. Instead, it must be converted into other forms, leading to:

1. **Forced energy conversion**: Rotational kinetic energy is transformed into thermal energy (localized temperature rise), pressure energy (high-pressure zones), or wave energy (acoustic waves, shock waves), resulting in strong nonlinear effects.
2. **Drastic flow field reorganization**: The vortex system undergoes fragmentation, ejection, or merging due to energy saturation, generating extremely small-scale regions of intense shear (e.g., elongated vortex filaments), where velocity gradients, vorticity, and dissipation rates reach extreme values.
3. **Local breakdown of the continuum assumption**: When energy concentrates at molecular scales, the local flow may deviate from continuum behavior, potentially leading to cavitation, ionization, or plasma formation.

Collectively, these phenomena constitute a **physical singularity** in turbulence. Crucially, this singularity is not a mathematical infinity, but rather an **extreme state of energy dissipation and redistribution under rotational constraints**. This interpretation aligns closely with experimental observations of turbulence intermittency, localized extreme gradients, and multiscale dissipation.

4.3. Analogy with Shock Wave Phenomena

To better understand the hypothesis of $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ and its associated singularity mechanism, we draw an analogy with shock wave phenomena.

When a moving object (e.g., a spacecraft) exceeds the speed of sound a in a medium (e.g., air), disturbances ahead cannot propagate fast enough (limited by a), resulting in abrupt compression and the formation of a shock wave. Across the shock, fluid properties such as pressure, density, and temperature undergo discontinuous jumps, accompanied by significant energy dissipation. The resulting sonic boom illustrates the generation of strong nonlinear effects in the fluid medium.

Similarly, under the $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ hypothesis, when a fluid element's angular velocity approaches its physical upper limit, further energy input cannot be accommodated as rotational kinetic energy. The excess energy is then forced into internal energy (local heating), pressure fluctuations, or wave energy (e.g., shock waves). This process also involves a rapid increase in local energy density and strong nonlinearity, forming a **physical singularity**.

Thus, both the **speed of sound a** in translational motion and $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ in rotational motion represent physical constraints that trigger energy dissipation under extreme conditions. This analogy not only enhances the intuitive understanding of $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ but also suggests new experimental approaches for validation.

Notably, while a governs translational dynamics, $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ governs rotational dynamics. Despite acting on different degrees of freedom, they share similarities in energy transfer and dissipation mechanisms—further supporting our assumption that $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ is fundamentally linked to a.

4.4. Contrast with Traditional Views on Turbulent Singularities

Traditional studies of turbulent singularities focus on the regularity of solutions to the **NS equations**—specifically, whether velocity or vorticity can blow up in finite time ("mathematical blow-up"). Work by Leray, Beale-Kato-Majda, Tao, and others (central to the Clay Mathematics Institute's Millennium Prize Problem) [1, 8] centers on this question. These studies emphasize mathematical consistency but often overlook the issue of physical realism.

The innovation of our hypothesis lies in:

1, **A physics-based perspective**: Singularities are interpreted as forced energy conversion due to saturation, not mathematical divergence.

2, **A clear mechanistic origin**: Attributed to an upper bound on rotational energy, rather than uncontrolled growth from the nonlinear term in the NS equations.

3, **Bridging the gap between mathematics and physics**: Our hypothesis does not deny the possibility of smooth solutions to the **NS equations**. Instead, it argues that even if such solutions exist, they may fail to describe real turbulent flows if rotational constraints are ignored.

Moreover, traditional explanations—such as "reaching molecular mean free path" or "chaotic self-organization"—can be incorporated into this framework: the former describes energy transfer to microscopic scales, while the latter reflects the system's response to the coupling of rotational constraints and nonlinearity. Importantly, this hypothesis does not contradict Kolmogorov's scaling laws: the latter describes the statistical average of the energy cascade, while $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ characterizes the energy saturation and forced dissipation at the smallest scales—corresponding respectively to the inertial range and the dissipation range.

4.5. Testability and Experimental Pathways

The core testable proposition of this hypothesis is: **"The angular velocity of fluid elements has a physical upper bound."** Experimental validation pathways include:

1. **Direct verification**: Use high-speed PIV/PTV, super-resolution LDV, or quantum sensing techniques to measure angular velocity distributions of microscale vortices in high-Reynolds-number turbulence, searching for a consistent upper bound.
2. **Indirect verification**: Monitor the relationship between energy input and local temperature/pressure, testing for a sudden increase in energy conversion efficiency after rotational saturation.
3. **Cross-medium comparison**: Test $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$ in gases, liquids, and plasmas, examining its dependence on fluid properties (speed of sound, molecular mass) to determine the universality of *Ki*.

While current technology may not yet allow precise determination of $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$, statistical analysis can reveal cutoff features in angular velocity distributions. The initial goal is not to measure the exact value, but to **confirm the existence of a physical upper bound on rotational angular velocity**—this is the fundamental criterion for assessing the hypothesis's validity.

4.6. Theoretical Value and Potential Impact

If validated, this hypothesis could have profound implications for fluid mechanics:

1. **Enhanced physical modeling**: Supplement the **NS equations** with rotational constraint terms, enabling a new "translation-rotation coupled" framework.
2. **Re-definition of physical singularities**: Treat them as finite but extreme dissipation states, aligning more closely with physical reality.
3. **Revolutionizing turbulence modeling**: Provide physics-based subgrid dissipation mechanisms (e.g., "angular velocity saturation") for LES and RANS models, replacing empirical closures.
4. **Unifying extreme phenomena**: Offer a cross-scale perspective for interpreting turbulence in fusion plasmas, angular momentum transport in black hole accretion disks, and other high-energy systems.

Future research should focus on advancing high-resolution measurement technologies and developing theoretical derivations to establish a universal expression for $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$, thereby advancing fluid mechanics toward a deeper understanding of its physical foundations.

5. Conclusions and Prospects

This paper investigates the physical completeness of the **NS equations** in describing turbulent flows, focusing on their inadequate representation of rotational degrees of freedom. To address this limitation, we propose the hypothesis of a **maximum rotational angular velocity for fluid elements**, denoted $\boldsymbol{\Omega}_{\mathbf{FluidMAX}}$, and construct a new physical interpretation of turbulent singularities based on this constraint. The main conclusions are as follows:

1. **Deficiency in rotational degree modeling in the NS equations**:
   The **NS equations** are derived under the continuum assumption, and their momentum equations do not explicitly include inertial torque terms associated with the rotation of fluid elements. This omission leads to an incomplete description of rotational dynamics. In strongly rotational flows—such as turbulence—angular acceleration can become significant, and inertial torques may dominate the energy transfer process. Neglecting these effects in the NS framework may result in inaccurate modeling of energy partitioning between translational and rotational degrees of freedom. This deficiency implies that all turbulence models based on the NS equations—such as RANS and LES—must rely on empirical parameters or numerical fitting to compensate for missing physics, thereby limiting their universality and cross-scale predictive capability in complex, non-equilibrium flows.
2. **The fluid element maximum angular velocity hypothesis (Ω_{FluidMAX})**:
   We hypothesize that, in a given fluid medium, the rotational angular velocity of a fluid element is physically bounded by an upper limit, $\boldsymbol{\Omega_{\text{FluidMAX}}}$. This limit is preliminarily expressed as:

   $$\Omega_{\text{FluidMAX}} = K_i \cdot \frac{a}{2\pi\ell}$$

   where a is the speed of sound in the fluid, ℓ is the characteristic length scale of the fluid element (e.g., the Kolmogorov scale), and K_i is a dimensionless constant dependent on molecular structure and thermodynamic state, to be determined experimentally. When the angular velocity of a fluid element approaches this limit and additional energy continues to be supplied (e.g., via pressure gradients or external forcing), the excess energy cannot be stored as rotational kinetic energy and must instead be converted into other forms—such as internal energy (local temperature rise), pressure fluctuations, or wave energy (e.g., acoustic or shock waves).
3. **Re-definition of physical singularities in turbulence**:
   Based on the $\boldsymbol{\Omega_{\text{FluidMAX}}}$ hypothesis, we define a **physical singularity** as a critical state in which energy undergoes forced conversion and extreme dissipation under rotational constraints. Mathematically, this state is characterized by sharply increasing—but finite—gradients of velocity, vorticity, and dissipation rate. Physically, it corresponds to localized high-temperature, high-pressure regions, drastic structural reorganization, or even the local breakdown of the continuum assumption (e.g., cavitation, ionization). This redefinition avoids the unphysical infinities associated with mathematical blow-up in the NS framework and offers a new pathway to reconcile mathematical modeling with physical reality.

If the $\boldsymbol{\Omega_{\text{FluidMAX}}}$ hypothesis is validated, it could have profound implications for fluid mechanics and related disciplines. Future research directions include:

1. **Experimental validation and parameter calibration**:
   There is an urgent need to develop high spatiotemporal resolution measurement techniques—such as ultra-high-speed PIV/PTV, quantum sensing, femtosecond laser spectroscopy, and X-ray phase-contrast imaging—to directly observe the angular velocity distribution of microscale vortices and test for the existence of a consistent upper bound. Cross-medium experiments (in gases, liquids, and plasmas) should be conducted to establish quantitative relationships between $\boldsymbol{\Omega_{\text{FluidMAX}}}$ and fluid properties (density, viscosity, speed of sound, molecular mass), enabling the determination of the universality of the constant K_i.
2. **Physical reconstruction of turbulence models**:
   Incorporate **angular velocity saturation dissipation terms** or **rotational-constraint closures** into existing models (e.g., LES, RANS) to replace empirical eddy-viscosity assumptions. Such physics-based subgrid-scale models could significantly improve predictions of near-wall flows, separation zones, transition processes, and shock-turbulence interactions.
3. **Unified explanation of extreme flow phenomena**:

The hypothesis may provide a new theoretical lens for understanding cross-scale extreme flows, such as turbulent transport in fusion plasmas, angular momentum loss in black hole accretion disks, and hypersonic boundary layer transition. It highlights the universal role of rotational degrees of freedom in energy cascading and dissipation across vastly different physical regimes.

4. **Theoretical challenges**:

   major challenge lies in rigorously incorporating the $\boldsymbol{\Omega}_{\text{FluidMAX}}$ constraint into the NS framework—or an extended version thereof—while preserving the system's closure, well-posedness, and consistency with conservation laws. This may require advances in partial differential equation theory, constrained dynamics, or the development of new mathematical formalisms for rotational fluid elements.

5. **Numerical simulation challenges**:

   Accurately capturing the energy conversion processes at the onset of Ω_{FluidMAX} in high-Reynolds-number, small-scale turbulence demands ultra-high-resolution direct numerical simulation (DNS) or novel multiscale coupling methods. New numerical schemes capable of resolving such **physical singularities**—without numerical instability or artificial dissipation—must be developed.

6. **Interdisciplinary integration with fundamental physics**:

   Investigating the microscopic origin of $\boldsymbol{\Omega}_{\text{FluidMAX}}$ —such as molecular collision frequency limits, quantum uncertainty constraints, or relativistic rotational bounds (e.g., in Kerr spacetime)—could foster theoretical convergence between fluid mechanics, statistical physics, quantum fluids (e.g., superfluid helium), and general relativity. This may lead to a unified understanding of rotational phenomena across scales, from molecular vortices to astrophysical systems.

Turbulence remains one of the most profound unsolved problems in classical physics. Its resolution will require not only advances in computational power and numerical methods, but also a deeper re-evaluation of fundamental physical mechanisms. If experimentally confirmed, the $\boldsymbol{\Omega}_{\text{FluidMAX}}$ hypothesis could provide a **physical regularization mechanism** for the NS regularity problem, shifting turbulence modeling from a paradigm of **empirical compensatory fitting** to one of **physics-driven prediction**. This transition will demand synergistic innovation across theory, experiment, and computation. While significant challenges remain, the potential rewards—deeper insight into nonlinear dynamics, energy transfer, and the nature of complexity in physical systems—are immense.

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